Condensed Matter Physics
Prof. G. Rangarajan
Department of Physics
Indian Institute of Technology, Madras

Lecture - 33
Superconductor-Worked Example

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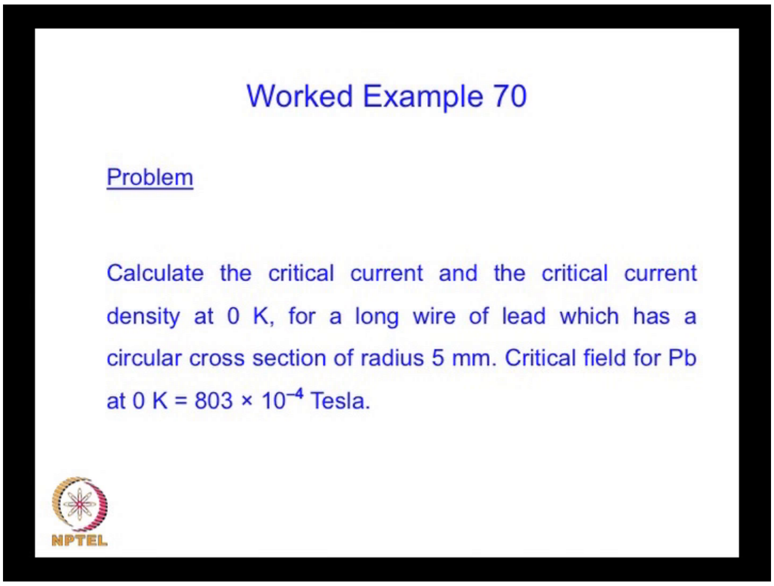

Today, we will consider some solved examples on the topic of superconductor. The first example, concerns the critical current and critical current density.

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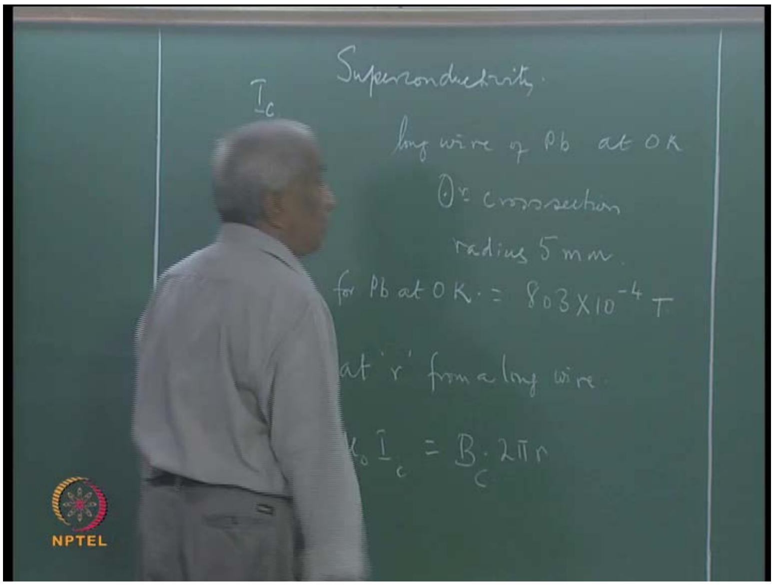

So, the critical current density and the critical current for a long wire of led at 0 Kelvin, long means the length is very long in comparison to the lateral dimensions. It is a wire of circular cross-section and radius 5 millimeter. We are told to the critical field at zero k for length is 803 into ten to the power minus 4 Tesla.

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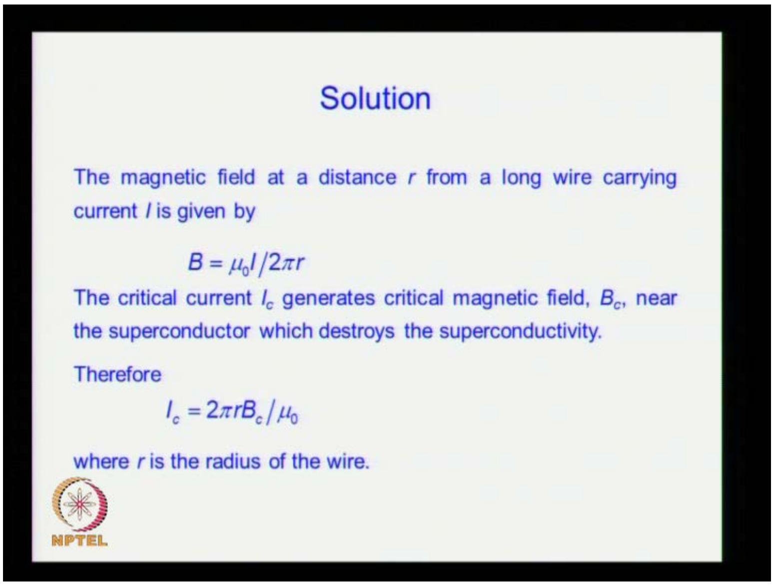

So, this is the data given and we have to find the critical current for which we go back to the amperes theorem which gives the magnetic field at a distance are from a log wire. So, this is given by amperes circuital theorem i is the current. And in this case, obviously, the d l is going to give you B is constant. So, this will be B times 2 pi r. So, this is the critical field this will be the corresponding article current.

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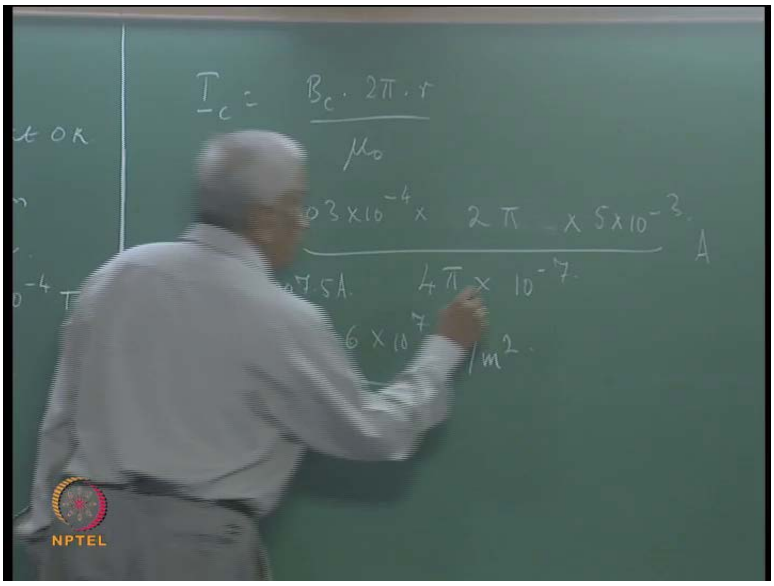

So, I c will be B c into two pi r divided by mu naught, so that we have to simply substitute the values the radius is given a 5 millimeters divided by 4 pi minus 7 and appears.

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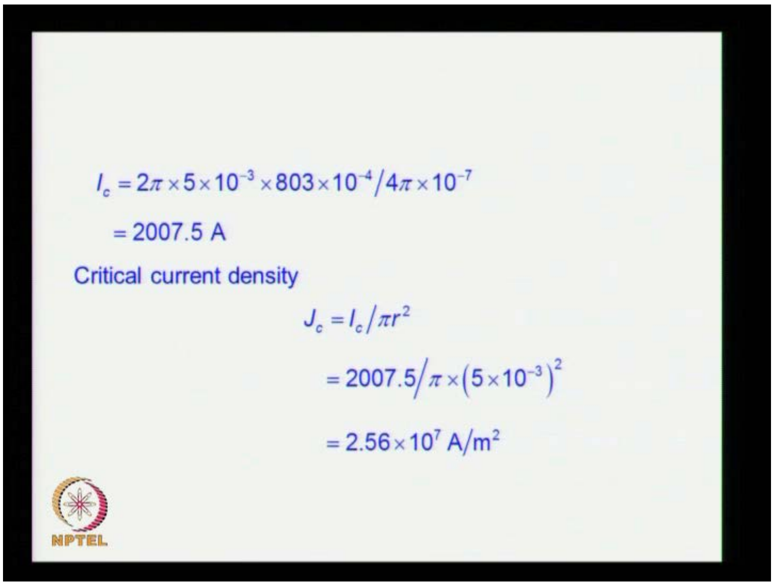

So, this in simplified works out to the critical current is 2007.5 amperes in the corresponding critical current density works to be this.

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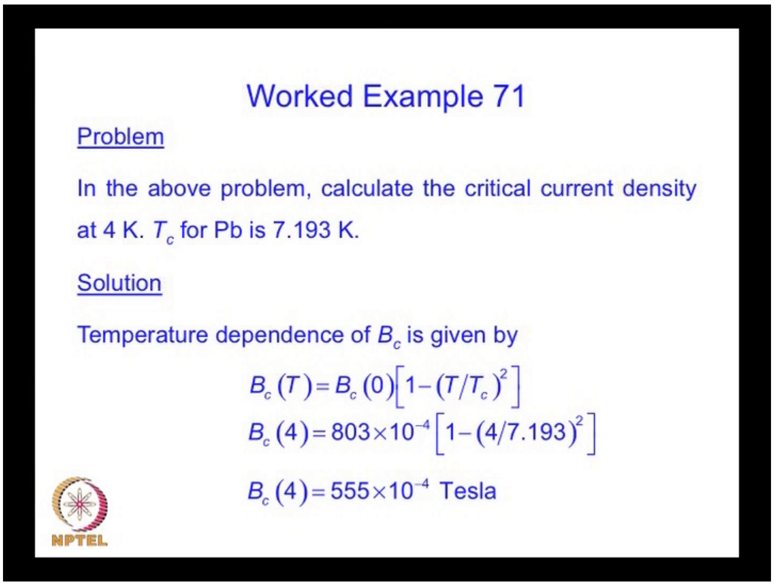

The next problem is a continuation of this where we are asked to calculate the critical current density at 4 k which was 0 k.

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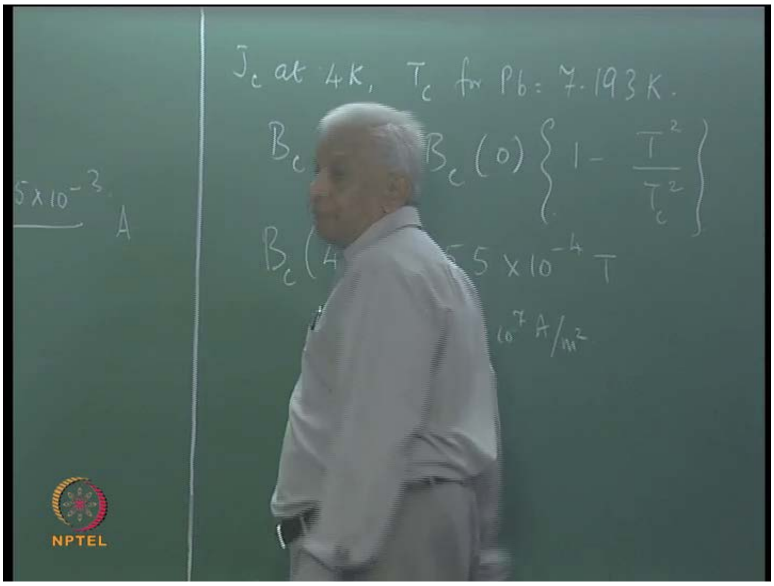

And for this we need the additional data regarding the transition temperature for led in that is given to the 7.193 Kelvin. So, we start from the expressions for the critical magnetic field at any temperature in terms of the critical magnetic fields at 0 k that is the parabolic law. So, substituting these values B c at 4 k works out to be 555 into ten to power minus 4 Tesla.

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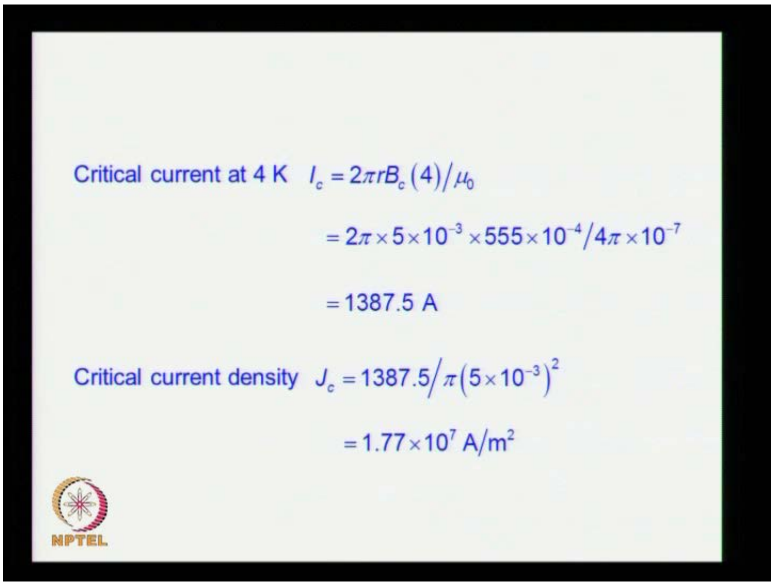

So, while working back at the critical current density in the same way as in the last problem beget this as 1.77 into 10 to the 7 ampere for mu square.

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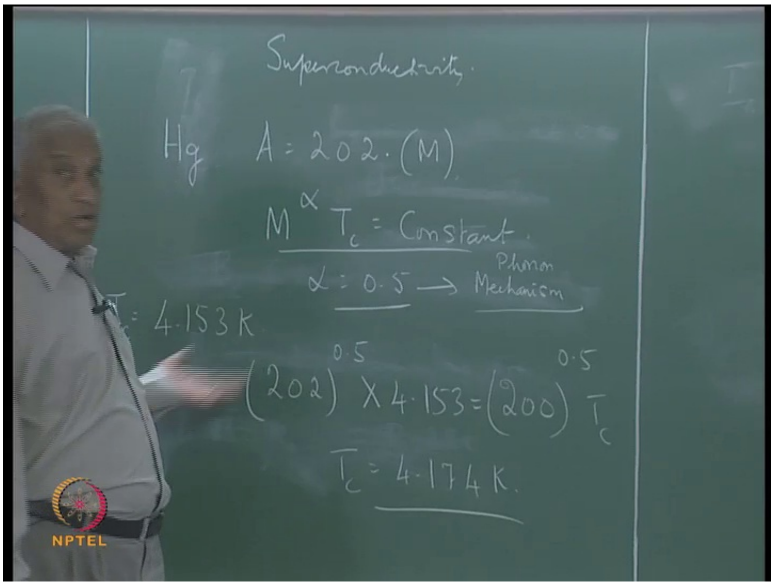

Next problem, these about this sample of mercury we are given the mass number is 202, and we are also told that there is an isotropic dependence of the form M to the power alpha let us use the same symbol m here isotropic mass T c is constant, where alpha is also given to the 0.5 the transition temperature is given as 5.153 k.

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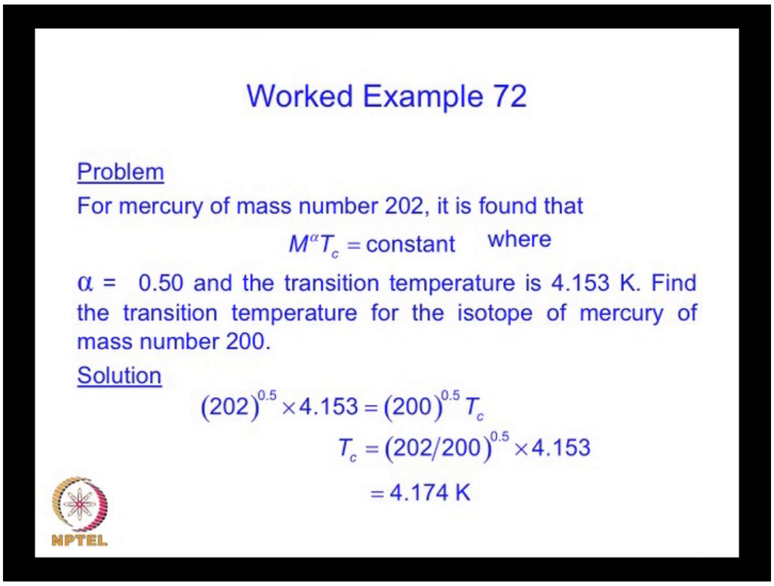

So, with all these values be as the relation 202 0.5 into 4.153 is equal to we are all as to find the transition temperature for the isotropic of mass 200. So, substituting the values we get a number there is a result is T c is 4.174 Kelvin. It is this isotropic dependence of the critical temperature which gave a clue especially in this value of alpha equal to point five which gave a clue to the mechanism of superconductivity the phonon mechanism. Because the phonon are involves then the mass there will be a mass dependence is given by this the simple harmonic oscillator model as 0.5.

Of course, there are many instances where this isotropic dependence is satisfied with the different value of alpha. This is because there can be many mechanism rather than phonon mechanisms and they can be and un harmonistic as well. So, this is not a very these not always the case that this relation is satisfied, but in the case of mercury this relation is found to be satisfied to a considerable degree of accuracy.

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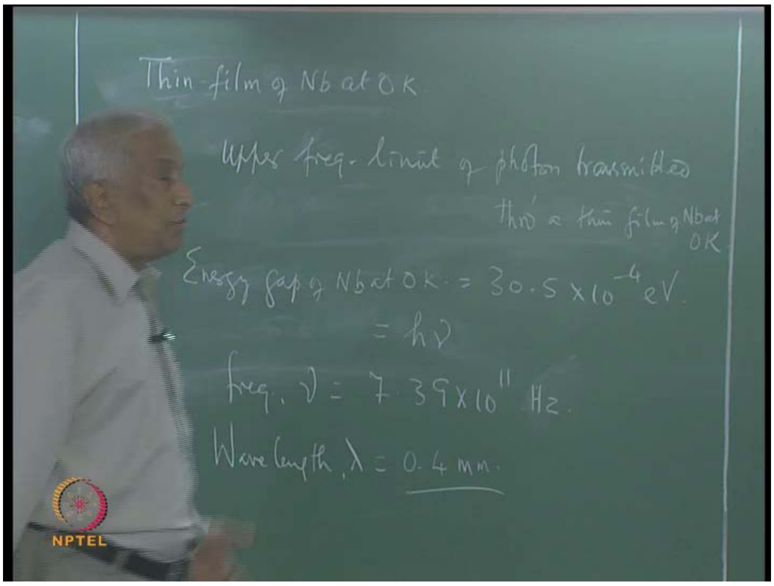

Then the next problem is about as thin film of niobium at zero k. Again we are asked to find the upper frequency limit of photon is transmitted through a thin film through a thin film at zero k.

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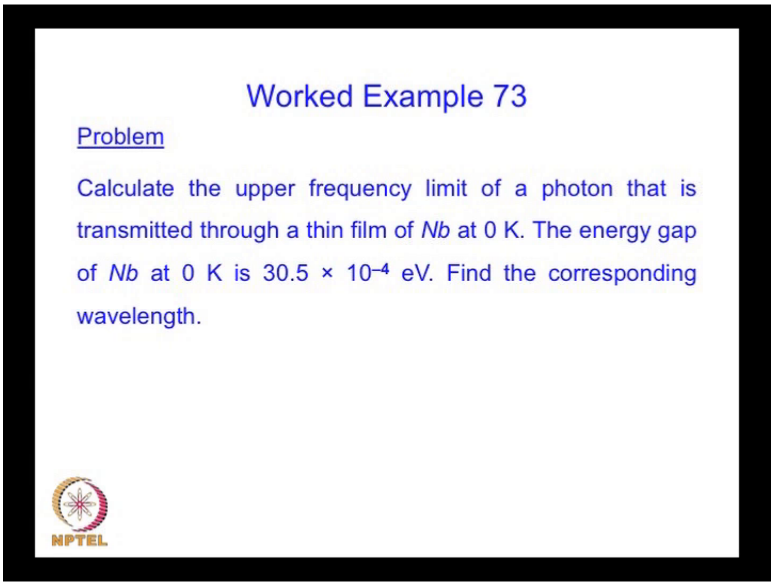

For which we are given the data the energy of niobium at zero k is given as thirty point five into ten to power minus four electron walls.

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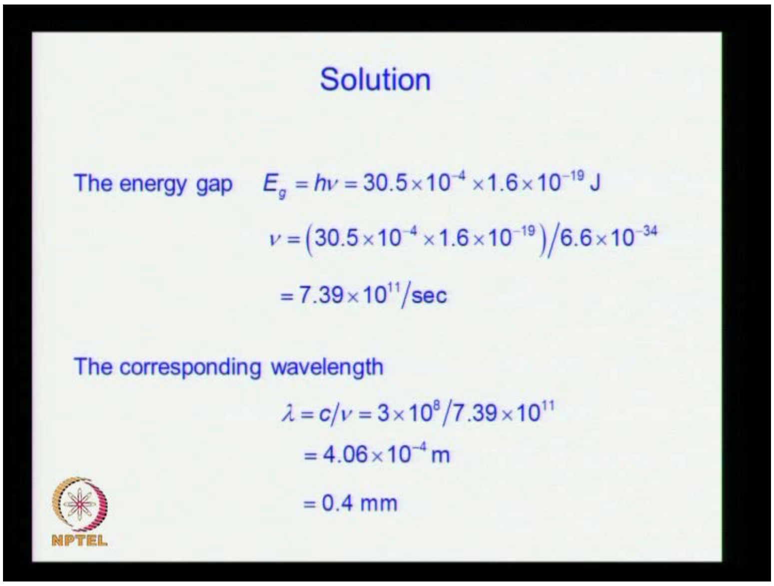

So, we are given this energy now we have to simply set is equal to h nu. So, the upper frequency limit their corresponds to the frequency at which the energy is the photon just equals that of the energy gap. So, the corresponding where h is the constant. So, that the upper frequency limit works out to be second inverse are hertz. We also has to find the corresponding wavelengths and that works out to be two point four millimeters which is in the microwave range. So, it is the microwave photon which can be transmitted.

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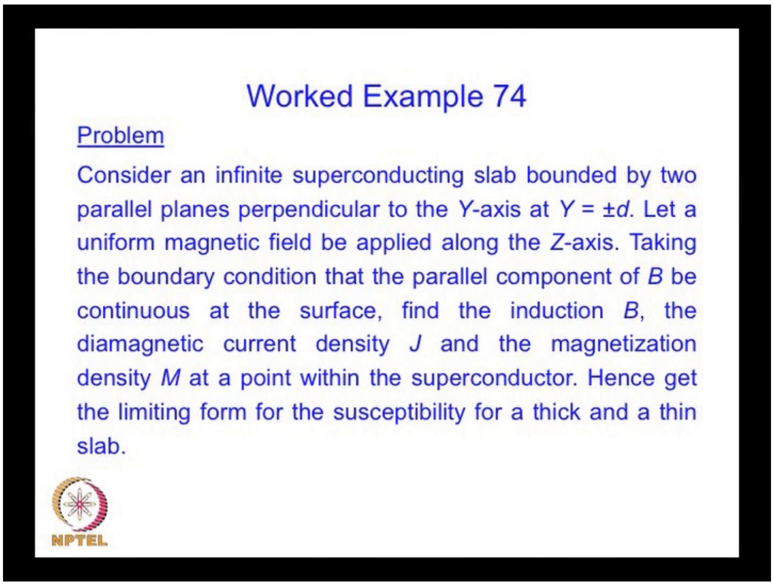

The next problem talks about a general superconductor and infinite superconducting slab.

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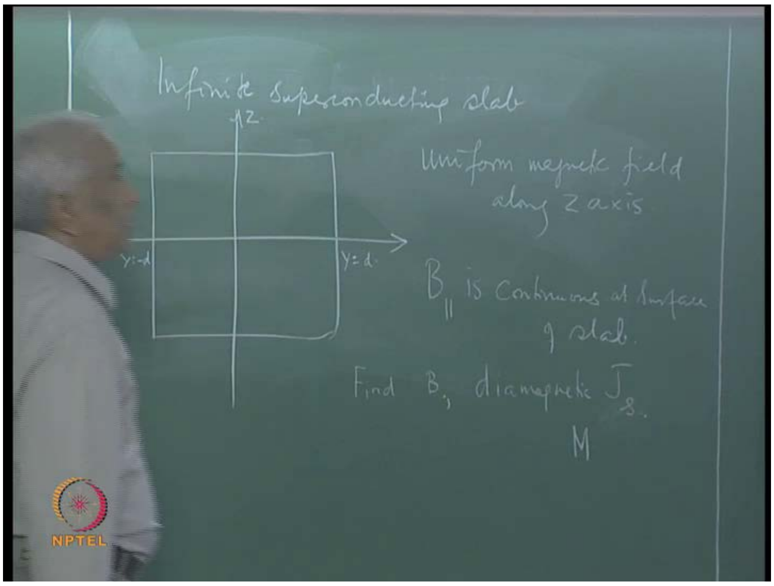

So, we have the next figure shows the geometry you have it is bounded by 2 parallel planes perpendicular to the y axis at y equal to plus minus d. So, this is y is the and this is y is the minus. So, there is a magnet uniform magnetic fields applied along the z axis.

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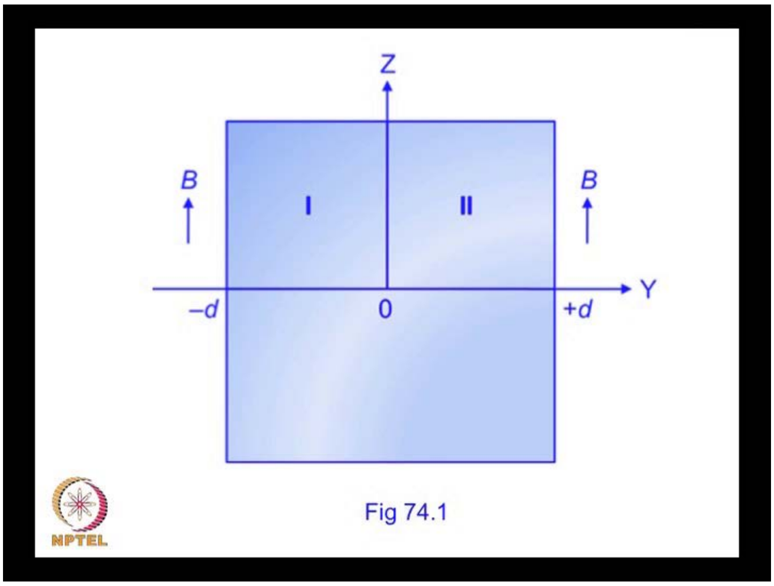

Fig 74.1

We are ask to take a boundary condition that the b parallel the parallel component of this field is continuous at the interface. So, that is the boundary conditions. So, we are asked to find the magnetic induction b the diamagnetic current density j s and the magnetization density at a point within this superconductor.

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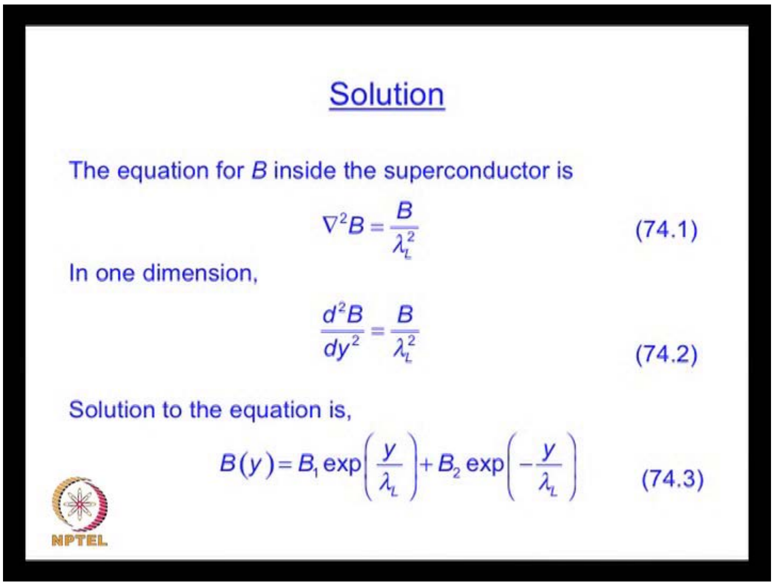

So, this is a problem which concerns the application of the London equation.

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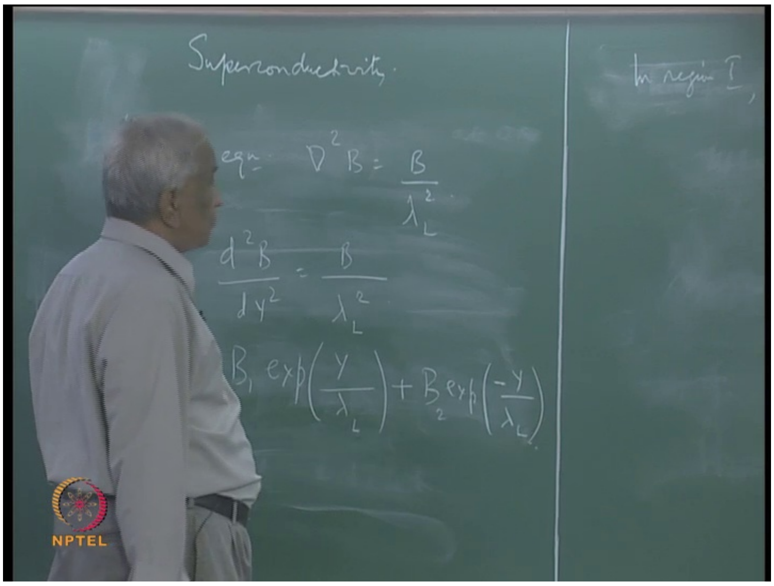

So, restart from the London equation del square b equals b by lambda l square the corresponding one-dimensional equation is e square b v d y square equals b by lambda l square. So, this is a second-order differential equation for which the solution is known to the b one exponential y by lambda l plus b two is equal to minus y lambda. So, that is the expression for the magnetic induction as a function of y within the superconductor now we apply the boundary conditions.

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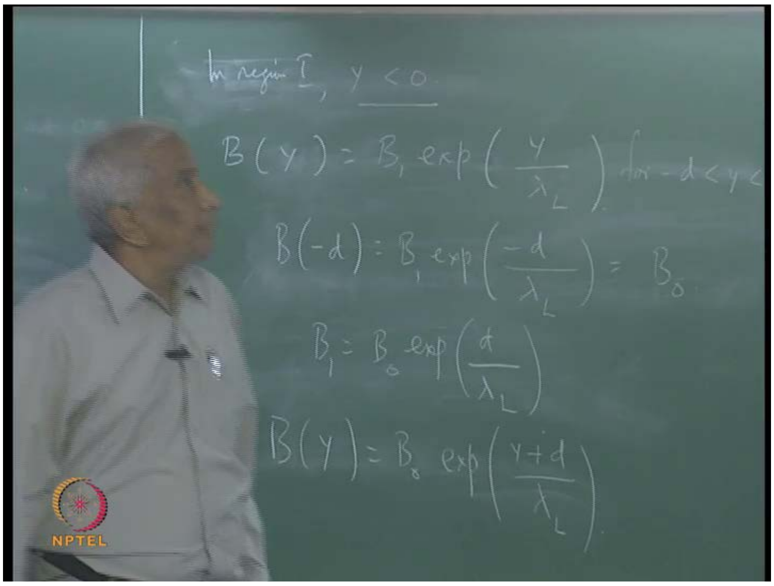

So, in region one in the figure y is negative in y is negative in region one y is less than zero. So, if that is. So, the first term in this solution will be bounded while the second term will not be bounded. So, this will go, so we have b of y you will have only b one exponential y by lambda two for minus b less than y less than zero. And they can also apply the boundary condition b f minus d is b one exponential minus d by lambda l equals b zero.

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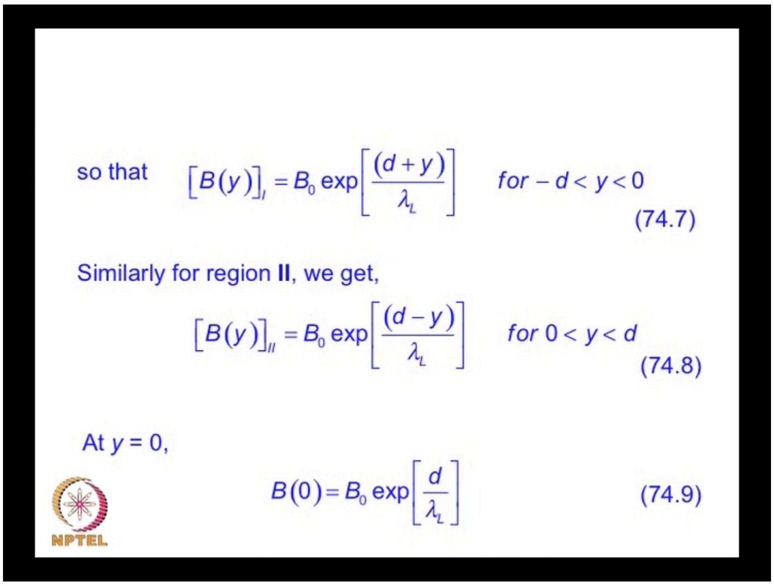

So, B 1, B 0 exponential d by lambda l by a similar argument, so that we can also write b f y as b zero here substituting exponential in this, so y plus d y lambda l we can follow a similar line of argument for region two.

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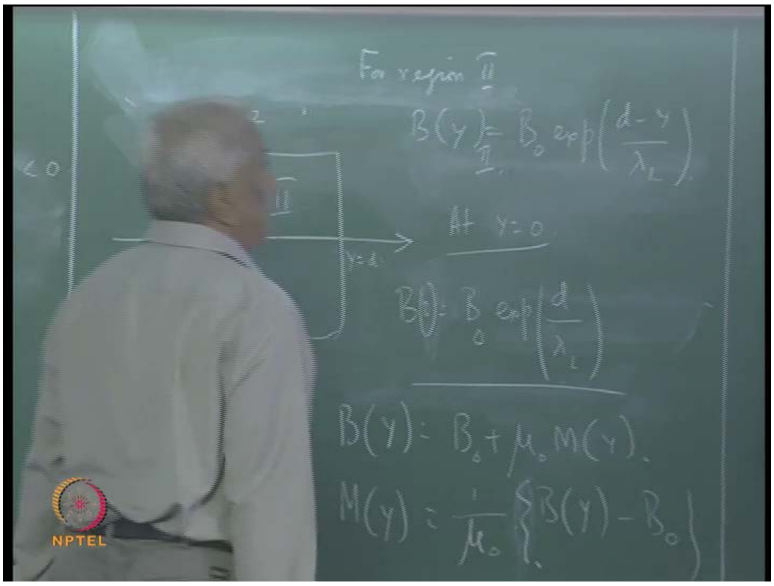

And get the solution as b f y is p zero exponential p minus y by lambda l. So, you have to know match it match these two solutions at y equal to zero. So, b zero is b f zero b at y equal to zero is two are exponential d by lambda l.

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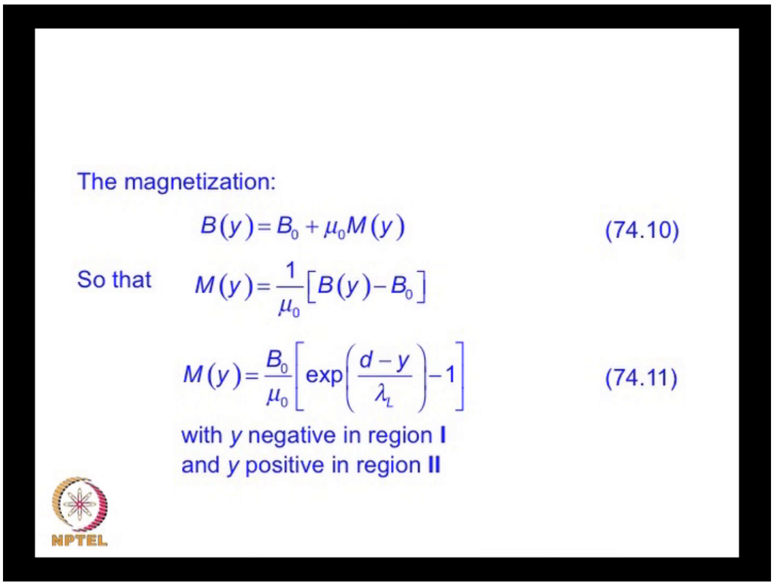

So, that is the magnetic induction which was asked to be found. So, correspondingly the magnetization is b f y equals b zero plus mu naught m of y. It is gives the standard result m f y from b f y is given off one by mu naught to e f y minus. So, plaguing the value by expressions for b f y.

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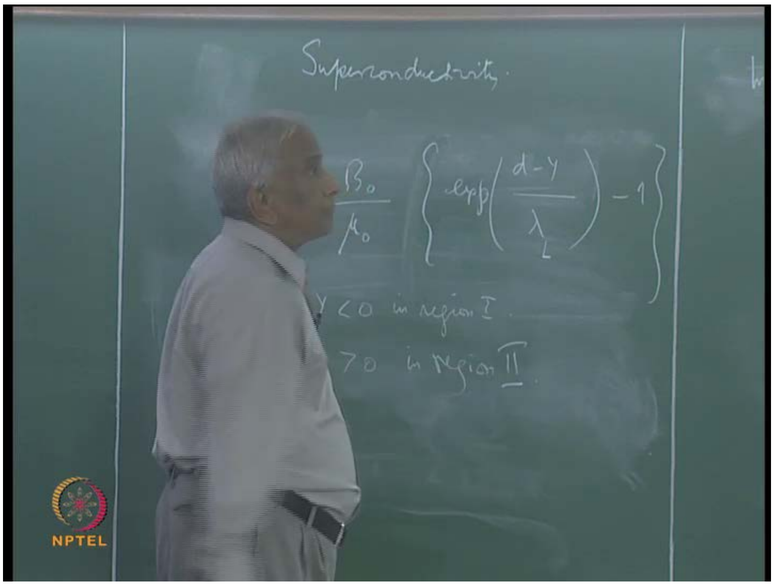

The magnetization m f y is b naught by mu naught is exponential d minus y by lambda l minus one where y is negative in region one when positive in region two.

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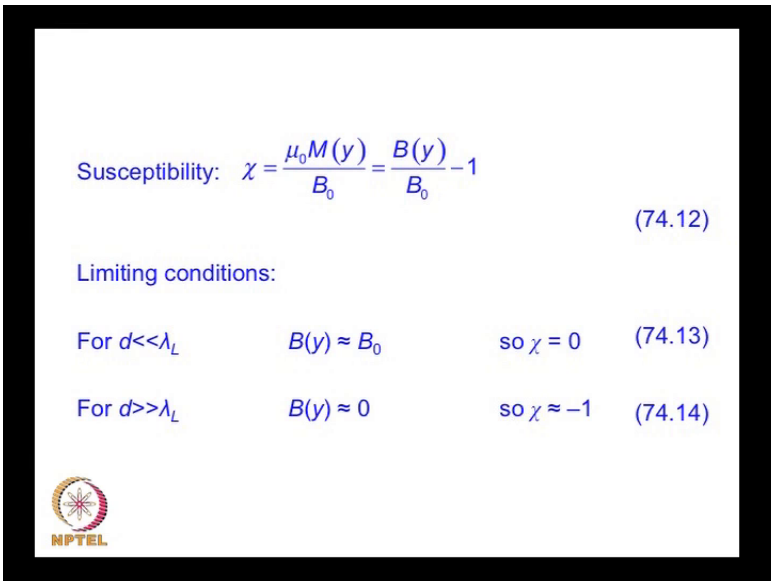

We also asked to find delimiting form for the susceptibility the magnetic susceptibility is related to the magnetization.

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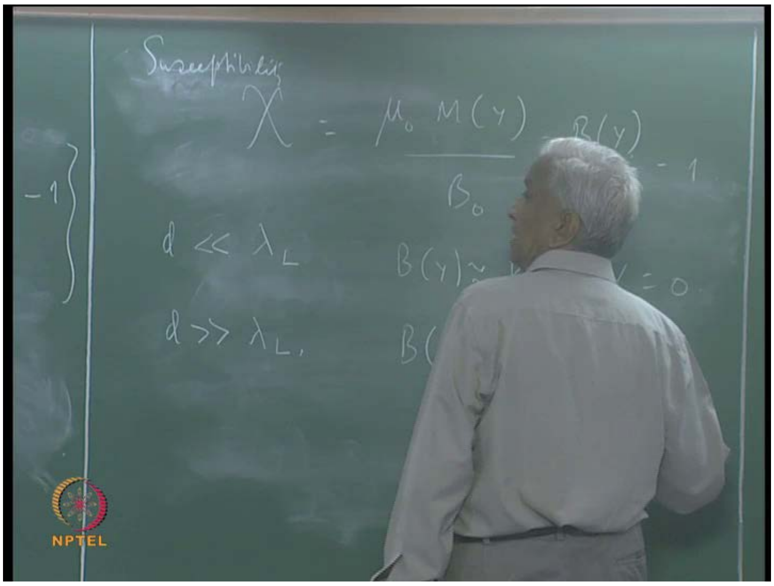

How the susceptibility is given by the standard relation mu naught m of y by b naught. So, that will be b f y substituting for m f y by b naught minus one. So, for d very small compared to the London penetration for lambda l e f y is almost same as b zero. So, xi is zero that is just outside just within short distance inside whereas, for d very large is compatible to London penetration have b f y b zero is minus that effect complete flex it fusion and xi chance out have a perfect diamagnetic value minus of minus one v p pass on to another question.

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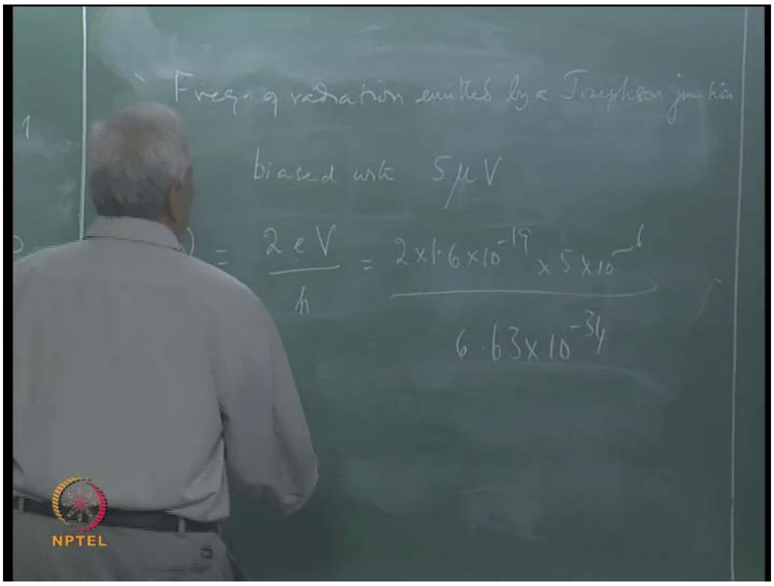

Which we are asked to calculate the frequency radiation emitted by Josephson junction when the junction is by a as two the voltage of five micro volts in solution has straight forward from the Josephson equation. The frequencies two e v by h, where v is the voltage and then e is electronic h, h is the Planck constant.

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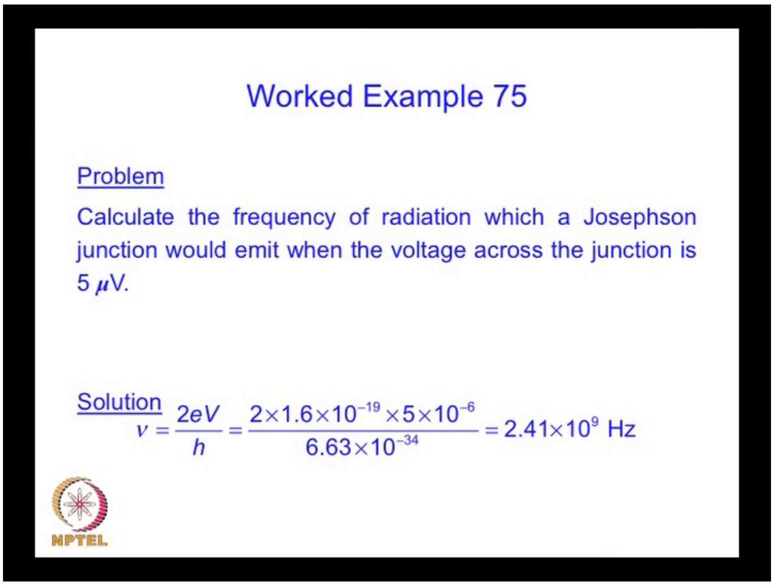

The substituting for all the numbers and that would be two point four one this will be a micro volts frequency gigahertz.

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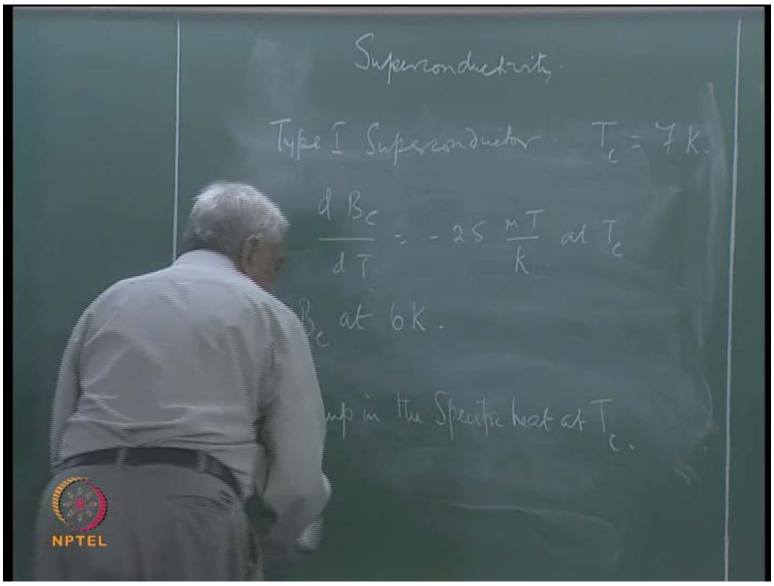

We next consider a type one superconductor in which transition temperature in given to be seven k probably late as a slope d B c by d T - the rate of change. The critical magnetic fields is minus 25 million meters law per k at T c with this data are the are all as to estimate the critical field that six k and then calculate the than jumping the specific heat at T c.

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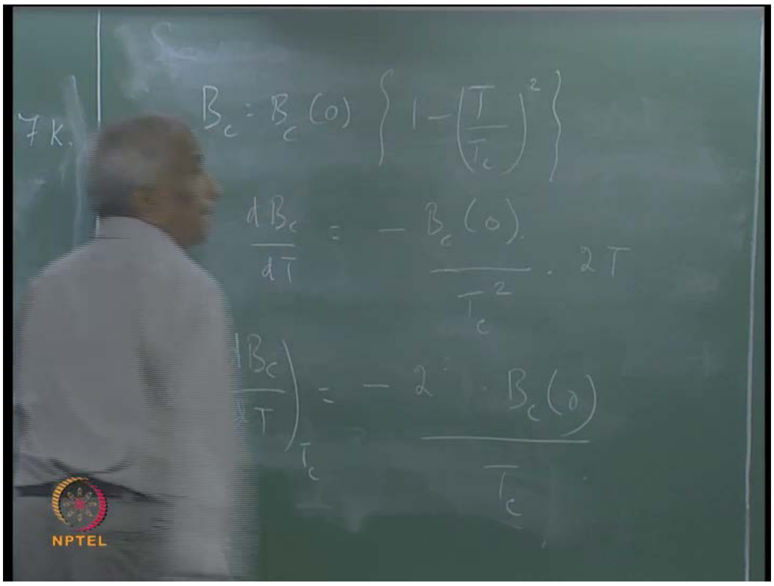

To do this we start from again the parabolic law and d b c by d t differentiating is minus B c 0 is T c square into 2 T, now d b c by d t at T c is given.

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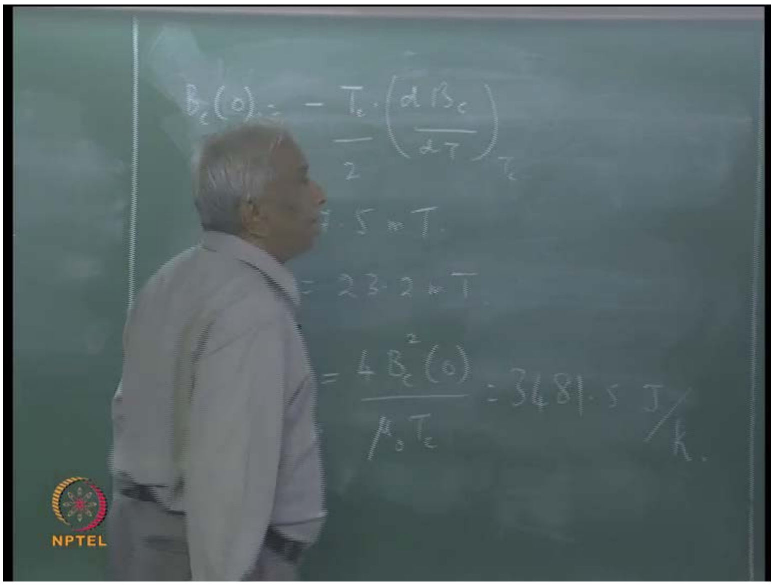

So, substituting this, so B c zero at absolute zero is just minus T c times d B c by d T c at T c divided by 2. So, substituting value this works as to eighty seven point five millimeters therefore, using the parabolic law it is quite easy to find the critical magnetic field at six k substituting the values is works out to be twenty three point two using this law the jumping the specific heat at T c.

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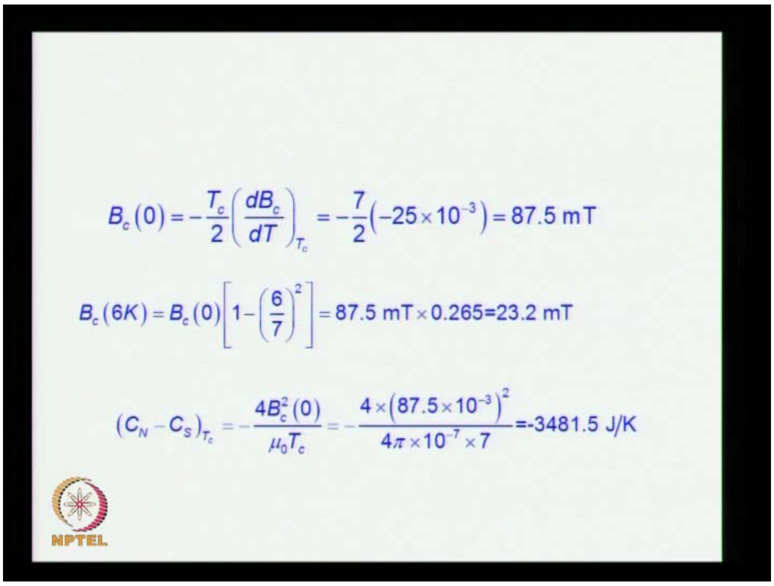

This subscript n and s means normal and superconducting and this was done in the lecture in given has . So, substituting all these values, this use the thirty four minus one five.

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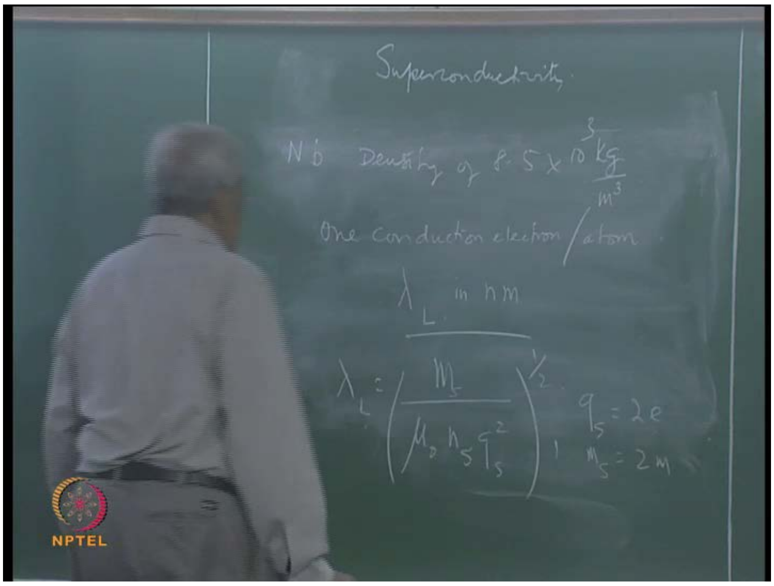

We next consider niobium. We already told that it has the density of eight point five into ten to power three kilogram per meter cube and it is donuts one conduction electron per atom. So, we are asked to calculate at the London penetration that lambda l of superconducting niobium in nanometers.

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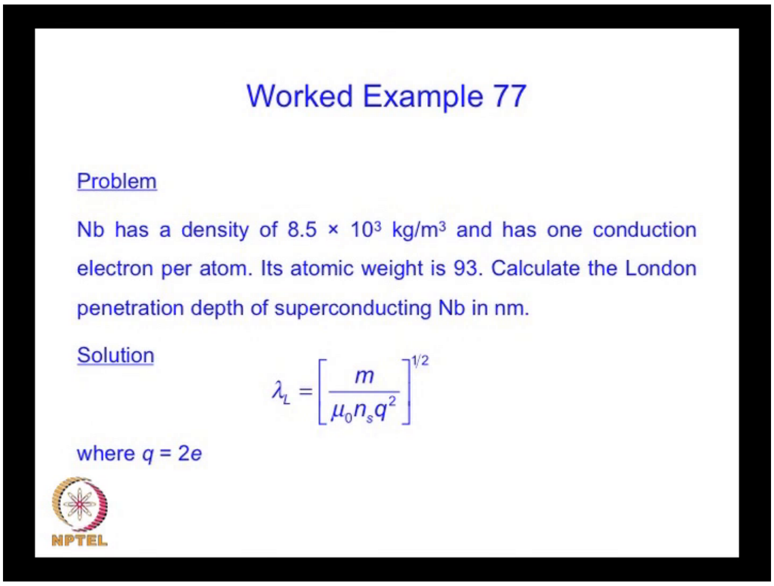

Again it is a substitution will problem in which we using expression for the London penetration that as where q is q s is to e and m is also two m where e m stands for the electronic charge in mass effectiveness, n is the concentration of super current carriers cooper pairs.

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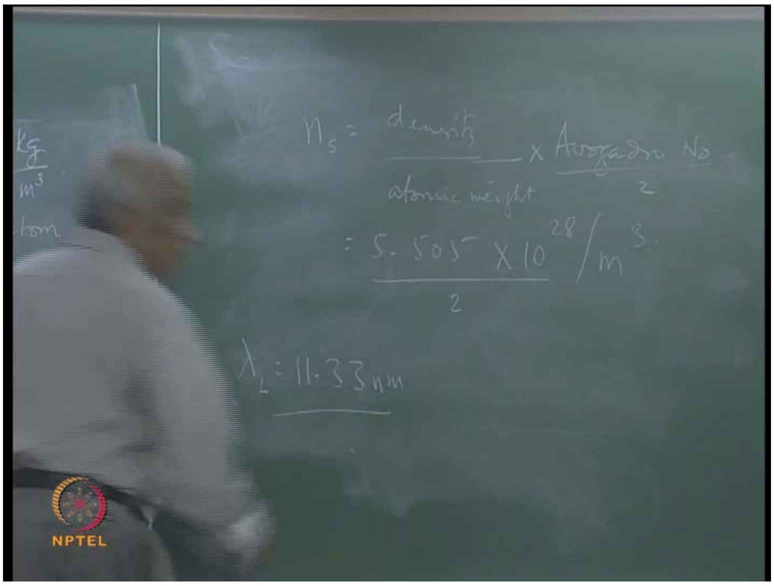

In order to find the concentration of cooper pairs, we just take the density by atomic weight times the other Avogadro number. So, that works out to be if I point five zero five into twenty four twenty eight per meter cube well it should strictly by divided by 2. So, the London penetration left works out to be 11.33.

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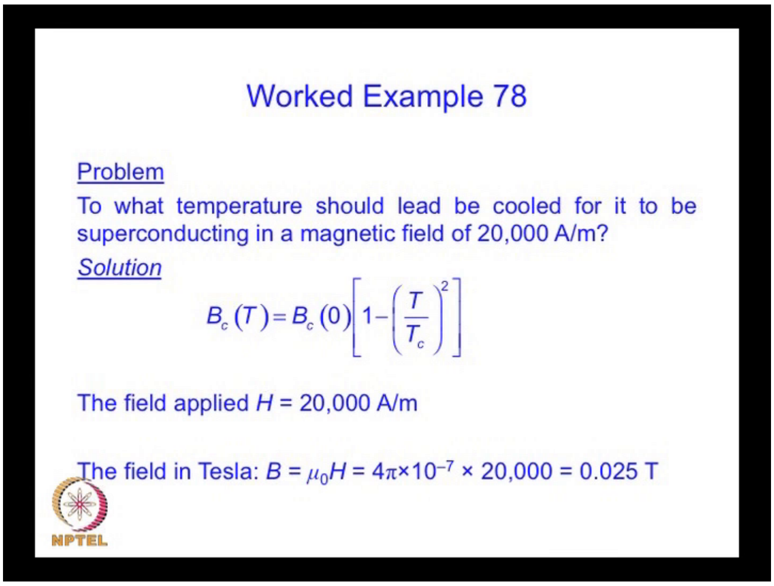

Next is equation and about a superconductor which is in a magnetic field superconducting led in a magnetic field of twenty thousand ampere per meters.

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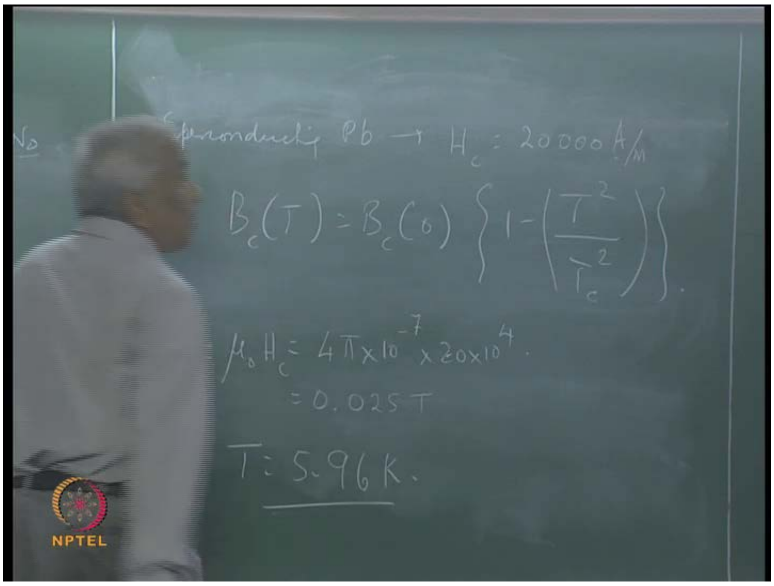

Because of the unit have used a H c i naught B c. So, what temperature should be cold for this to be superconducting in this magnetic field saw gained the critical magnetic field. So, we know the critical finance pressure and temperature of led and we have to find the t that is the temperature should which you should be cold and the field is twenty thousand ampere per meter which corresponds to b c corresponds to in Tesla mu naught h c. So, it will be four pi into 10 to power minus seven into twenty two into twenty thousand. So, that works out to be naught point naught two pi Tesla.

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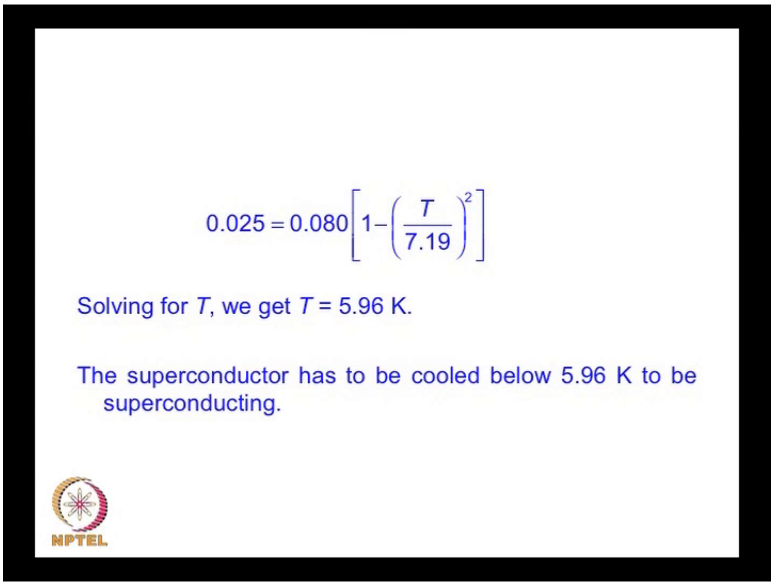

Substituting this value is readily found to be 5.96 k, we have to cool the led sample below five point nine six k in order that to be superconducting.

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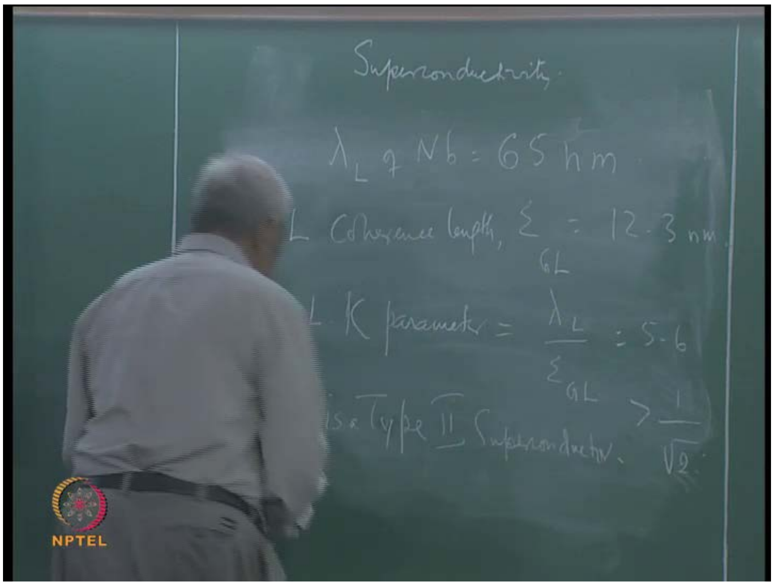

Next we are given the London penetration is that of niobium as sixty five nanometers and its Ginsburg Glandau coherence length standard for symbol for s t c l it is given to be twelve point three nano meters. So, the Ginsburg Galndau kappa parameters which is crucial for determining other it is a type one or type two superconductor is the ratio of. So, that works out to be 5.6.

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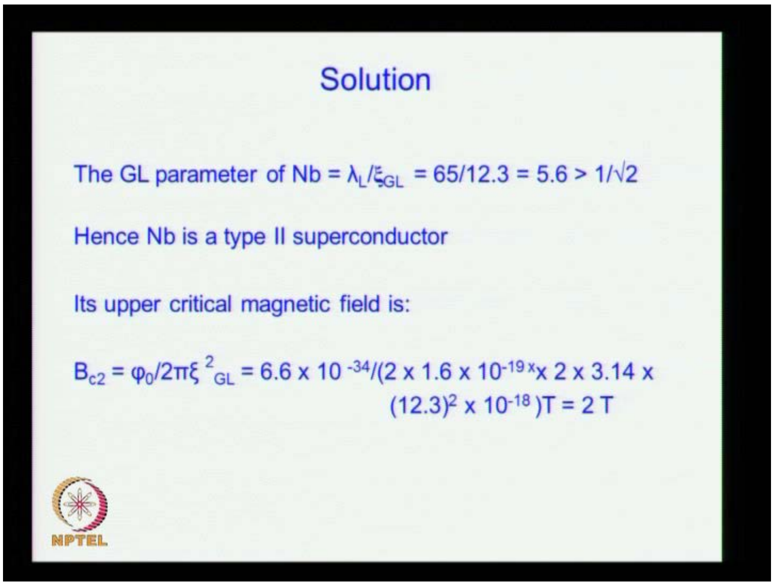

In this case, which is greater than 1 by root 2, so the Ginsburg Glandau criterion for a type two behavior is that this kappa should be greater than 1 by root 2 which is the case so it is a type two superconductor that are also ask to find the magnetic field for which the flexible completely secluded from the type two superconductor.

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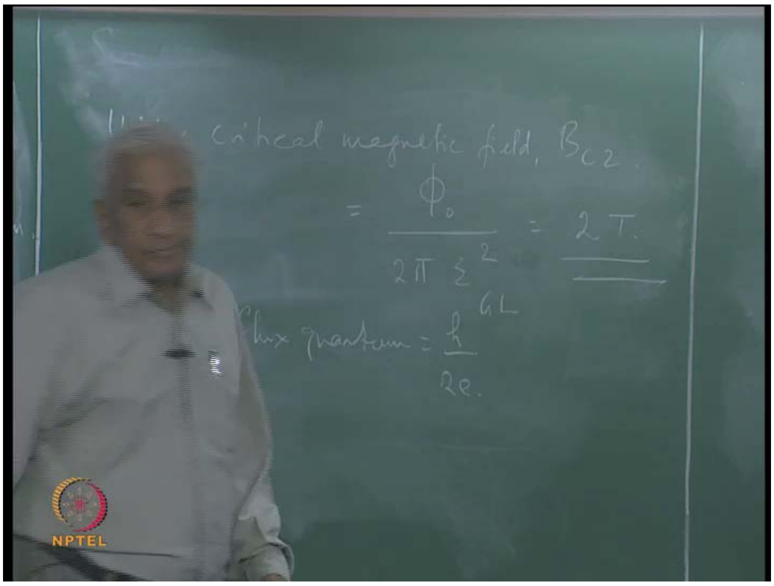

And the magnetic field b the upper critical field b c two and that was found from the discussion on the Ginsburg Glandau theory a five naught by 2 pi psi by 2 pi f square where five naught is the flux quantum equal to h by 2 e and size g l is already given. Now plugging these number in obtain the critical magnetic field upper critical magnetic field, so that turns out to be to Tesla.

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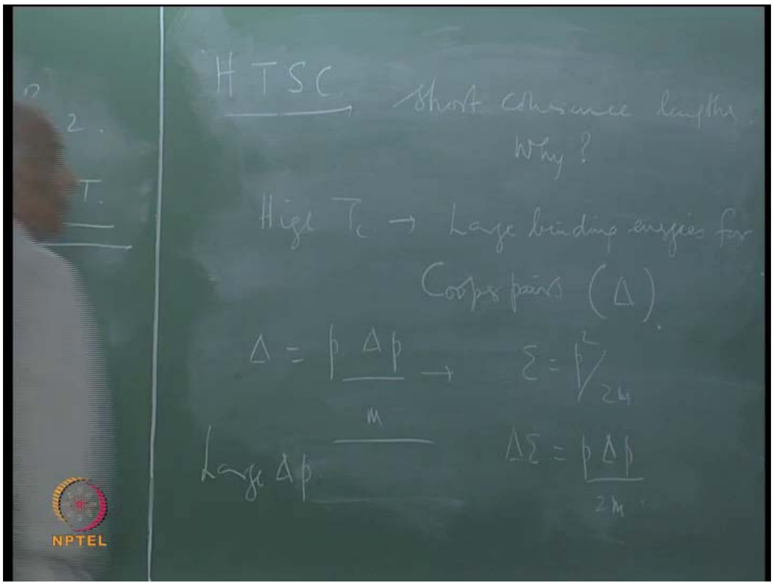

Next we go to the question about high temperature superconductors, they have very short coherence link. This is an experimental finding they are asked to explain why we know on the b c s theory a that if we have a high t c which is the case in the superconductor this means large binding energies for cooper pairs standard symbol is delta. Now the delta is given by p delta p by l because we know the kind attic energy is p square by 2 m. So, delta e e s p delta e by 2 m. So, this is the relation between delta and. So, this large value of the delta this is a large uncertainty in delta p.

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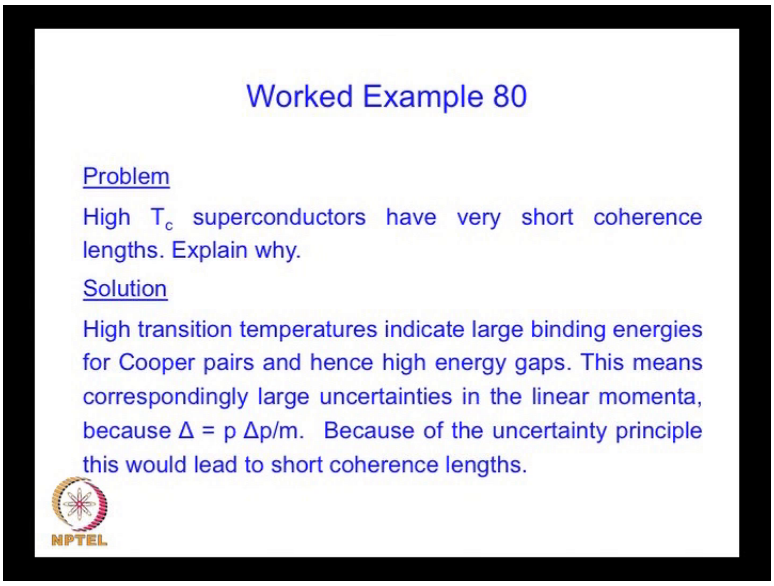

And the uncertainty relationship tells us that the product of the uncertainty in the moment and the position delta x delta p in order of h cross 2.

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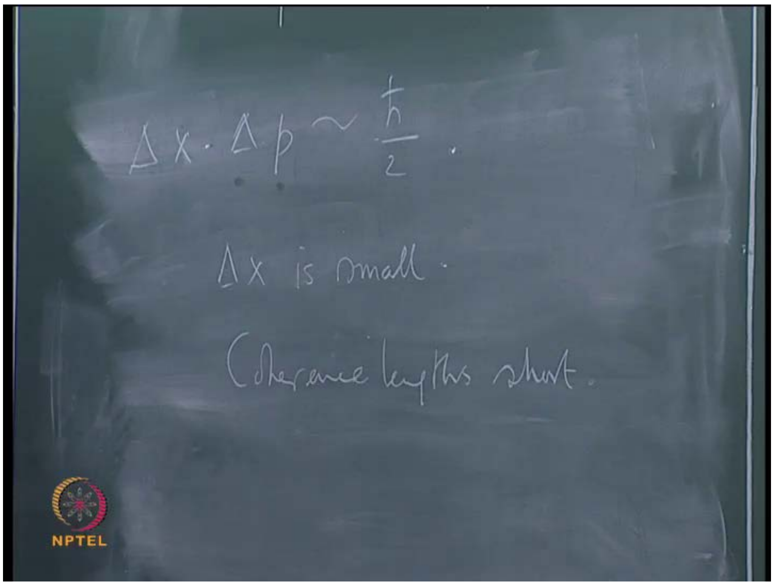

So, if the uncertainty the momentum p is large, the uncertainty the portion is small this explains y the coherence links short.

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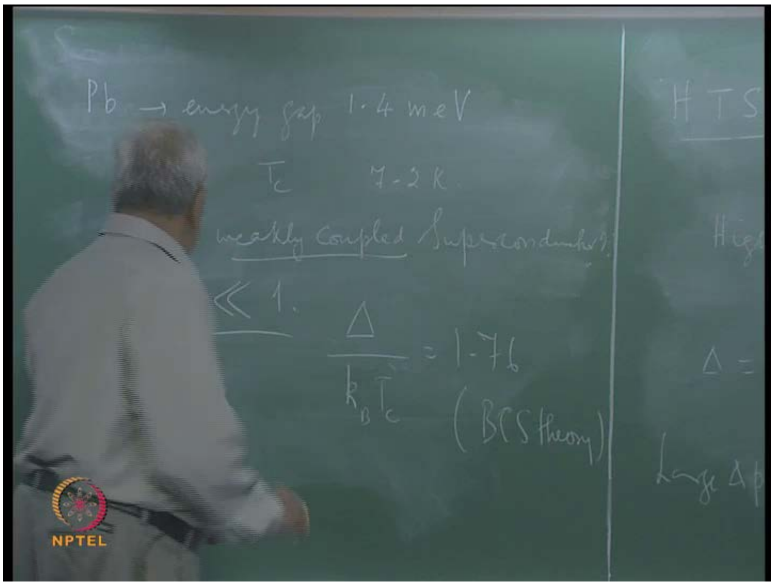

Now, consider a question again regarding P b we are given the energy gap of led as 1.4 milli electron volt.

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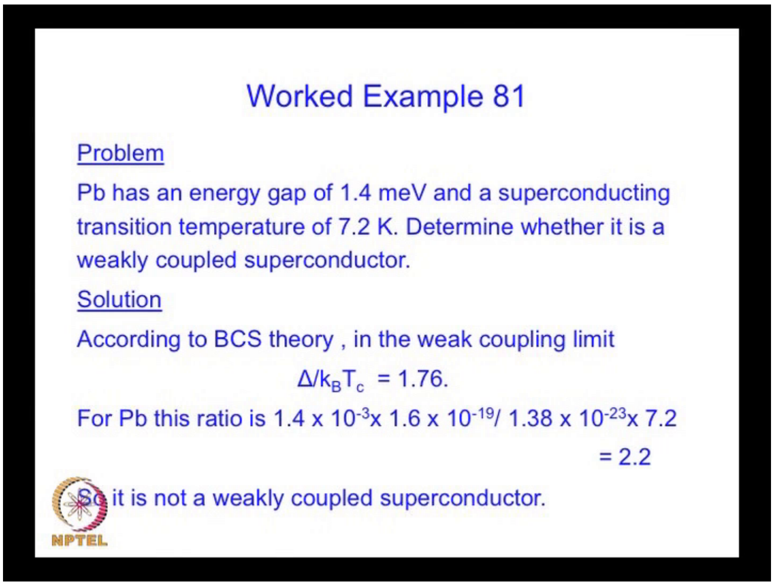

And the transition temperature is 7.2 k as we already know. So, we are ask to find is it a weakly coupled superconductor. We know the condition for the v coupling if the b c s theory is that the condition for the v coupling. And the v coupling limit, we also know that the ratio of weld are the energy gap to k B T c is 1.76 according to BCS theory. So, we are given the values of delta and T c. So, it is a simply checking how is values compare.

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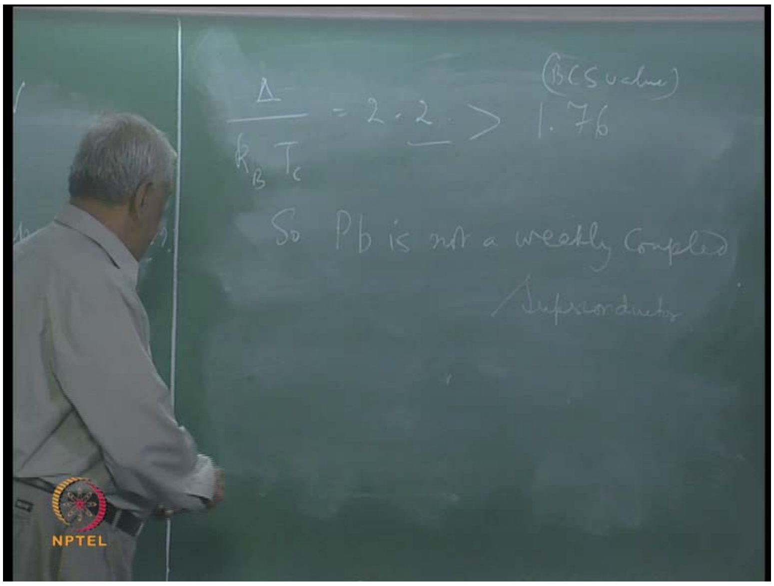

So, taking this ratio delta by k B T c for led, so the given data is comes to be 2.2 certainly much higher than 1.76 BCS value. So, the conclusion is led is not their weakly coupled superconductor, it is what is called the intermediate coupling.