Condensed Matter Physics
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Lecture - 33
Josephson Effect (continued); High temperature superconductors

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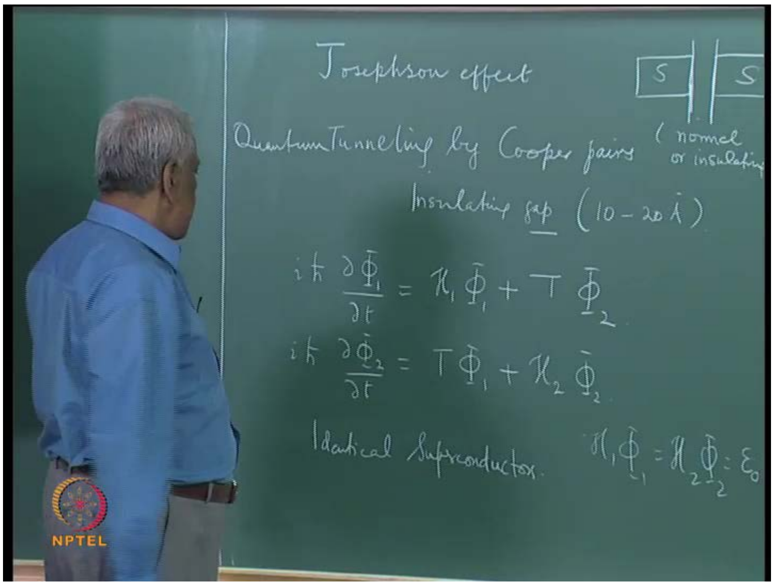

Yesterday we talked about the Josephson effect the case which was discovered by Josephson in 1962, and it is the phenomenon of tunneling by cooper pairs quantum mechanical tunneling by cooper pairs across a gap across an insulating layer, which is a very narrow insulating gap of about 10-20 angstroms thick extremely narrow.

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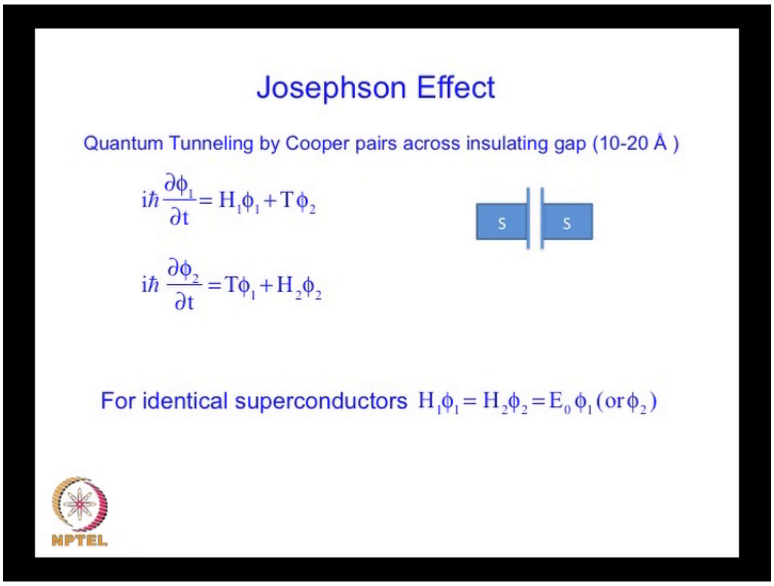

So, we have a supper conductors a junction of super conductors, a pair of super conductors with a gap between them which can be normal or insulating gap. So, the Josephson tunneling demonstrates the quantum mechanical nature of the super conducting state and this tunneling is described by considering the two super conductors as two quantum systems and writing the Schrodinger equation.

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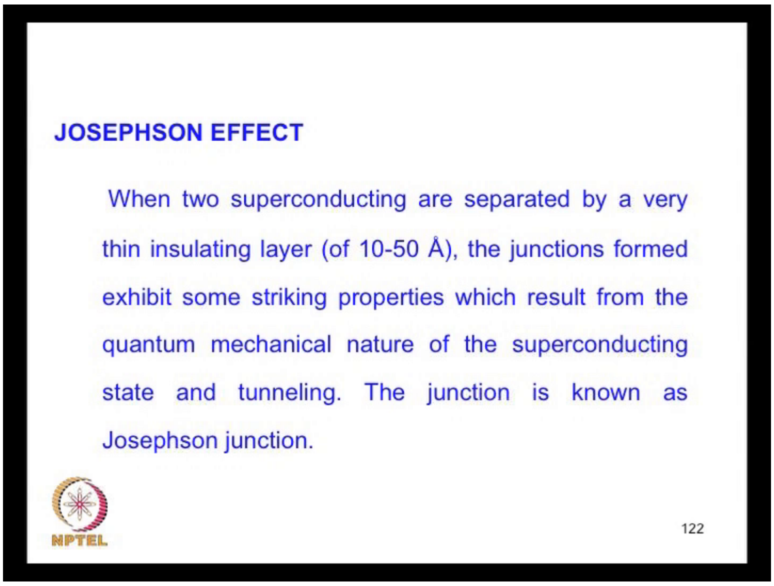

Where phi 1 and phi 2 are the quantum mechanical wave functions, and a similar equation for the second super conductor and the time dependence of this wave function which is given by… So this are the basic equations and if we have identical super conductors for simplicity then H 1 phi 1 equal to H 2 phi 2 equal to e 0 phi 1 or phi 2. So, that is a common energy, so which can be ignored.

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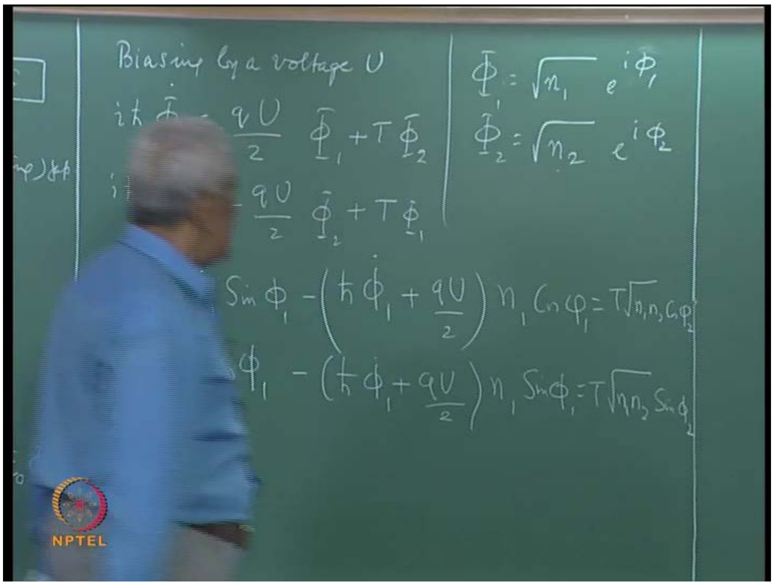

And if this junction is biased by a voltage U, then the equations get modified to taking the 0 of energy in the midway of the gap and a similar equation for phi 2 tau.

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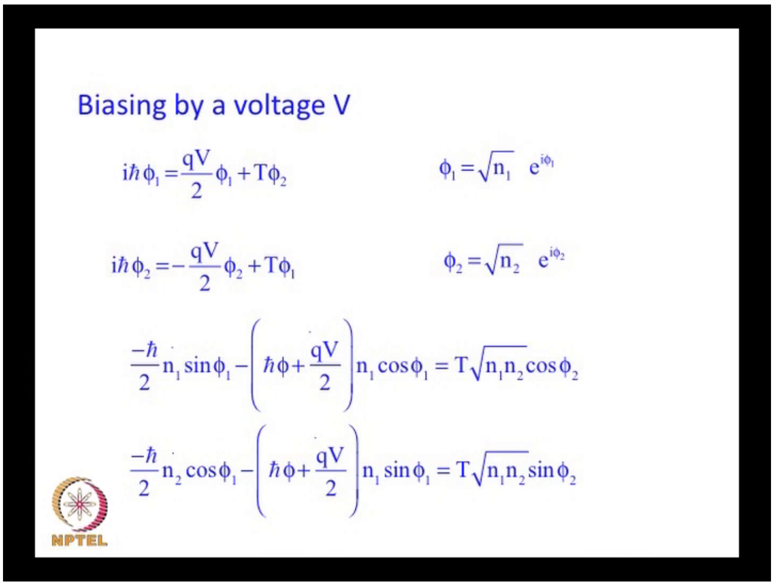

Where phi 1 and phi 2 are of the form, now this phi 1 and phi 2 are the quantum mechanical parses, and n 1 and n 2 are the densities of the cooper pairs in the two super conductors. So such that, so with this we get this gets modified to under similar equation and then we just multiply this equations by cost phi 1 and this by sign phi 1 and then subtract.

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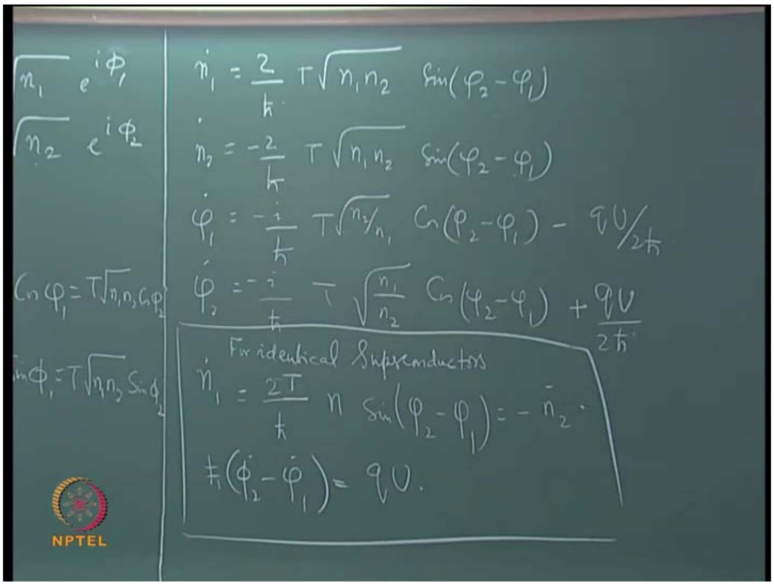

Then we get the final equations for the time dependence of the charge densities, and similar equations for the time variation of the phi's, the phrases minus q u by 2 h cross here plus q u by 2 h cross.

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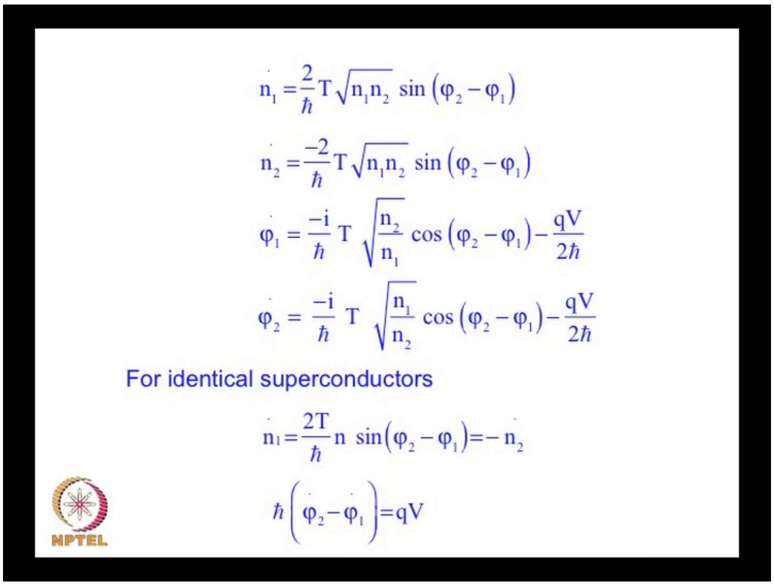

So, per identical super conductor, we can take n 1 to be equal to n 2 and so on, so that we get finally, per identical super conductors. We get the particularly simple equations of the form and so these are the governing equations for Josephson effects, which tell you that even if the absence of a voltage, you have a Josephson current given by the time variation of the charges. So, this is this represent the flow of cooper pairs across of the gap the Josephson current is determined by the phase difference the quantum mechanical phase difference and the tunneling Hamiltonian in the coupling between the two super conductors and the phrase itself in the presence of an applied voltage biasing the junction the phase changes at a rate given by q u by h cross. So, those are the principal results of the Josephson effects.

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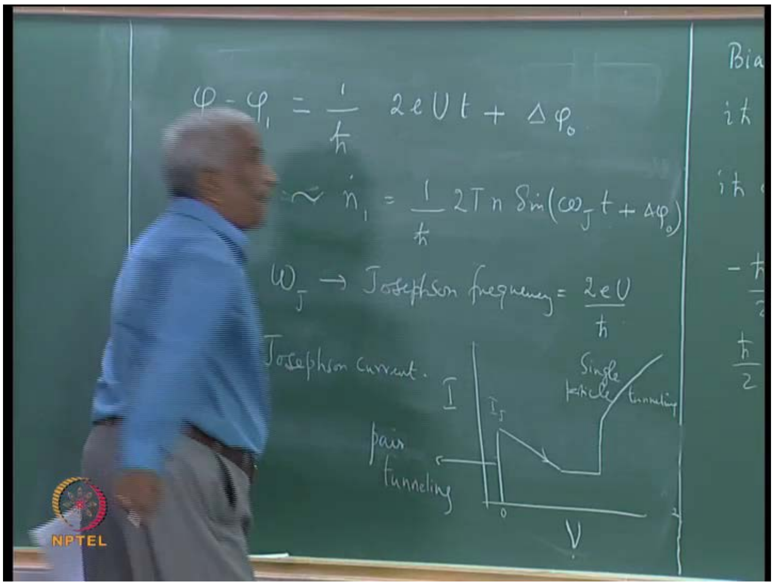

So, this means that the quantum mechanical phase difference phi to minus phi 1 between the two super conductor is just one by h cross into q is twice the electronic charge because of pairing as we know in e u times r plus any initial phase difference which might already we present. So, the Josephson current which goes as n 1 dot is 1 by h cross to t n sign omega j t plus delta phi 0, where omega j the Josephson frequency.

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$$\phi_2 - \phi_1 = \frac{1}{\hbar} 2eVt + \Delta\phi_0$$

$$I_J \sim \dot{n}_1 = \frac{1}{\hbar} 2T\, n \sin(\omega_J t + \Delta\phi_0)$$

$$\omega_J \rightarrow \text{Joesphson frequency} = \frac{2eV}{\hbar}$$

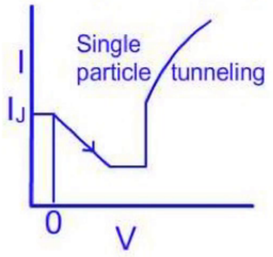

Fig 33.1 Josephson Current

Which is given by 2 e u by h cross and this is the Josephson current. So, this means that the as I said even in the options of a voltage across the tunnel superconductor tunnel junction there is a current which is known as usually known as the d c Josephson effect and in the presence of a biasing voltage u, there is an a c Josephson effect which tells that this point mechanical phase changes in proportion to the applied biasing voltage and correspondingly the a c Josephson effect gives you the Josephson current which changes which is a sin aside function of the Josephson frequency which in turn dependence is equal two e u by h cross.

So, this is givens schematically by the I V characteristic. So, that gives the this is the zero bias and that is the Josephson current and then this settles down to a constant value till it should up again and goes up like this. So, this is single particle tunneling when the cooper pairs are broken at sufficiently large biasing voltages are equal to the gap voltage the cooper pairs are broken and this corresponds to the tunneling current due to the single particle tunneling whereas, this is the Josephson or pair of tunneling and if we have in addition to a d c bias.

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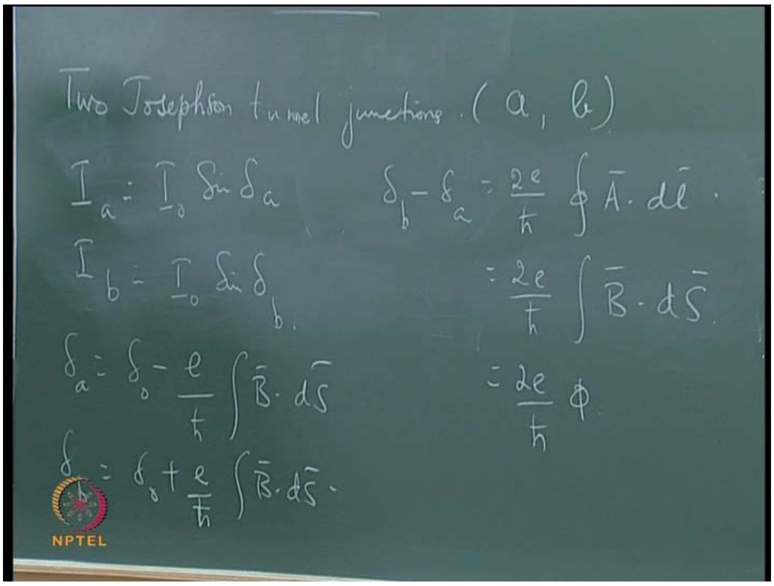

If we have u if it is u naught plus a small modulation a sinusoidal voltage cooper post on the biasing voltage substituting here defined the Josephson current source a frequency modulation, so which corresponds to this sum and different frequencies corresponding to the well-known side bands in a frequency modulated correct. So, this is the basic functioning mechanism of the functioning of the Josephson junction and as I mentioned yesterday the importance of the Josephson junction is inconsistent the fact that you can use to Josephson tunnel junction, which are identical.

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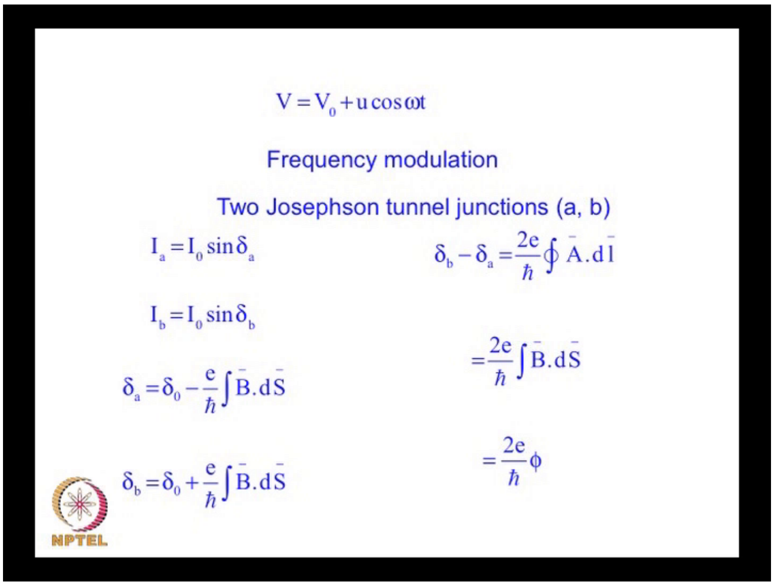

And then you have let us call them a and b then following the original treatment, we can write I a is some I zero sign delta i-a whereas, I b is I zero sign delta be where delta b minus delta I a is truly by h cross integral A dot dl because as we explain earlier the quantum mechanical phases will depend on the line integral of the vector potential or by Stokes theorem. And this is nothing but the magnetic flux. So, this means that I can write delta a and delta b as some delta naught minus e by h cross integral B dot d s and delta b as some delta naught plus e by h cross. Therefore, the current depends on the phase difference the Josephson current across such a device where there is the current flowing through this pair of Josephson junctions.

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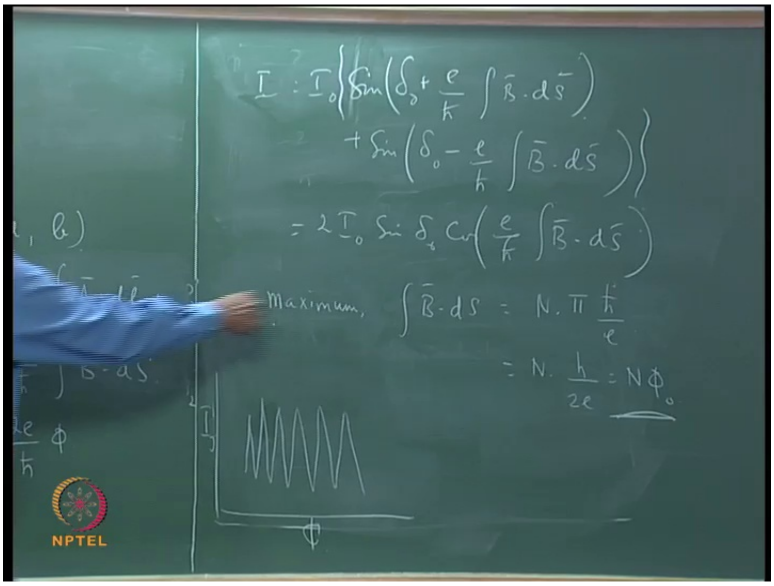

Becomes I equals I naught sign delta naught plus e by h cross integral B dot ds plus sign delta naught minus e by h cross integral B dot ds.

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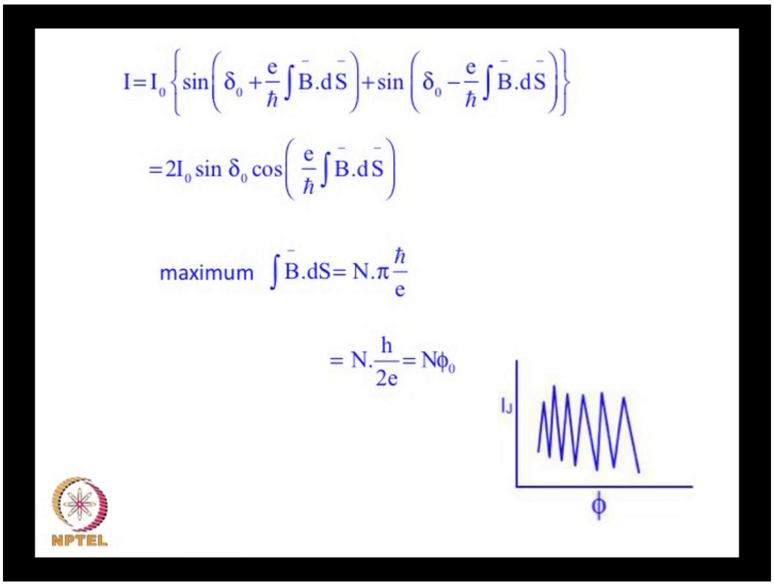

And therefore, that has the form 2 I naught sign delta naught cos e by h cross integral B dot d s. So, this current across the pair of Josephson junction will be a maximum when e by h cross or integral B dot d s equals n times pi h cross by e and that is n times h by 2 e or n phi naught. So, there will be a maximum in the Josephson current flowing across this pair of superconducting junctions this maximum will occur, whenever there is a flux quantum which is enclose in the superconducting ring consisting of two identical super conducting tunnel junctions.

So, this is the basic… So, by controlling this magnetic flux you can control the phase factor and you will get maximum in current like this the Josephson current. Whenever there is flux quantum, you get a maximum in the Josephson current and by measuring these maxima the flux corresponding to this maximum one can measure the flux quantum very sensitivity. So, that completes our discussion of the Josephson effect which is a very important application of super conductors in superconducting electronic devices. Now we pass on to a very important development which took place in the late 1980s as we already saw the phenomena now super conductivity was discovered in 1911.

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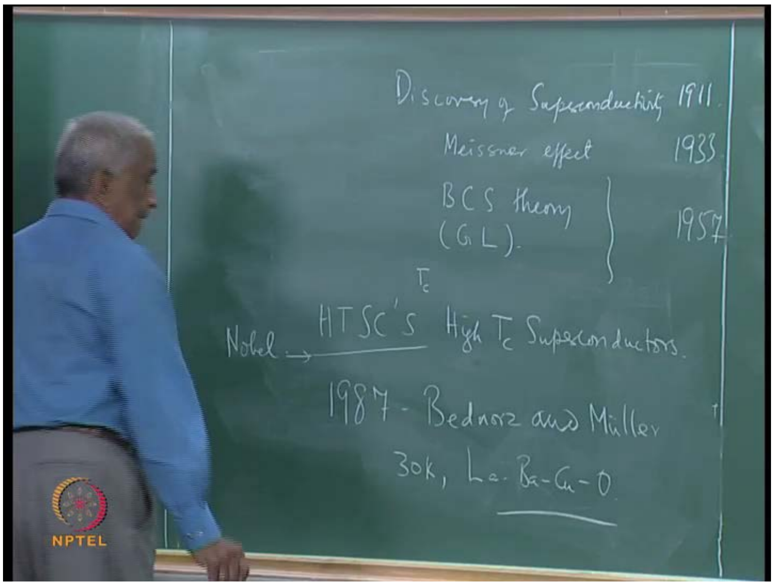

Then the Meissner effect which is an important aspect of super conductivity was discovered in 1933. Then the BCS theory the microscopic theory as well as the Ginsburg Glandou theory were in the 1950s, but the BCS theory while explained everything about superconductivity the equation for the transition temperature T c, let people to think that it is a unlikely that one can discover super conductors which T c is in excess of thirty Kelvin. Now this is a serious drawback because all applications of super conductors if this is the true will take place only if the super conductor is cooled below say something like 30 Kelvin and this refrigeration cost offsets most of the advantages of super conductors in devices.

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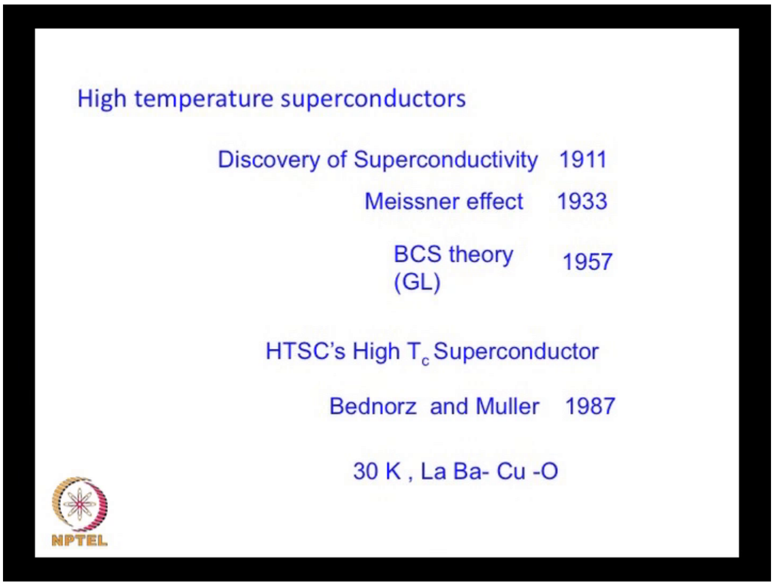

So, there was an imperative need for discovering some may in which a super conductor can become superconducting at a relatively high transition temperature high t c superconductor there was always a quest for our high t c superconductors they are all. So, written in strap as HTSC's. Now there was a amperes for quite some time till in 1987 two person Bednorz and Muller discovered a relatively high transition temperature of about thirty Kelvin in an oxide of lanthanum calcium barium copper oxide now this was in nineteen eighty-seven and then, but this was in an oxide not in a metal. So, this is itself by was a surprise and they got the noble prize in 1987 for this discovery.

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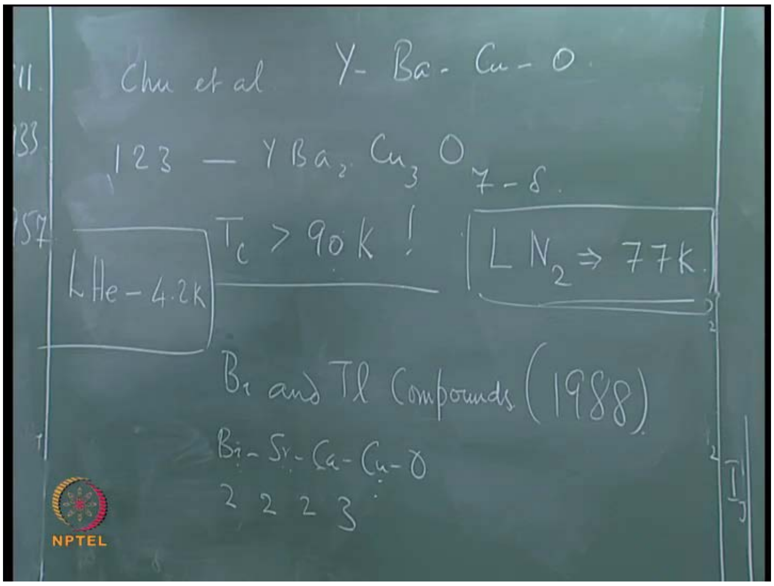

So, now, in the very next year Chu and co-workers discovered made a very stapling discover Chu et al discovered superconductivity in a compound consisting of atrium barium copper and oxide now the actual formula goes like Y Ba 2 Cu, 3 O seven minus delta. So, this is one, this is two, this is three, this 20 Kelvin. Therefore, it is known as a one two three superconductor and the oxygen has as 20 Kelvin naught seven, but slightly less than seven by a certain factor known as the oxygen deficiency delta and that was crucial for the answered super conductivity the T c in this case was greater than ninety Kelvin. So, that was a fixed k big jump in the discovery of high temperature superconductors now since the boiling point of liquid nitric n is 77 K as a boiling point of 77 Kelvin.

So, one can use liquid nitrogen instead of the more expensive and more difficult liquid helium which boils at 4.2 Kelvin. So, this problem can be a work that and one can use the much more readily available and cheaper liquid nitrogen as coolant for superconducting devices based on such one two three super conductors now this was very soon followed by discovery super conductivity in bismuth and thallium compounds this was in 1988 and the 20 Kelvin was bismuth calcium calcium copper oxide. So, this was known as 2 2 2 3 or 2 2 1 2 that gives the 20 Kelvin of this ternary elements in the compound similarly thallium based. So, these were discovered and they had much higher superconducting transition temperatures.

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Table 33.1

High T_c superconductors	Critical temperature (K)
$La_{1.85}Ba_{0.15}CuO_4$	36
$YBa_2Cu_3O_{10}$	90
$Tl_2Ba_2Ca_2Cu_3O_{12}$	125
$HgBa_2CaCu_2O_{6+}$	133

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Now, this can be seen in table shows high T c superconductors the newly discovered representative compounds and their critical temperatures in degree Kelvin. So, you can see that the lanthanum barium copper oxide as a critical temperature of 36 Kelvin while yttrium barium copper oxide as 90 Kelvin and thallium barium calcium copper oxide as a temperature transition temperature of 125 Kelvin the mercury barium calcium copper oxide has even higher temperature of 133 Kelvin.

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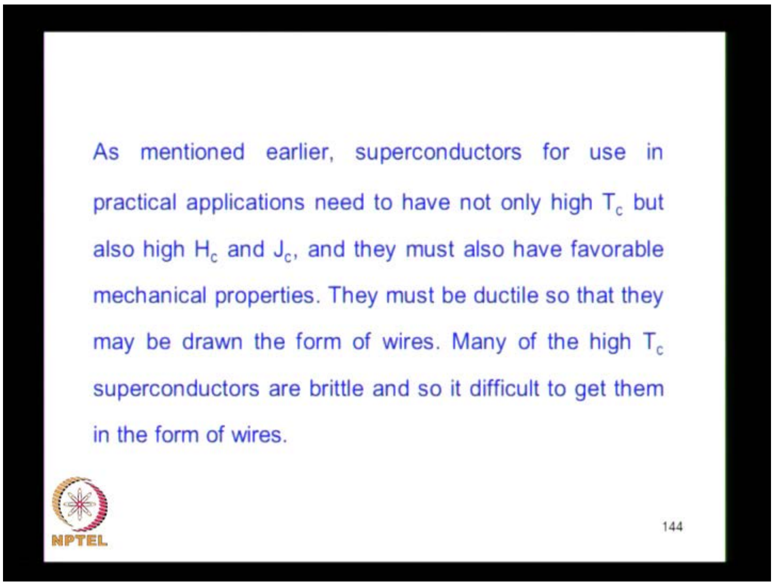

So, a number of materials which were more or less ceramic where discovered in quick succession which pushed up the transition temperature to something like 133 Kelvin. But as we already mentioned superconductors for using practical applications need to have not only high transition temperature, but also a high critical magnetic field and a high critical current density. Now the newly discovered super conductor also satisfy this criterion of having a high enough the upper critical magnetic field. They must also have favorable mechanical properties in order to be drawn into the form of wires to be wound in magnets and so on, they must be ductile. Many of the high T c superconductors have the disadvantage that they are brittle and so it is rather difficult to get them in the form of wires. Flux quantization experiments carried out in the super conductors showed evidence for pairing.

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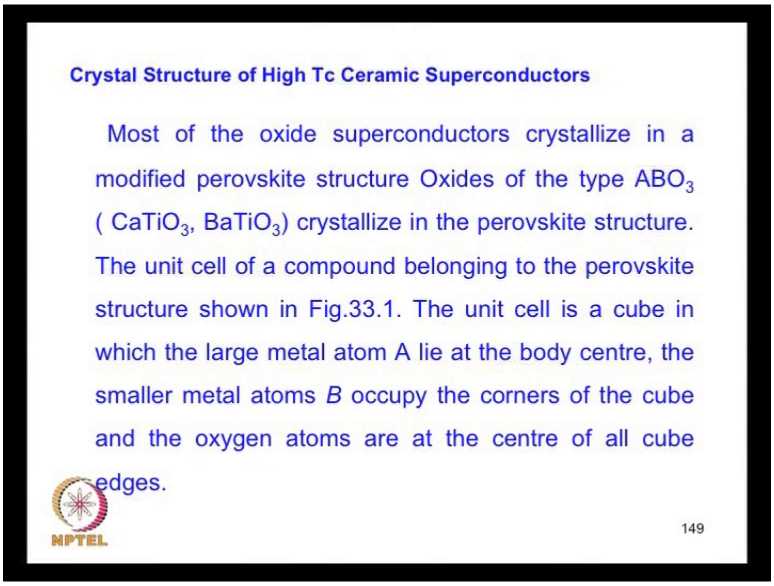

The measurement of the flux quantum tells you whether they are the parley accurse or not. So, the mechanism of superconductivity in this is again based on some form of pairing which takes place not between electrons, but between holes in the super conductor. We will discuss what is a Hall in the next few lectures when we come to discuss semiconductors because of this the whole nature of the superconductivity whole pairing they are known as p-type superconductors, and this is also confirmed by the sign of the Hall constant. We will discuss these things when we discuss semiconductors.

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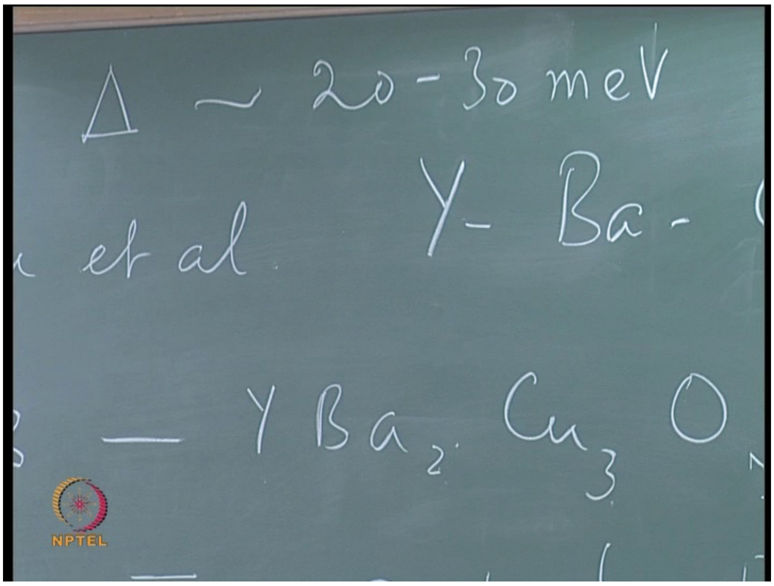

The energy gap in this case is in the range 20-30 million electron volts and in order to understand the mechanism of pairing whether it is d c s type.

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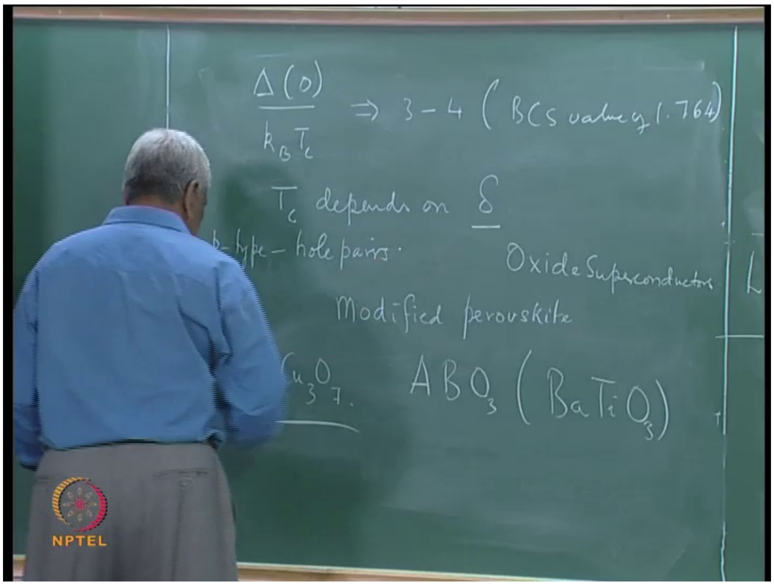

It is necessary to find the ratio delta 0 by k B T c. The energy gap at 0 Kelvin divided by k Boltzmann constant times the transition temperature. This is the range of 3 to 4 as against the BCS value of 1.764. Now the dependence there is a crucial critical dependence T c dependence on the oxygen deficiency parameter delta very critical. These are some the factor which has to be understood in order to understand the mechanism of superconductivity in these compounds. The crystal structure for example, this compounds most of these oxide superconductors these are all oxides not metallic upper conductors they crystallize in a modified Perovskite structure. For example, the oxides of the type A B O 3 for example, barium titan ate a typical member of this oxides in barium titan ate, which is well known as a ferroelectric material. They crystallize in the Perovskitz structure the unit cell of compound belonging to the Perovskite structure is shown in the figure.

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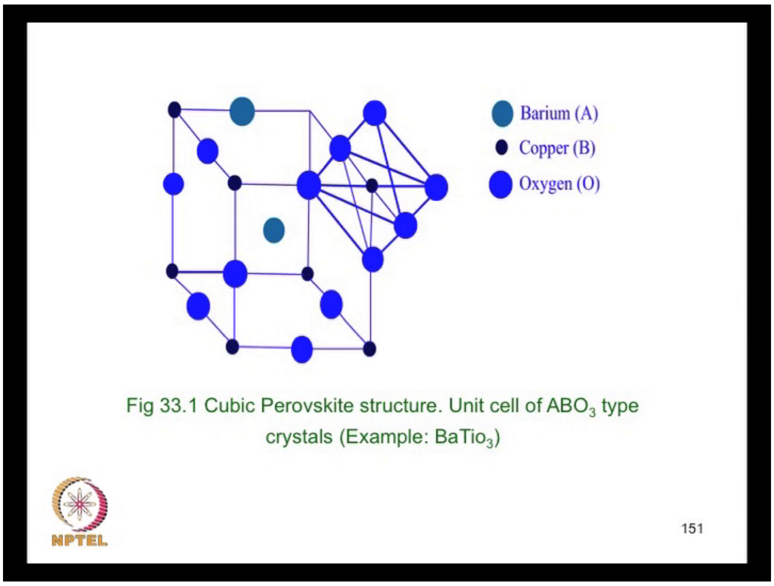

This is modified perovskite structure for the barium titan ate. So, the barium atoms are the big fears a atoms and the copper is the b atoms and the oxygen's are at the middle of the edges. So, the barium atoms sits of the body center and the oxygen's sit at the middle mid points of the cube edger's and the copper atoms sit at the vertices of this cubic unit cell. So, that is the basic structure. So, you have a cube in which the large metal atom a lies at the body center the smaller metal atoms b occupy the corners and the oxygen atoms are at the center of the cube edger there is one molecule in the unit cell the smaller metal atom b metal atom is surrounded by the oxygen atoms 6 oxygen atoms forming on octahedral in an octahedral coordination

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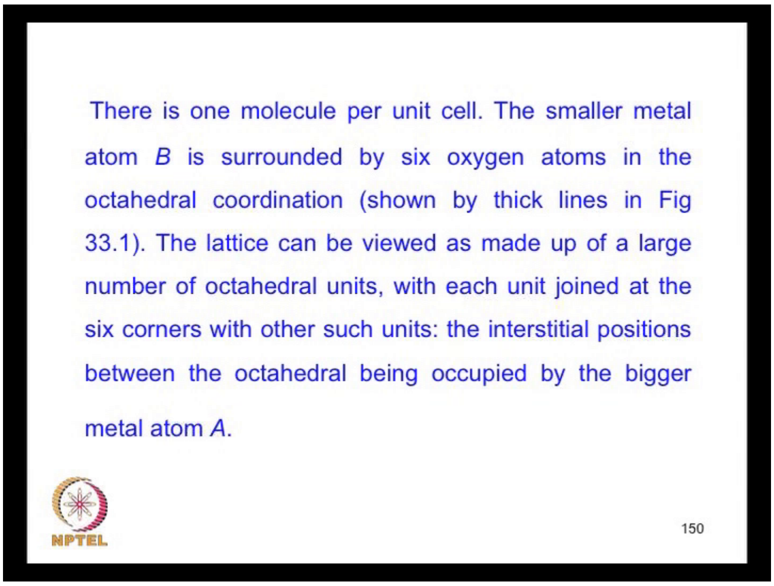

So, you have the lattice which can be viewed as being made up of a large number of such octahedral units with each unit joined at the six corners with other such units at the six corners of the octahedral and the interstitial positions between octahedral are occupied by the bigger metal atom a. So, that is the basic cubic structure now the yttrium barium copper oxide is a modified version of this.

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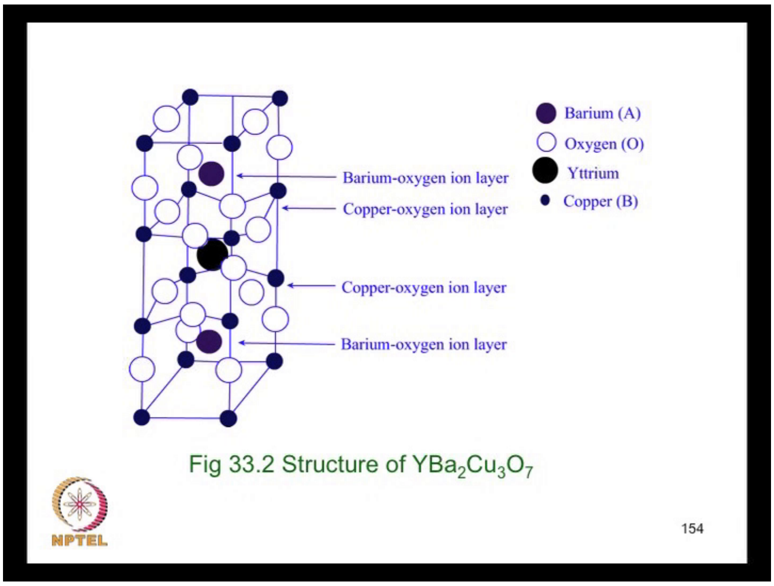

Fig 33.2 Structure of $YBa_2Cu_3O_7$

Now, this is shown in the next figure. Now you have a modified version in which we are showing the unit cell is bigger with three perovskite you read the cubes arranged along the c axis you can see it is tetragonal crystal not a cubic one and you have a c axis like this.

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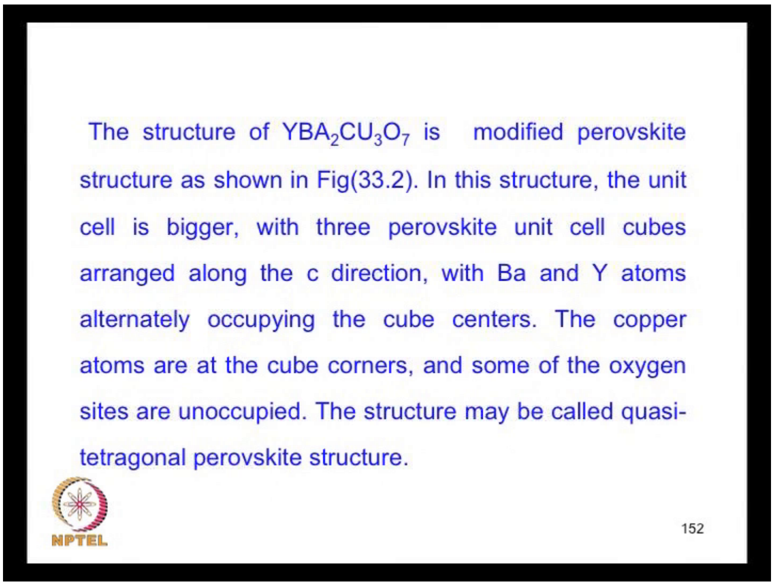

And the three cubic unit cells are stacked along the axis with the barium and yttrium atom alternately occupying the body centers of the cube. The copper atoms are at the cube corners and some of the oxygen sides are not occupied. So, it is a kind of quasi tetragonal perovskite structure which is a laid structure with alternating copper oxygen ion layers and barium or yttrium oxygen ion layers. The copper oxygen ion layers form conducting layers.

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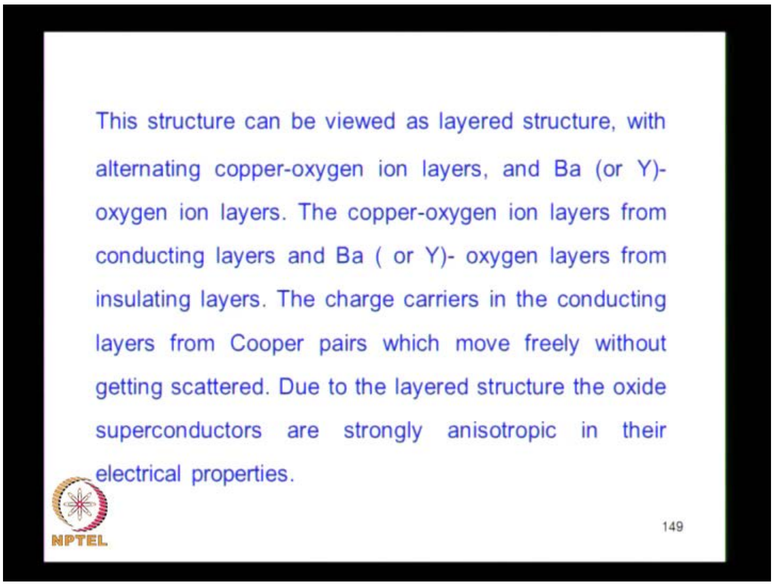

So, you have a kind of quasi two-dimensional charge transport, these copper atom are responsible for the superconductivity. The barium or yttrium oxygen layers form insulating layers between this conducting playing is the charge carriers in the conducting layer form upper pairs which move freely without getting scatter. Due to the layer structure the oxide superconductors are strongly anisotropy. So, that is the basic crystal structure.

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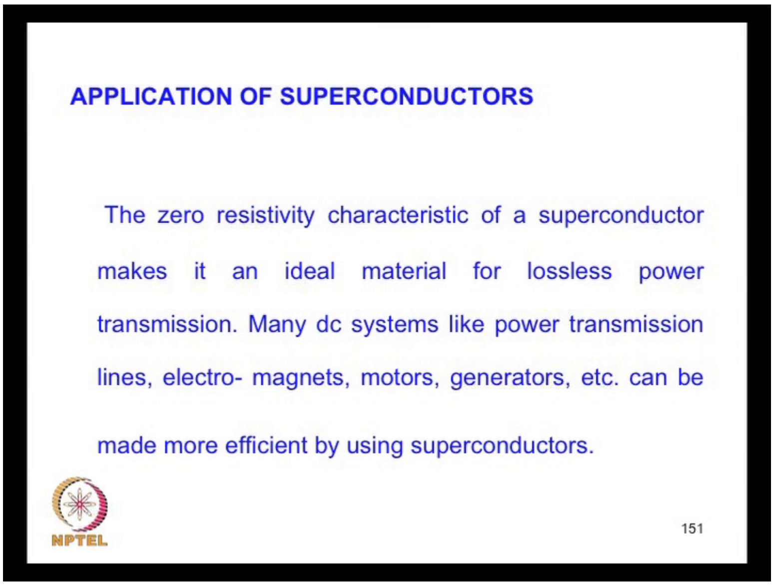

So, you have also in addition to these are p-type, because of hole pairs.

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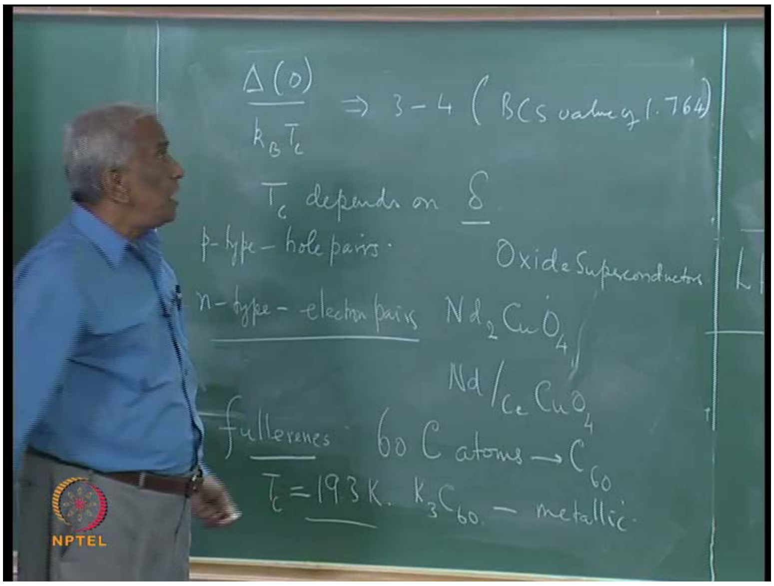

One can also have n-type superconductors which are due to the formation of an electron pairs they were also discovered zone and such as material such as neodymium copper oxide or neodymium cerium copper oxide, so these are n-type superconductors. In addition we also have the interesting new family of superconducting compounds based on the fullerenes. The fullerenes are consist of 60 carbon atoms formed in the form of a ball of diameter 7.1 angstroms. Now this is known as C 60 and if you take a crystalline film of carbon fullerene carbon and evaporate potassium atoms in. So, you have a potassium vapor then you have the formation of K 3 C 60 where the potassium form go into this ball.

So, then becomes a metallic material with the superconducting transition of 19.3. So, these are in more recent development in the area of ITC super conductor the elucidation of the mechanism of super conductivity in these material remains to be the discovered. Similarly, there are lot of interest in developments in regard to the technological exploitation of this newly discovered high-temperature super conductor; well, this whole feel these in a set of flux. So, we leave it at this stage.

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