Condensed Matter Physics
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Lecture - 31
Microscopic (BSC) Theory of Superconductivity

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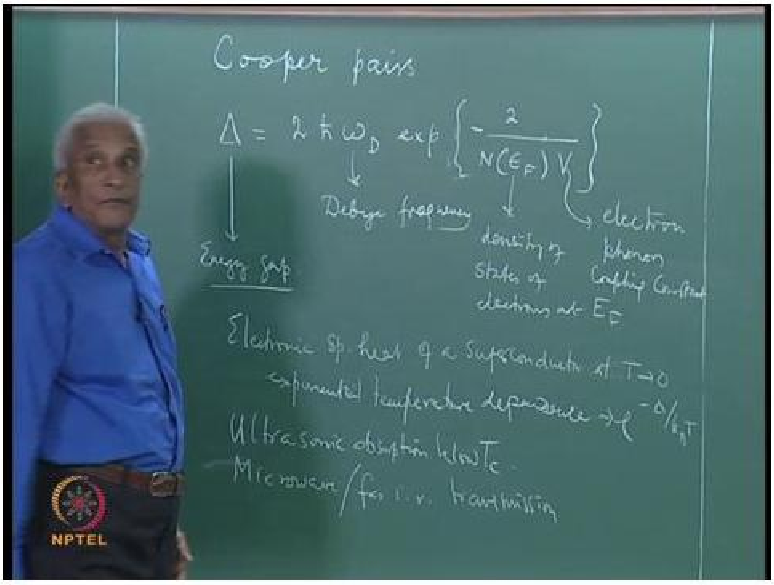

In the last lecture we discussed the quantum mechanical problem of founding a bound state for a pair of electrons, when there is a net weak attractive interaction between them, which is mediated for example, by the coupling of the electrons to phonons. This was a problem which was handled first by cooper, and the electron pairs thus forming the bound state are known as cooper pairs. And we saw that in the presence of one electron phonon coupling, which brings about a net attractive interaction between the pair of electrons forming the cooper pair. We get a net lowering of energy given by an amount where omega D with debye frequency, this is the density of states off electrons at the fermi energy E f in this is the electron phonon coupling constant.

So, the energy of the bound pair is lower than that of the separated normal state electrons, and therefore energetically these favorable to form cooper pairs for these two electrons. Now, the existence of such a gap which opens at the fermi level is experimentally supported by techniques, such as electronics specificate of a superconductor at very low temperatures. So, this is an exponential temperature dependence very similar to what you find in system such a semiconductor we will discuss this shortly, where there is a energy gap. So, similarly the exponential temperature dependence indicates an activation of the carriers responsible for thermal transport in this case. So, there are excited across this gap delta and that is why there in order to do that they have to have an activation energy of this order, so this goes a exponential minus delta by k B T.

So, that is the temperature dependence which is observed you can determine the energy gap. The second experiment is that of ultrasonic absorption below T c, and the third experiment is microwave or for infrared transmission in superconducting films. So, these are experimental evidences for the existence of the energy gap.

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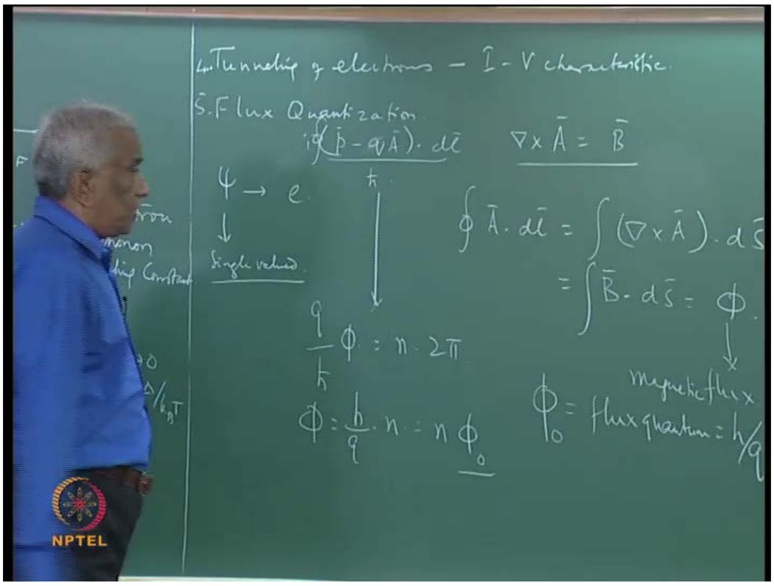

So, we discuss this already also an tunneling of the electrons in the I V characteristics, which shows the onset of current when the biasing of voltage of the tunnel junction exceed that of the energy gap then current starts for ((Refer Time: 06:07)). So, that is again, so these are the various experiments to which we know that there is the energy gap in the excitation spectrum of the electrons, when the material become superconductor. The other important experiment associated, this is flux quantization which we discussed already the quantum mechanical phase of a superconductor goes as e to the power i p minus e A dot dl is integral by h cross. So, the or q A, the q is the charge. So, this is the form of the wave function. So, in order to that this wave function is single valued when there is a magnetic field, who’s A is a vector potential. So, when we apply a magnetic field, the phase factor is related to the line integral of A by stock theorem this becomes del cross A dot ds, and this is just nothing but B and this is what we call the magnetic flux trading the low.

So, the phase factor goes as q by h cross times phi, the phi is the magnetic flux. So, this in order to psi is single valued, when one goes through close loop inside this superconductor, this should be a integral multiple of 2 pi. In other words phi should be h by q times n or n phi naught, where phi naught is the flux quantum, h is flux constant, in q is the charge carried by the carriers in the superconductor.

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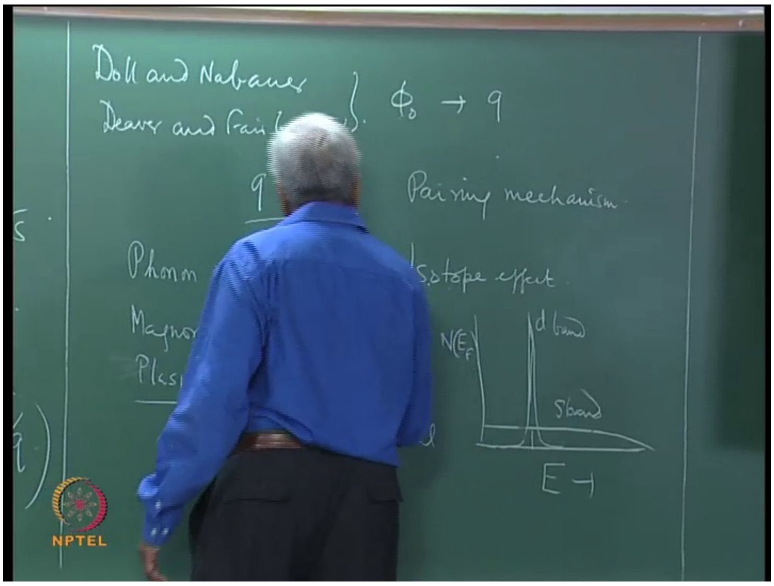

Now, in the experimental measurement of the flux quantum by doll and ((Refer Time: 09:06)), as well as Deaver and fair bank independently determine the flux quantum, such that q can be evaluated from this. Since h is a constant determination of the flux quantum enables as to know the value of q, and both are them found independently the q is twice the electronic charge. And that is a lynching evidence for the existence of cooper pairs which a pair of electrons carries a charge of twice the electron charge. So, the determination of flux quantum, gave conclusive evidence regarding the paring mechanism, which is a responsible for the superconducting behavior.

So, essentially nowadays if you want to account for the superconducting nature of a material new material, one looks for the pairs, and the immediate question is how does the pairing arise? So, the theory which was which was discussed first by cooper is based on the phonon mechanism of pairing. This is also supported by the experimental observation of the so-called isotropic effect, since this phonon mechanism depends on the vibration of the crystal lattice. So, the harmonic nature means that there is a dependence on the mass for the coupling. So, they found that the transition temperature depends on the mass of the ion involves, so this again indicated at phonon, so somehow involved in bringing about the pairing, but it is not necessary that you should only be the phonon mechanisms anything that which the conduction electrons are coupled, any excitation. For example, magram are there, exciton are there or plasmon's are there, and so on so. All these can be pass any of these elementary excitation in a crystal lattice, which can couple to the electronic systems can provide parrying mechanisms.

So, it is not necessary should be the phonon mechanisms from the phonon mechanisms is the most obvious and simple is of the possible mechanisms, but other mechanisms cannot also exists what is crucial is there is a pairing of the electrons. So, the formation of the cooper pairs is crucial to be superconducting the feature of any material. and because of the phonon mechanism which indicates that the energy binding energy goes as exponential minus 2 by n e f v this is the three axis for free exponential factor this is only weakly dependent on the free exponential factors, the main temperature dependent's occurrence of the argument the exponent's.

So, they density as state the product of the electronic density of states, and the coupling; this is the crucial factor which decides how much the energy is lower and how strong is the binding between the cooper pairs, and that is also the reason why when we extra large, the electron phonon coupling is strong. Usually we have already discussed the mechanism of electrical resistivity the normal metal, in the presence of a strong phonon electron coupling, the material is going to be a poor conductor having high resistance electrical resistance in the normal stay. And that is also the condition of the occurrence of superconductivity, which is against the experimental fact, that it is usually the poor were conducting metals such as led or aluminum, which becomes superconducting rather than the very good conductors like silver or gold or copper for that metals.

So, this is also experimentally go now. Similarly the electronic density a sate is another crucial factor determining extent of winding of the cooper pairs, and this is again seen experimentally in the sense at most to the superconductors I T c superconductors, which have a high binding energy which indicates high binding energies for cooper pairs are all involving a transition metal. And transition metals because of the D band electrons, so niobium all these of transition. So, there all here high pick, because of the narrow D band they are strongly peaked at the fermi level and therefore, the electronic density of states at the fermi level tends to be very large.

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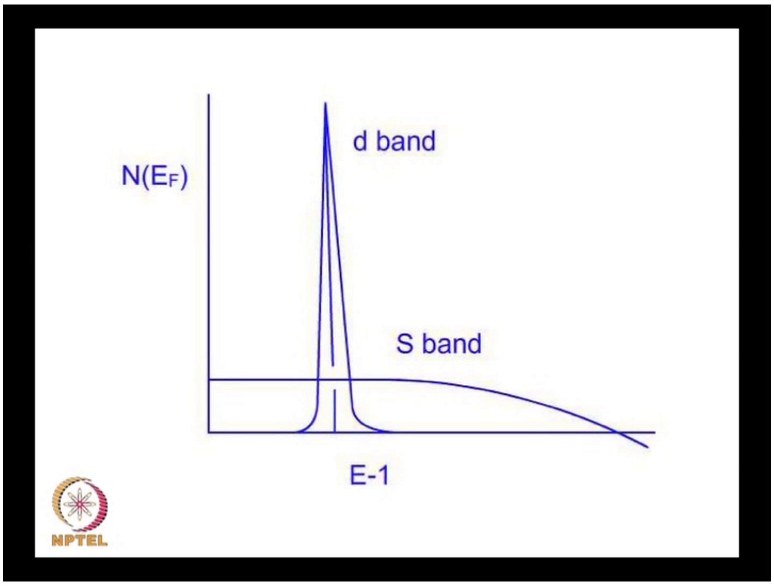

So, if you see, so these are narrow for example, a conventional metal like this. So, you can see a sharply peak d band metal, this is an s band metal. So, this is the fermi level. So, you can see that a d band metal has a larger density of state, therefore there are more prone to have a higher T c the transition temperature the conventional s band metal that is also experimentally seen.

So, these are all strong dedication that basic mechanism of being propose padding mechanism as the essentially correct. However, in case of the ground state of BCS superconductors and I have only discussed the formation of a pair by being coupling between the two electrons, but a BCS superconductor consist of a very large number of electrons something like that 23 electron.

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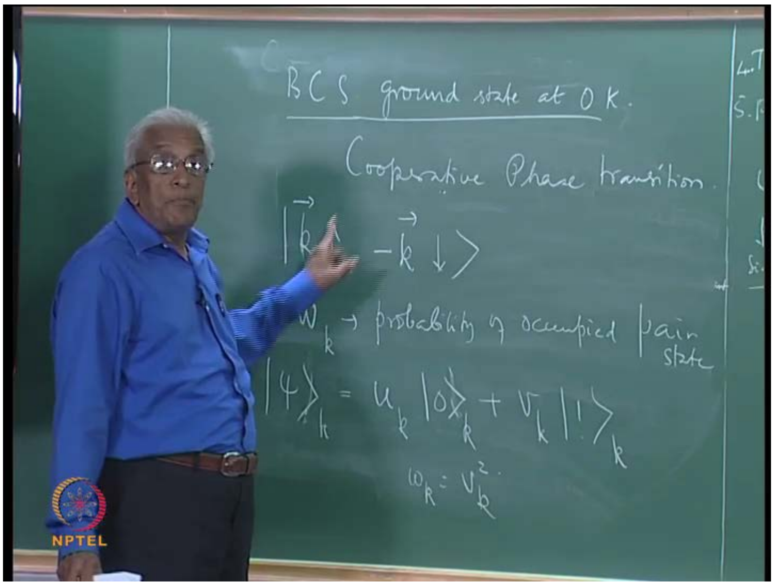

So, we are in order to discuss the energetic of the BCS ground state at 0 kelvin, we have to discuss how all the 24, 23 electrons form pairs, this is and we also know the superconductor transition is a cooperative transition which takes place in a very narrow temperature interval. And this is because when the energies is lower than the formation of cooper pairs, it is energetically favorable this leads to the formation of additional pairs and so this becomes a cooperative phase transition. So, the entire T f conduction electron forms pairs. So, this is the essence of the cooperative phase transition, and we have to talk about how the assembly of large number of electrons forms pairs, this was discussed by BCS and for these we have to discuss the formation of the cooper pairs state, which is characterized by this, the cooper pairs wave function is characterized by this.

So, we have a state in which both states with plus k and minus k with an up stiff spin and a down spin; these two states of the normal electrons are both completely occupied or completely empty. So, the wave functions, if you want to write the probability that a pair state occupied that the state is occupied by a pair of electron. If w k is the probability, then we have the wave function can be written as u k 0 k plus v k 1 k, where w k just a the square of the v k, we assume u k and v k to be both real for simplicity. And the state 0 k and one k indicate that the state with the wave vector k is simultaneously occupied by a pair of electrons which momentum k and minus k wave vector k and minus k with up- and down spin. So, then this is called one k. So, it is a state with two electrons occupying such a whereas if it is 0 k; that means this state is completely empty. So, u k is the probability that the state is a state with k is completely unoccupied, whereas v k square is the probability whether it is completely occupied.

So, the states are either occupied together in pairs or completely unoccupied, and v k square is just the probability of transition where probability of occupation, which is w k. So, this is the essential we function of these two electrons state paired state with wave vector k. And the BCS wave function of the ground state of a BCS superconductor can be written in the form of a product of such paired states in which the electron, the states are either completely occupied by a pair of electron or both are are completely empty.

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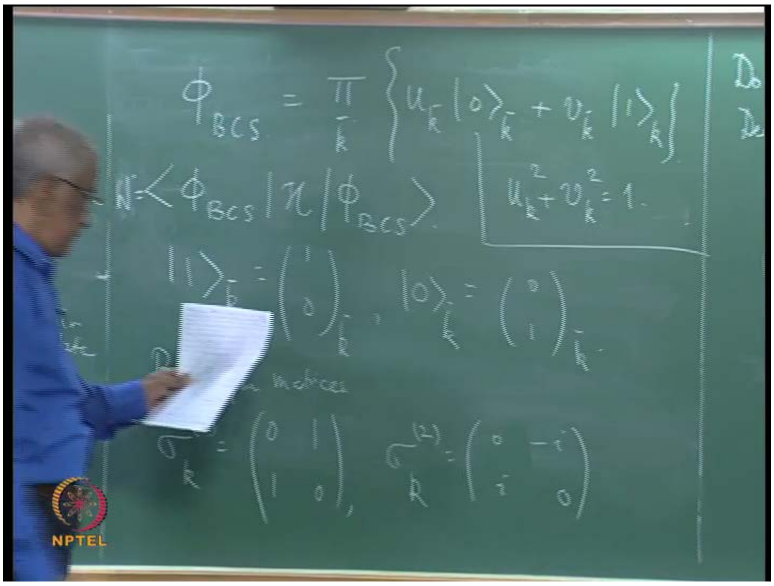

So, we write the phi BCS the ground state wave function a consist of a product of such states pi k u k 0 k plus v k 1 k. So, that is the downstate we function i k and this total B c s state is formed of products of such one such estates pared states, which means that it assumes that the pairs are non-interacting. Now in order to find they energy of this ground state we have to write Hamiltonian, and then find this energy that is the Hamiltonian of the electrons. So, this is the energy w, and we minimize this with respect to the variation in the para meters u k and v k, we have u k square plus v k square equal to 1. So, these are not independently vary if you fixed u k then v k gets automatically fix and vice versa, therefore enough to vary with respect to one of; these two para meters in order to do that we represent these states 1 k 1 0 by matrixes k, and 0 k is 0 1 k. In this case we can use the pauli spin matrices to calculate the expectation values, what are the pauli spin matrices? We have they characteristics sigma k one as they form 0 one one 0 and sigma k 2 is 0 minus i 0, if we have this combination of this can be used to represent the analyze assign and creation of pairs in the following way.

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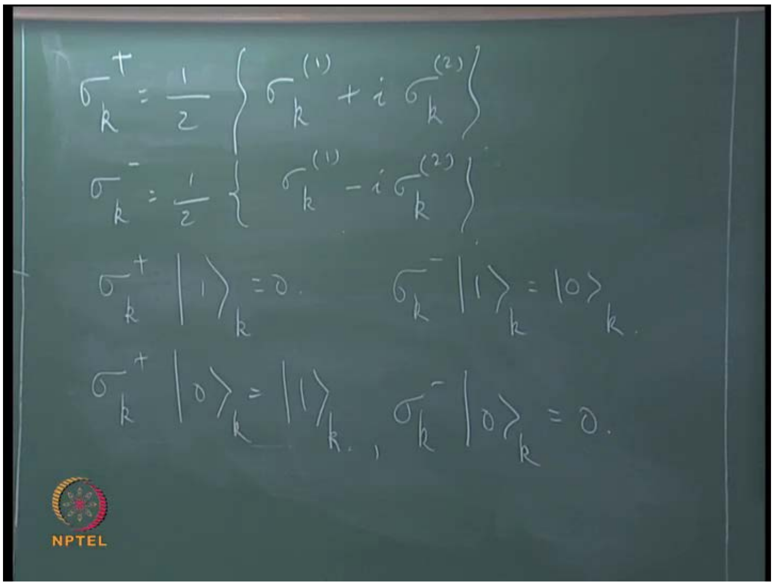

We write sigma k plus is half of sigma k 1 plus i sigma square k 2, and sigma k minus is half of sigma k 1 minus, in terms of sigma k plus and sigma k minus. We can show that sigma k plus operating 1 k is 0 sigma k plus operating on 0 k 1 k and sigma k minus operating on k 0 k, and sigma k minus operating on 0 k is 0 k, in terms of these we can write the electron phonon interaction Hamiltonian.

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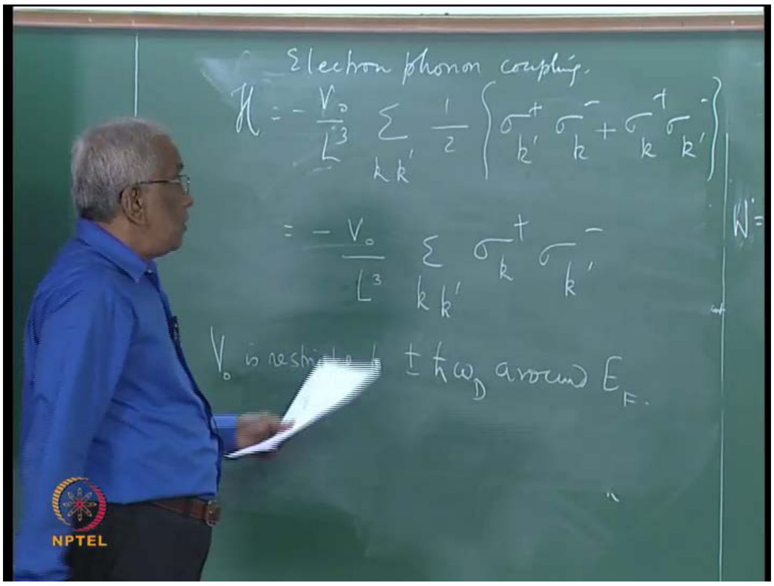

And then find the energy expectation value it turns out that the Hamiltonian, the electron phonon interaction, the Hamiltonian for this turn out to have the form l by v naught l cube v naught sigma k k prime half sigma k plus sigma k minus plus sigma k plus sigma k prime minus 1, which means that we create an expectation in the state k prime, this term where simultaneously removing an excitation. So, this can be written as… So, using this, now v naught is respected to plus or minus h cross omega D around ferny energy, with this we can calculate with this, and the BCS wave function we are now in a position to calculate this in times out.

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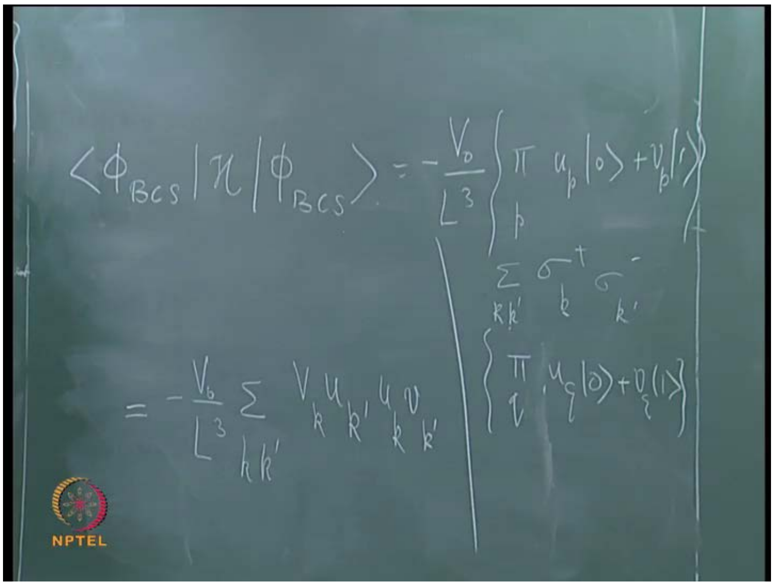

That we have the result is equal to minus v naught by L cube pi p u p 0 plus v p 1 times sigma k k prime sigma k plus sigma k prime minus times pi cube u k 0 plus v cube 1. So, we can calculate this using the properties of the Powli spin matrices, and therefore we have this as minus v naught bi l cube sigma k k prime v k u k prime then u k v k prime. So, we have calculated the net energy value Eigen value of this.

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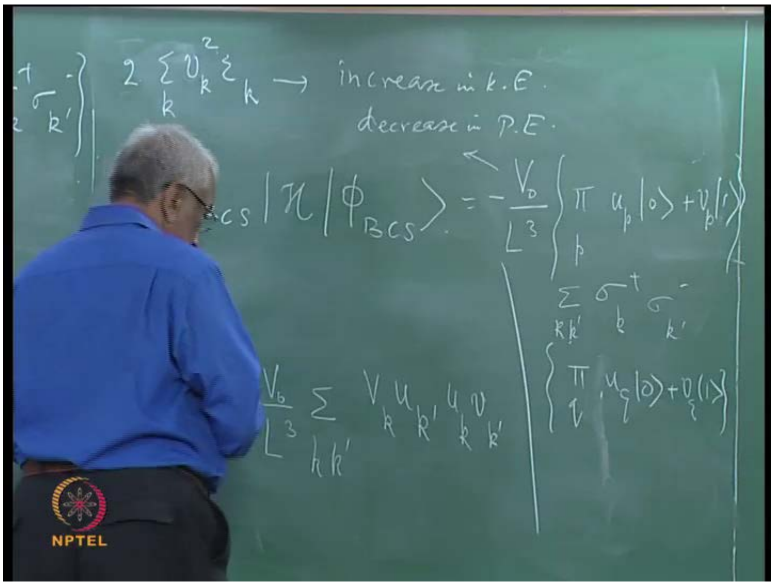

And we have also the formation of pace also increasing the kinetic energy by an amount psi k per pair times v k square, which is the the probability of the forming prize this because there is a page. So, that is the increase in kinetic energy is due to pay formation, this is the decreasing in potential energy due to electron phonon interaction. So, the net BCS energy is a some of these two and increasing the kinetic energy plus lowering of the potential energy.

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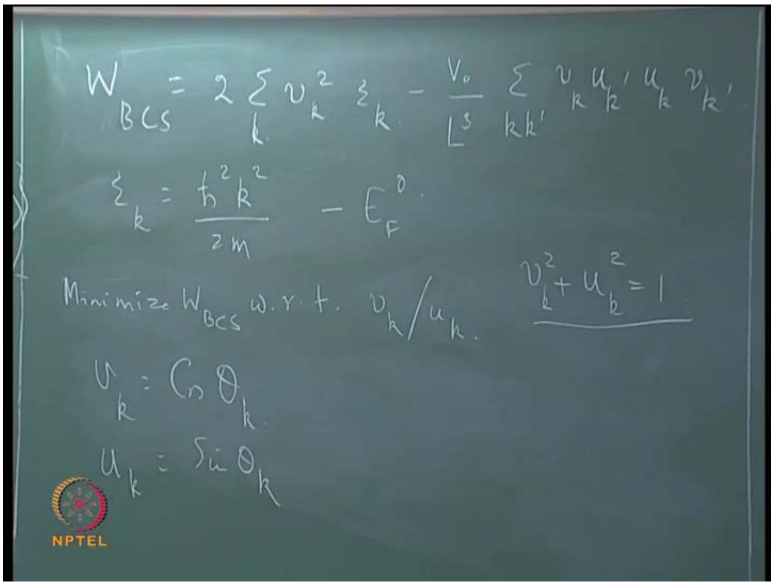

So, W BCS ease some of these two sigma v k square by k minus v naught by l cube sigma k k prime v k 2 k prime u k and v k prime. Here psi k is the kinetic energy of a single electron with respect to the ferny energy at absolute zero. So, we have to minimize this w B c s with respect to v k or k. So, this is done such that v k square we must remember that v k square plus u k square equal to 1. So, we choose v k to be cos theta k. So, that and u k to be sin theta k. So, this condition is automatically satisfied.

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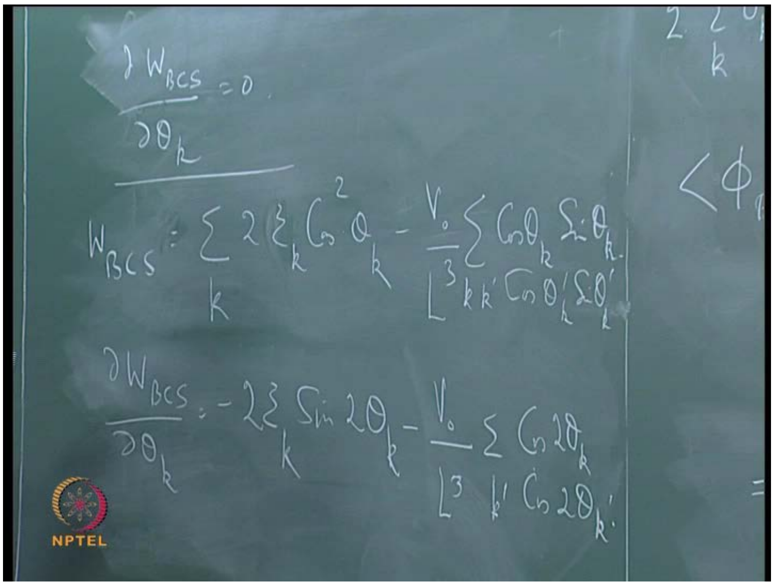

So, it is enough if we minimize it rewrite this in terms of the theta k and then minimize it with respect to variation in theta k. So, this is what we will do now. So, we demand that the ground state energy is minimum when D W BCS by D theta k is 0, where W BCS in terms of the theta is sigma to psi k cos square theta k minus v naught by L cube sigma cos theta k sin theta k k k prime cos theta k prime. So, this is what we have to minimize with respect to theta k, and this gives 2 psi k sin2 k minus by v naught by l cube sigma k prime cos theta 2 k cos theta 2 k prime.

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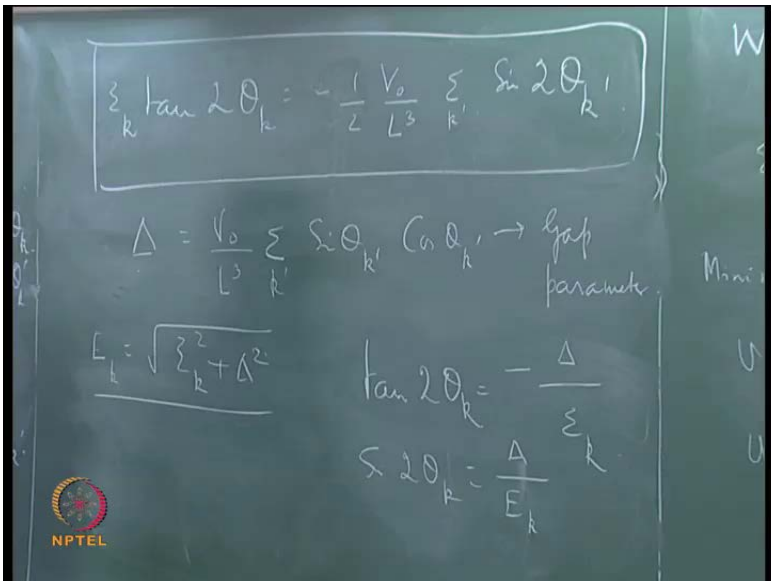

So, that the condition for minimum this is equal to 0 gives the important result psi k, and two theta k equal to minus half v naught by l cube sigma k prime sigma k prime sin2 theta k. So, that the essential condition for a minimum of the energy. Now we designate a parameter delta as a short tan this v naught by L cube sigma k prime sin theta prime cos theta k prime with that short hand, we have and also be right E k equal to root of psi k square plus delta square, delta is known as the gap parameter. In terms of these this condition reduces to tan 2 theta k equal to minus delta by psi k which also mean sin 2 theta k is delta by k were v k is defined by this. So, this is the condition for minimum and we use this to calculate the minimum energy, which is the energy of the ground state of the BCS superconductor at absolute 0.

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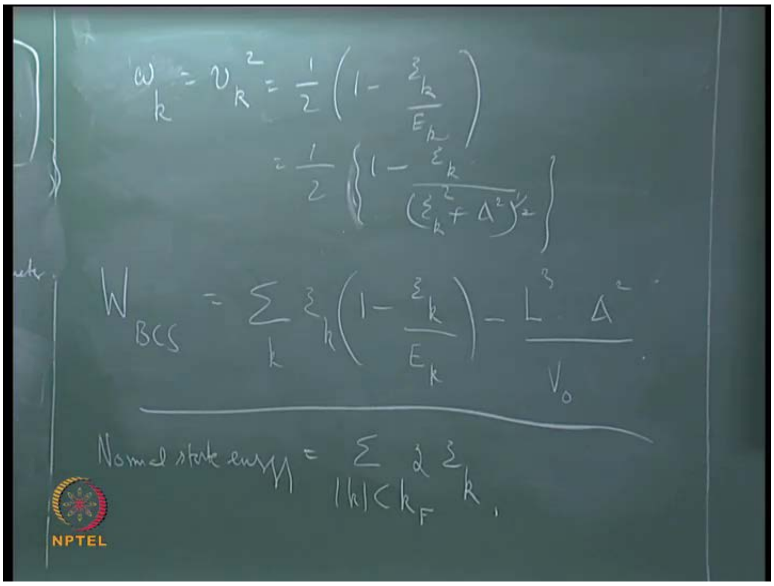

So, we get w k, this is the probability half of 1 minus psi by k is equal to half of 1 minus psi k by psi k square plus delta square to the power half. So, the w BCS with this minimization turns out to be… So, this is the energy of the energy of the BCS super conducting state at absolute 0, and if he wish to determine whether the super conducting state as the lower energy than the normal state. We have to subtract this energy from the normal state energy, which is sigma mod k less than k f where these fermi wave vector of 2. So, if we remove this energy if we find the difference between these two, and if it is negative; that means that the BCS super conducting state is energetically lower. And therefore, it is favorable to for the electrons to be compared and form a superconducting state at absolute 0, we will consider this to determine the condensation energy of a super conductor in the next lecture.