Condensed Matter Physics
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Lecture - 28
Type I and type II Superconductors

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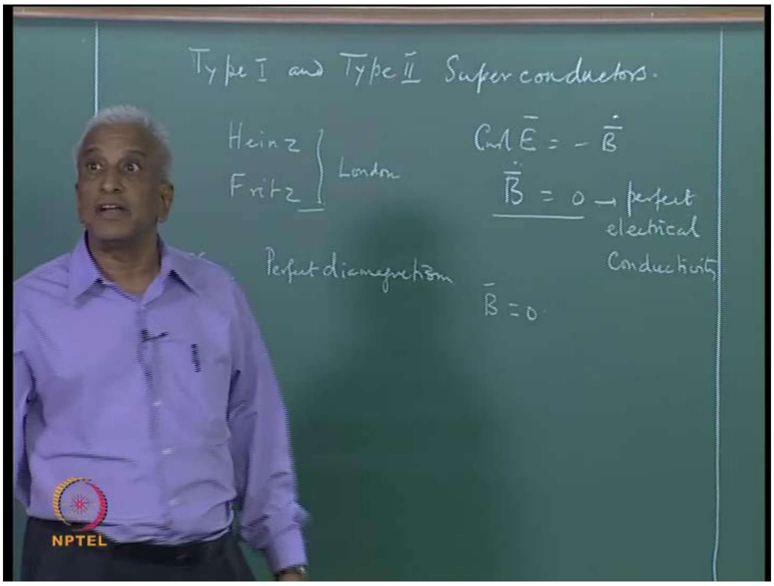

In the last lecture, we talked about type one and type two superconductors table for list some other typical type two superconductor such as niobium tin which is an inter metallic compound or niobium germinate or niobium aluminum, niobium titanium and niobium zirconium are alloys of niobium and titanium. So, the T C values superconducting transition temperature and the upper critical fields at absolute zero the values are given in this table.

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Material	T_c(K)	$B_{c2}(0)$ (Tesla)
Nb_3Sn	18.5	24
Nb_3(Ge,Al)	21.0	43.9
NbTi	10.0	12
NbZr	11.0	9

Table 28.1 T_c and $B_{c2}(0)$ values of some Type II superconductors

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And one can easily see that with a NbTi superconductor it is possible to cool it down below 10 Kelvin and realize upper critical fields closed absolute zero which is as high as 12 Tesla. So, and if you want larger values most stronger electromagnets using superconductor then one use as the typical material is niobium tin which has a transition temperature of 18.5 Kelvin and a it has an upper critical field of something like 24 Tesla. So, it can be used to really produce really very high field superconducting magnets.

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Compare these B_{c2} (0) values with the $B_c(0)$ values (in milli Tesla) of elemental superconductors.

The famous current-field- temperature diagram of high field high current superconductivity in certain Nb alloys is shown in Fig.28.1 (after Kunzler, 1961)

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For example, if we compare these upper critical field values with the critical fields which are of element in superconductor one can see that the type two superconductors are ways all much superior in performance from the point of view of construction of high field magnets superconducting magnets. So, the three parameters of crucial which are crucial for the performance of a superconductor are the critical current, the critical field, and the critical temperature.

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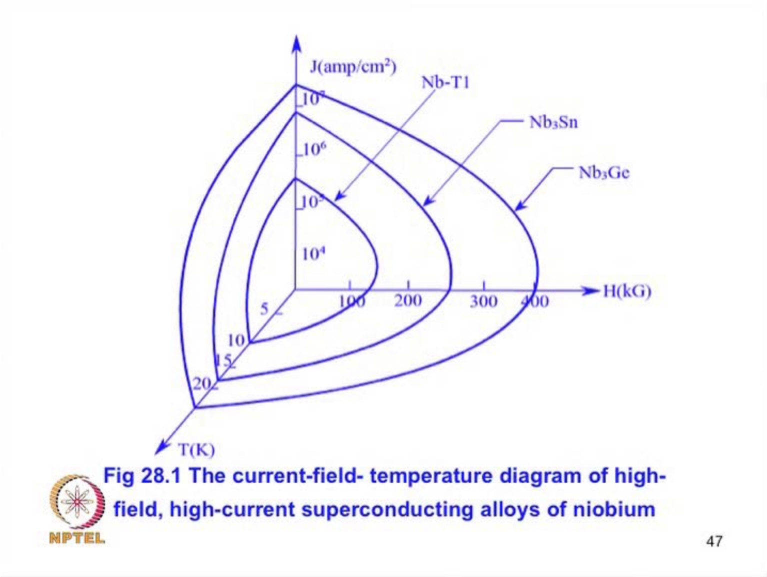

Fig 28.1 The current-field- temperature diagram of high-field, high-current superconducting alloys of niobium

So, one can see that all these values are shown in a graphical form in figure, where the current-field-temperature diagram of high field high current superconducting alloys of niobium are shown graphically.

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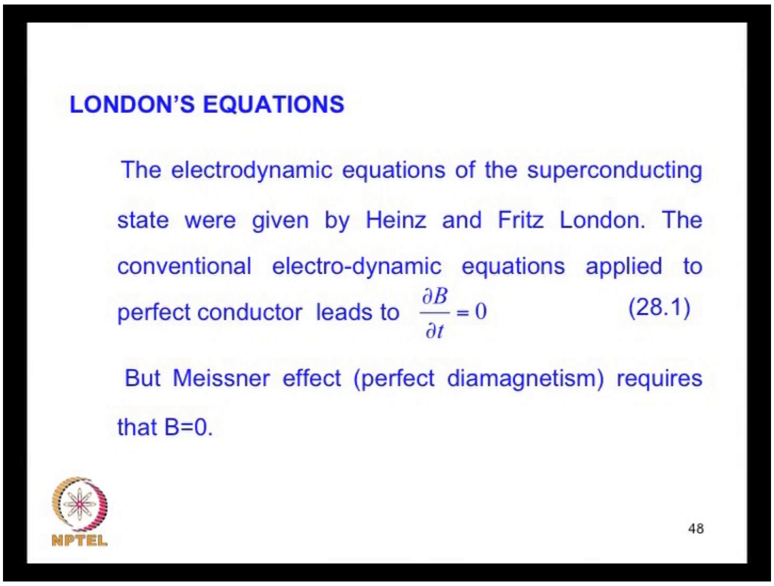

Now, all this, of course, means that we have to understand the electro dynamic behavior of a superconductor. This was first attempt at by the brothers Heinz and Fritz London, there are brothers H London and F London. So, their approach was as follows. If one sees from the Maxwell equation for a typical metal, it says that the curl E is minus B dot this is really nothing but a parade law. In a superconductor since the electric field deep within a superconductor is 0 because of high conductivity perfect conductivity, so curl E is also 0 and therefore, it automatically leads to the situation that B dot 0. In other words the magnetic induction is a constant in time, but this is just a property which flows from perfect electrical conductivity predicts that the B field, the induction field inside a superconductor does not change in time. But Meissner effect which was discuss also earlier it predicts perfect diamagnetism. So, the discovery of Meissner effect showed that a superconductor is also a perfect diamagnet and this requires that not just B dot is 0, but B is 0 the induction field inside a superconductor vanishes.

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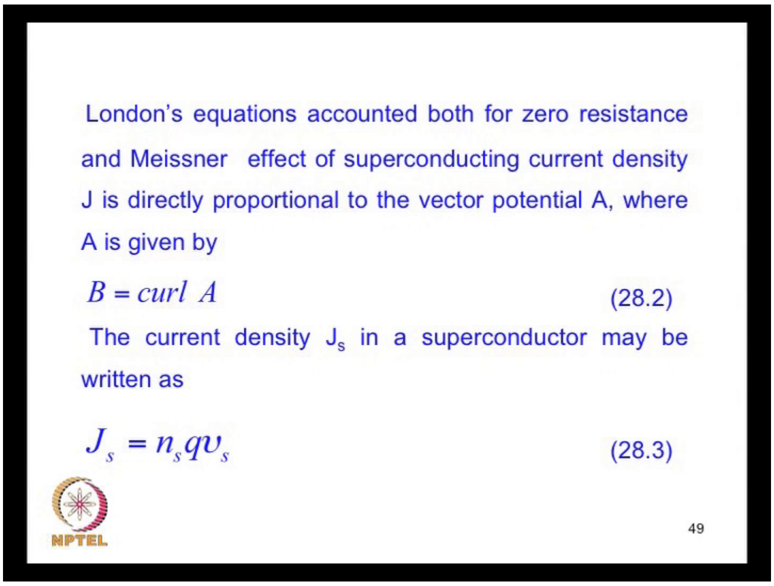

So, the magnetic field is completely is excluded by surpass superconducting current shielding super currents which make it a perfect diamagnet. So, London's equations proposed a new behavior electro dynamic behavior in which the super current density J s this is super current density.

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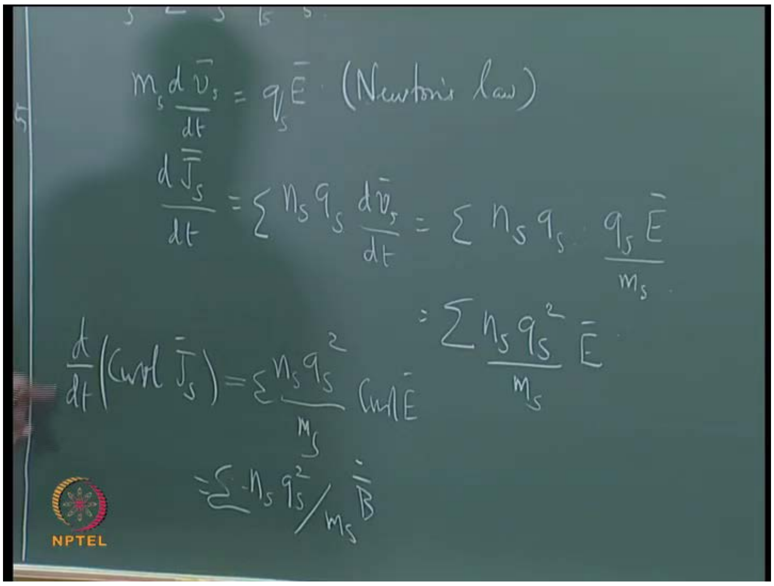

The London postulated that this j s the subscript s means it is in the superconductor. So the super current density is directly proportional to the vector potential A of the magnetic field where B is l cross A as usual. So, this is the definition the vector potential and they proposed that B is curl J s and this was the proposal of London's to account for the electro dynamic behavior of a superconductor which implied in addition to perfect electrical conductivity also perfect diamagnetic behavior. In general, of course, the super current density may be written income for the charge carriers whatever be the charge carriers inside a superconductor that is the concentration of charge carriers in the superconductor, q is the charge carried by the charge carrier, and v s is the velocity of the charge carrier.

So, if I take this and sum over all the charge carriers then that gives you the super current density. We also know that because of Newton law m d v s by d t is q E this is Newton's law. We can even put q s, so that they are super current carriers. Similarly, m goes with m s; the subscript s denotes that they are the charge carriers are super current carriers. So, because of these, I can write d J s by d t is n s q s d v s by d t. And since this is n s q s in to q s E by m s. So, this go there n s q s square by m times E. So, if I take curl and inter change curl and d by dt this gives me n s q s square by m s curl E. And since curl E is minus B dot. So, you can see that this d by dt.

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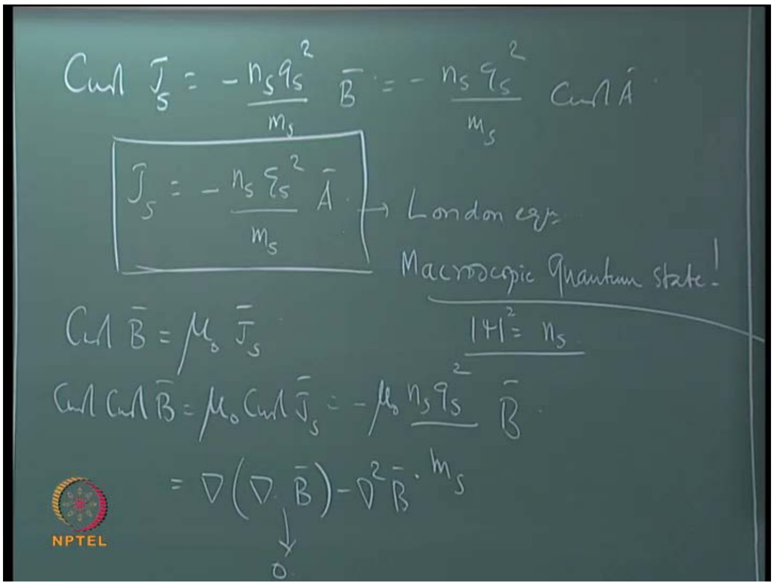

So, I can simplify for the case of one carrier the current contribution is minus n s q s square by m s times B, and since B is curl A, and therefore, I can write and this is precisely a London relation.

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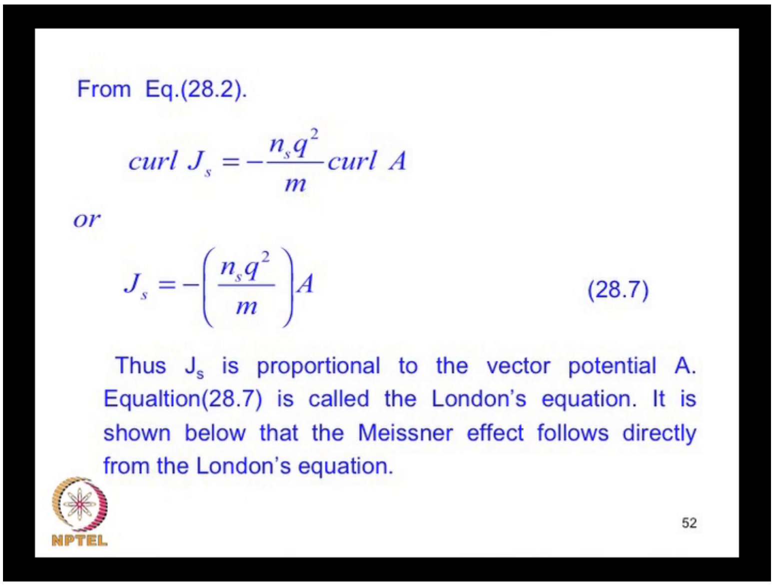

The relationship that was proposed by London; this is known as the London equation for a super current carrier. Now if this is the case, we can see that curl B is mu naught J s by Maxwell equation, therefore, taking curl on both sides curl curl B is mu naught curl J s. And curls J s, it gives is given by mu naught m s q s square by m s B. And the curl curl B is dell off del dot B minus dell square B and del dot B is 0 by Maxwell equation.

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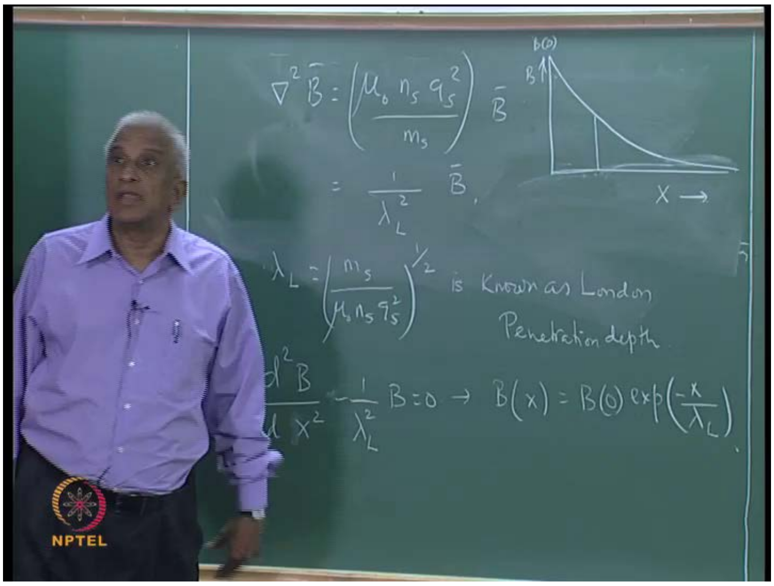

So, we arrive at the following fundamental relationship namely del square B is mu naught n s q s square by m s B - this is predict.

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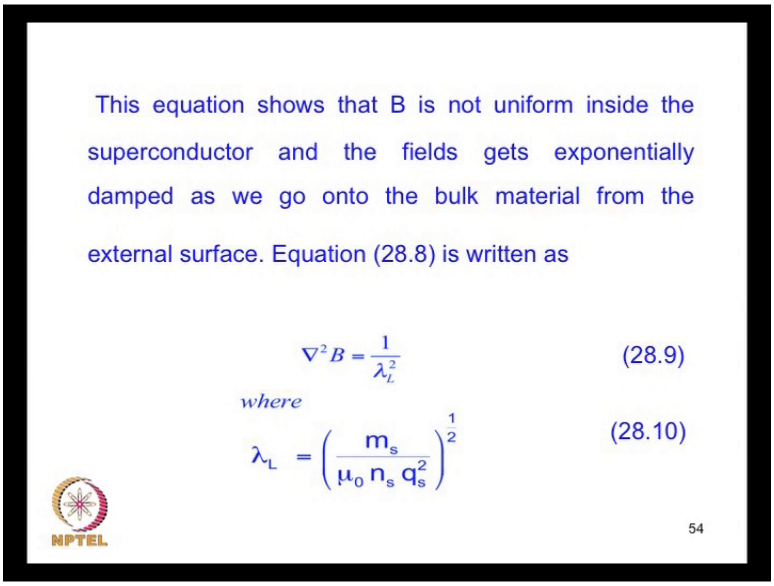

This is what suppose I define this as 1 by lambda L square B, where lambda L is m s by mu naught n s q s square to the power half is known as London penetration depth . So, that is why there is a subscript L here. And with that I can rewrite this as, this and this can also be in the suppose we considered a one-dimensional conductor with a component of B. Suppose, B is a just applied along z axis there something at then I can write d square B by d x square instead of del square B and that is equal to or minus one by lambda L square B equal zero. Now that is an equation, which is a simple second-order differential equation with the solution B equal to B x equal to B 0 exponential minus x by lambda L. So, that is a simple solution for such a equation.

So, if we have what does this mean, this means that the magnetic field within a magnetic induction field within a superconductor within a one-dimensional superconductor which extends along the x-axis is given by the value of the magnetic field at the edge of the superconductor corresponding to x equal to 0. And it times exponential minus x by lambda by l; that means, the induction field decays in a superconductor if I plot induction field as a function on the distance with x equal to 0 here then the value here decays exponentially. So, with a characteristic decay length of lambda L, lambda L is the distance within the superconductor in which the induction field decays to one by e of its value at x equal to 0. So, it is a characteristic length or which the induction field decays to 1 by e of its value. So, that is the significant of the London penetration depth.

It describes the extent to which the field penetrates the superconductor and it gives you the value the distant, the characteristic distance over which the induction field false exponentially. And so deep within a superconductor, the induction field the really fall to a negligibly small value which nearly close to zero. The London penetration depth one can have an idea of this from substituting the typical values assuming that the charge carriers are electrons or some combination of electrons, as we will see, pair of electrons to be precise. Then if you put in these values then you get the penetration depth for a atypical superconductor elemental superconductor these are the order of fractions of a micron.

So, it means that the induction field within a superconductor decays really very fast within a distance of the order of a fraction of a micron, and then it decays to 0. Now the form of this equation the London equation, also lead London to say something more about the nature of the superconducting state. The nature of the superconducting state can be inferred from if we assume the London following London that the superconducting state is a macroscopic quantum state such that the wave functional such a quantum state is given by psi square equals n s. If you make this identification mod psi square is enough then this gives the typical diamagnetic response have such a quantum state.

And therefore, London was leads to postulate that the superconductor is nothing but a quantum state operating over a macroscopic distances. This was a very start link prediction, which was verified much later. In other words, a chunk of superconductor a long length of superconducting wire is something macroscopic dimensional is quantum mechanical states; unlike atomic and molecular system, which are of strangely small dimensions atomic dimensions and quantum effects operate at this scale. And we cannot view this effect, realize this effect on a daily basis, but superconductor is a state which enables us to observe quantum effects on a macroscopic everyday scale. So, that is the importance of the London form of the London equation.

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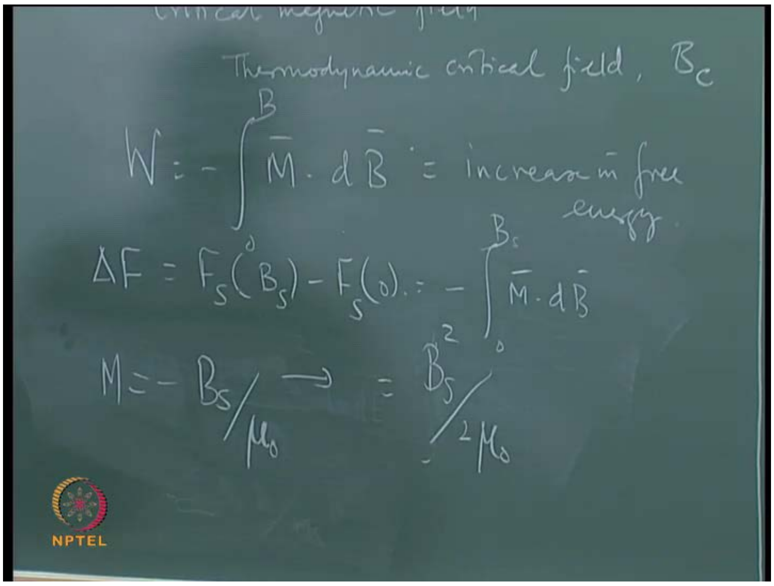

Next we pass onto a consideration of the thermodynamics of the superconductor this is the electrodynamics behavior. In addition we said that the superconducting transition is some kind of phase transition, which is neither structural not magnetic, but we would like to know what kind of phase transitionity. And so we would like to discuss the thermodynamics behavior, in order to do that we are given a clue from the existence of the critical magnetic field, which is also known as the thermodynamic critical field. Suppose this is B c at a particular temperature T then the suppose we take a superconductor and bring it to a region of zero field, and from there we take it to a region of magnetic field B, then we were done on the superconductor per unit volume is given by where m is the magnetization.

Now, this work done may be equated to the increase in the free energy. In magnetizing a superconductor, in applying a magnetic field superconductor we get an increase in free energy. So, the change in free energy can be written as delta F and that is F s of B s minus F s of zero and that is equal to minus integral M dot d B. The subscript s stands for the superconducting state. Now for a superconductor M is minus B s by mu naught, we saw this how the magnetization is just given by this. So, substituting this I get integral. So, B s square by 2 mu naught.

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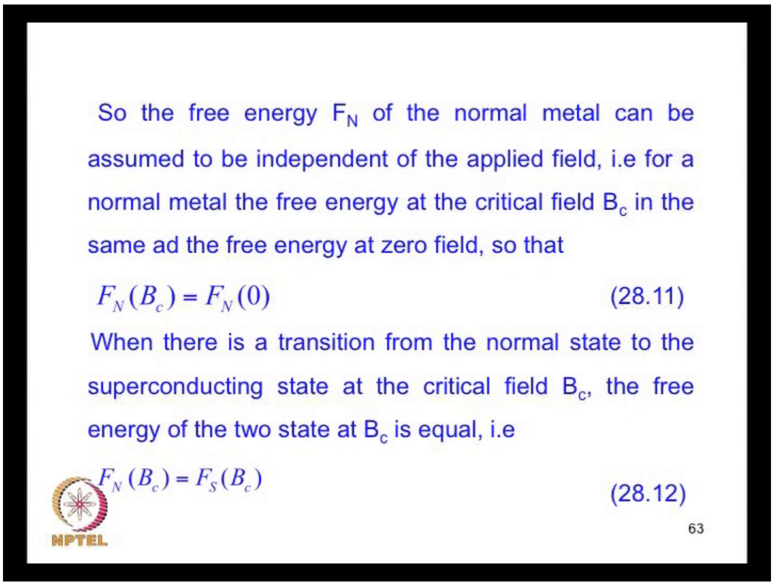

So, this is the increase in the free energy of a superconductor. We neglect, of course, the magnetization due to the weak paramagnetism in the superconductor in the normal state. So, the free energy in the normal state can be taken to be 0.

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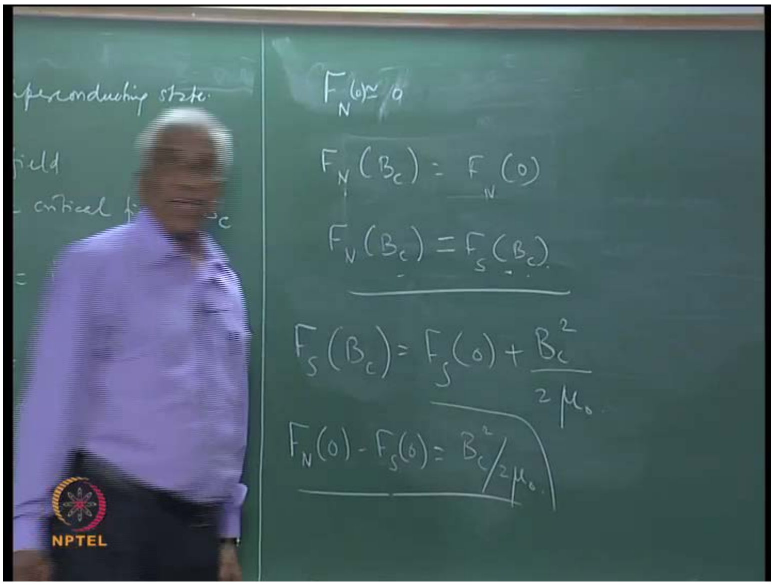

So, the free energy in the normal state is approximately zero, if we neglect the if on neglect the weak paramagnetism. So, the F N of B C of course, is the same as a F N of because it is non-magnetic. So, no energy change in free energy when you magnetize it. Therefore, this gives you a F N of B C minus F of B c. This is not zero, but this all right.

So, F N of B c this is the difference in a magnetic field. So, you have the normal state and the superconducting state and when there is a transition from this in a critical field then the free energy as a two states will be equal at the critical field. Therefore, from this the consequence is that we have F s of B c equal to F s of 0 plus B c square by 2 mu naught. Therefore, F N of 0 minus F s of 0 equals B c square by 2 mu naught. This tells us that the free energy difference between the normal state and the superconducting state is determined by the thermodynamic critical field, its square divided by 2 mu naught. So that means, that when you apply a magnetic field this is the difference in free energy

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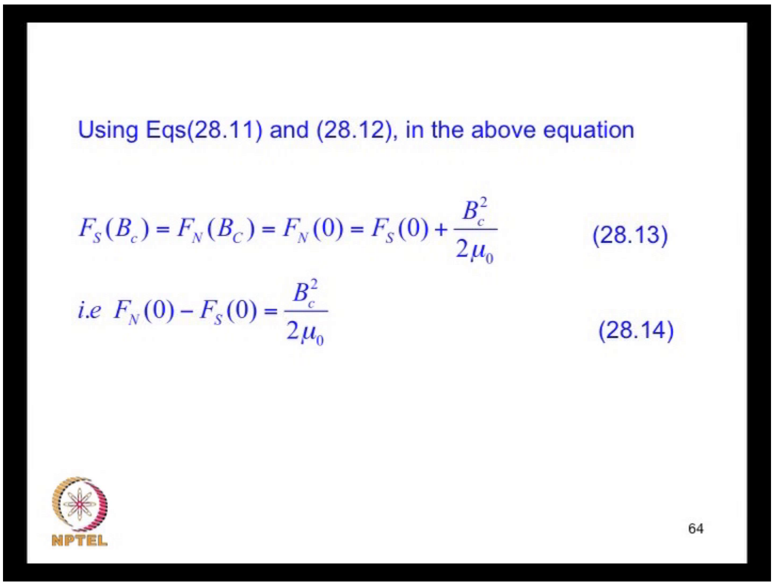

So, this gives a clue to describing the thermodynamic behavior of a superconducting state as follows. We know how the critical field varies with temperature.

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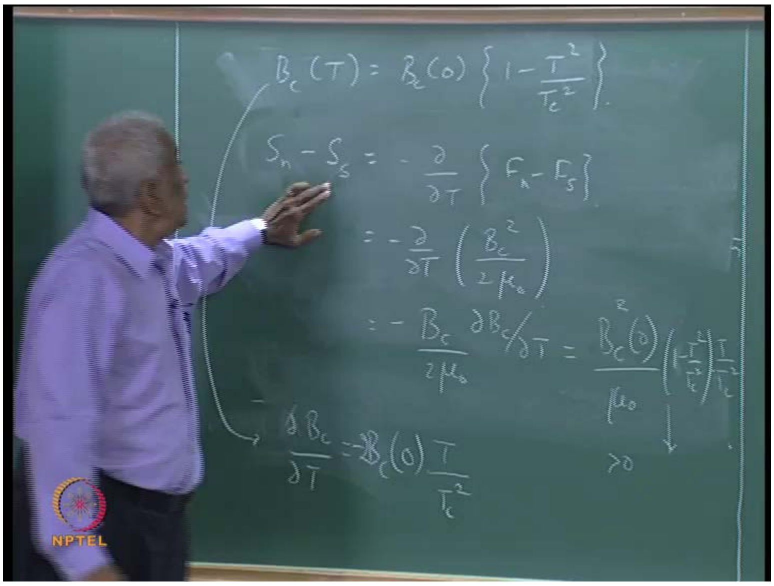

We already saw that there is a parabolic law. Now we can equate the free energy difference to as related to the difference in entropy in the normal and superconducting state by writing minus d by dt of F n minus F s because of the connection between the free energy and the entropy. And therefore, now replacing this and that will be minus B c d B c by dt divided by 2 mu naught and what is d B c by dt from here gives me B c 0 minus T by T c square twice. So, replacing here I get B c square by 2 mu naught mu naught into one minus t square by T c square times t by t c square. So, that is the entropy difference.

And since I have a positive definite quantity here and everything else is positive. So, the free energy, the entropy difference is positive. In other words, the normal state is a higher entropy then the superconducting states of lower entropy then the normal state. In other words, the superconducting state is more orders since the entropy is connected to the amount of disorder present in a system thermodynamically that the interpretation of entropy. So, it says this relationship. So, this says this is greater than zerp showing that the superconducting state is a more order state then the normal state.

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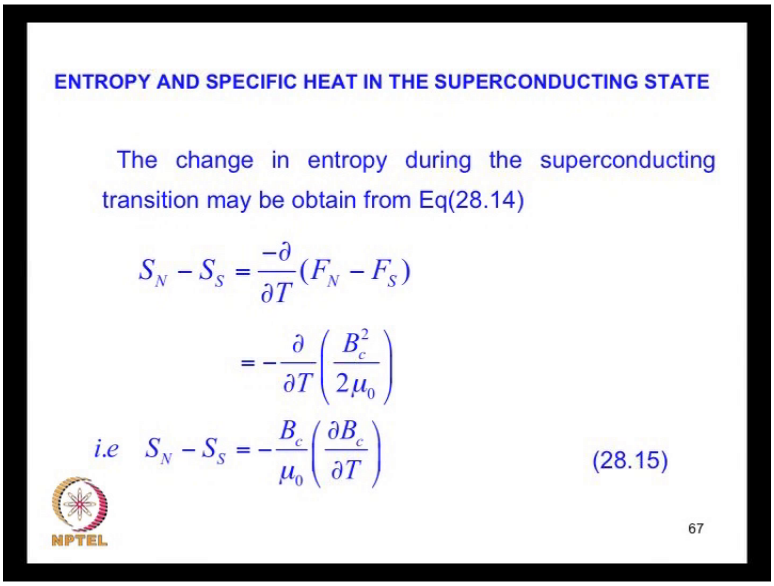

Once we have the entropy different, we can also write the difference in specific heat; specific heat is a quantity, which one can measure it experimentally.

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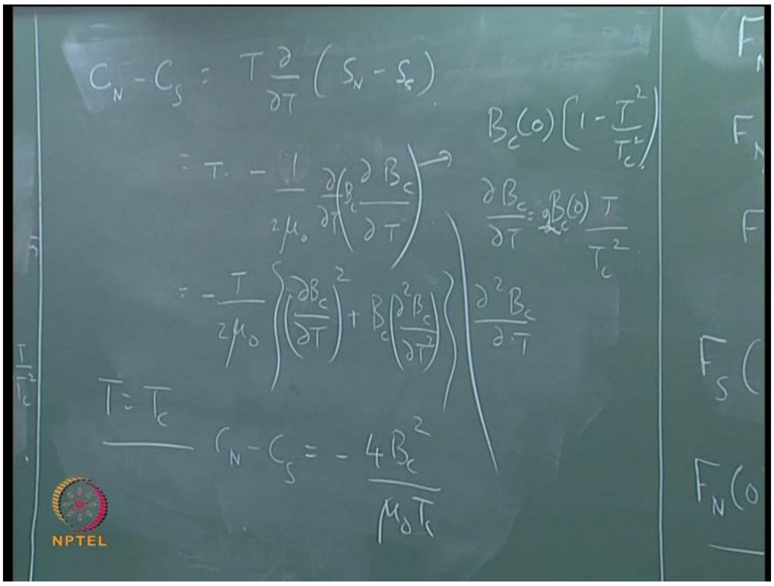

So, let us go to this specific heat C N minus C S and that will be T d by dt of S N minus S S. And therefore, making a further differentiation we get t into minus B c by 2 mu naught into d B c by dt d square B c by dt square. So, this will have because I know that B c is B c 0 into one minus t square by t c square therefore, d B c by d t is B c 0 into minus two noise t by t c square and d square B c by dt square. So, this will be minus T by

2 mu naught d B c by dt whole square plus B c d square B c by dt square. So, we can find this and since we know the d B c by dt wholes square will be one can find d square B c by dt square from this and therefore, you will find that a at d equal to B c at T equal to T c. There is edge in this specific heat. So, this is negative indicating that there is an increasing specific heat at the superconducting phase transition from the normal to the superconducting state.

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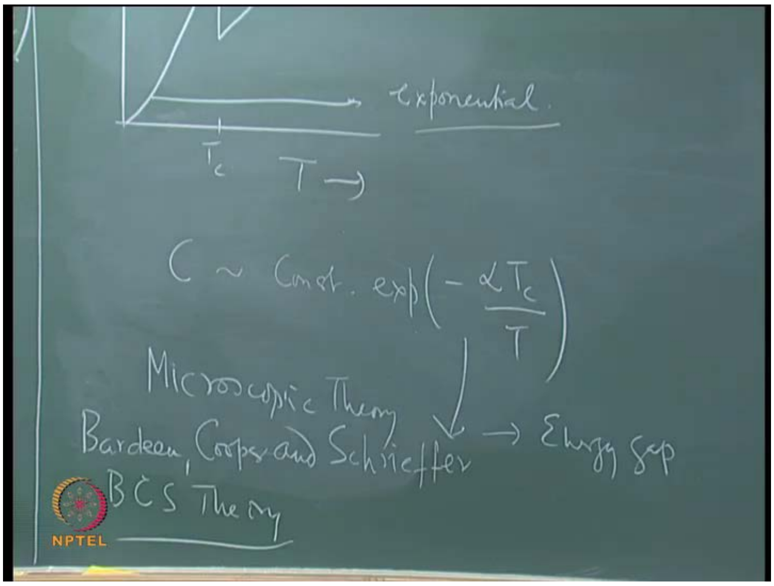

So, if we plot this specific heat from the normal state downwards as a decrease the temperature C is suppose this is the T c. Then we know that is a linear specific heat for a metal for a normal metal, and it decrease and then at the transition temperature there is a jump in this specific heat of the superconducting state and then it falls exponentially down. So, this exponential fall is something close to absolute zero, this specific heat falls exponentially to zero. This is something that cannot be understood in the light of this, this theory and this specific heat goes a constant time exponential minus alpha T c by T.

So, that is a typical experimental variation and this cannot be understood in the light of this thermodynamic theory one has use the microscopic theory regarding the superconducting state which predict and energy gap in this excitation spectrum of a superconductor in this microscopic theory was proposed by three people Bardeen, Cooper and Schrieffer. So, this is known as the B c s theory this was for this they got the Nobel Prize in 1957. So, the microscopic theory of the superconducting state prediction energy gap in the excitation spectrum which enable us to understand why there should be an exponential decay of the specific heat.

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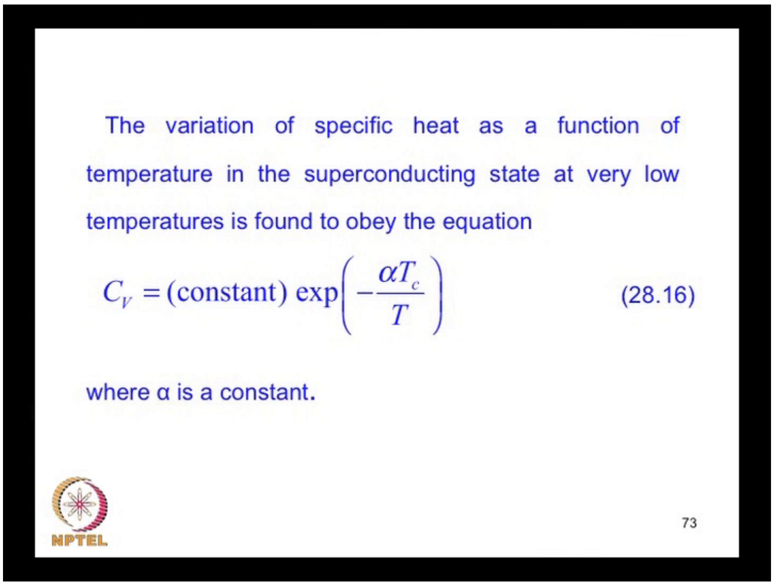

We will see the microscopic theory a later at right now we will try to understand the thermodynamic behavior in terms of another celebrated theory, but it is a thermodynamic theory namely Ginsburg Landau theory.

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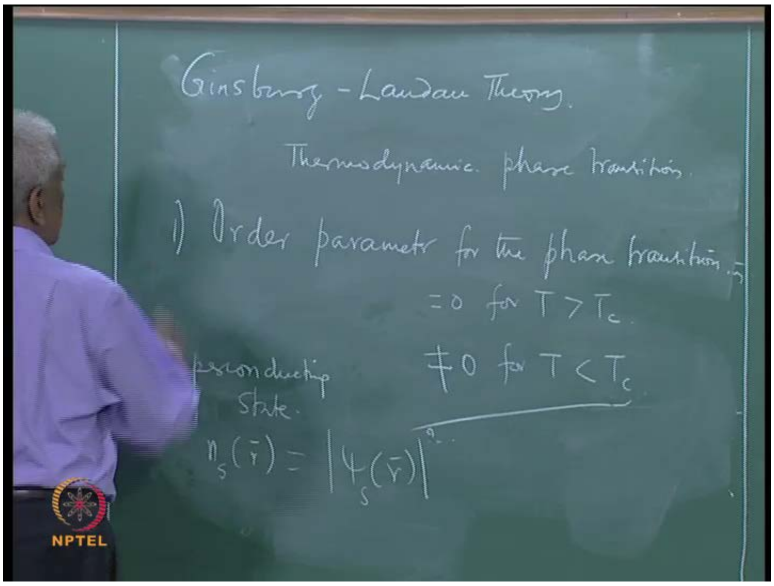

We will discuss this theory because of its a important from a thermodynamic point of view of the superconducting state as a thermodynamic phase transition. Landau proposed a general theory of a phase transition, which was applied by Ginsburg and landau to the case of a superconductor. Now this general theory always proceeds by defining an order parameters of the transition. Now what is an order parameter? An order parameter is defined by the fact that this is zero for temperatures above the transition temperature and not zero for temperature below the transition temperature. In other words, it can be any parameter, which appears only at the transition temperature above it has a 0 value there is no order.

And therefore, there is a order parameter is zero, and below the temperature transition phase transition temperature the order parameter is non-zero. Any parameters associated with the ordered state, which satisfies this behavior can be taken as an order parameter. For example, in the case of a ferromagnetic the order parameter will be the magnetization, this spontaneous magnetization. There is no spontaneous magnetization in the paramagnetic state, but in the ferromagnetic state, there is a nonzero spontaneous magnetization.

So, this spontaneous magnetization can be used as the order parameter for a magnetic phase transition. Similarly, we introduce an order parameter for the superconducting state Ginsburg and landau introduced this by taking the order parameter as the charge density, this superconducting charge concentration n s, which is related to mod psi s r square. In other words, there is a wave function because the already saw that the super conducting state is a quantum mechanical state. So, there is a wave function associated states such that the square of the modulus of this wave function gives you a measure of the charge concentration. Of course, multiply by the appropriate charge.

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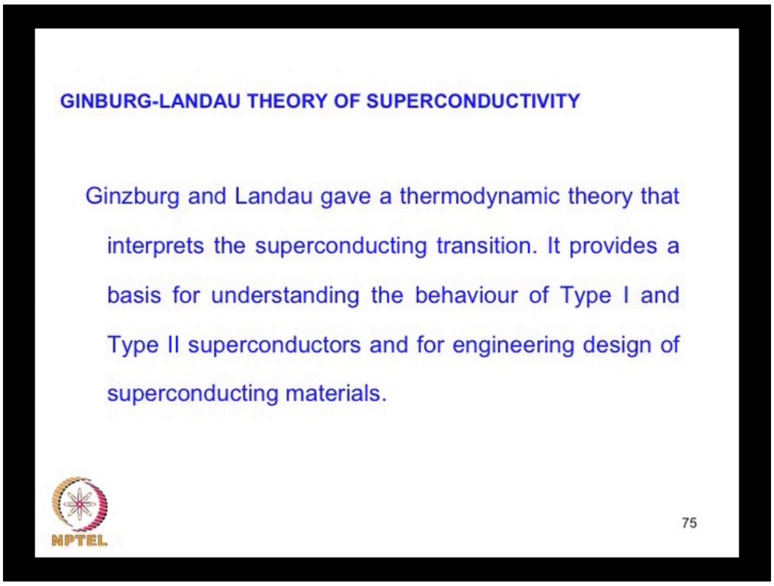

So, now, the next step polarisability this is step one in the Ginsburg Landau theory. The next step is to expand the free energy, because the any thermodynamic theory, start with the assumption that the phase transition is the result of a reduction in the free energy.

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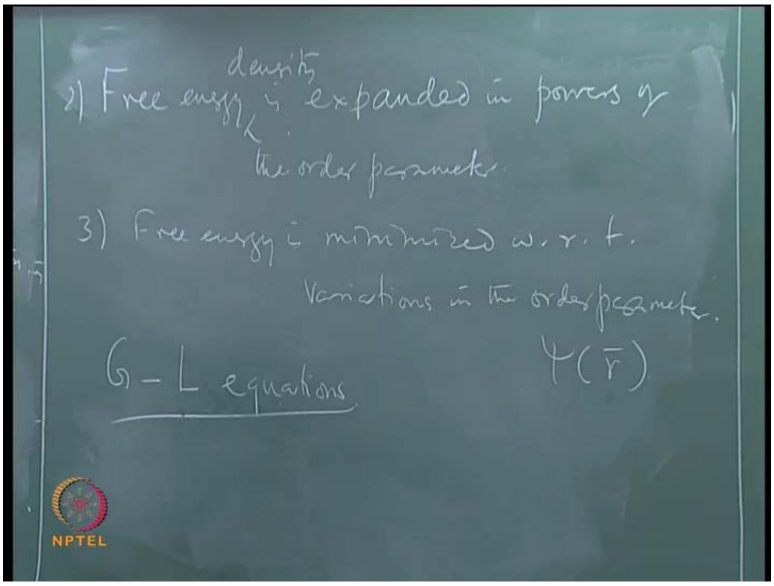

So, this appropriate free energy is expanded in powers of the order parameter. In this case, the wave function psi and its higher power. Then it is minimized with respect to variations in the order parameter. So, this minimization procedure enables us to discover the phase which has a free energy minimum and therefore, it is stabilized thermodynamically. So, the free energy density with respect to the unit volume is expanded and then minimize with respect to variations in the order parameter psi of r. There can also be changes here because of an applied magnetic field. So, the free energy have to be written in terms of it will have a term due to the magnetization, and then it is minimize also with respect to variations of the applied magnetic field.

This procedure results leads to before so-called G L - Ginsburg landau equations corresponding to the free energy minimization process with respect to variations spatial variations, so the order parameter as well as variations due to the field applied field. These results in the so-called Ginsburg landau equations, which characterized a superconductor thermodynamically and the Ginsburg landau equations provide the starting point for all thermodynamic explanations of the superconducting state, which are at the bottom of form the basis for the application of a superconductor in technological context. So, for technical application the G L equations provide a convenient starting point. We will describe these equations in greater detail in the next lecture and then go on to the microscopic.

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