Condensed Matter Physics
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Lecture - 29
Ginsburg-Landau Theory, Flux Quantization

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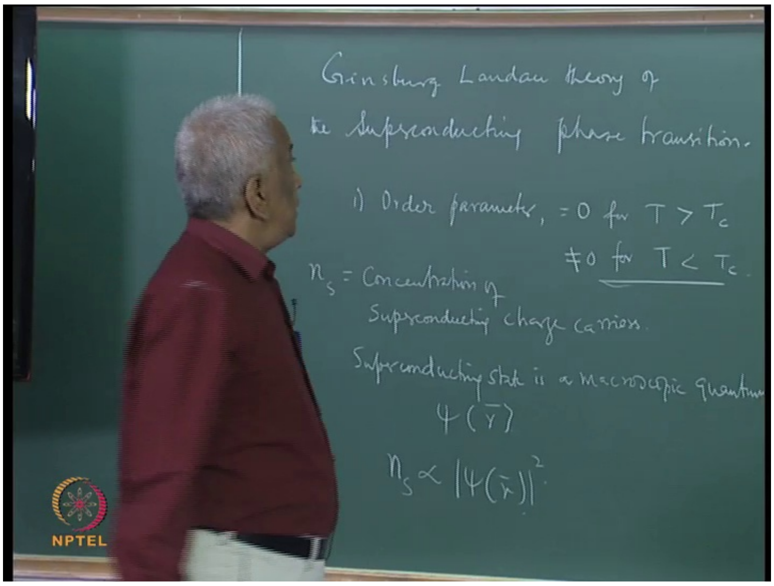

Last time we started with a discussion of the Ginsburg landau theory of the superconducting phase transition. Today we will discuss this in some are greater detail, because as I already said a Ginsburg landau theory is of great important whenever 1 considers the thermodynamic aspects of the superconducting phase transition under these are aspects are of pheromone important whenever we consider as super conductor for technical applications. Now, the Ginsburg landau theory is a variation of the much broader theory propose by landau in order to explain any phase transition in different thermodynamic systems, the basic formalism goes by discovering and order parameter of the transition. So, this step 1 in all these theories is shows specify an order parameter it is 0 for temperatures above the transition temperature, but not 0 for temperatures below the transition.

So, this is the basic definition any parameter, which have this property can be used be explained last time that the magnetization in a ferromagnetic can be for example, this spontaneous magnetization can serve of an order parameter. So, in the early spontaneous polarization in a federal electric or in the case of the super conducting state we know the concentration of superconducting electrons, concentration of superconducting current carriers charge carriers can serve of an order parameter, because there are no superconducting charge carriers on the concentration is the zero for temperature above the superconducting transition temperature. And it is non-zero for temperate below the transition temperature. And be already said that super conductor is a quantum mechanical state superconducting state is a microscopic quantum state, therefore with a view function psi which may vary as a function of this special coordinate. So, this is such that this concentration is proportional to merge psi of r square, this square as a models of the quantum mechanical wave function is a measure of the charge concentration. So, the Ginsburg landau theory start by identifying this the order parameter as a microscopic view function is modules square.

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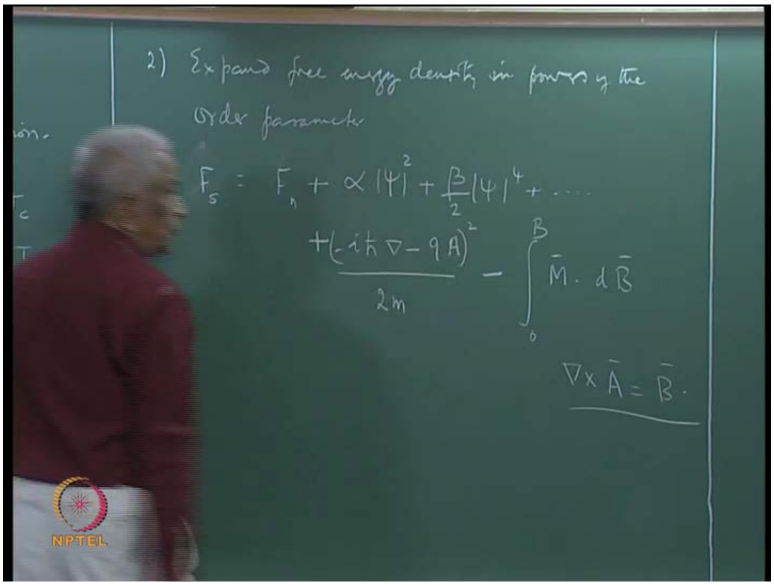

And then expanse, the second step is expand free energy density in the order state in powers of the order parameter, that is if F s is the free energy density in the superconducting state be this is equal to F m. If the free energy density in the normal state plus alpha psi square plus beta by 2 psi power 4 plus, etcetera. To be clear this since psi of a function of the position vector are you can also have a kinetic energy density, which is given as, this is the expression for the momentum in the presence of an applied magnetic field, whose vector potential is a the applied magnetic field by max equation is given by the carve of the vector potential. So, this is the kinetic energy density, and then you can also have in the presence of a magnetic field a magnetization a magnetization term. So, all these together give you the total free energy per unit value L I can write this, like this. So, the keep the discussion simply let us first to go in steps and remove the field dependent terms, and consider this situation in zero magnetic field.

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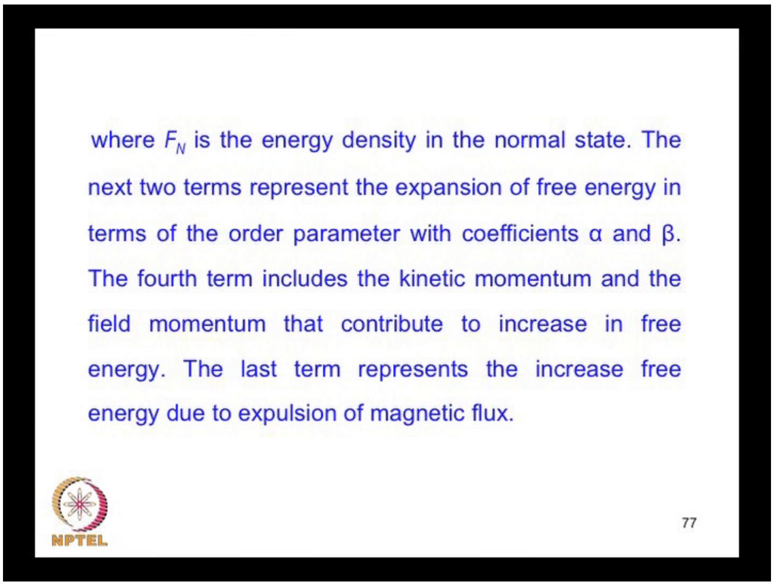

So, in that case I write lower case let us to show the d c c energy density, now alpha and beta are constant to be shows an by the theory. So, if we remove these terms involving the analytic energy as well as the magnetic field term.

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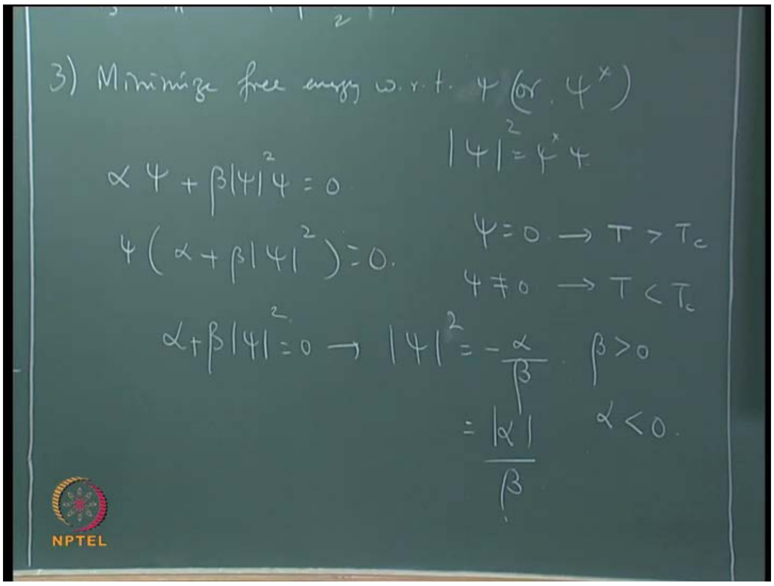

Then we simply have F s minus F n equals alpha psi square plus beta by 2 psi power 4 neglecting higher order terms. So, if you take this and then third step is to minimize the free energy with respect to the order parameter in this case psi are equivalently psi star since psi mode psi square mode psi square this psi star psi. So, we can minimize it with respect to the variations psi are equivalently with respect to psi star. So, if we do this we end up with an equation like this minimize it means said the derivative with respect to that parameter equal to 0.

So, this gives me… So, this as 2 solution either psi is the 0 in other word there is no order. So, this corresponds to the state situation in for temperatures above the superconducting transition whereas, below the superconducting transition psi is not zero. And therefore, therefore, alpha plus beta psi square equal 0, which gives me psi square as minus alpha by beta this automatically says this is related to the conservation of charge carrier. And it is a positive definite quantity therefore, the if beta is greater than 0 alpha is less than 0 you know I know that this is positive definite. So, I can write there for this as model for by beta having got the condition for minimum free energy minimum.

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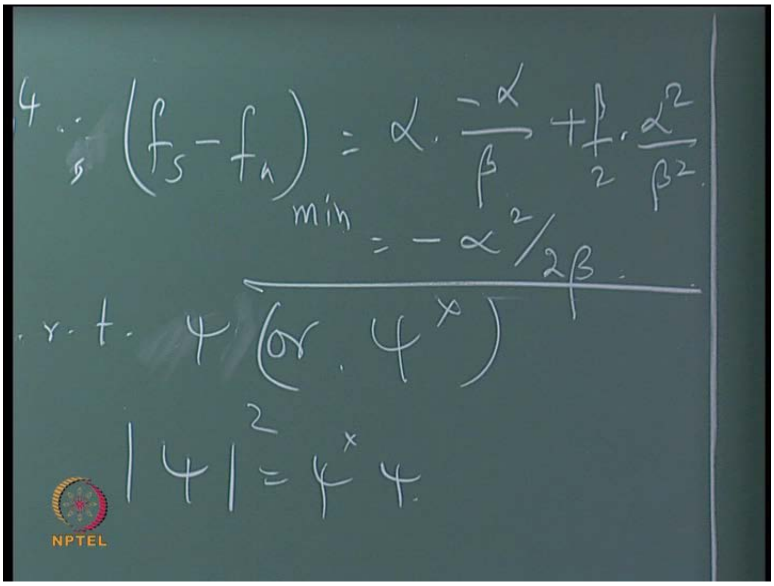

That has plague this is the free energy minimum expression, and this becomes f s minus f n corresponding to the minimum if alpha into alpha l. Let me write minus alpha by beta plus beta by 2 into alpha square by beta square which gives me minus alpha square by 2 beta. Now we know the free energy minimization takes place why as the critical thermodynamic critical magnetic field.

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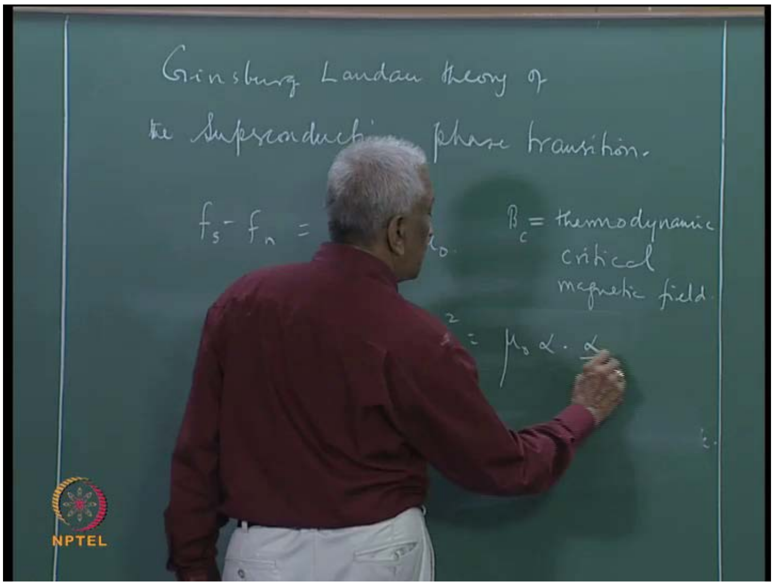

And we know that the f s minus f n is actually B c square by 2 mu naught. We already discuss this where B c is the thermodynamic critical field critical magnetic field. So, in terms to the Ginsburg landau parameter the therefore, we get alpha square by beta equals B c square by mu naught that gives me an expression for B c square as mu naught alpha times alpha by beta. And we also we know the landau penetration depth isms by mu naught m s a square.

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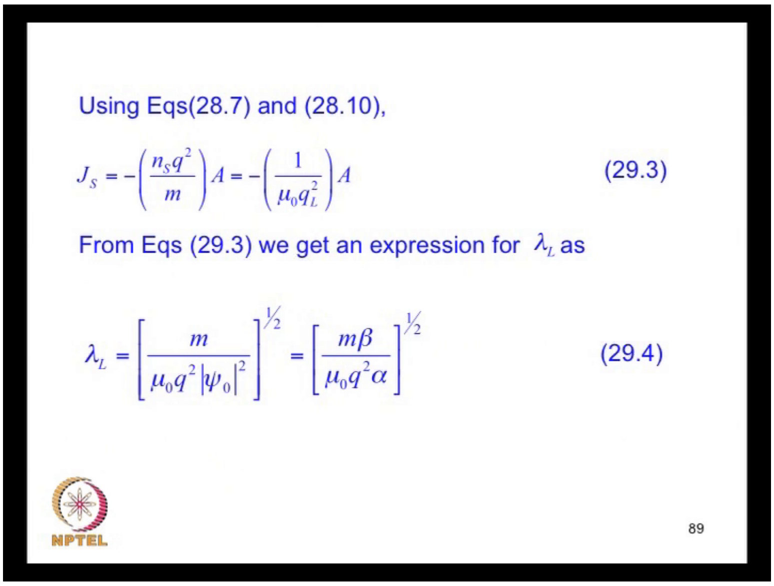

So, we can see the thermodynamic critical field as given by the Ginsburg landau theory can be related to the landau penetration depth the Ginsburg landau theory goes further, and talks about also the variation of the spatial variation of the order parameter. And we can minimize the free energy also with respect to the spatial variation of the order parameter.

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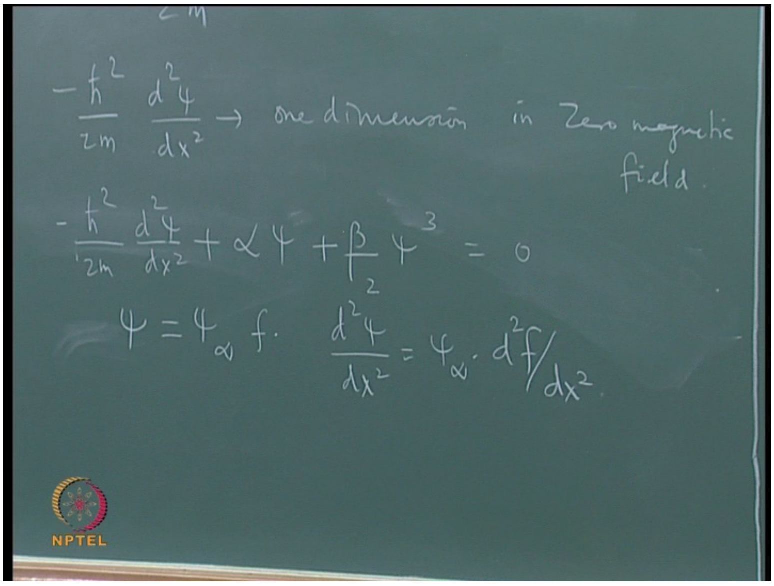

For example our discussion here can be regarded as the situation deep within a super conductor where the order is already fully establish, but when we come to the interface between a normal and superconductor or the surface of the superconductor then the order develops, and it may be expected to develop over a certain length in space. So, there can be special variations to the order parameter with associate changes in the kinetic energy density as given by. So, this is the kinetic energy density, if is still take be as zero the applied magnetic field this simply gives me minus h cross square by 2 m d square psi by d x square in 1 dimension for 1 dimensional superconductor in zero magnetic field.

So, if I take this then the equation becomes this is the difference in free energy and if we take this, and minimize it with respect to the order parameter then I get any equation like where as psi is different from the value of psi deep with us superconductor, let us called this psi psi infinity. So, this means let us say the actual wave function here is psi infinity times some parameter s. So, that psi infinity is constant deep within this. So, d squares psi by d x square really becomes psi infinity times d square f by d x square.

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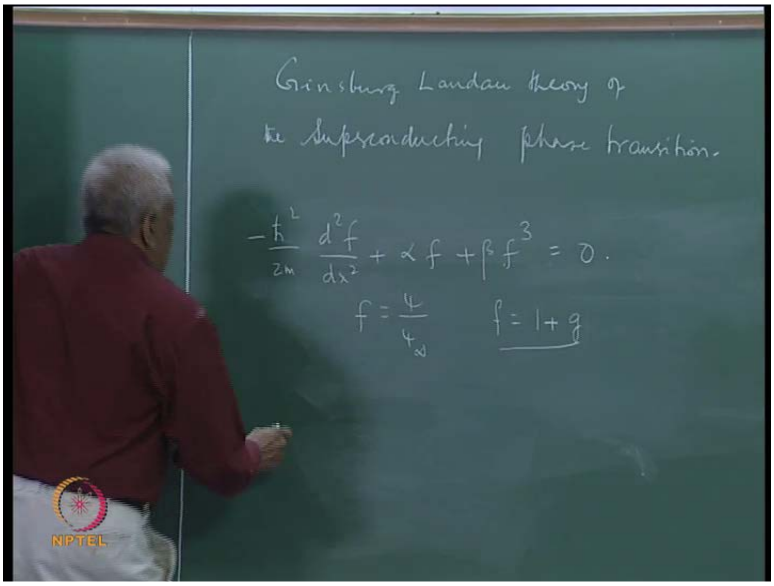

So, I end up with an equation like have to minimization now this canceling psi infinity throughout therefore now let us say f is slightly different from this is really psi by psi infinity. So, deep within the superconductor this will become f equal to 1. So, let us say f at the interface is something like 1-plus g where g is a small compared unity. So, that g is a measure of the order.

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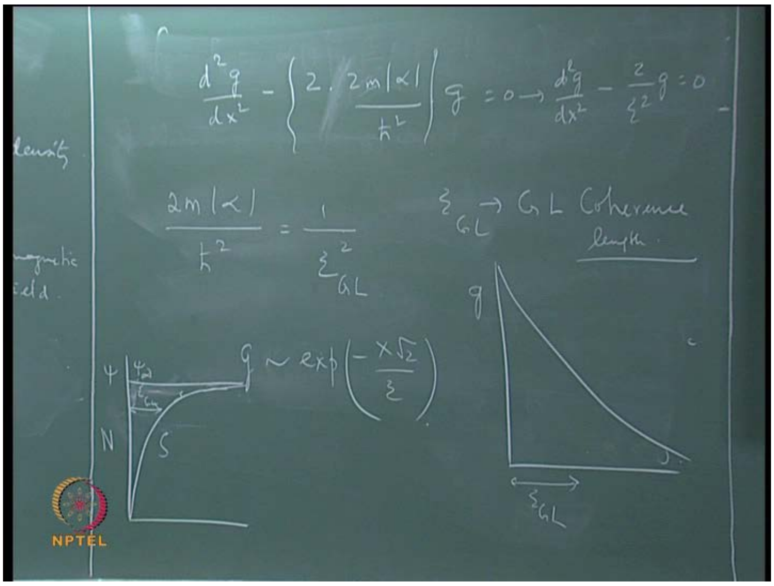

So, that in terms g we get an equation for g which is the firm d square g by d x square minus 2 g equal into 2 m alpha by h cross square g equal zero. So, all these are constants. So, let us say 2 m model for by h cross squares since these an energy for this as the dimension reciprocal length. So, let us call it with subscript g l to show the it is a length defined by the Ginsburg landau theory this psi g l is known as the Ginsburg landau coherence length. So, with that normal cloture I can write this as d squared g by d x square minus 2 by psi square g equal zero.

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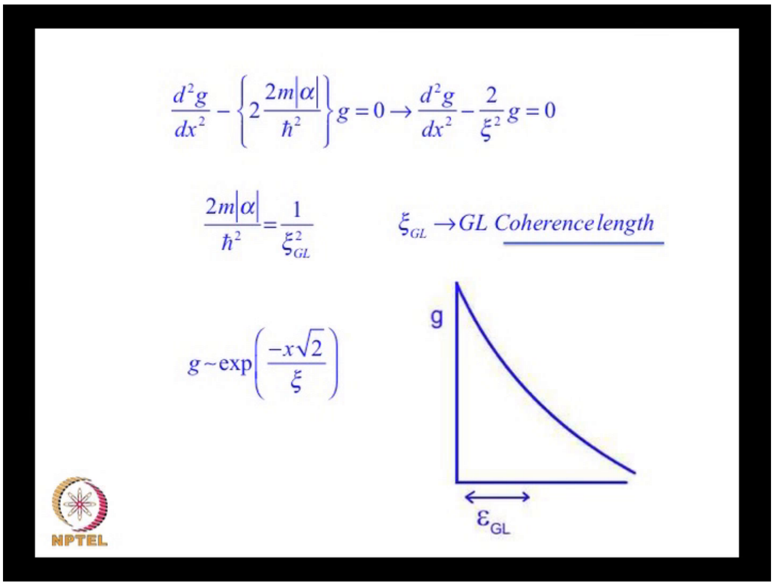

For which the solution will be g equal to g is at the farm exponential minus x root 2 by psi. So, what is the physical meaning of this this means that the order given by this develops and the case with this term in a way in a character with the characteristics length of the order of the Ginsburg landau coherence length. So that means, at the the interface of a super conductor normal to superconductor interface we have the order building up there is no other here. So, this zero and then the order builds up over a characteristics length of the order of the coherence length.

So, this is how the order develop and then this goes to its full value psi infinity here deep with the super conductor. So, the Ginsburg landau theory deals with the situation the cheese at the surface is the superconductor where the order develops from zero to and builds up to its full value deep within the super conductor and the length scale over which these order develop is called the Ginsburg landau coherence length. So, the ginsburg landau coherence length is another length scale which is discover by this discussion and a in addition to the landau penetration there this is a very interesting situation.

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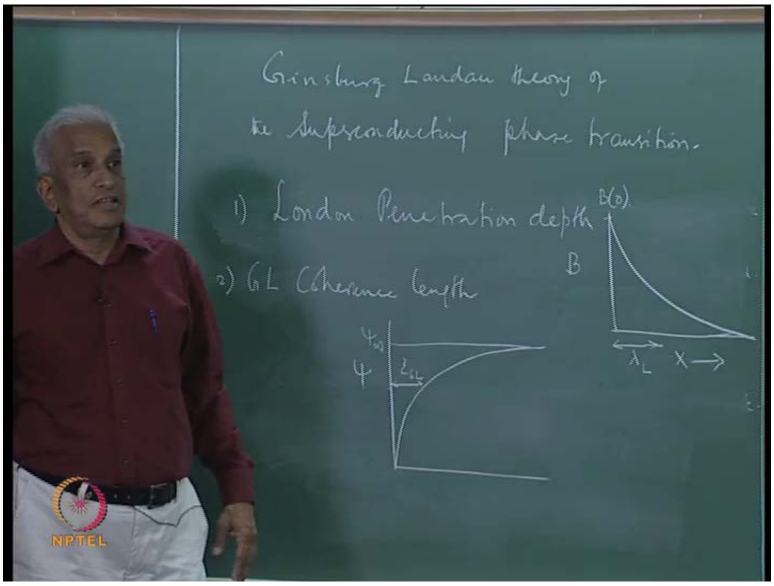

Because we know how to 2 length scales 1 is the London penetration depth we says how the magnetic induction field goes, and penetration super conductor and how this is. A trough depth. So, this is at the full value and it d k's like this overall length scale with a characteristics length which is known as the London penetration depth. So, this is the full value, and this is how it is d k's that is 1 length scale the second length scale is the ginsburg landau coherence length. And it describes how the superconducting order at the interface develops, and bulls to its full value deep within the superconductor and the length scale characteristic of that is the Ginsburg landau coherence length. Now when I have magnetic field caner state the magnetization use raise to a increase in the free energy whereas, the order developed overall length scale of the order of the Ginsburg landau coherence length gives you the free energy decrease, because of the evolutionary the order. So, these 2 competitive process 1 contributions to an increase in the free energy another contributes it decrease in the free energy.

So the total of this 2 contributions decides their relative balance of the normal, and free a superconducting free energy densities therefore, suppose you have a material this gives a convenient way to classify the so-called type 1 and type 2 superconductors. Now we already saw that a type 2 super conductor is 1 in which there are there is partial penetration of magnetic flux in the feel the region between the lower, and the upper critical field.

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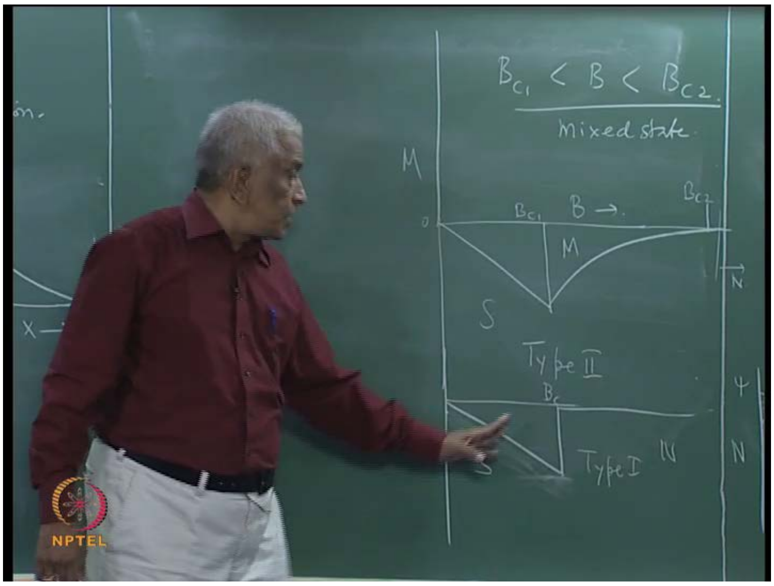

We saw the magnetization graph goes like this in the superconducting state and then goes like this. So, this is B. So, this is zero is a negative value.

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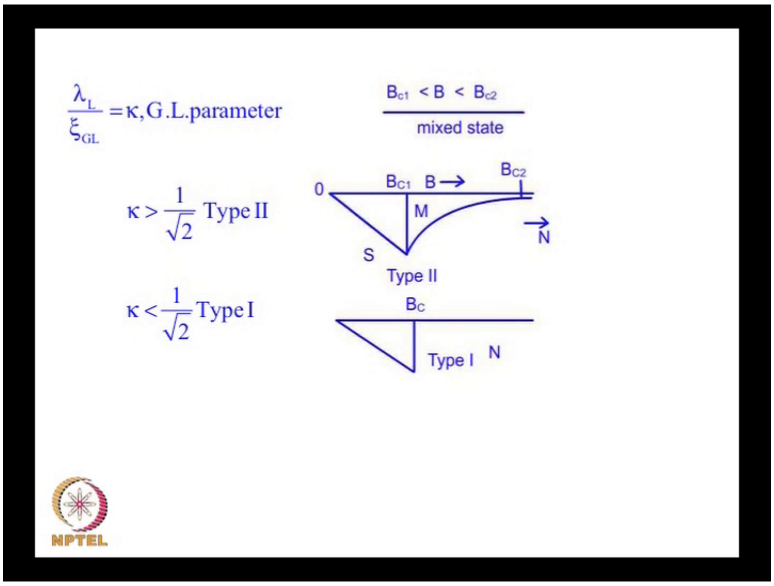

So, this is B c 1, this is B c 2. So, in the region between B cone less then B less then B c 2 this is the mixed state, this region is the superconducting state this is the mixed state and the region beyond this is the normal state where the penetration of flux is complete. So, in the mixed state this is the behavior of a type 2 super conductor there is the behavior is just this for a type 1 super conductor. So, this is B c this is type 1 behavior this is the superconducting state this normal state. So, at the flux is exploded in this region with a negative magnetization, and then add the thermodynamic critical field the flux penetrates completely and it the martial is given in the normal.

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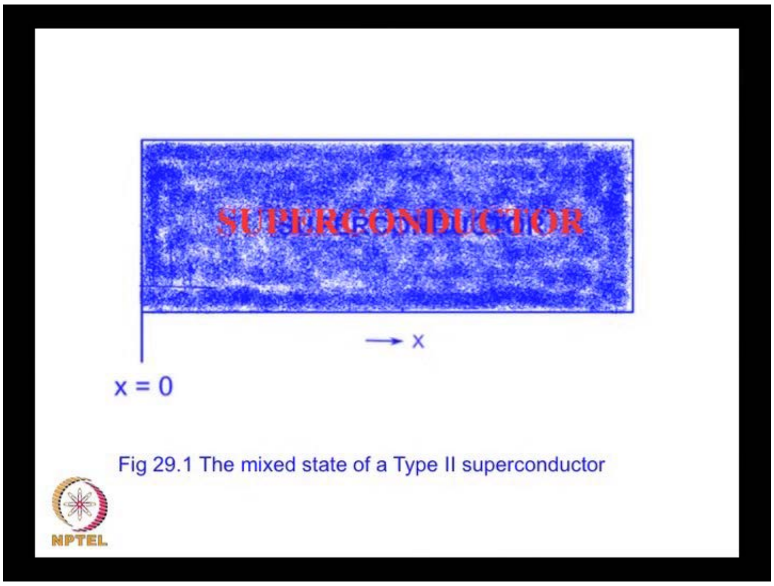

Fig 29.1 The mixed state of a Type II superconductor

Whereas here you have a region of rearrange a field values between B cone and B c 2 in which this superconducting and normal region coexist. So, there is partial penetration of magnetic flux. So, the Ginsburg landau theory now gives as way to understand how the super conducting an normal region can queasiest in the state, because it basically reduces the problem 2 1 of the free energy minimum.

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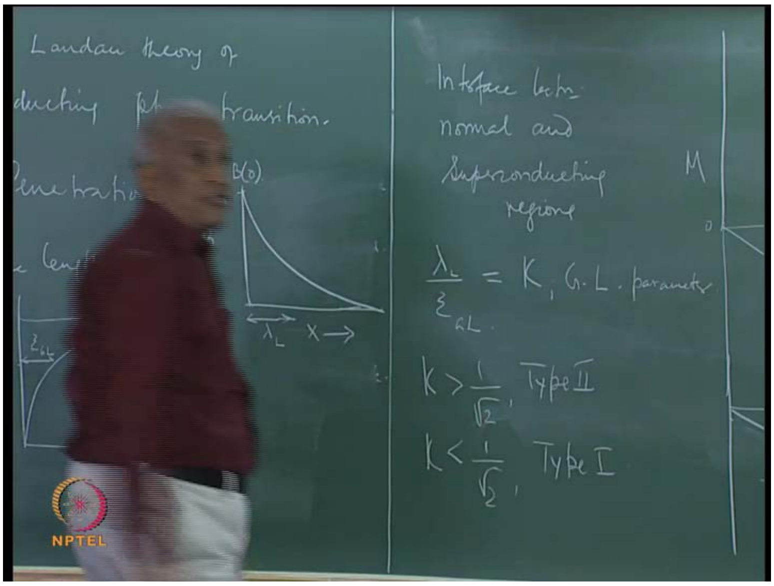

When you try to produce a interface between normal, and superconducting regions a mixed state of this kind will be possible only if it is energetically favorable to produce, such interfaces where the normal and superconducting the region's or separated by a surface. This is possibly thermodynamically only if the free energy is lower in effect by the creation of such an interface. So, essentially we saw the free energy balances is the determined by the 2 length scales lambda l and psi g l, the London penetration depth and the Ginsburg landau coherence length.

So, the Ginsburg landau theory gives as a criterium by v h a materiel can exhibit type 1 or type 2 to behavior Ginsburg landau theory gives a ratio of the London penetration depth to the Ginsburg landau coherence length, this is defined as a Ginsburg landau parameter at it is usually denoted by the letter kappa. And if kappa is greater than 1 by root 2, then the material it is energetically favorable to have such interfaces between normal. And so there can be mixed state for kappa less than 1 by root 2 the material in exhibits type 1 behavior. In fact, both the London penetration depth and the Ginsburg landau coherence length or experimentally can be experimentally determined. And therefore, this ratio the kappa parameter the g l parameter they can be the actually determine, and it is found this criteria is valid in most known cases of type 1 and type 2 superconductors. So, apart from all this the Ginsburg landau theory also gives a very interesting method to determine quantitatively the upper critical free for this we should discuss what happens when you have an applied magnetic field in the presence of an applied in an applied magnetic field.

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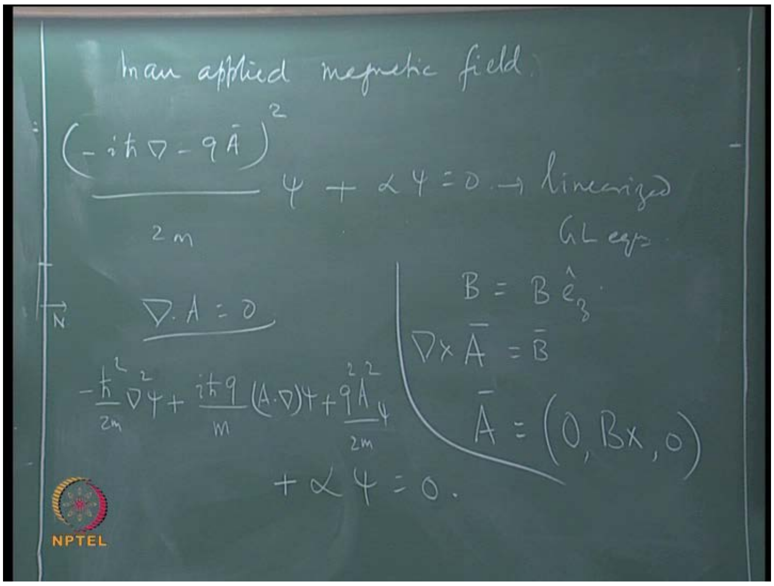

The Ginsburg landau equation becomes we are neglecting the higher order term involving beta, and other this is known as the linearized Ginsburg landau equation in which all terms except the linear term alpha psi is neglect.

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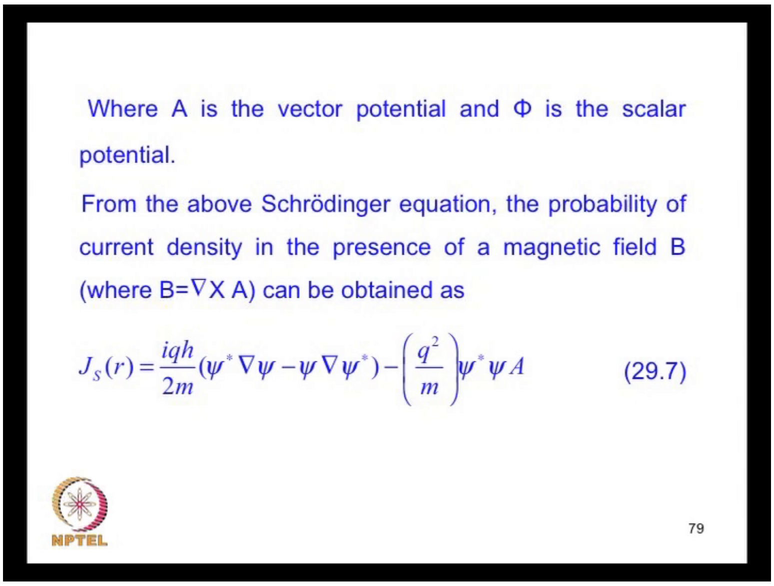

So, if you do this here a is the vector potential. Now if B is taken along the B equal be u z along the z axis, it is a constant uniform field directed along the x-direction then we can choose a, because a is del cross a is B, you have various possibilities one possible choice is zero B x 0.

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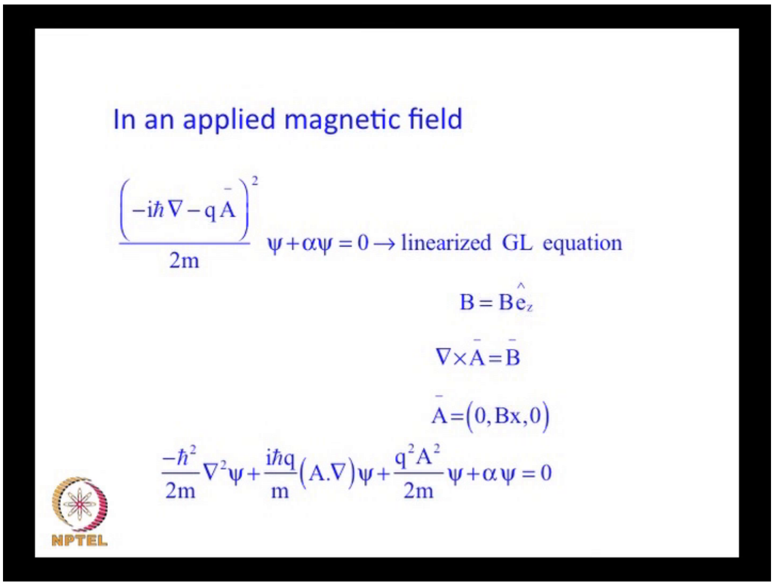

Suppose we take this and rewrite this equation this becomes, where we have used the condition the work in the coulomb gauge. And we assume del naught A u 0 with that assumption this work be arrive at that can written.

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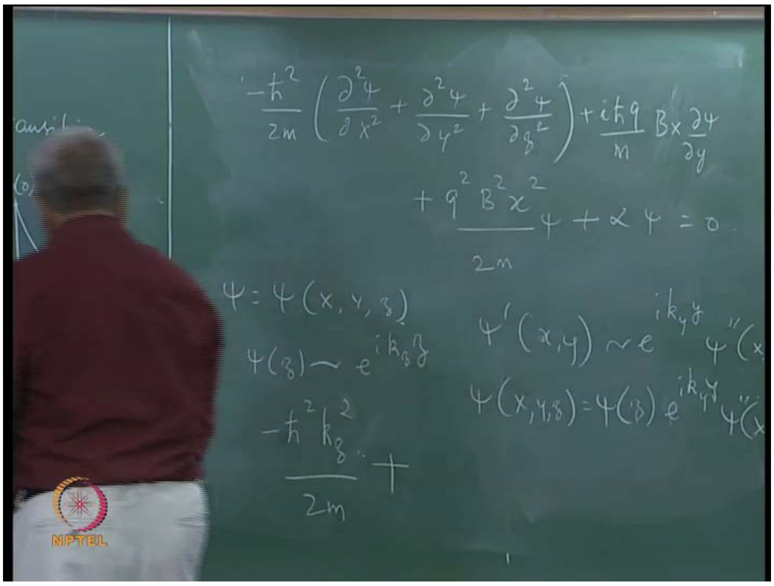

Because we have only B x in this in this vector potential term expanding, this we arrive at in koppies inquired net well I have a x though by though x here by though by though y is that though by z of which only a y is non-zero.

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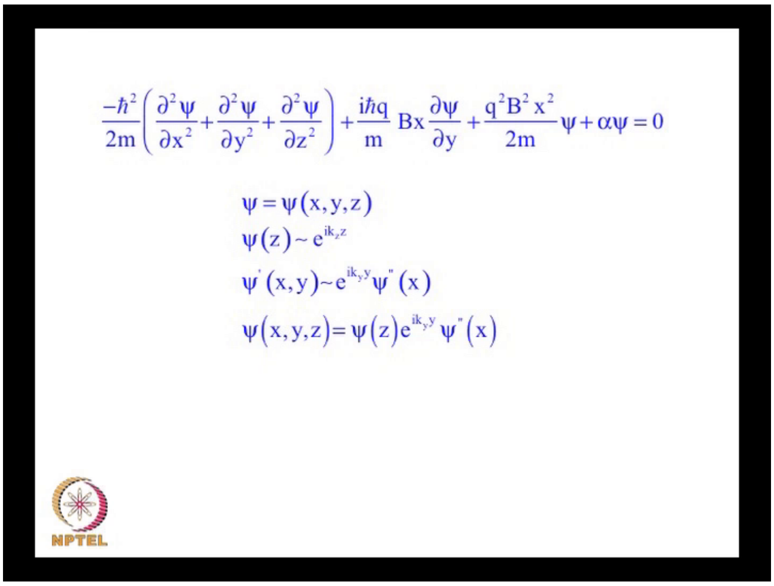

So, I will write also this. So, the a y is B x times though by though by plus q square B square x square by 2 m psi plus alpha psi equal 0, where psi is psi of x y and z since we can do this by the method of separation variables. And since this tuff not involved this z coordinate except here becomes striate away right this psi of z goes e I k z. And therefore this term can be written as minus h cross square k z square by 2 m. And then we can write the remaining as i psi prime x y can be written as e to the power of I k by y and primes psi w prime x, where the primes are just used to say the psi x y z is a product psi z will also this form. So, with that we can differentiate and write the rest to the question governing the psi double prime.

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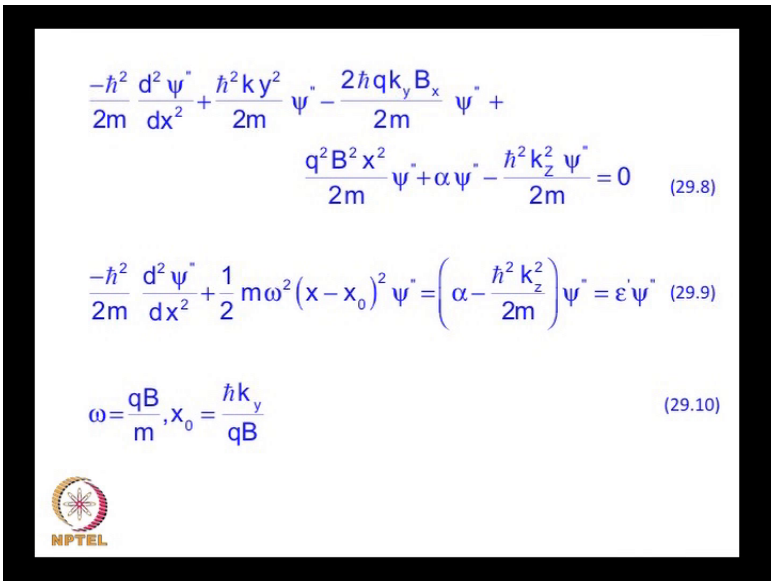

Therefore we will have minus the equation the part involving k z factors out, and we as the rest of it return the equation governing psi double prime can be written by the standard method of separating in the variables I will write the final result.

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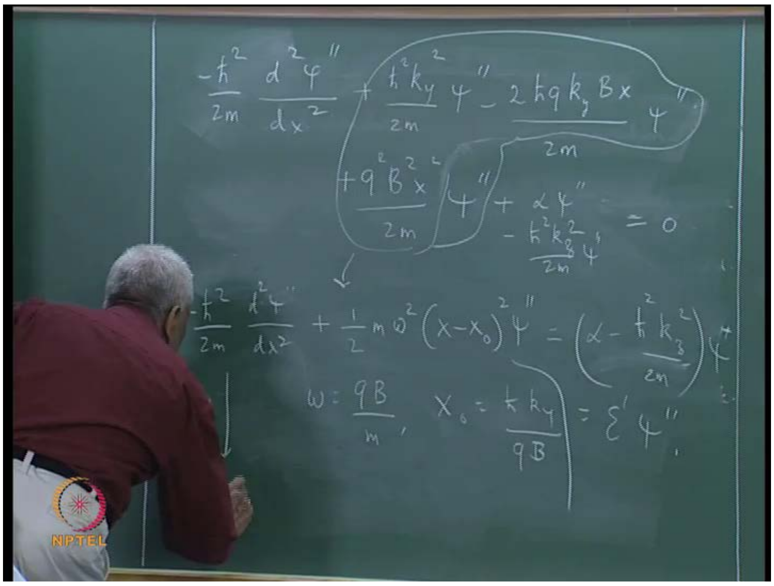

The equation governing now these can be written out in the form of a perfect square. So, this can be written out and the firm. So, I will have minus h cross square by 2 m d square psi w prime by d x square, which is in the form of a kinetic energy operator operating on psi double prime than the rest of it can be written in the form of m omega square x minus x naught square psi double prime equal to alpha minus I have left out. So, alpha minus h cross square k z square by 2 m. So, where omega is q B by m, and h naught is h cross k y by q B. And in the form you can see, but this is the energy, which is which I can write as equals e dash psi w prime, where e dash is the energy total energy minus. This therefore, and this is in the form of the standard Schrodinger equation for a one dimensional harmonic oscillator of frequency omega equal to cube m which is displaced in centered will by an amount of h cross q a by q b. So, the Eigen values of energy of this 1 by oscillator or well know these the exactly solved problem in quantum mechanics.

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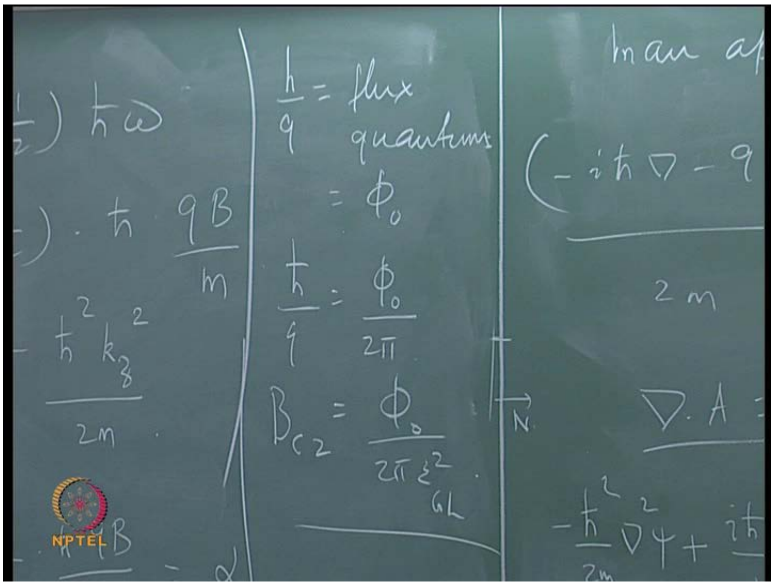

So, we know the Eigen value e n or n plus half h cross omega. So, we can write now this is n plus half h cross into omega u q B by m, and that would be a alpha minus h cross square k z square by 2 m.

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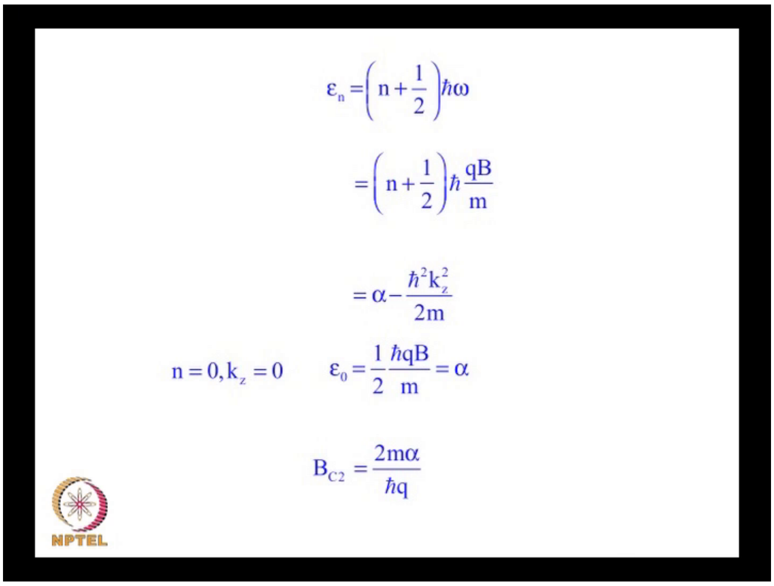

So, the highest Eigen value of this will correspond to n equal to zero and k z equal to zero then it becomes e zero equal half h cross q B by m equal to alpha. So, this means the corresponding magnetic field which is the highest allowed magnetic field is 2 m alpha by h cross q. So, this is the highest value of the magnetic field which can be first range super conductor according to be g l theory. So, this is by definition the upper critical field and this we know that 2 m alpha by h cross square is already the Ginsburg landau coherence length the inverse of the square of Ginsburg landau coherence length and h cross by q. We will see h by q is special quantity for a super conductor, it is known as the flux quantum as we will see shortly it as a special notation if I naught. So, h cross by q if phi naught by by 2 pi.

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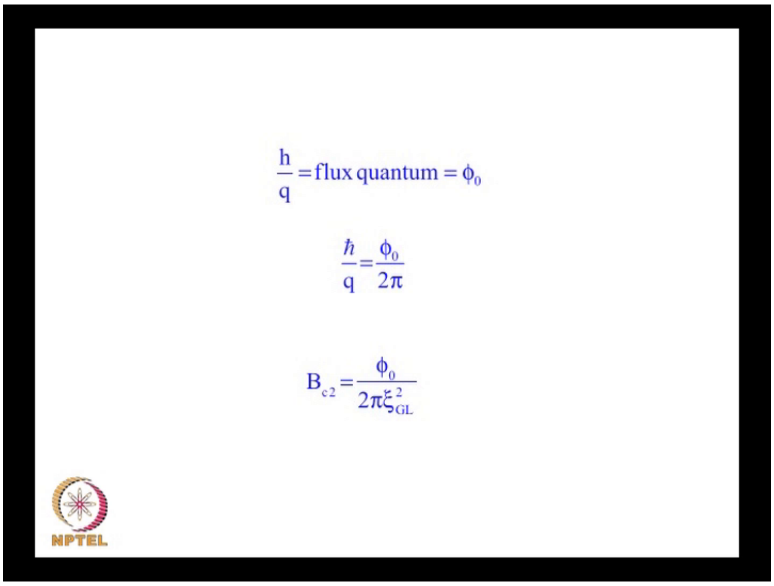

And therefore the Ginsburg landau theory gives an expression for the upper critical field a psi naught by 2 pi psi g l square. So, that gives an a expression for the upper critical field if type 2 superconductor in terms as the flux quantum phi naught, and the Ginsburg landau coherence length phi g l what is this truck quantum we can just see it a minute.

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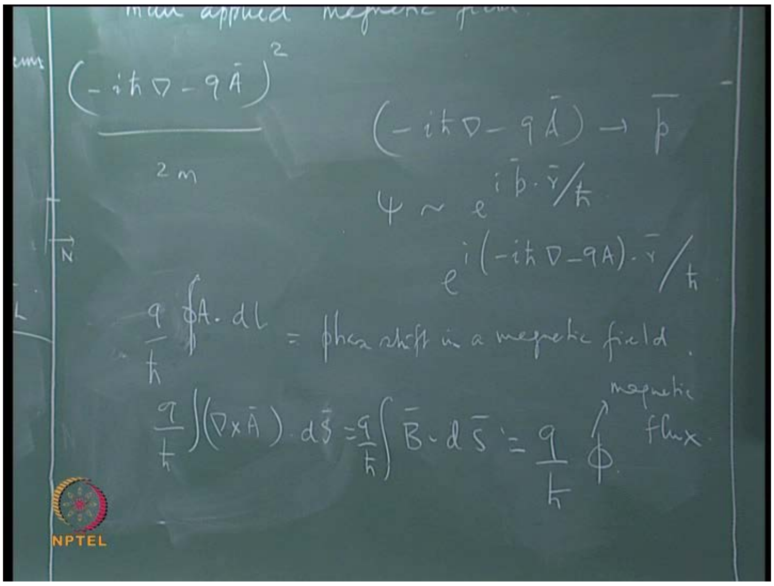

We see that the moment operate in the coherence length magnetic field is minus I h cross del minus q a, this is the canonical momentum for a superconductor in an applied magnetic field who's vector potentially is A. And we also saw the wave function of the super conductor the superconductor is a microscope quantum state with a wave function which goes as I p data r by h cross. So, this will have writing this inside of p i times minus I h cross del minus q a dot r by h cross.

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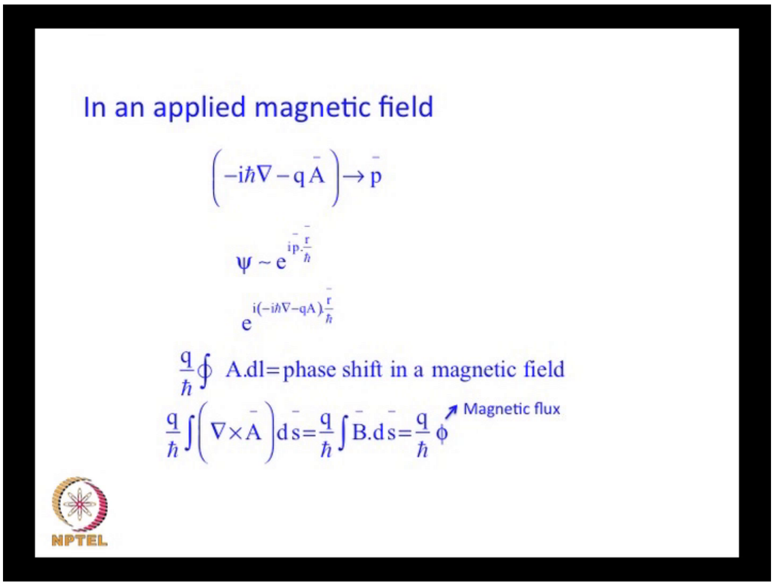

So, we can see the quantum-mechanical phase is given by the line integral in the presence of a magnetic field, the phase factor due to the magnetic field can be separately written as q integral a dot d l. The line integral of q by h cross this is the phase phase shift in a magnetic field and the line integral of a dot d l has a special significance in in any system we can transform by stokes theorem into del cross a dot d l, and we know del a is b. So, d s the surface integral. So, this becomes B dot d s. So, the line integral. So, q by h cross and we know the line integral of the vector potential is nothing but the surface integral at the magnetic induction by Stoke's theorem and the surface integral is nothing but what we know as the magnetic flux threading this circuit. So, this is the magnetic flux this is a very profoundly interesting situation, where we have phase change due to group of current carrying a super current group super conducting group.

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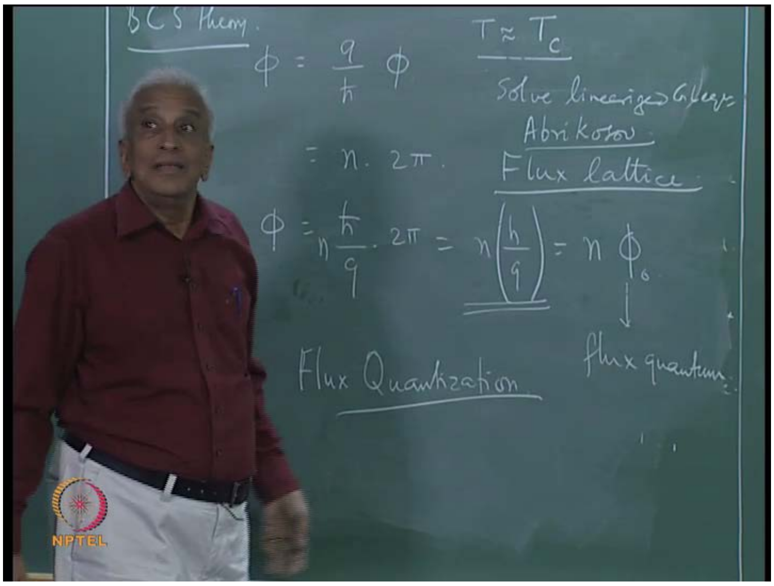

The phase change the quantum-mechanical phase change is given by q by h cross times phi the phi is the magnetic flux. And we know at the phase change because this is the fears that the quantum mechanical wave function, if we go to through complete circuit and come back to the same point the wave function should be single valued. So, this phase change should be an integral multiple of 2 phi. So that means, at phi is h cross by q n times into 2 phi when h cross 2 psi is just an h by q, this is a very performed result in requirement the wave function of a superconductor should remain single valued results in a quantificational the magnetic flux threading a superconducting globe in units of a quantity called h by q. So, this quantity is known as the fundamental flex quantum this is the first 1. So, the flux quantization is a very interesting automatic result of the fact that the super conductor is a microscopic quantum state, and this is what we have used in here. So, that is the significance of these.

So, the mixed state is 1 in which the normal regions in which flex is penetrating content a quantum of magnetic flux spreading the superconducting waves. So, these 2 co exist together with normal super conducting in interfaces separating done and the Ginsburg landau the landau, and penetration depth is length scale characterized how the magnetic field penetrating normal region and the Ginsburg landau coheres length tells us how the order developed in the super conducting region. So, this the microscopic understanding of the mixed state of a type 2 super conductor.

In fact, the Ginsburg landau theory can be pushed further and the linearized Ginsburg landau equation can be returned in the neighborhood of t c when the order is very small. So, we can linearized the Ginsburg landau equation because this the change in the order parameter is extremely smart. So, it is a small quantity. So, I have expansion and thank gas from the linear term is perfectly justified. So, if this solve the solve linearized g l equation this solution was carried out by a person called up the course of, and he this solution he show results in the creation of a flux plat is in a type 2 super conducting in the mixed state. And a he show the condition for the stability of this flex lattice and all an electro dynamic behavior and characterization of a type 2 superconductor for applications, such as making I feel superconducting magnets we have know the nature the flux lattice. And how the flux the periodical is the flux lattice and the structure will be flux lattice in order to go further.

So, this is the reason why I said the Ginsburg landau theory provides an insight and a guideline for describing as a real practical type 2 superconductor, which is really impressed in technical applications well. It is not just the full story as I already said we have also soon afterwards a microscopic theory was developed by body in cooper, and B for and we called BCS theory, and for a long time the Ginsburg landau theory was developed by Ginsburg landau inside The soviet union. And the BCS theory was develop in the united states, and for a long time because of cold war situation there nothing known on either side.

And curtain about what happens in the other side and then nobody nu whether the Ginsburg landau equation is correct are the BCS theory is correct. And it was left be a person called work of to show that both theories are really mean the same thing head out to the green function formalism. In order to establish the essential equivalence of the g l theory with the BCS microscopic theory. So, the now call the GLtheory the as GL the a g theory Ginsburg landau abri course of gork of theory. So, in that form we have a fairly unified understanding of both the microscopic, and the thermodynamic behavior an electro dynamic behavior for super conductor. We will considered the microscopic theory in the next lecture.