Condensed Matter Physics
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Lecture - 30
Cooper Pairs

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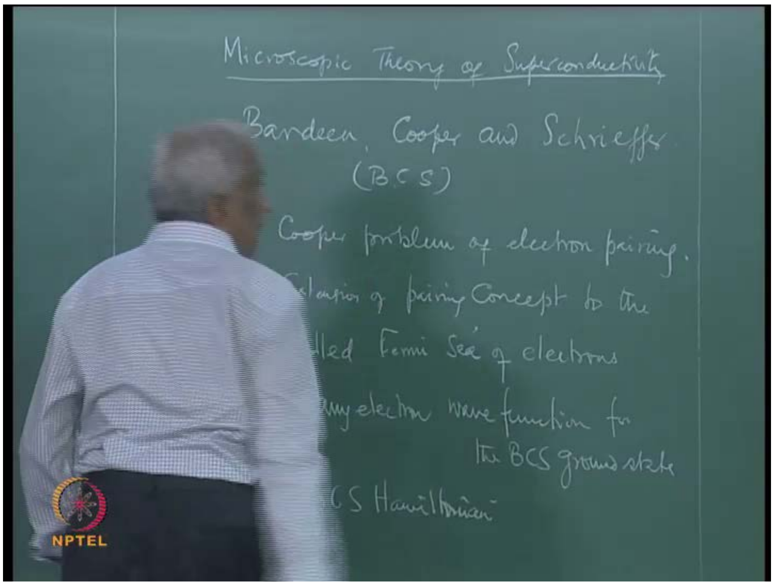

Last time we spoke about a microscopic theory of superconductivity due to bardeen cooper and schrieffs sharp it is bees it is known as b c s theory. So, today we will starch on this microscopic theory of superconductivity we will do it in several steps today we will do step one this step one is what is known as the cooper problem of electron pairing step two will be you extended to extension of pairing concept to the filled fermi see of electrons. And here we defined the many electrons wave function for the b c s ground states we also define b c s Hamiltonian have been defined these we go to step three which is using this Hamiltonian and diagonalize the b c s, Hamiltonian to obtained the ground state energy eigen value of a b c s superconductor than use this formalism this is the theory at zero k for to explain behavior of a b c s superconductor not at zero k, but at finite temperature this will be our program for the next few lecture.

Today we will talk about step one namely the cooper problem the problem of forming cooper pairs this is the problem which was discussed that by cooper; obviously, the figure shows the situation of such a superconductor at absolute zero where the spherical portion of the wave vector space flows the filled fermi see is the represented by the sphere of radius k f where k f is the fermi wave vector defined by e f equal to h cross by a k f square by two m this is the fermi energy. So, that what is show in this figure and. So, cooper consider this situation of all the conduction electrons forming a the filled fermi see in which each state is occupied by one electron according to the pauli exclusion principal. So, he considered an electron which is just an outside this filled fermi see one electron just outside the filled fermi see.

So, this electron we consider the interaction of this electron with another electron this electron electron interaction between one pair of electrons just two electrons this is what we shown in the figure now these two why they have taken just outside is that a if they are inside the filled fermi see they cannot interact cannot go into another state because of pauli principal. So, they just interact between themselves such that all the states inside the sphere of radius k f or filled or. So, are not available.

So, now this interaction depends on the electron states and we know that these are planes we states at the farm e to the power i k dot r where k is the wave vector and. So, we have one electron with wave vector k and another electron with wave vector minus k both corresponding to the same energy because energy is h cross by h k square by. So, it is depends on the square of the wave vector and therefore, both k and minus k correspond to the same energy eigen value for the given pair of electrons now what is the hamiltonian of these two electrons the system of two electrons. So, let us write the hamiltonian the hamiltonian consist of the two electron hamiltonian is given by two e k where e k is the individual kinetic energy of a electron kinetic energy p is given by this two e k plus these two electrons are supposed to interact and the interaction potential energy is taken as v.

So, if I take the interaction potential matrix’s elements of the interaction potential, it is v k k prime and this is again the total system is this, but let me just. So, it is two e k plus sigma k prime v k k prime. So, this is the v k k prime the interaction potential between the two electrons and they different electrons in this k k prime can be consider, but in our case of course, will finally, restricted to k and minus k now we are keeping a quite general that there is an electron and two electron here and this is the v k k prime between them let us just a put v k minus k there k prime equals minus k let see two electron Hamiltonian. Now we have to find a Eigen value of these when there non-interacting this term is absent and we know that the appropriate Hamiltonian we such that h naught psi k is equal to e k psi k this is the zeroth order hamiltonian zero th orders states and these zero th order states schrodinger equation these zero th order states will in general be a pair of electrons linear combinations of the plane wave states e to the power i k dot r and the e to the power minus i k dot r. So, there can be symmetric and anti symmetric linear combinations.

So, the symmetric linear combination goes as cosine dot r while they anti symmetric in a linear combination goes as sign k dot r it is quite clear to see that it is the symmetric linear combination corresponding to the cosine function is concentrates charge being the few electrons therefore, we take the symmetric linear combination spatial linear combination and an anti symmetric spin combination of the wave function. So, because the total wave function cheese just the product of these two should be anti symmetric not to satisfy pauli exclusion therefore, we can write this states charge k minus k and then anti symmetric spin formula and. So, that would be a kind of functions which we are talking about.

So, this is the wave function and now we want to solve this general h psi equal to e psi we want to find the energy eigen values e corresponding to the two electron hamiltonian the presence of the electron electron electron interaction, and the wave functions here or just linear combinations of the zero th order. Therefore, sigma over k e k psi k. So, plugging this back into the and using this values we get e k sigma g k two e plus sigma v k k prime g k prime you can take this out.

So, this is over both k and k prime and that is equal to e sigma k g k. So, bringing these two together we can write ringing these two together we can write g k into e minus two e k equal to sigma v k k prime g k prime g k prime now taking this we can just tried g k as sigma v k k prime g k prime divided by e point two e now we some over the case and we know is some just one and this will be now we go over from a summation over the case takes to an integration changes sigma with an integral because the case states are form a quasar a continue we all.

So, restrictive a interaction matrix's elements v k k prime to a specific value namely v k k prime is equal to minus v for e f zero less than e k and e k prime and which is in turn left there e f zero plus h cross omega d what this means basically this means that this interaction matrix element is attractive in a negative sign shows that it is an attractive value attractive interaction and as a constant value v over an energy range it will see this this is e f zero the super script zero means it is at zero k we are discussing everything at zero k and this is a cut-off frequency h cross omega d. So, that is a typical cut-off frequency the beyond which the interaction potential is not effective. So, within this range it does not change, but it as a constant value.

It is negative for e k and e k prime in the region that is the significance of this defeat is outside range than it is zero; that means, we will be justified in taking the factor in this out and writing one equals to minus v and then converting this summation to integration n of e e d e by e minus let me write the energy eigen value as e should distinguish between two. So, this is between e f zero and e f zero plus h cross omega d to make further progress we will make a simplifying approximation will say the density of states n of e is nothing, but the density of states of electrons. And we will assume that this is a slowly wearing function of energy so; that means, that its value at the fermi energy is n of e f and it stays more or less the same the entire range the neighbor wood of fermi energy.

So, we can take it as the constant and take it out of the integral and. So, doing that the finally, arrive at one equal to minus n of e f v is integral d e by e minus two e e f zero to e f zero plus h cross omega d and that gives me substituting the limits after performing the integration this will be just and this can be simplify even further therefore, since I have a logherithm here I can take exponential two n by f by e bringing all these factors to this side and exponentiating Isimply get my idea is to get at energy eigen value therefore, I will simply write two h cross omega d by e minus e f zero equal to one minus exponential two by n p. So, cross multiplying I get e equal to yeah two e f zero plus two h cross omega d into and that can be simplified. So, e minus e is the energy eigen value and the original energy value was y is the ferny energy two e f zero because he take the electrons to be at the ferny energy.

So, this is the original energy now this is the energy eigen value in the presence of interaction. So, we get finally, e minus two e f zero is the change in energy due to the interaction, and that can be further simplified and written as a simply multiplied and divided by exponential minus two by n f e f one e this because particularly transparent if we make the simplifying approximation that n f e f times v is very small compared to one this is this means that this is a weak coupling this is called the weak coupling limit in this case this will become delta we write delta has a difference in energy negative of that and that is the binding energy of a pair if a pair is formed this will be the bending energy this is the final energy and this is the original energy the difference between these two is the change in energy negative of that gives a binding energy of the open pair we call it delta and that is just minus two h cross omega d exponential minus two by.

So, we are arrived at our desired result this is a remarkable result this is the essential solution of the cooper problem. So, this says that if v is not very high even in general and if it is extremely small as long as it is attract you. So, that this is first you definite then there is a binding energy which is positive in other words the two electrons which form a bond pair which is known as the cooper pair of electrons. So, it will just pairing will have is happen because it is energetically favorable to form a bond pair as long as v is attractive no matter how week never are less let us, now we look at the electron electron interaction let us go to the figure. So, we have that two electrons and they form the bond pair because the two electrons normally have a coulomb repulsion and this coulomb repulsion of between the two electrons is screened screened coulomb repulsion that is one of the main obvious source of interaction between the two interactions. So, this screening is because of the interaction of the positive ions in the metal.

So, they provide a screening and. So, we have not a long-range infinite range coulomb repulsion, but it is a screened short train coulomb repulsion. So, and then in addition and we have a another mechanism which gives a phonon mediated electron attraction between electrons between the two electrons well this is the new ingredient of the b c s theory which is introduced this is saying that in addition to this screened coulomb repulsion there is also an attractive force between the two electrons due to the mediation by a phonon we already know a what is a phonon and we have talked about the electron phonon interaction.

So, let us represented schematically suppose i have a an electron with wave vector k and then which gets scattered in to a state k prime and then emitting a phonon here of wave vector q equal to the difference between these two and then we have another electron minus k which goes with this will be k minus q and this will be k minus k plus q and. So, i have one electron or wave vector k emitting a phonon wave vector q and getting scattered into a state minus k minus q and there is a another electron with a wave vector minus k which observes this phonon of wave vector q and goes into a get scattered in to a state of wave vector minus k plus q. So, this is the feynman diagram which represents the electron electron interaction which is mediated by the phonon there is a effective interaction between these two electron which is mediated by the emission and absorption of a phonon value in practice this only means that if you have a lattice is off ions positive ions like this and if I have an electron somewhere here and if there is an electron phonon interaction. So, this lattice a positive ion here.

So, effectively there is a coulomb attraction of this electron from its equilibrium position towards this positive ion and therefore, another electron of wave vector minus k which sees this changing the position of the positive ions from its position of equilibrium because of the interaction by the first electron it ceases and therefore, it also get attracted towards essentially this constitutes an interaction between these two electrons which is mediated by the displacement of the positive ion from its equilibrium position and that is what we call a phonon mechanism of electron, electron interaction.

Now this screened coulomb interaction is a certain magnitude and if this electron electron attraction just overcomes this screened coulomb repulsion then the net attraction this will be the the net interaction will be attractive if the phonon mediated attraction just overcomes the these screened coulomb repulsion. So, it just compensate a little bit more remains. So, that the net interaction will be attractive and that will be a very small amounts where it will be a very week attractive interaction and cooper theorem says that no matter how week these interaction is as long as it remain the net interaction is attractive there will be a tendency for two the electrons to form a bond pair. So, the cooper pairing gives way by which the two electron can form a pair with equal and opposite moment a and equal anti parallel spins.

So, that is paired by bound state of a cooper pair and the b c s theory emissions the entire superconductor the conduction electrons in the normal state to become paired in this way to form a sea of capital n electrons becomes a c of bound cooper pair of n by two per pairs at zero k. So, that is the s n of the p c s pairing mechanism which is at the of the b c s theory. So, i found to the a conditions under which the paring will tack place we will now go to the next step of finding how the entire super conductor of say ten to the power twenty four free electron they would be get become paired at zero k and form an ordered super conductor at obsolete zero. So, this will be our next step.