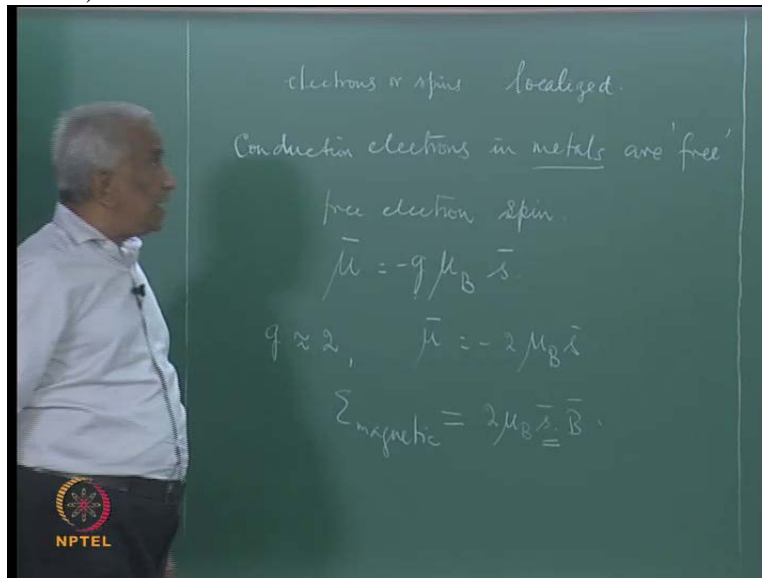**Condensed Matter Physics**
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Lecture – 25
Pauli paramagnetism and Landau diamagnetism

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So far, in our discussion of magnetism we have always talked about electrons as if or spins as if they are localized in space inside the crystal lattice. So all our treatment has been under the assumption that the spins, which are responsible for the magnetic behavior are all localized in space and stay fixed in a particular point or space. For all time, we know that this assumption is not justified in the case of metals in which conduction electrons in metals are somewhat free to move around and so they are not fixed in space. So this assumption is certainly not justified when discussing the magnetism due to spins which are in metals. So if you want to discuss magnetic behavior of metallic systems, we have to have a different approach, the assumption breaks down of a fixed position in space, and therefore we have to allow for these in our discussion.

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So this is what we will do today and we will find that very interesting effects take place when we do this. So the main contribution to paramagnetism, let us discuss paramagnetism first is from the free electron spin magnetic moment; free electrons spin associated with this spin angular momentum there is a magnetic moment. And we know this magnetic moment is given by… Well we write a negative sign to show that, that is electron. And where here, this is the g due to free spin, and g is known to be very close to two; it is the ((Refer Time: 03:10)) factor. And therefore, we can write the magnetic moment as…

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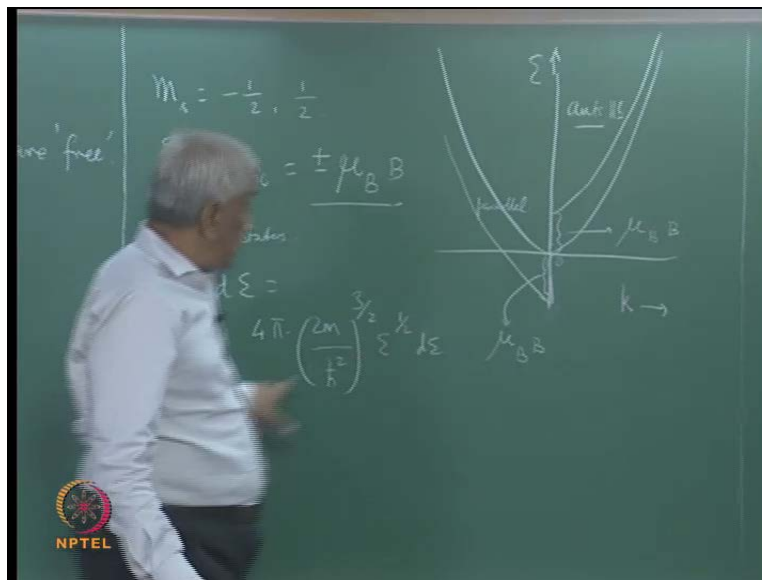
So that is the magnetic moment of an individual free electron which a when we apply an external magnetic field acquires an energy - magnetic energy, which is this magnetic moment times…

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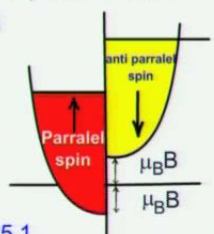
So and we know that in a magnetic field, then electron has two orientation, which are parallel and anti parallel to the field which are given by the spin magnetic quantum number minus half, plus half. So we have corresponding to the two different configurations, the magnetic energy turns out to be plus or minus mu B B.

- When an external magnetic field is applied, it is seen from Eq. (25.2) that the energy of those electrons with spin quantum number $-\frac{1}{2}$ will decrease by $\mu_B B$

- The energy of those electrons with spin quantum number $m_s = \frac{1}{2}$ will increase by $\mu_B B$

- This is indicated in Fig.25.1

Fig.25.1

So when an external magnetic field is applied, the energy of the electrons with spin quantum number minus half will decrease by an amount mu B B. While the energy of the electrons whose spin quantum number is plus half will increase by the same amount mu B B. So this is what we show in this figure, so we just plot e versus k, but we conveniently split the band into two, so this is 0. So positive k is we have a parabolic energy depends as we know already and this is show on both sides. So these are the two spins sub bands corresponding to m s equal to minus half and plus half.

So now this the once with anti parallel orientation will go up in energy now. While those with parallel orientation will go down in energy, so that will be the situation, so these will be the new configuration corresponding to parallel and anti parallel, anti parallel spins and these are parallel spins. This is the sub band corresponding to the parallel spins, so that would be the situation and the amount by which this change take place is what we wrote as mu B B. So this is the same as mu B B here. So this is the situation in an external magnetic field.

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Fig.25.2
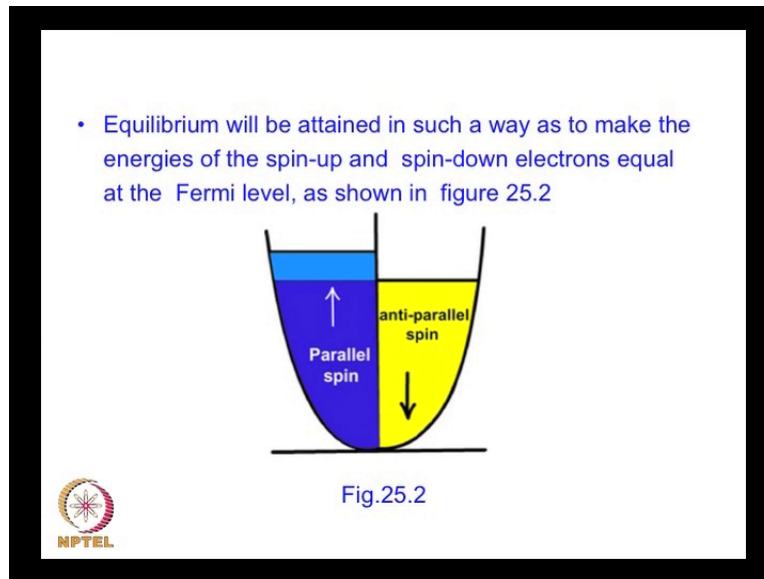
So equilibrium will be attained so as to make the energy of the spin up and spin down band electrons equal at the Fermi level, so this is what we shown in the next figure. So we have a net magnetization, because the density of states of the electrons D of e d e depends on this is equal to 4 pi into 2 m by h cross square to the power 3 by 2 into e power half d e. So if this is e minus mu B B or e plus mu B B the density of state will be different.

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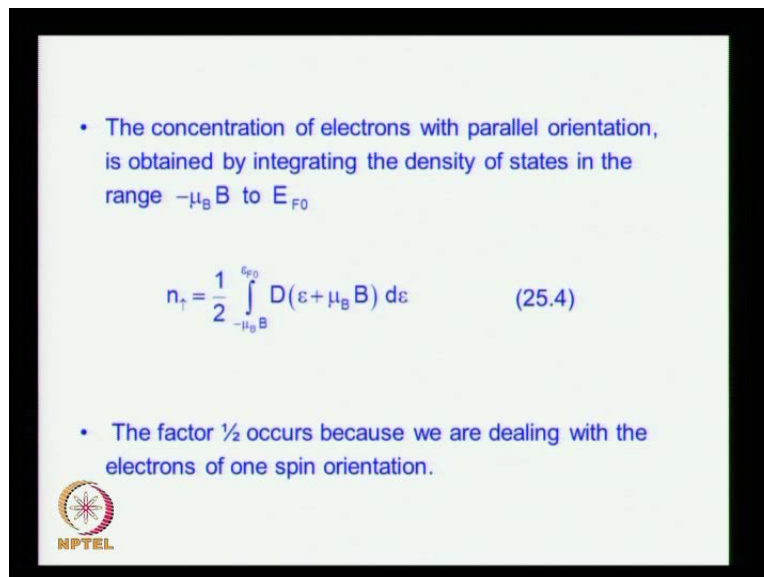
So, the consequence of this will be that the electron concentration.

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If you consider the electron concentration in the spin up sign band that will be half integral... In the presence of an applied magnetic field, the strength B, the density of states will be given with by this and we have the factor half, because only the one half of the number of electrons will occupy one spins of band.

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- Similarly the concentration of electrons with antiparallel orientation $n_{\uparrow\downarrow}$ is obtained by integrating the density of states in the range $\mu_B B$ to ε_{F0}

$$n_{\uparrow\downarrow} = \frac{1}{2} \int_{+\mu_B B}^{\varepsilon_{F0}} D(\varepsilon - \mu_B B)\, d\varepsilon \qquad (25.5)$$

Similarly, n down or n anti parallel, n down spins will have half integral... So this is the concentration or different because the density of states are different corresponding to different energy.

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- From Eqs (25.4) and (25.3)

$$n_{\uparrow\uparrow} = \frac{1}{2} \int_{-\mu_B B}^{\varepsilon_{F0}} 4\pi \left(\frac{2m}{\hbar^2}\right)^{3/2} (\varepsilon + \mu_B B)^{1/2} \, d\varepsilon$$

The lower limit may be replaced by 0, since $\mu_B B \ll k_B T$ and the number of states near $\varepsilon=0$, is very low, so

$$n_{\uparrow\uparrow} = 2\pi \left(\frac{2m}{\hbar^2}\right)^{3/2} \int_0^{\varepsilon_{F0}} (\varepsilon + \mu_B B)^{1/2} \, d\varepsilon \qquad (25.6)$$

So, because of that there in the absence of the magnetic field, the concentrations were equal, but now there is an unequal concentration and therefore we have a net difference and which we write as the difference between the two configurations.

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Similarly we get from the Eqs (25.5) and (25.3)

$$n_{\uparrow\downarrow} = 2\pi \left(\frac{2m}{\hbar^2}\right)^{3/2} \int_0^{\varepsilon_{F0}} (\varepsilon - \mu_B B)^{1/2} \, d\varepsilon \qquad (25.7)$$

• The net concentration of electrons oriented parallel to the field.

$$n_{\uparrow\uparrow} - n_{\uparrow\downarrow} = 2\pi \left(\frac{2m}{\hbar^2}\right)^{3/2} \left[\int_0^{\varepsilon_{F0}} (\varepsilon + \mu_B B)^{1/2} d\varepsilon - \int_0^{\varepsilon_{F0}} (\varepsilon - \mu_B B)^{1/2} d\varepsilon \right] (25.8)$$

So we have this net can easily be calculated and that is given as so that would be the net difference in concentration.

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And this can be simplified for small values of the applied magnetic field, we can write this by using binomial theorem.

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So expanding the square root using binomial theorem, we get this population different, the final difference in population is turns out to be, so that would be the difference.

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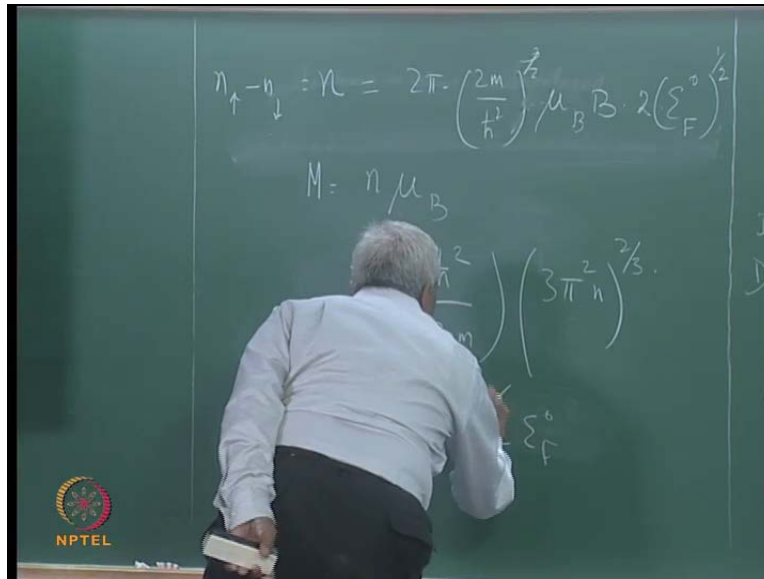
And the corresponding magnetization is just this times mu B; this is nothing but…

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So this magnetization due to the difference in orientation of the ((Refer Time: 11:10)), it turns out to be times well.

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We can the simplification of the stage using the expression for the Fermi energy is… So using this, we finally arrive at the expression for the magnetization in the form. And therefore, the susceptibility is just M by B. So this susceptibility turns out to be… And this is known as the Pauli paramagnetic, Pauli who first determine this. So this is the paramagnetic susceptibility as seen of the positive sign and this has the magnitude of the sign. Now the question is this is the difference in electron concentration, this is the Bohr magneton, the square of the Bohr magneton and this is the Fermi energy.

Now the question arises can be experimentally observe this, well the answer is it is rather hard, because this is complicated by additional effects, it is by the overall diamagnetism of the ion cores in the metal lattice also as we will see. There is another effect called Landau diamagnetism which will consider presently, so both these effects the diamagnetism of the ion core as well as the Landau diamagnetism susceptibility complicate add to this and therefore the net susceptibility that one will observe in metallic specimen is the resultant of all these three. So that we already saw in the previous lecture on magnetic resonance, we talked about the NMR – Nuclear Magnetic Resonance in a metal.

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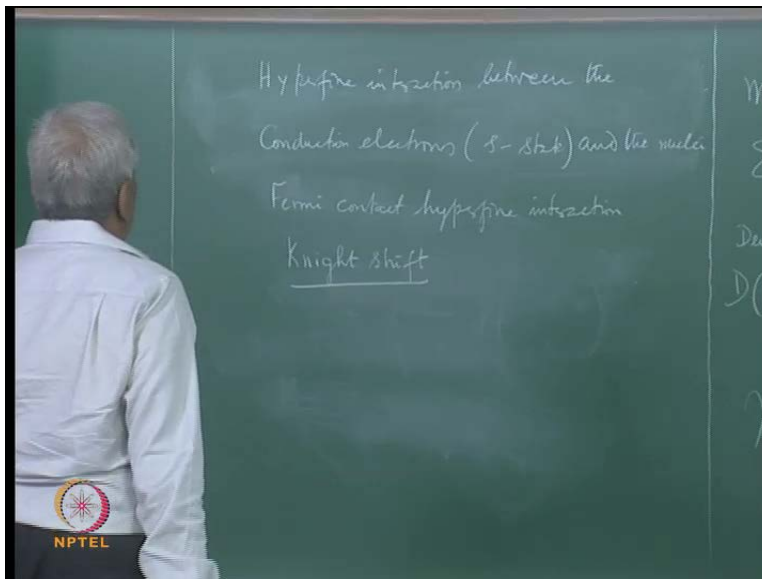
We talked about the conduction electrons and their interaction, the hyperfine interaction between the conduction electrons, which are in the s-states and the nuclei, so this provides so called the Fermi hyperfine interaction, which leads to establish went of a local magnetic field which adds to the field applied, because the resonance.

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• In metals this produces a shift known as Knight shift. The hyperfine coupling also enables the electronic magnetization to be transferred on to the nuclear levels, thus enhancing the strength of NMR absorption.

• This method of observing a relatively weak nuclear resonance via a change in the electronic magnetization is known as Electron Nuclear Double Resonance(ENDOR).

• Double resonance can also be of different origins and due to different coupling mechanisms.

And therefore, there is a shift which is known as the knight shift, we discuss this in the lecture on the magnetic resonance. Now this knight shift is also related to the Pauli paramagnetic susceptibility and so both these effects are related.

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Landau diamagnetism

In an applied magnetic field whose vector potential has the components (0,Bx,0) the Schrodinger equation becomes

$$\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial z^2}\right)\psi+\left(\frac{\partial}{\partial y}-i.eB\frac{x}{\hbar^2}\right)^2\psi+\left(\frac{2m}{\hbar^2}\right)\psi=0 \quad (25.16)$$

Taking

$$\psi=\exp\left(ik_y\,y+k_z z\right)u(x) \quad (25.17)$$

We get

$$\frac{d^2u}{dx^2}+\left\{\left(\frac{2m}{\hbar^2}\right)E'-\left(k_y-\frac{eBx}{\hbar}\right)^2\right\}u=0 \quad (25.18)$$

where

$$E'=E-\left(\frac{\hbar^2}{2m}\right)k_z^2 \quad (25.19)$$

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Now we move onto discuss this other effect, which is known as Landau diamagnetism.

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Now what is this effect give to, this is the name implies this is due to the not the spin angular momentum, but the orbital motion of the electron in a magnetic field, and when we say electron

we mean the the conductional effect, not the orbital electrons, but the conduction electrons which is free. So, when you apply a magnetic field, we usually write B equal to del cross A, where A is the vector potential, magnetic vector potential. So, if we apply a uniform magnetic field along the z component then this can correspond to a vector potential which has the components…

So in the presence of such a vector potential, we can write the Schrodinger equation for the electron, which is d square by dx square plus d square by dz square psi plus d by dy minus i e B x by h cross square operating term psi plus two m by h cross square psi equal zero. That would be the Schrodinger equation, which has this additional term due to the magnetic field. So we have to solve this by the method of separation of variable, so obtain the energy Eigen value in the stationary states.

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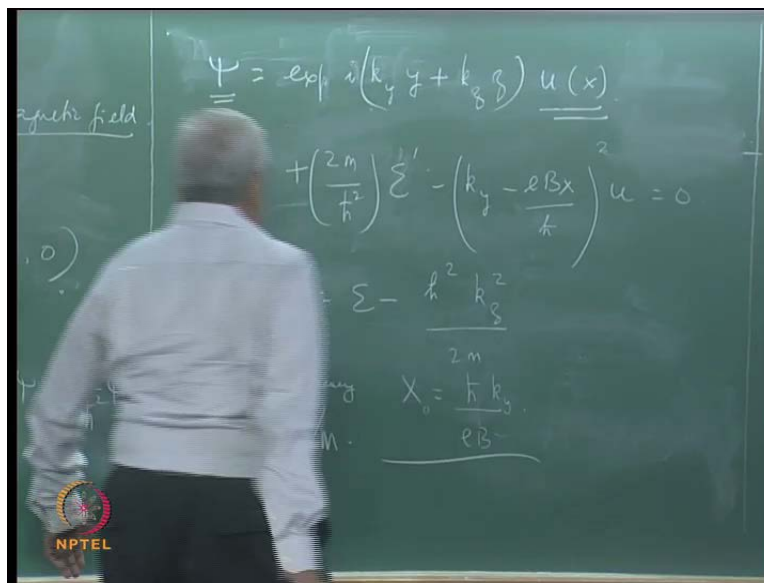
So following the method of separation of variable, we know that these two parts correspond to the free electron, therefore we can write times U of x, because there is a extra dependent term. So the x part of the wave function will not have the free electron form, and so this U of x is the function corresponds to the x part of the wave function. The y and z parts are free electron wave functions, as in the case of the free electron in the absence of the magnetic field. So substituting this back here, we can write the equation which is satisfied by the function U here, that would be the equation satisfied by the function U where e prime is e minus h cross square k z square…

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Defining the cyclotron frequency as
$$\omega = eB/m \text{ and } x_0 \text{ as } (h/2\pi\, eB)\, k_y$$

we get the Schrodinger equation for a displaced harmonic oscillator of frequency ν and centred at $x = x_0$. Its energy eigenvalues are given by :

$$E_n = (n + \tfrac{1}{2})\, h\nu \quad \text{where } n \text{ is an integer.}$$

Thus $E(n, k_z) = (n + \tfrac{1}{2})\, h\nu + \left(\dfrac{\hbar^2}{2m}\right) k_z{}^2$

Now we define two quantities the cyclotron frequency which we are already familiar, omega equal to e B by M. And we also define X 0 as h cross by e B into k y.

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Defining the cyclotron frequency as

$$\omega = \frac{eB}{m} \qquad (25.20)$$

$$\text{and } x_0 = \left(\frac{\hbar}{eB}\right) k_y \qquad (25.21)$$

we get the Schrodinger equation for a displaced harmonic oscillator of frequency ν and centred at $x = x_0$. Its energy eigenvalues are given by :

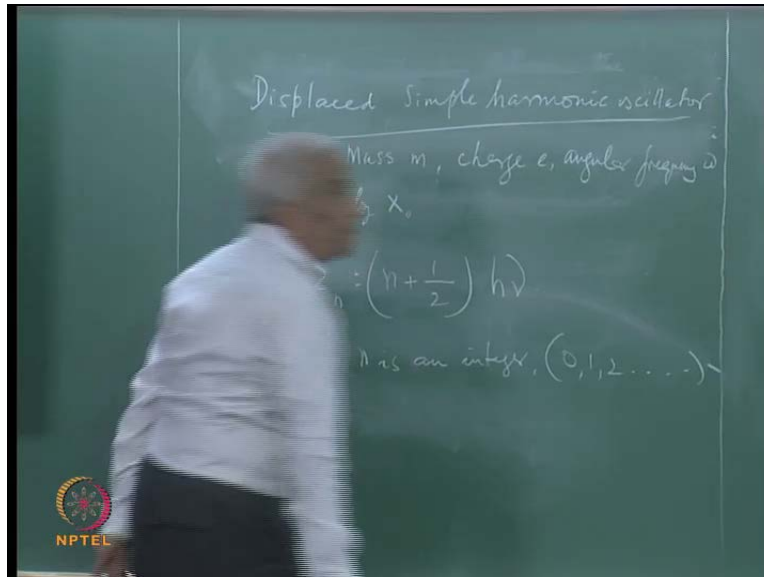$$E_n = (n + \tfrac{1}{2})\, h\nu \qquad (25.22)$$

where n is an integer.

Thus $E(n, k_z) = \left(n + \dfrac{1}{2}\right) h\nu + \left(\dfrac{\hbar^2}{2m}\right) k_z^2 \qquad (25.23)$

With these two substitutions, this equation can be simplified to the following form.

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The Schrodinger equation, it turns out that the equation is identical to that of a displaced harmonic oscillator of mass-m, charge-e, angular frequency-omega, and displaced amount –X naught, which is defined there. So the energy Eigen values of such a displaced simple harmonic oscillator are well known from quantum mechanics, they are given by… Where n is an integer, say zero, 1, 2 etcetera.

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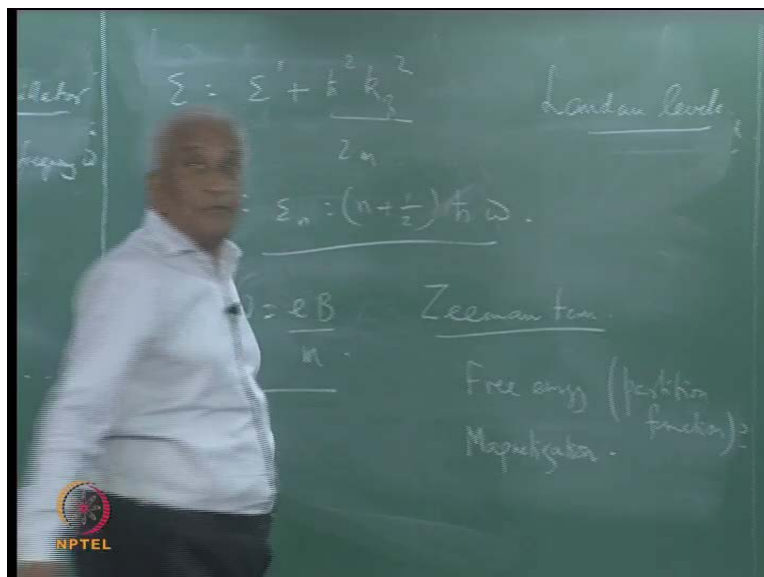
So that the total energy Eigen value for such a electron orbiting around in the presence of a magnetic field turns out to be… Where e prime e n given above. So what is the result of all these, the motion now of the free electron or the conduction electrons which originally had a plane wave like states, you know all the three direction, it means that they are free electron states. And now because of the application of the magnetic field mu is so this omega depends on this magnetic field. So because of the applied magnetic field, energy levels only along the z direction or free electron plane wavelike while the energy level in the other plane perpendicular to the z axis, the direction of the field direction are quantized the units of the cyclotron frequency omega, h cross omega.

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These levels are known as Landau levels.
Introducing the Zeeman term $\sigma \, \mu_B \, B$ we can calculate the free energy and hence the magnetization in the presence of an applied magnetic field of strength B along the z direction. This leads to the magnetic susceptibility
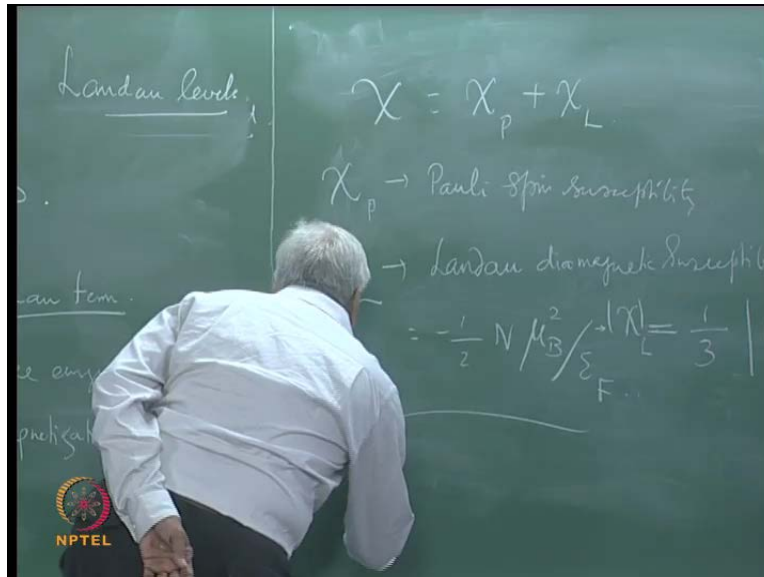
$$\chi = \chi_P + \chi_L \qquad (25.24)$$

where χ_P is the Pauli spin susceptibility discussed already and χ_L is the Landau diamagnetic susceptibility given by :

$$\chi_L = -\frac{1}{2} \frac{N \mu_B^2}{E_F} \qquad (25.25)$$

So, this means that we have these energy levels are known as Landau levels, because Landau first describe this. The result of this is that now we have to add the Zeeman term, well-known Zeeman term to this and find its effect. So we can usual procedure is to calculate the free energy in terms of the partition function, once we know the energy level Eigen values, we can write the partition function and so calculate the free energy. And then the magnetization can be calculated from the free energy as the first derivative of the magnetic, with respect to the magnetic field. So the magnetization can be calculated, the calculation of these algebraic somewhat complicated, I will not go through it here, I will simply give the result of this calculation.

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It turn out that this magnetization corresponds to a susceptibility, which is of the form chi P plus chi L. Here chi P is the Pauli principal spin which we already discussed. And chi L is the Landau diamagnetic susceptibility. It is found to have a value, minus half N mu B square by e F. So the Landau diamagnetic susceptibility, the magnitude is just one-third, the Pauli susceptibility is 3 by 2 with a positive sign, so this is equal to one-third of chi P. So the Landau susceptibility has a magnitude which is just one-third of the Pauli susceptibility, so the comparable to the Pauli spin susceptibility.

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In addition, it is also found that the susceptibility has an oscillatory component and which has period, which is given by delta 1 by B equal to 2... So that is the period, the period of these oscillation. These susceptibility oscillation are known as de Haas-van Alphen oscillations.

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It is an effect which was discussed first by discovered by de Haas-van Alphen, so it was named of them. So this gives you of the effect of this oscillatory susceptibility. This has a very important consequence that one can determine the topology of the Fermi surface using this,

because the exact calculation gives delta one by B as 4 pi square mod e divided by h cross A. This is this A effective is the effective cross sectional area of the Fermi surface. So the period of these oscillation is related to this cross-sectional area and therefore determining this period as the function of the applied magnetic field gives us the information about the topology of the Fermi surface. So this is an extremely interesting effect which help us in determining the exact topology of the Fermi surface in real metal.

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Fig 25.3

de Haas van Alphen effect in gold showing
oscillations in the magnetic susceptibility
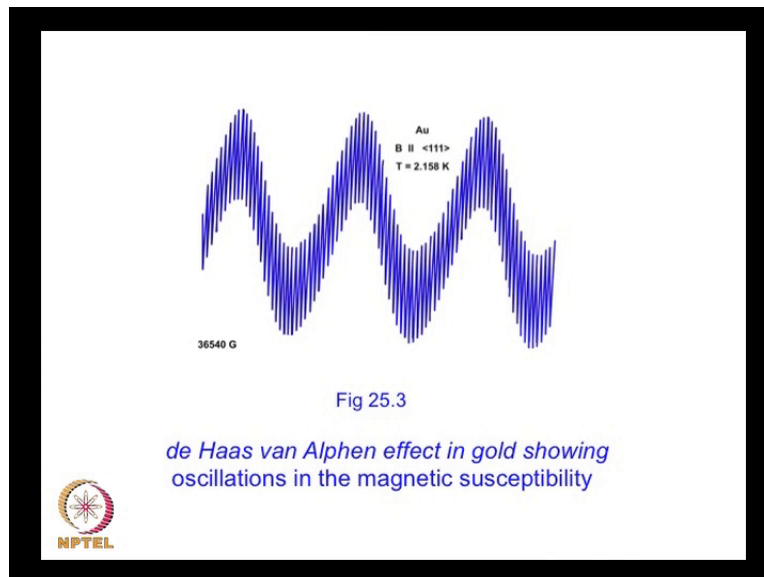
The next figures shows the experimental recording of a such quantum oscillation in the susceptibility of a sample of gold as a function of magnetic field strength. So these are interesting experimental results associated with conduction electron magnetic in metals. This also leads to a similar interesting effect due to the cooperative magnetic ordering of conduction electron in a metal and leads to what is known as the band magnetism. We will discuss this in a separate lecture.