Condensed Matter Physics
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Lecture - 23
Hysteresis and Magnetic Domains
Spin Waves and Magnons

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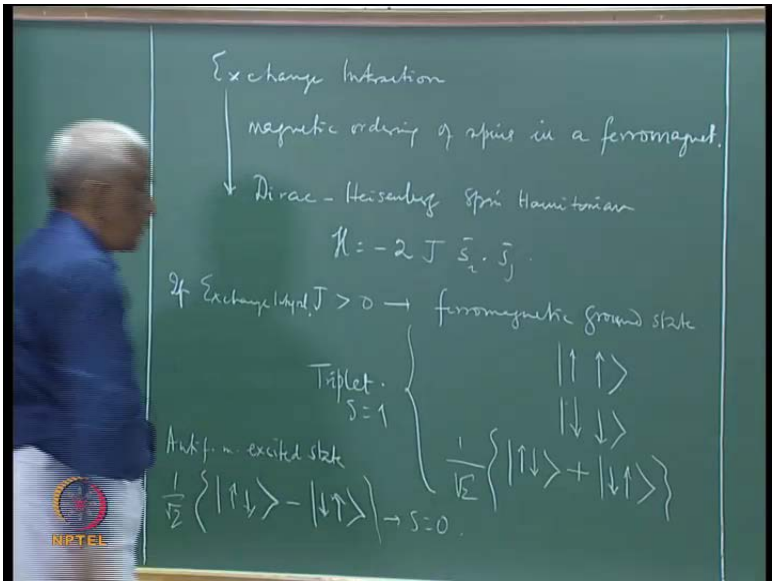

In the last lecture, we discussed the exchange interaction and showed how this interaction is a responsible for magnetic ordering of spins in a ferromagnetic. In particular we pointed out that this exchange interaction has the form due to Dirac and Eisenberg, it is known as the spin Hamiltonian, because this interaction energy is written in terms of spin operators only, and it as the form minus 2 at i J i J s i dot s J and we will just right j. So, if J is greater than 0, that is if the exchange integral J, if J is positive then ferromagnetic ground state is energetically favored.

So, it becomes the ferromagnetic alignment means that the two spins I and J are lined up parallel. So, the two spin wave function has the form of the parallel spin are can also be like this both of or both down or it can be symmetric linear combination of this. So, all these three spin states spin combination are energetically degenerate. So, this is known as the triplet, whereas the excited state will be anti ferromagnetic excited state, which has

the form, which is an anti symmetric combination all these have s equal to1, this as s equal to 0.

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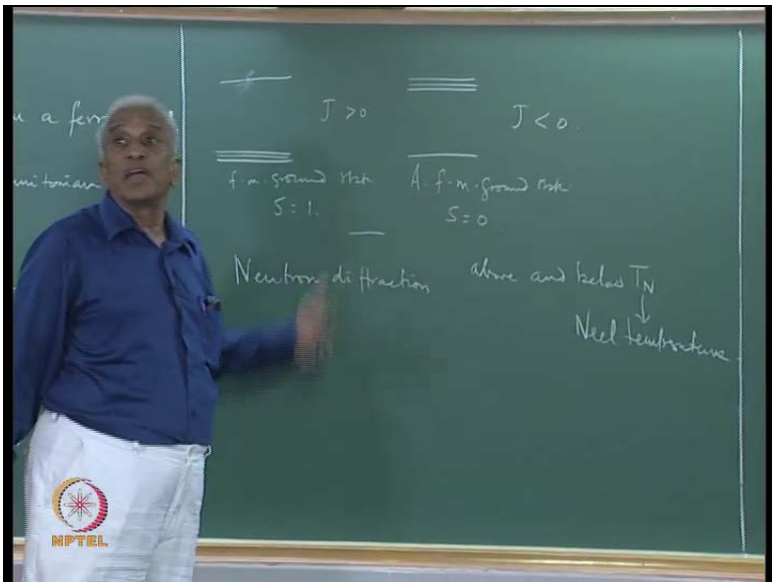

So, you have a ground state which is triplet, and excited state this is the situation, which is anti ferromagnetic, this is the ferromagnetic. So, this the situation for J greater than 0. We also said that it will be reversed and the triplet will live higher while the singlet lower, if it is an anti ferromagnetic ground state for J less than 0, if the x J integral is negative this is what will happen? So, this will have s equal to one this will have s is equal to 0.

So, this is what we discussed last day last time, and so we have both ferromagnetic and anti ferromagnetic ground states are possible depending on whether the exchange integral J is positive or negative. Therefore, what happens in a real crystal consisting of a very large number spin's all these spins will be lined up parallel then it becomes a ferromagnetic, and all the spins are anti parallel in pairs then it becomes a anti ferromagnetic. So, these are the different ways of magnetic gaudery spin ordering, but in the case of antiferro magnetic.

Since you have half the number of spins lined up in one direction and the other half lined up in the other with equal, and opposite movement the magnetization due to one sub lattice cancels completely the sub lattice magnetization due to the other therefore the net magnetization of anti ferromagnetic will be 0. So, if you measure the magnetization it

will not be at clear whether it is an anti ferromagnetic or it is a non-magnetic material. So, what is the best way to detect anti ferromagnetic ordering. So, that must be an experimental method to distinguished an anti ferromagnetic material a non-magnetic material, because both have 0 magnetization.

So, the anti ferromagnetic becomes anti ferromagnetic below the nil temperature. So, also we would like to know what be the nil temperature. So, the best way is to carry out the experiment call neutron diffraction, if one performs neutron diffraction above and below nil temperature, this is the nil temperature at which the material becomes anti ferromagnetically ordered. So, if your carry out neutron diffraction measurements both above and below the nil temperature. Then it is possible to fair weather a material as is an anti ferromagnetic or not this is, because neutrons have a magnetic moment even though they are electrically neutral particle. But still they carry a magnetic moment, because they have a magnetic moment the neutrons can interact with this spin magnetic moments of the ordered spins in the crystal lattice.

Therefore, because it is interaction they can be defected just like x-rays get distracted by atoms in the crystal lattice only thing is the interaction here is magnetic, whereas in the case of the conventionally x ray the electromagnetic waves interact with the charge cloud of the electrons in the material. So, the neutron diffraction intensities will be determined by the nature of the ordering of the spins risk which differs potentially from the atomic or molecular ordering, because the atomic or molecular ordering comes because of the periodicities in the electron charge distribution whereas, here the ordering neutron diffraction intensity arise from the periodicities in the magnetically ordered spin configuration. So, the magnetic ordered lattice may not be the same as the crystallographic lattice therefore, the diffraction intensities can be substantially different from that of the x-ray diffraction intensities.

So, that is why performing the neutron diffraction we can detect spin ordering anti ferromagnetic crystals provide the most striking neutron diffraction patterns, that is because the sites are occupied by the magnetic irons in these materials are all crystallographically equivalent. So, they cannot be distinguished crystallographically, but as the crystal is cool through nil point extra lines appears in the neutron diffraction pattern. because of the ordering into magnetic sub lattice. Introduce additional periodicities in the lattice, which was not present before about the nil point.

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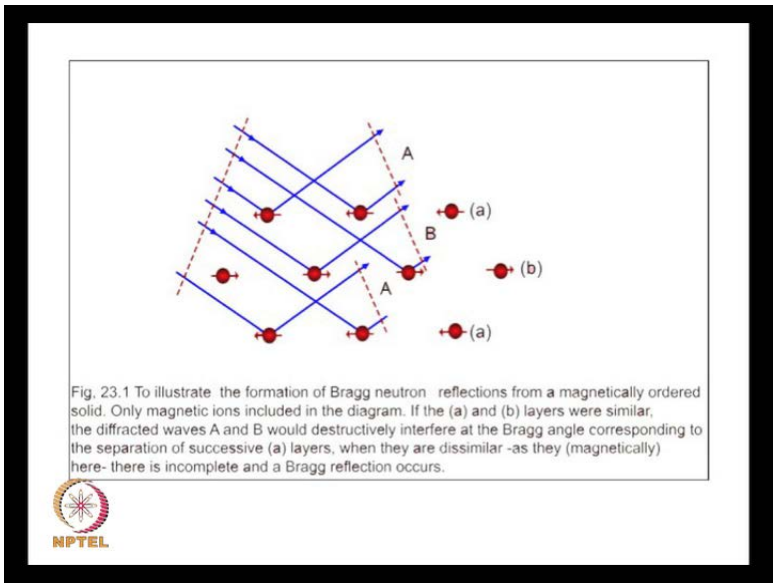

So, the magnetic periodicity gave rise to what are known as super lattice line due to magnetic ordering. So, the appearance of the super lattice lines below the nil temperature is due to the magnetic scattering of the neutrons, when there magnetic moments interact the magnetic moment of the ions examples of such abound in nature such materials, but a example is for a crystal shown in the next figure.

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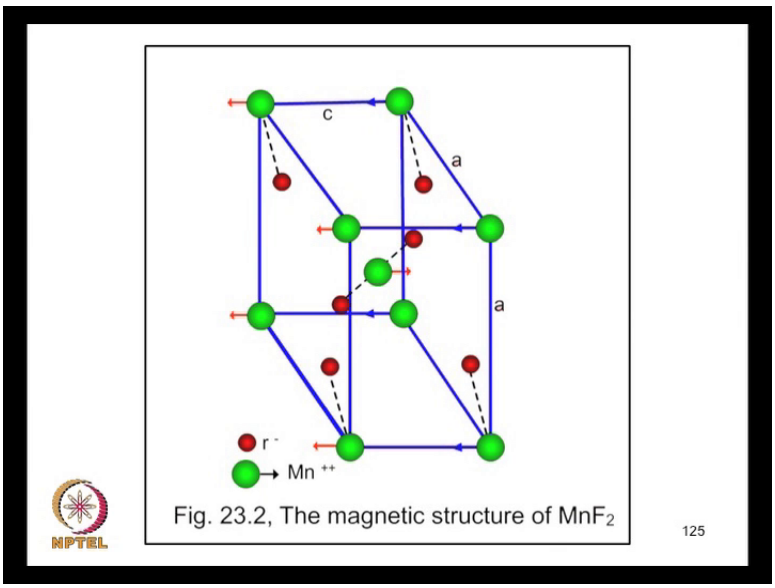

Here you have the Bragg diffraction from a magnetically ordered solid this lattice is a body centre lattice, and if there are no magnetic moment on the ions; there are two

different layers in this figures the a plane ions. And cause constructive interference and they interfere with the wave friend from the neighboring b planes with the neighboring a planes, but there would be also destructive interference from the wave friends from the neighboring b planes, which are through the body centre. Because of that no Bragg reflection would normally occur, because of this cancellation, but if the ions are magnetic ions amplitude of the wave reflected from the a ion planes differ substantially from that of the plane of the b ions because of the additional scattering contributions. They depend on the angle between the movements the spin moments of the ion and incoming neutron beam.

So, destructive interference will then be incomplete, and therefore you get the diffraction line at that particular angle whereas if even though the ions are magnetic, but if we are above nil temperature when there is no order. Then they will not give this extra line therefore you can one can distinguish easily the onset of magnetic ordering anti ferromagnetic ordering in particular at the nil temperature by the appearance of this magnetic super lattice lines. The position of the super lattice line are index, just the way the x-ray diffraction line for index according to the plane from, which they are reflected that is the position than the next information is their relative intensity which tells as about the nature the kind of ordering pattern present.

Then you have also the nil temperature one can determine at which this extra lattice super lattice line occur, this will give an indication of the nil temperature. Next figure for example, shows a typical case of magnetic structure of a compound called manganese fluoride m and s two where, such an situation exit where the ions occupy both the corners of the cube basic cubic cell unit cell as well as the body centre and therefore, there will be extra super lattice line at the nil temperature

So, there will be this clearly shows that the there are the body centre the ion occupying the body corners and ions occupying the body centre they are anti parallel therefore, there is an anti ferromagnetic alignment and this causes a non-zero Bragg reflection from one 0 0 plane. So, this extra line which is not normally present in the above nil temperature will start appearing below the nil temperature.

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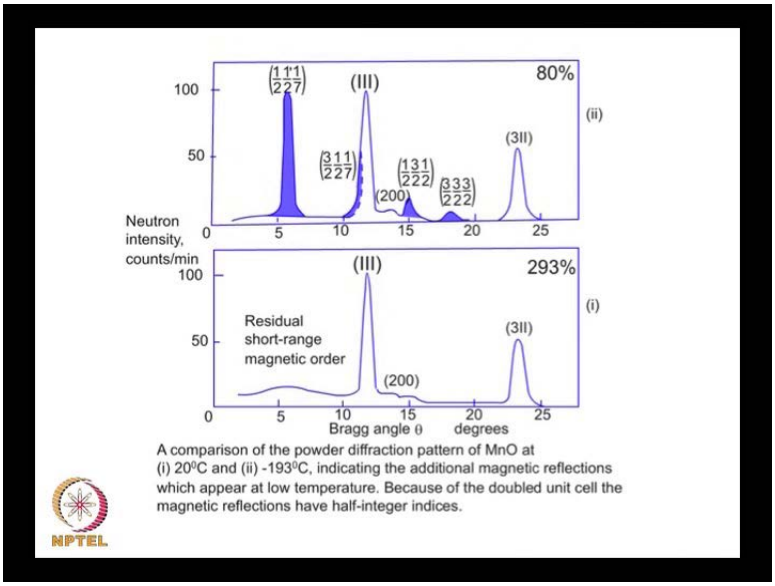

So, one can determine the nil temperature also in this way next figure shows the similar situation in the oxide material manganese oxide, where you have ordering above and below in the similar fashion in this case. Of course, the magnetic reflections are from the 111 planes indicating that corresponding ions on successive 111 plane are oppositely aligned. So, this immediately tells as that the type of magnetic ordering we can get totally different point and space group pattern in the case of magnetic diffraction.

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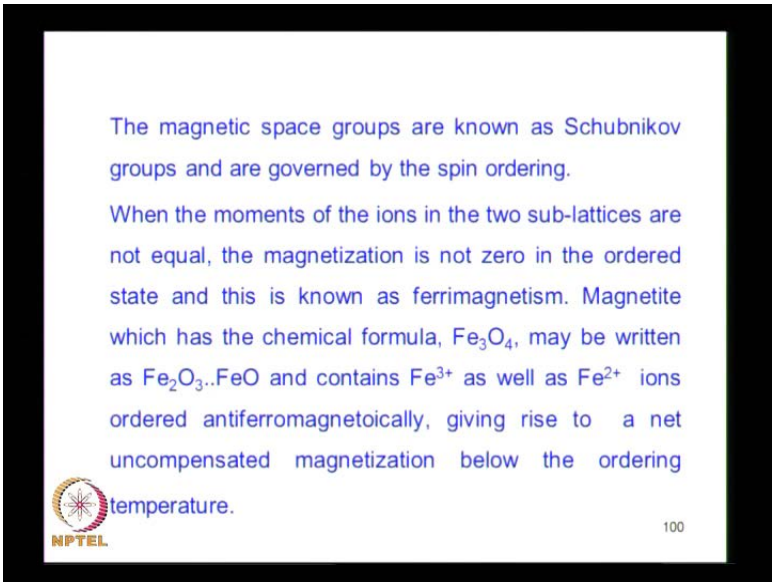

The magnetic space group are known as Schubnikov group and adamant by the spin ordering now we talked about anti ferromagnetism then the movements of the ion in the two sub lattice are exactly equivalent opposite. So, cancel each other there are also situation with the moments of the ions are not the same then opposite, but not equal in which case the magnetization will not be 0 is not completely cancelling each other and therefore, in the ordered state, there is a net magnetization which is non-zero. So, there is a net non-zero magnetization in the ordered state. So, that is known as ferrimagnetism here it is a Fe 2 plus ion and this an Fe 3 plus ion.

So, even though ion is the basic magnetic entity it contains ion in two different balance state 3 plus and 2 plus which corresponds to different magnetic moment. So, when the ion ions Fe 3 plus and Fe 2 plus are aligned anti ferromagnetically, you get a net uncompensated magnetization below the ordering temperature in this case. So, that is why. Hayrides have what is known as spinner structure of magnate tine, and these are very interesting magnetic properties and applications.

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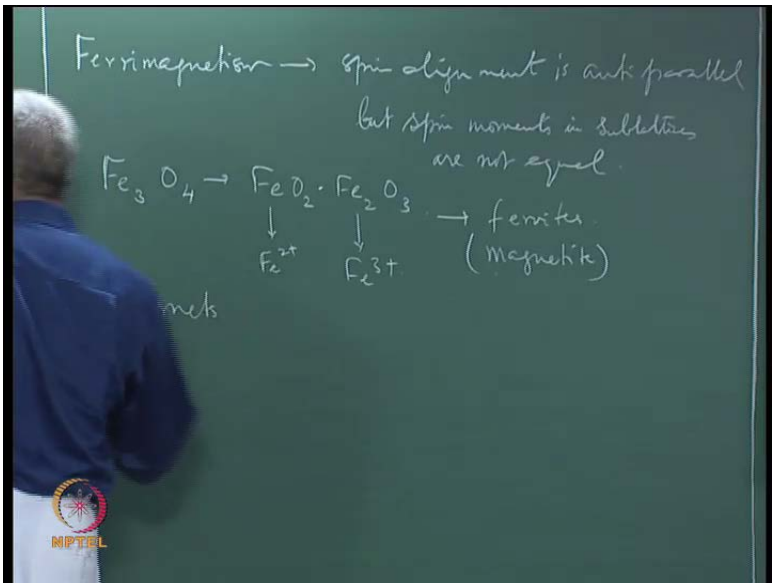

Now, in addition to ferrites these are known as ferrites this member is typically known as magnetite, it is one of the oldest and non-magnetic component it was known as lodestone in ancient times. So, in addition to ferrites you also have what are known as garnets in which the sub lattices are not just two in number, but there are three sub lattices with anti parallel element. And because of the odd number of sub lattice cancellation exact

cancellation and therefore, they also behave like ferrimagnetism in addition to ferro anti ferro and ferrimagnetism, we can also have other kinds of magnetic ordering pattern like helical magnetism which has been discovered by neutron diffraction studies. So, the magnetic ordering mechanism provide a rich variety of phenomena and materials.

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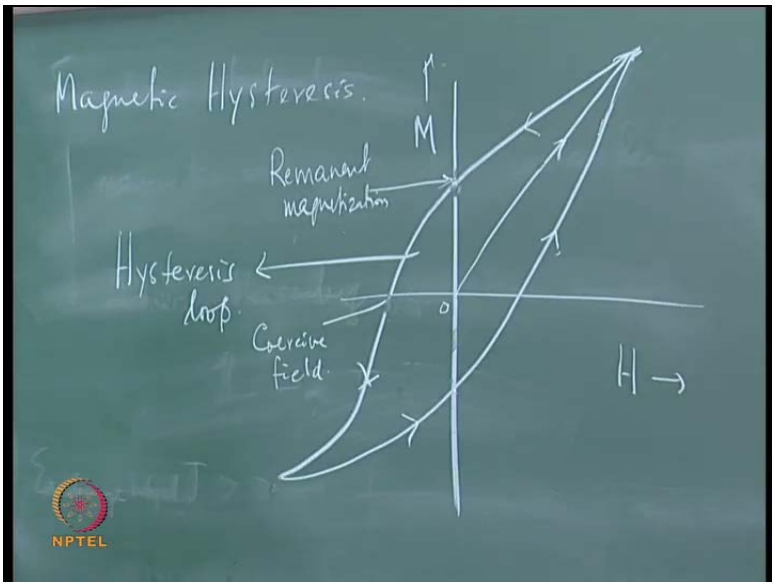

We now turn to another important characteristic of ferromagnetic which is magnetic hysteresis that' a very important characteristics of ferromagnet. Now this is a based on so-called magnetization versus field plot and starting from the origin you go to a point where up to it getting magnetized, and then come here like this and then goes back. So, this is the magnetization pattern the arrow here. So, the direction in which the magnetizing current and the magnetizing field goes. So, you start magnetizing this specimen up to a particular value of the applied field. And then decrease a applied field gradually till you reach 0 magnetization before that through 0 field, and then this is the 0 magnetization point, and then you go in the negative in the equal but opposite direction you go up to this point.

And then you come back increase it again goes through again are 0 field from negative value and then a 0 magnetization and then it goes back to the original. So, that is known as the hysteresis loop in which an initially unmagnified specimen and then it is demagnetized by reducing the field and making the field 0. And then it is demagnetized in the opposite direction and then again the field is increase through 0 value to positive

value till the original state is reached. So, that describes the complete hysteresis loop now this a of hysteresis loop gives an idea of the energy dissipation when the ferromagnetic material is used for example, in the transformer core in order to concentrate the magnetic flux.

So, are and using the Ferro magnet to produce a magnetic field you pass a current and then you get electromagnet in which magnetic field is produced. So, in all these cases the energy needed or stored in the Ferro magnet is given by the area of this hysteresis loop now this field the magnetization when you come back from 0 to 0 field from positive value it is seen that the magnetization does not become 0, there is a remanants magnetization in 0 field. And then when you reduce you need to have a failed in the opposite direction which demagnetize make this remanants magnetization 0.

So, you coercive the field you you need a coercive field, this is known as the coercive field which coercive the magnetization the remanants magnetization to become 0 again. So, these are the two important characteristics the point at which the magnetic field is 0 and then the remanants magnetization is there and the point at which the remanants magnetization is cancelled by a coercive field. So, these two values together determine the area of the hysteresis loop. So, for example, if you have an electromagnet we want a soft iron core for which the b h loop for m h loop is as a small area and it has a relatively small remnant magnetization after demagnetization and that can be removed by a relatively small coercive field.

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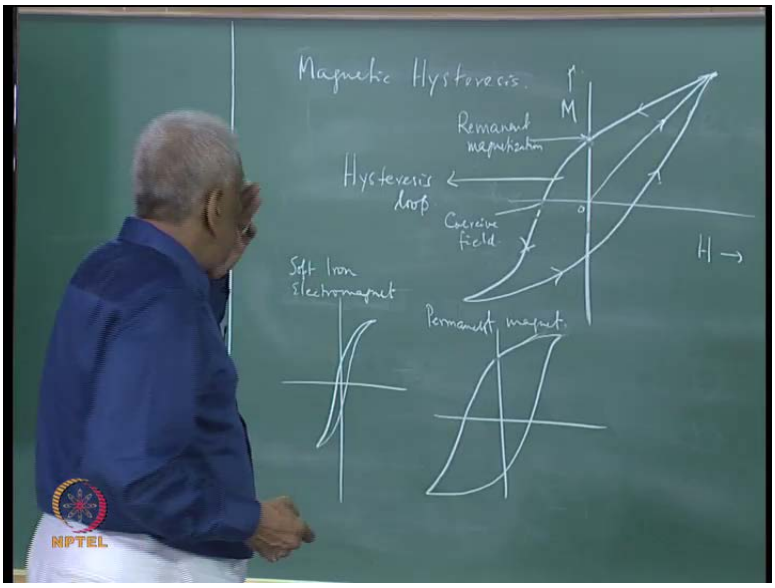

So, the loop will look narrow somewhat like this. So, that is how it will look for a soft iron used in electromagnets because you would like the fields to become 0 as soon as possible you would not want one would not want a large remnant magnetization to stay. And this remnant magnetization is removed by the application of an extremely small magnetic field in the opposite direction on the other hand. If you want a permanent magnet this is used in a electromagnet hysteresis loop is has a very small area whereas, if you have a permanent magnet, if you would like ferromagnet to construct a permanent magnet you want a high remnant field which can be removed only by a large coercive force.

So, that would be the nature of the hysteresis loop for a permanent magnet. You can see that the areas b h loop or m h loop is correspondingly large. Now how do you produce such a high remnant field. And this is achieved by precipitating impurities like carbon atom, which impede the motion of the so-called magnetic domain.

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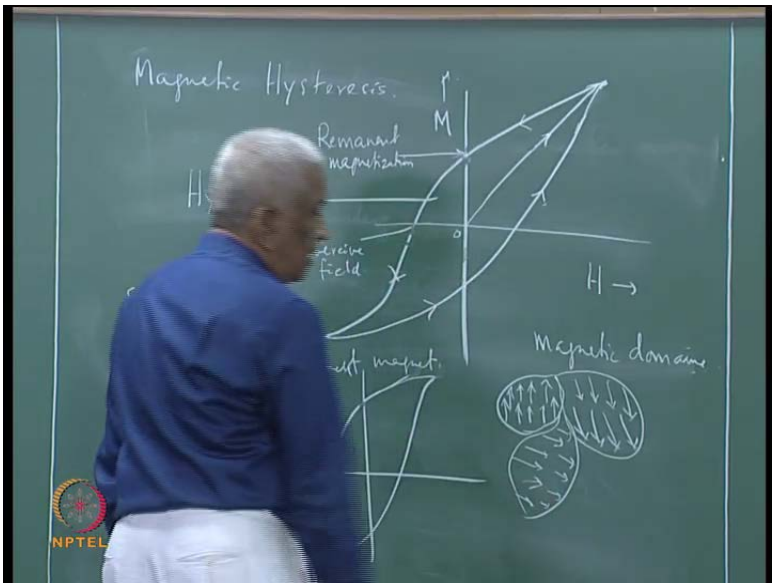

The behavior of a ferromagnet or any ordered magnetic solid is such that there are domains inside in which for example, in a ferromagnet, there is parallel alignment of the spins. And then there is another neighboring magnetic domain in which this may be completely different, but still parallel. So, these two within a domain the alignment is parallel, but in neighboring domain the alignment is not necessarily parallel. So, on the whole you have a large number of magnetic domains within which there is ferromagnetic parallels spin alignment, but between which there is no correlation.

So, on the whole all there is completely random distribution of this magnetic domain for example, a third domain here. So, they are all aligned in completely random fashion and therefore, giving rise to a net net 0 magnetization even though within the domain there is ferromagnetic alignment. Now when you apply a magnetic field these domains are all reoriented parallel to each other reducing our net magnetization now in a permanent magnet material the remnant magnetization exist, because of because these domains are prevented from relaxing back to this random configuration by the inclusion of precipitates such as carbon. So, if you have carbon steel for example, is a very good permanent magnet.

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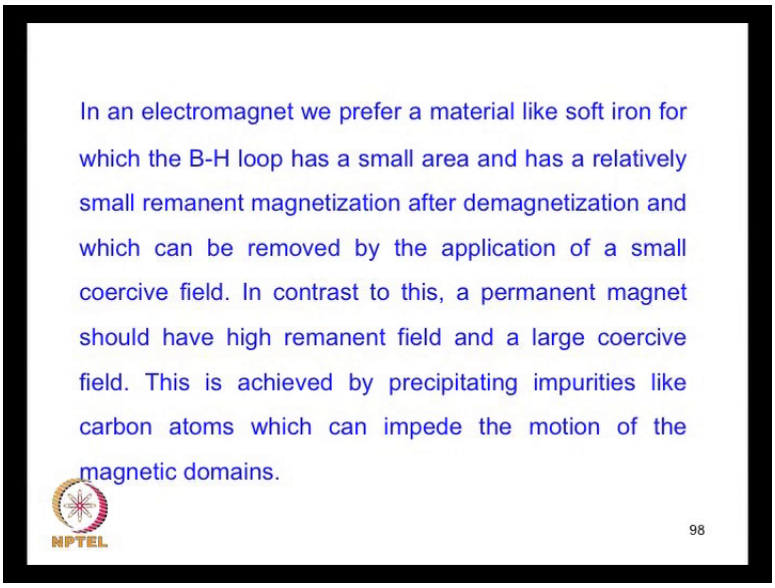

Let us now discuss the ground state of the Heisenberg ferromag net in some detail.

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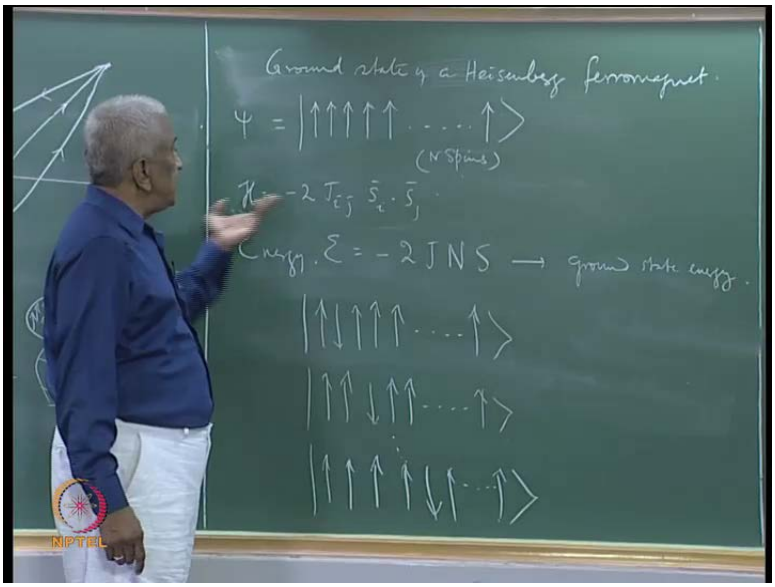

We now know what is Heisenberg a ferromagnet a Heisenberg ferromagnet, he has neighboring spins line the parallel. Now in a given ferromagnet is consist of a very large number ten to the power 23 are. So, number of spins all of them lined a parallel in I really. So, you have a large number of them n spins. So, this is a product of the wave function the of this state is the product of the individual parallel spins functions. So, you

have something like this the total wave function is of this kind up to n. So, the this all of them are of spins.

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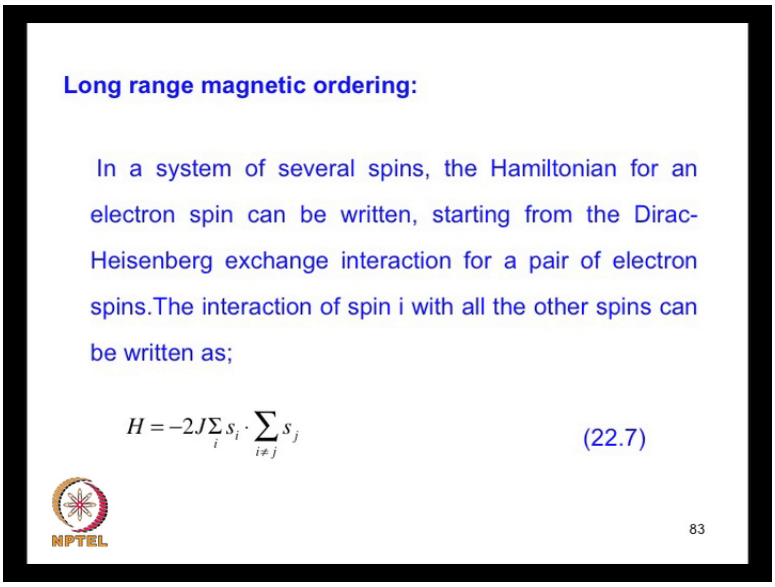

Now, this state will have an Eigen value determined by the Hamiltonian minus J i J s i dot s J that is the Hamiltonian. So, the energy is minus two J assuming that the J i J for all n spins between any spare of spins is the same and has the value J therefore, n times s where n is the number of spins. So, that will be the ground state energy when all the n spins are lined the parallel to each other how do you know what nature of the excitation. Spectrum now we can create a low lying excited state by flipping one of this spins.

So, that is the easiest things to do I can flip this spin anti parallel and while keeping all the others same that would be a typical situation, this flipping can be here or it can be here or it can be any any of this combination of any one of the n spin can anti parallel. So, all these are equivalent in which anyone spin is tip down. So, all these are energetically equivalent. So, they are degenerate.

So, the wave function is the typical wave function is of this form, where the spin can be a anywhere in any one of the j'th J sites where J runs from one to n, but this will not be very, very easily you can see that using this Hamiltonian on such a wave function. You can show that this is not a Eigen state of the Hamiltonian heisenberg hamiltonian. So, we have to construct an Eigen. State how do you construct by super posing all this various possibilities. So, make their special path.

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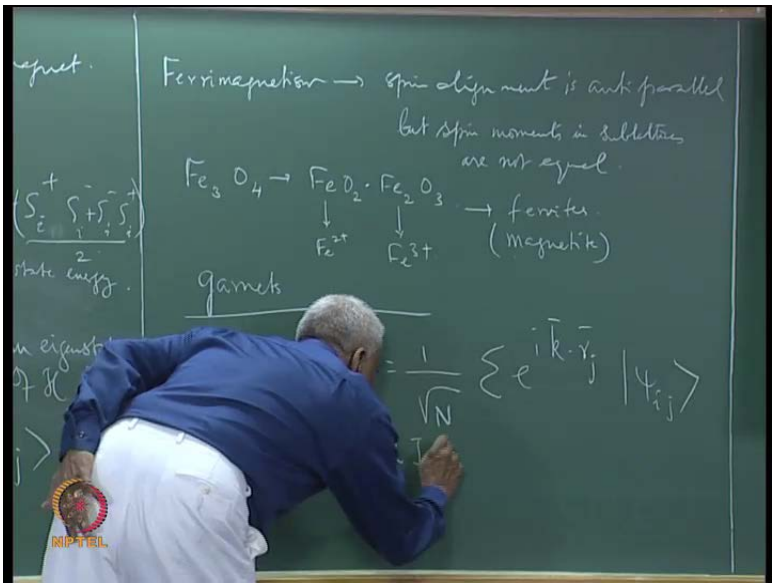

So, the real superposition where there are of the form 1 by root 10 for normalization sigma e to the power i k dot r k dot r J si i J where si i J is any one of them.

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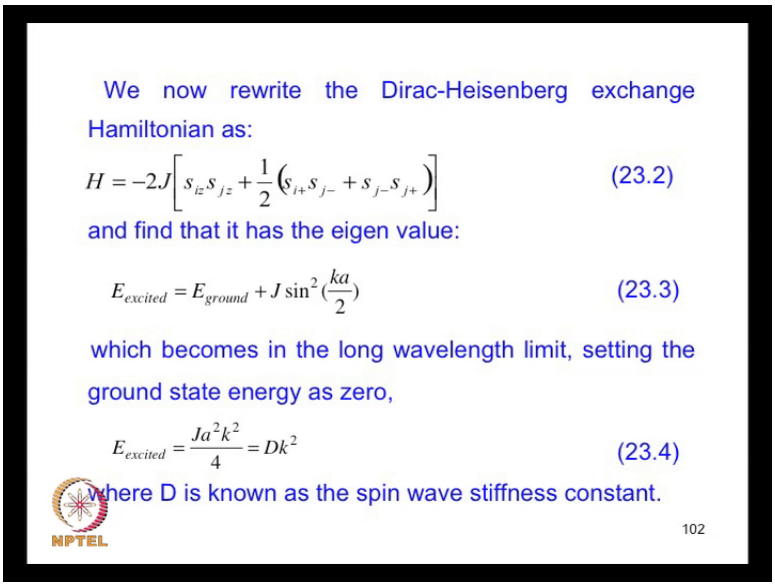

This can be shown such a superposition can be shown to have a an Eigen state it should to be an Eigen state, because we can rewrite this as minus 2 J s i z s J z plus s i plus s i minus plus s i minus s i plus by 2. So, we can rewrite like this and show that the Eigen value in of energy for such a state for this excited state, is it lies about the ground state by a value minus 2 J sin square in the long wavelength limit sin square k a is

approximately equal to k a square. So, in the long. Delta e is J sin square k a, and therefore since k a is 2 phi by lambda the wave number the long wavelength limit if lambda is very long k tends to 0.

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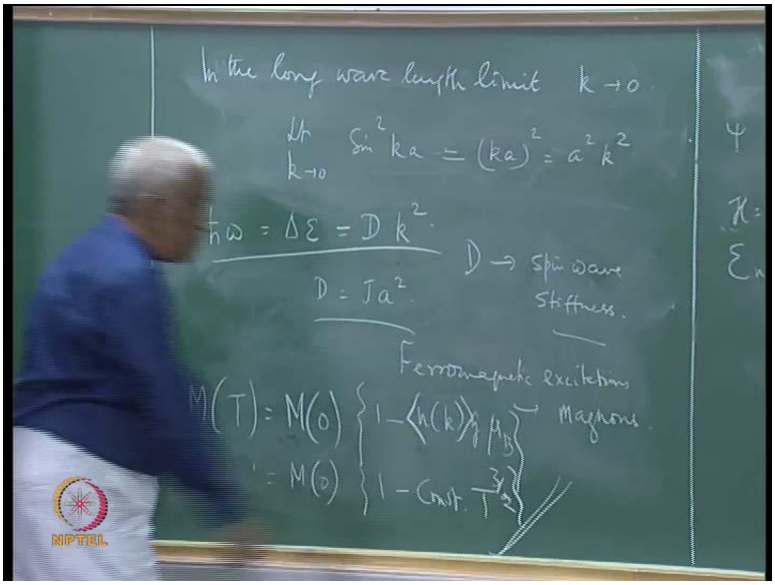

So, the limit k tends to 0 sin square k a is just k a square for equal to a square k square therefore, the energy difference between the ground state, and first excited state of Heisenberg ferromagnet becomes d k square. So, d is known as the spin wave stiffness and this gives you h cross omega. So, the omega versus k relationship is a quadratic function omega is a quadratic function of k. So, this is the dispersion relationship we have already seen how phonon have a liner dispersion relationship in the in the long wavelength element that is at the centre of the Brillivan zone.

Now we have spin wave dispersion equation in which the omega is a quadratic function not a linear function. So, we have quadratic dispersion relation for the ferromagnetic excitation these excitation are known as are called magnons there are known as magnons and this is known as the magnons dispersion relationship like phonon magnons are the elementary excitation, which described the spin wave modes which have wave function of this form given here. So, these are just the normal modes of the propagation of the so-called the spin deviation in this case the spin deviates by flipping one spin. So, it become less by one it is s minus one. So, spin deviation is one. So, it is the spin deviation which propagates through this crystal lattice, because of the flipping of the various spins.

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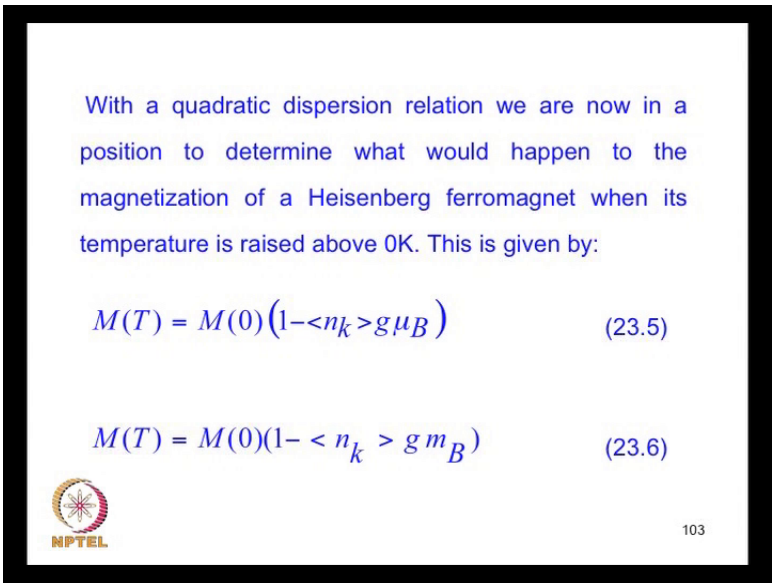

So, this is the low-lying excitation whose excitation spectrum is given by this. How do we know that such an excitation spectrum is there we know it by measuring the magnetization. So, suppose we have Heisenberg ferromagnet, and we raise its temperature about 0 k at absolute 0 all spins are lined a parallel and the ferromagnet is in its ground state which we take as the 0 of energy. Now, we slowly warm up the Heisenberg ferromagnet about 0 k and then we get a magnetization which is a function of temperature this is describable in the terms of m 0 times one minus n of k times g mu b where n of k is the number of magnons of wave vector k. So, this is the 0 temperature magnetization corresponding to all spins lying parallel, and then out of this magnetization the number.

If there are N number of excitation each corresponding to a spin deviation of one corresponding magnetic moment G mu B. Therefore, the average value of N of k it is the statistical average times G mu B gives you the magnetic moment, which will be decrease from the ground state magnetization. Because of the excitation of n spins from the ground state into the excited state with the spin deviation of one we can use the Magnons like phonon obey both Einstein statistics. And so they there are number can be determined in exactly the same way as we did for the phonon.

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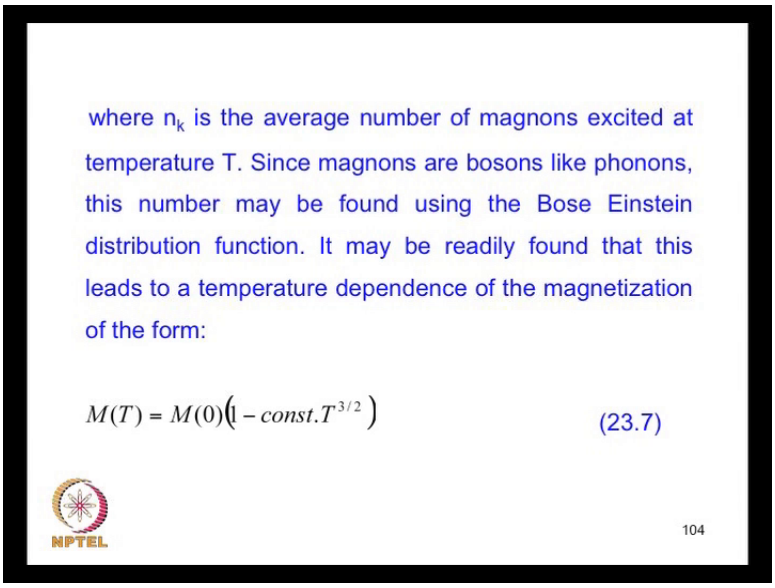

And therefore this can be calculated and one can find easily that this m magnetization constant times T to the power 3 by 2. So, this means that the temperature dependence of the magnetization at any temperature t is given by a relationship of this form. So, the magnetization follows a T to the power 3 by 2 dependents in temperature in the same way we can also calculate the magnetic specific heat.

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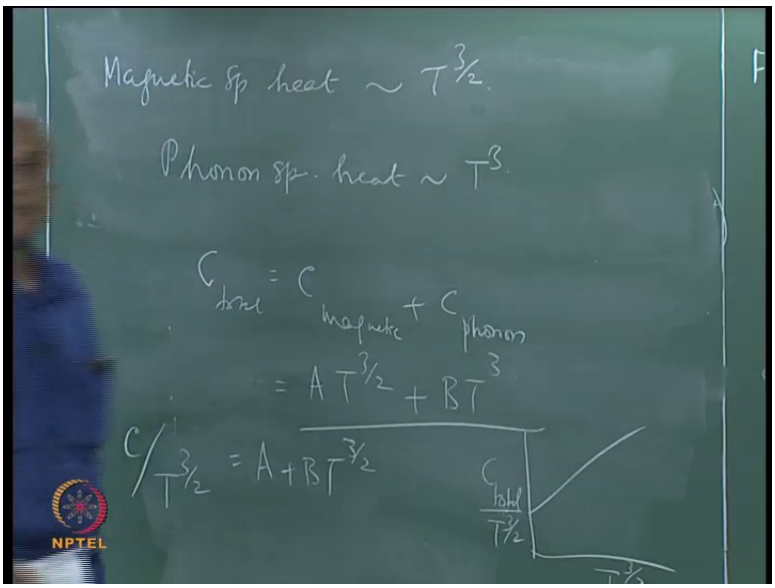

So, the magnetic specific heat also has a similar t to the power three by two it can be calculated using same kind of statistical approach. So, if we can separate it from the

phonon specific heat the phonon specific heat if we recall is given by the d by t cube large low temperature. So, this goes as T cube while this goes as t to the power 3 by 2. So, you have two terms c total is c magnetic plus c phonon for ferromagnetic insulators which behaves like Heisenberg ferromagnet, this has two terms; one A T to the power 3 by 2 plus another term which is B T cube. So, we can separate out these by taking c by t to the power 3 by 2 is a plus b t to the power 3 by 2.

So, if we plot the total specific heat divided by t to the power 3 by 2 versus t to the power 3 by 2 we get this straight line. So, that is the signature of a Heisenberg ferromagnet in the same way we can make similar calculation for anti ferromagnet it and. So, on I would not like to go into this detail, but there are many such method by which you can experimentally determine the nature of the dispersion relationship and their hands nature of the temperature dependence which follows from it.