Condensed Matter Physics
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Lecture - 19
Dia- and Paramagnetism

Today we will start an important section of Condensed Matter meant physics namely magnetism. We have just finished discussing the mechanism of dielectric polarization in dielectric materials, how an applied electric field polarizes a dielectric medium and produces a polarization with various effects such as dispersion, phase transition into the ferroelectric field and things like that. We will, now go on a very similar process which takes place in magnetic materials. In the case of magnetism instead of a dielectric or electric polarization, we will have a magnetization, which is caused by an applied magnetic field. So, the analogy is very clear just as the dielectric polarization is established by an applied electric field a magnetization is produced in a magnetic material by an applied magnetic field. The process by which this magnetization is established goes on lines, which is very similar to that of electric polarization. Of course, there are very significant and important fundamental differences also.

So, in the course of this lecture, and in few subsequent lectures, we will be discussing some these processes and mechanisms. We start with magnetism is a well-known phenomenon of nature which has attracted the attention of people from time immemorial, immemorial. We know that people have talked about load stones in Sanskrit, people have talk quietly they have talked about [FL], so they have talked about how a magnate attracts iron. Similarly, people talked about lodestone, mariners people who go voyages in the sea have uses the magnetic compass to know the direction in the sea. So, these are all known for a very, very long time since ancient times.

So, magnetism is a phenomenon which is very well known even Aryabhata has talked about magnetism. So, the phenomenon of magnetism occupies a central position in condensed matter physics. There are reasons for this, not because magnetism in particular is a special topic. It is similar to many other topics, but the entities which produce magnetization which magnetize a material namely we are known as spins, magnetic spins, spin angular momentum, it is something which is new to classical physics. It is a totally a new degree of freedom in the case of electronic atomic and molecular system. But the spins are probably the cleanness to physical entity treat theoretically as well as experimentally. The spins give rise to magnetic effects and can be studied by a variety of techniques and can be described theoretically we have very considerably extends with great success. Now all magnetism mainly arises from the electrons of the atoms and molecules; of course, the nuclei also produce magnetic effects, we will discuss this later. But for recent which will become clear later on magnetism as a phenomenon is mainly due to the electrons in atoms and molecules.

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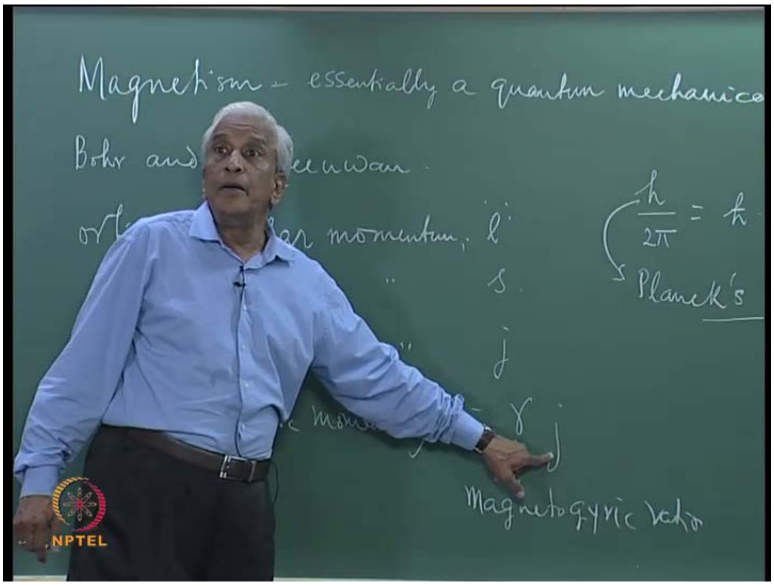

There is no classical way of describing magnetism. In fact, there is a theorem by there is theorem due to Bohr and von Leeuwen if states that if you applied classical statistical mechanical considerations to an assembly of electrons which produces magnetism you will find that the average magnetization vanishes. So, you cannot account for any magnetism using classical theories. Magnetism is essentially a quantum mechanical phenomenon. So, this is the first point to be understood that in order to understand magnetism, we have to learn quantum mechanics or at least be familiar with the fundamental principles of quantum physics, because we are talking about the orbital motion of electrons as well as spin of electrons.

Spin is a completely quantum mechanical effect, relativistic quantum mechanical effect. Even the orbital motion in a finite sample the macroscopic magnetic moment produced by the orbital angular moment of electrons vanishes identically by Bohr and von leeuwan theorem. Therefore, in order know how this magnetism arises and in order to quantitatively, describe the behavior of this magnetic moment one has to apply quantum mechanics and quantum statistical physical concepts. So, this is the first point be realized next we will ask how there is the magnetic moment is produced for an electron. In the case of an electron, we all know that the electron orbiting around the nucleus of an atom and similarly in the case of a molecule there are molecular orbital's in which the electron is oppose to the moving.

So, the cause of this motion this orbit to the motion whenever the charge is moving a current is produced in that loopso this orbit is like a current carry in loop and the current carry in loop always produces a magnetic moment as orsted describe first discovered. So, we know that the orbital motion it is very easy to see that the orbital motion of an electron will produced magnetic field because the orbit of an electron around the nucleus serves as a closed current loop in which the current don not vanish.

So, this is theoretically described by saying that the orbiting electron has an orbital angular momentum which is usually denoted by the letter l in books on quantum mechanics or j this is orbital, but usually there is also a spin angle of momentum which we will discuss later and that is represented by the letter s and. So, there is a total angle of momentum disguise as by the letter represented by the letter j. So, all these angular momentum are quantized according to the rules of quantum mechanics in other words the orbital angle of momentum can only be in units an integral multiples of a fundamental unit which is you fundamental usually the fundamental unit of angle of momentum is h by two pi or h cross where h is the Planck's constant.

So, the angle of momentum is measure as j h cross when we say that the angular momentum is j what we mean is the angular momentum is j h by two pi if this is. So, then quantum mechanics cause or tell us that there is a corresponding magnetic moment due to this angular momentum which is represented usually by the letter mu and that is proportional to this angular momentum orbital angular momentum. So, mu equal to gamma j where gamma is known as the magneto gyric ratio. In other words, it is the ratio of the magnetic moment to the gyro or angular momentum in world or books this should have been written as gyro magnetic ratio gamma is the magneto gyric ratio. So, this only means that the magnetic moment vector both mu and j are vector quantities.

So, the magnetic moment is parallel to the orbital angular momentum or the total angular momentum. So, this is what this expression says you would like to understand how this magnetic moment arises as I said this has been to be done in the framework of quantum mechanics.

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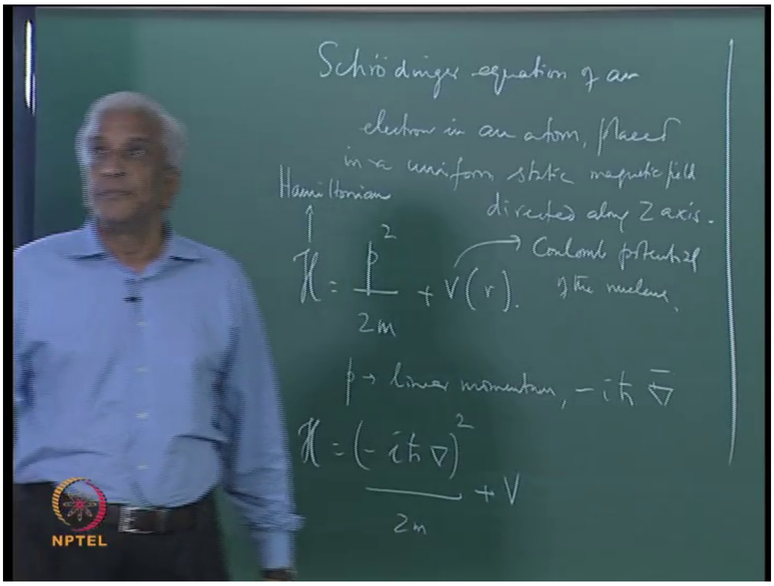

So, let us do some basic quantum mechanics by writing the Schrodinger equation of an electron in an atom placed the magnetic field in a uniform magnetic field uniform static magnetic field whose direction is taken as the z direction. So, how do we write the equation the Schrodinger equation for such an electron we know that this electron is in the potential of the nucleus. So, we have the standard way in the absence of an applied magnetic field the electron Hamiltonian this is the Hamiltonian which is the starting point of writing the Schrodinger equation is just equal to p square by two m plus b f I where p is the linear momentum.

And in quantum mechanics p is an operator which is represented by minus I h cross tell m is the mass of the electron b f r is the coulomb potential of the nucleus it is written as b f for because we know that the nucleus exerts a centre central coulomb potential. And therefore, it is function only the distance between the electron and the nucleus.

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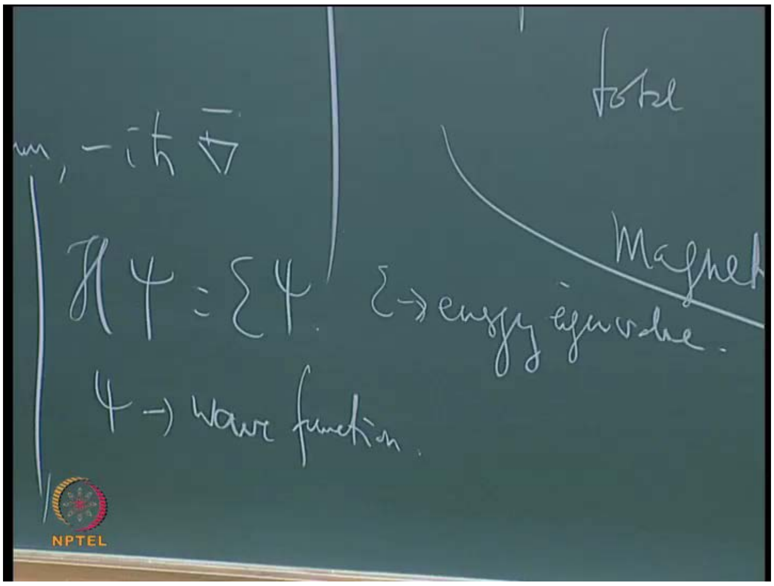

So, this gives me the Hamiltonian has minus I h cross del square by two m plus b and the Schrodinger equation h psi is just h psi equal e psi where psi is the wave function and e is the energy eigen value this Hamiltonian gets modified when an magnetic field is applied again quantum theory tells us that.

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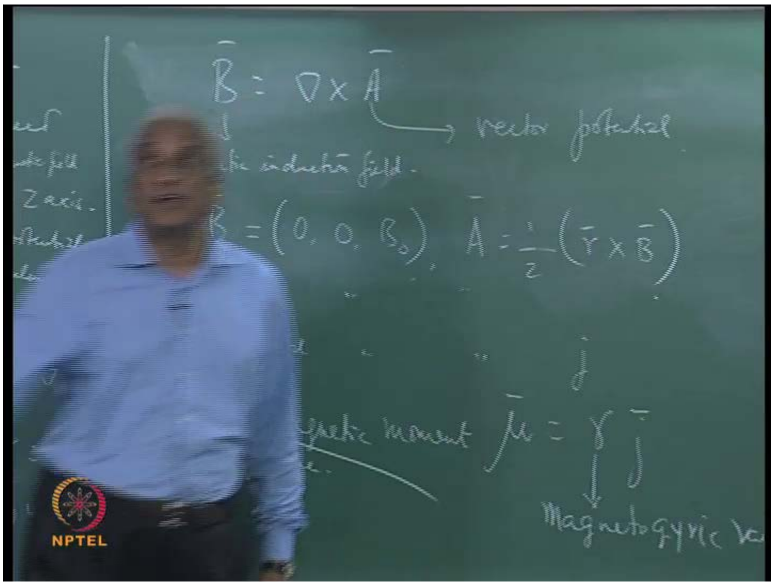

In fact, these an electro dynamic result that when there is an applied magnetic field this magnetic field has an induction which is given by a vector potential a. So, this is the magnetic induction field and a is the vector potential and b o and a are related in this form, so if b is constant and uniform directed along is that direction we can write between this form and if this is. So, we can choose the the form of the vector potential which will give you a b like this can be chosen as half r cross b this is the standard result from electro dynamics which can be written in the form.

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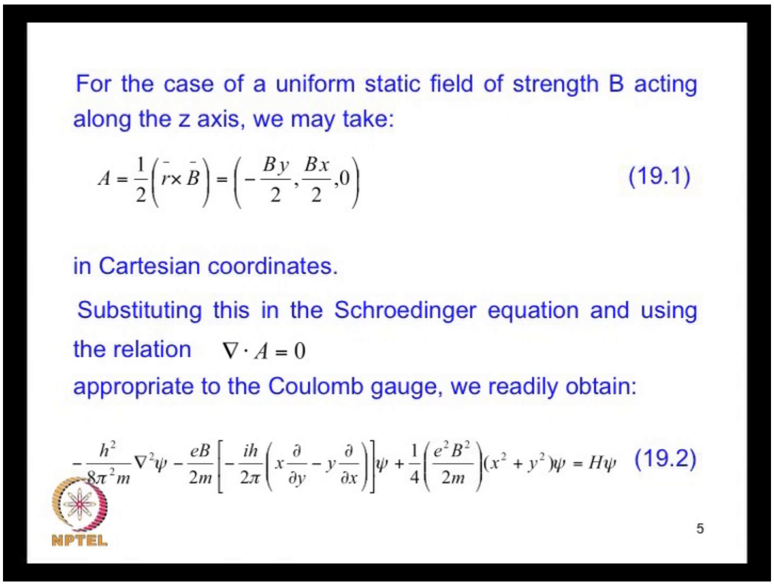

So, these are the components of the vector potential in Cartesian coordinates, now when such a magnetic field is applied the Hamiltonian gets modified in the following manner.

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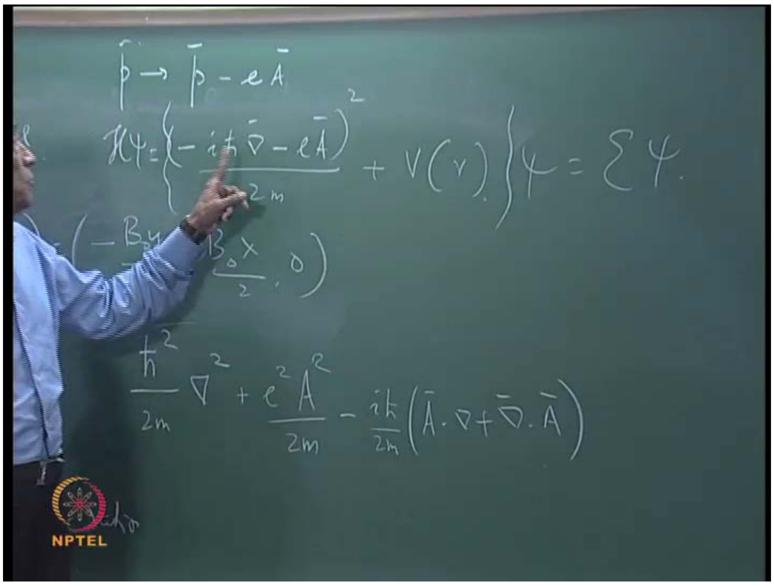

If p is the operator p is replaced by the operator p minus e A. So, that is the result of classical physics. So, p goes to p minus e A and therefore, the Hamiltonian in a magnetic field is written as minus i h cross del minus e A by whole square by two m plus v of r. So, this is the Hamiltonian and therefore, h psi equals and this is the equal to e psi that is the Schrodinger equation. And we can now replace A by these component, and rewrite this. When we do this do this we get h cross square by two m del square, that is the first term from squaring this. And then we have another term plus e square A square by two m and then we have cross term minus i h cross by two m into a dot del plus del dot A. We cannot just write two del dot A, we have keep the sequences separate.

So, we have two such term one from this del dot A, another from A dot del. Then we have v of r. So, i z equals psi where we can now substitute for A square and A. So, that is the Schrodinger equation which as to be solve in order to find the eigen values of energy and that is how we can find the magnetic moment. So, if we do this arrive at this result we arrive at this result we have the minus h cross to a del square by two m which is the kinetic energy term that is the kinetic energy term of the electron. Then we have minus i h cross by two m and then we have a dot del plus del dot a in order to a go further we choose the gauge we has the freedom to choose the gauge in which we work and we choose the coulomb gauge in which del dot a equal zero. If we apply this condition, we can show the a dot del and del dot a become the same.

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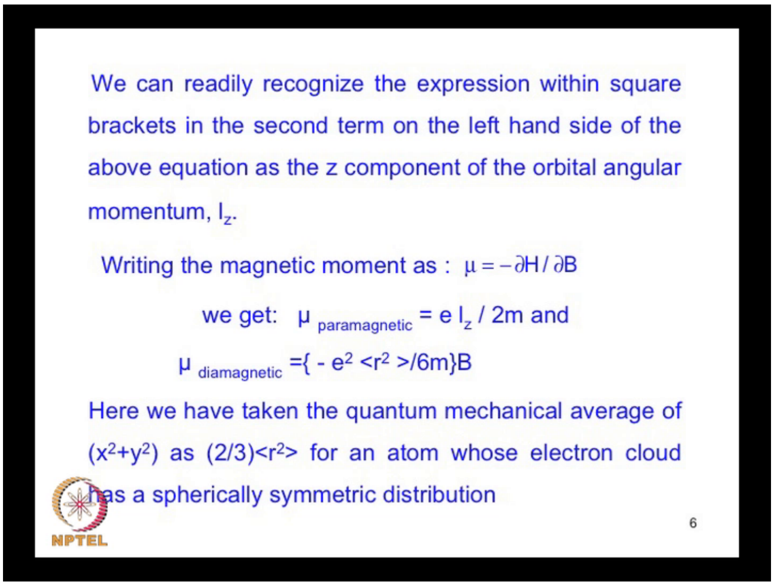

So, that this becomes then we have well this becomes plus and then this because has a minus sign, where we can now replace a square a square is nothing, but we have there b naught square by four into x square plus y square is a square. And similarly for A, we have those results there. So, A dot del will simply become minus b naught by two into minus y d by d x plus x d by d y and remembering that minus I h cross del is the moment operator this can be written as where p x p y are the components of p x and y components. So, this is nothing, but the z component of the orbital angular momentum l z if l is r cross p which is the definition of arbitral angular momentum l z will be plus x p y minus y p x therefore, the Schrodinger equation can now b rewritten a very simple form.

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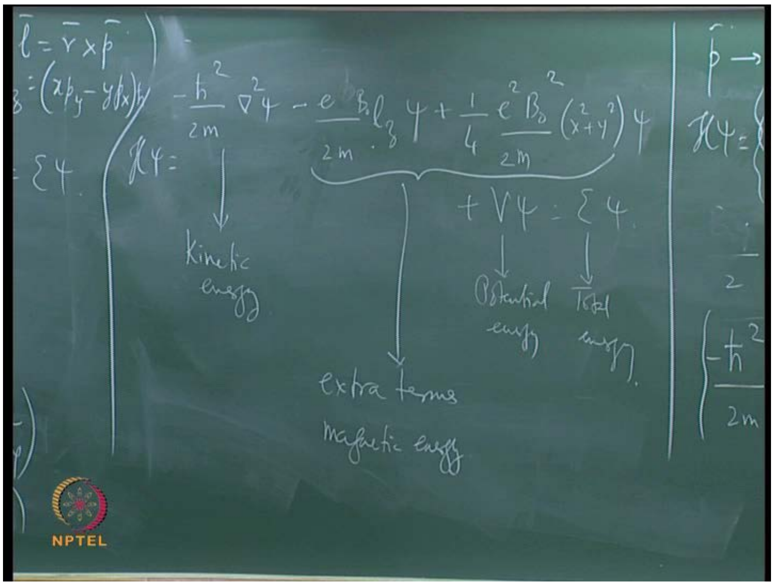

I have forgotten e here. So, e h cross by m into l z psi plus one four e square b naught square by m two m into x square plus y square psi plus b psi equals psi. Now in we cannot need not a and solve this Schrodinger equation to understand what this mean we can just look at it by inspection we can interpret the various terms in this energy Hamiltonian is nothing, but the energy. So, what is the Hamiltonian operator this term is the kinetic energy and this is the potential energy and the Schrodinger equation simply says this plus this plus these term is equal to the total energy. Obviously, these are the two-terms which arise which contribute to the energy on account of the application of the static magnetic field study magnetic field.

So, this means we have forgotten b naught by two right. So, this h cross is observed in a l zl z is really h cross, because l z is measured in units of h cross. So, this is these are the two terms extra terms which represent a magnetic energy the additional contribution to energy from the application of the magnetic field one of them is negative, another is positive. That is the really reveals what do you mean by a negative energy and a positive energy for this we have to just go back to classical electro dynamics.

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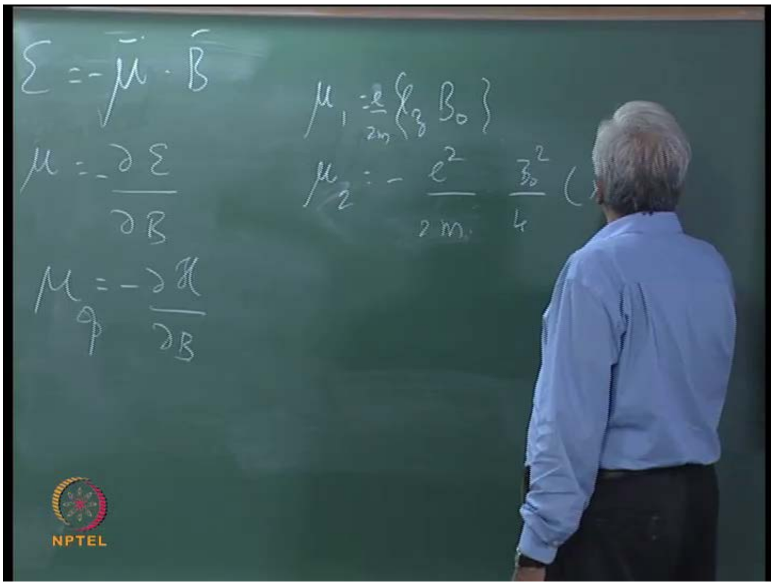

And see what happens when a dipole of magnetic moment mu displays at an applied magnetic field then the energy is just therefore, we know that the magnetic movement is just nothing, but b e minus b e by e b where. So, by looking at the Hamiltonian operator if you write it in a and operator form this is just mu operator this minus b h by d b. So, if you look at these two terms in the Hamiltonian that has looked what we get the negative term gives me mu one is l z b naught that is the contribution. Now similarly the other terms is going to give me this is going to give me minus e square del there is A one by two m and e by two m. And here it is e square by four into well it should be to have square right as we will see about that this square by two m into one by four b naught square into h square plus y square.

So, these are negative term a negative magnetic moment whereas, this is a positive moment and what is interpretation of this, whenever we have a magnetic field magnetizing a material. If you get a magnetic moment which is parallel are in the same direction as the applied magnetic field we say that the material is paramagnetic a paramagnetic material is only one in which the magnetic movement is parallel to magnetic field whereas, the magnetic moment is anti parallel then we call it a diamagnetic material. So, we have two cases here one giving a magnetic moment is parallel to the applied magnetic field which is also directed along the z direction. And the other gives a term which contributes to a magnetic moment which is anti parallel to the magnetic field because this is a positive definite everything is a square and therefore, you have a negative sign.

So, it gives a negative magnetic moment which indicates that the magnetic moment is anti parallel to the applied magnetic field. So, this is a paramagnetic term this is a diamagnetic term. So, by simply looking at the Hamiltonian of the electron an a applied magnetic field and looking at the various terms in the presence of the field here in a position to say that there are two new contributions to the energy arising from the application of the magnetic field one of these contributions is a paramagnetic contribution. And the other one is a diamagnetic contribution the paramagnetic contribution comes from the alignment of the angular momentum vector in the direction of the applied magnetic field.

So, just like in the case of the dielectric polarization these are that there was an electric dipole which gets lined up in the direction of the applied electric field creating dielectric polarization. In the same way, here we have an electron magnetic moment which is like a dipolar magnetic moment, there is no monopole in nature this lowest order magnetic moment is that of a dipole. Therefore, the electronic dipole lines itself in the direction of the applied magnetic field and gives rise a paramagnetic contribution which as this form and this is the gyro magneto gyric ratio e by m is the magneto gyric ratio.

So, this is equal to gamma and in addition the orbital motion of the electron because the electron is orbiting round this as a classical explanation. So, if you have an orbiting current loop a close current loop in which as charge is circulating this current loop when an applied magnetic field is produced A by lends as law there is a back e m f induced when therefore, there is a resistance to this motion. And this change is changes the acceleration this entry plate is the acceleration of the electron in its orbital motion and therefore, this produces the change in the angular momentum. And therefore, induces a magnetic moment which is of diamagnetic now this is of course, for an individual electron if you have a macro scope example in which there are a large number of ten to the twenty three are. So, of atoms are molecule each have which contain several electron then the makeup quantum mechanical and statistical average now for this the average is x square plus y square average. So, the average magnetic moment quantum mechanical average is just this this these bracket represent average. So, the quantum-mechanical average can be easily figured out.

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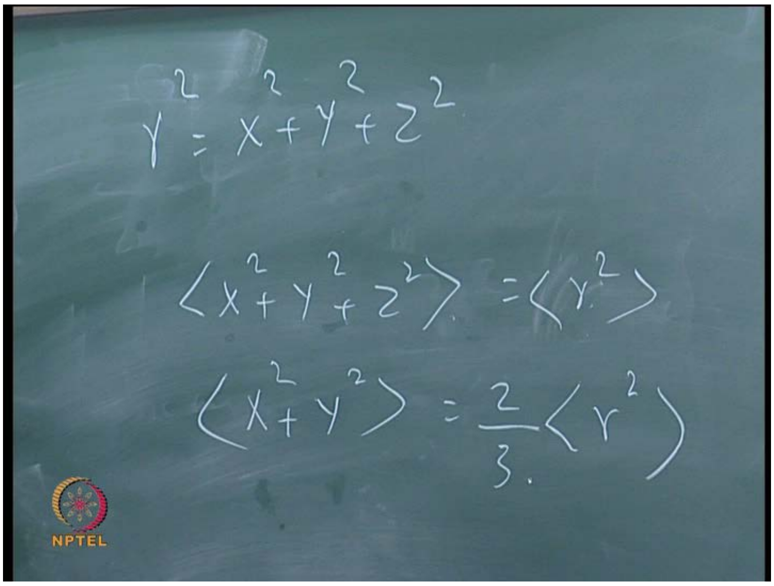

Because we known that in Cartesian coordinates r square is x square plus y square plus z square, where x y z are the compliments of r for any general direction position vector r. Therefore, if you take the average with all three directions x y z are equally probable the quantum mechanical average of this x square plus y square plus z square is. So, substituting this we can calculate the diamagnetic moment using this result.

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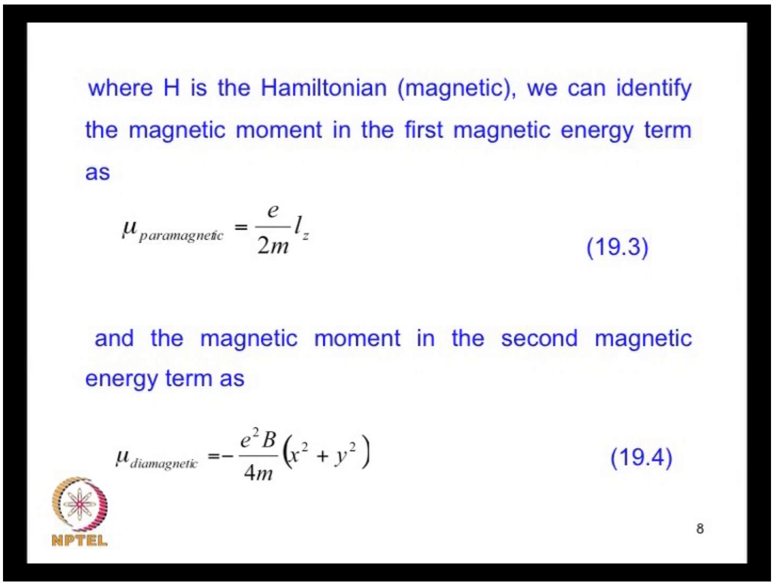

So, now we go to the magnetic moment terms and look at them closely. So, we have the paramagnetic moment.

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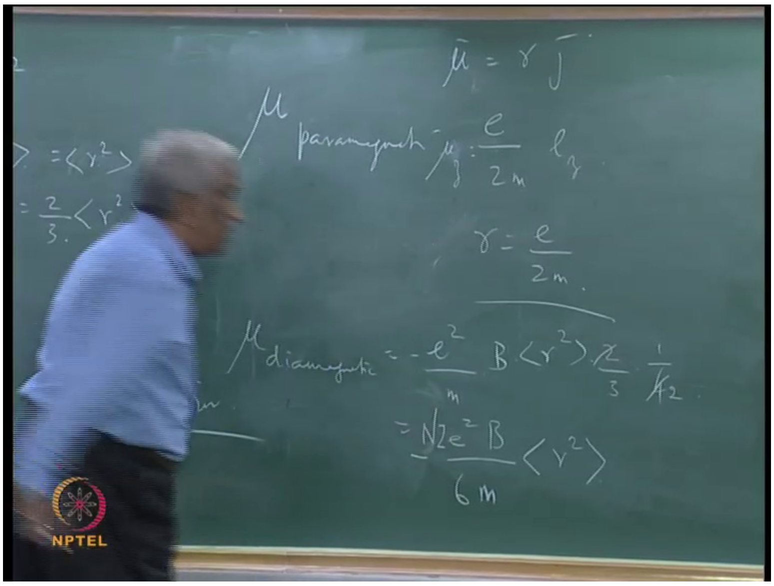

This is the paramagnetic energy the energy due to this because this is the Hamiltonian operator. So, the magnetic moment is just got by differentiating and this will will remove this because they are differentiated the linear term in the Hamiltonian. So, the b goes off. So, we simply have paramagnetic moment is e by two m into l z. So, that is mu z since it is directed along is that direction. So, this is the z component on the paramagnetic moment and that is why comparing it with our earlier equation mu equal to gamma j. We arrive at the result that gamma the magneto gyric ratio is just e by two m and the case as the diamagnetic term the result the corresponding result is e square by two m into b square will give you another two to be. So, b times r square into two by three into one by four which is one by six. So, this will give me minus e square b bb by six m times r square where r square is the quantum-mechanical average of the square o f the orbital radios of the electron since this is if the electron cloud spherically symmetric. So, this is the basic theory due to Langevin theory and this gives you the diamagnetic moment this is the contribution to the diamagnetic moment give to one orbiting electron if there are z orbiting electron this as to be multiplied by z if there are n atoms are molecule then it has to be further multiplied by n.

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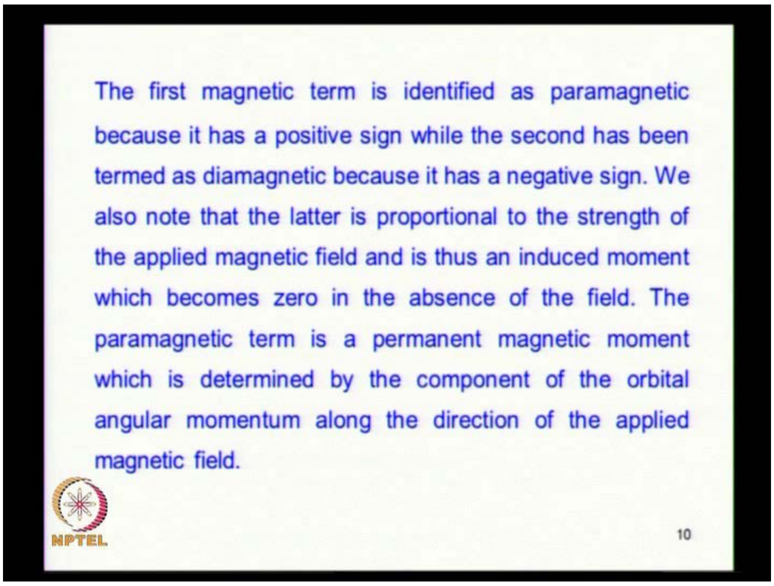

So, will give you the total diamagnetic moment in an assembly n atom each containing z electrons, so this is the basic theory due to langevin, which explains the diamagnetic moment which as a negative sign. The all matter which contain atoms are molecules and therefore, orbiting electrons around nuclei or molecular centers because of that every material is basically diamagnetic. There is a diamagnetism associated with every atom for molecule in nature there is no material which is not diamagnetic this diamagnetism is present only when there is a field if the field is removed it vanishes.

So, this is an induce effect diamagnetism is induced and is a reaction to the the applied magnetic field which states only as long as the applied magnetic field exists. So, the diamagnetic moment vanishes when up field is removed whereas, the paramagnetic term is a permanent magnetic moment which is determined by the component of the orbital angle of momentum along the direction of the applied field.

So, this situation is very similar to the polarizability of an atom in the presence of an applied electric field in the case of dielectric polarization this polarizability is zero once the field is applied electric field become is removed. Similarly, the polarizability as well as the diamagnetic moment are induced effects whereas, there are permanent dipoles electric dipole like water in the case of dielectric polarization. These dipoles get aligned along the electric field giving rise to a polar depolarization polarization associated with a polar nature of such a dielectric. In the same way, we have materials, which can become paramagnetic because there is there are there is an angle of momentum, and associated with this angle momentum there is a dipole magnetic magnet and these dipole lines up in the direction of the magnetic field. And therefore, this is parallel to the applied magnetic field the energy proper contribution is negative because the energy is minus mu dot b the paramagnetic moment is present all the time for a material and this will be their only if the orbital angular momentum is not zero. So, these are the main difference.

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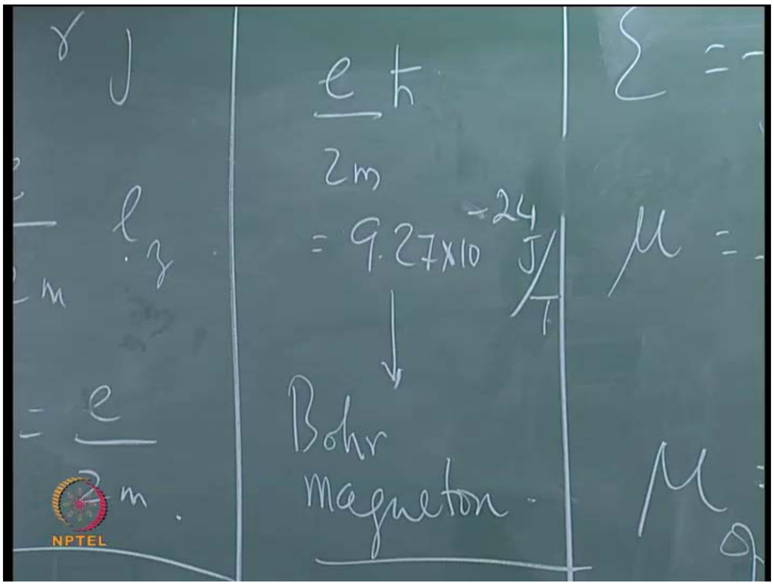

Now, this e by two m e by two m and as I already told you told you l z is an integer times h cross. So, e h cross by two m this as a value 9.27 into 10 to the power minus 24 joules per tesla. So, this has a special name that is the unit of magnetic moment in quantum mechanics in all electronic material. Therefore, this as a special name it is called the Bohr magnetron. So, we measure magnetic moments of electron in units of the Bohr magnetron. Now this immediately tells us why the the nuclear contribution is not very important in the case of magnetism, because of the presence of the mass term here. So, the nuclear mass is 2000 times more when that of the electron. So, the nuclear magnetic moment is going to be weaker by that factor. So, it is three orders weaker therefore, the nuclear magnetism is not seen easily. The thing that is seen generally is the electronic magnetic moment. We will stop at this point, and continue next time.