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Lecture - 18 Optical Properties of Metals; Ionic Polarization in Alkali Halides; Piezoelectricity

(Refer Slide Time: 00:21)

is frequency dependent = $E(\omega)$
 $E=m^2$.

We have discussed the mechanism by which the dielectric constant of an insulator becomes frequency dependent. So, we represent this by the function - the dielectric function epsilon omega. We also noted that this epsilon since this is equal to this square as a refractive index, so this frequency dependence also leads to a frequency dependence of the refractive index. In particular, this leads to the phenomenon of dispersion and absorption of electromagnetic waves when propagating through a medium, which is characterized by this dielectric function epsilon omega. So, this is related to the real part of the dielectric constant, while absorption is related to the imaginary part which in turn leads to an absorption. So, we discuss the theory of this in a dielectric media.

(Refer Slide Time: 02:36)

Just by way of digression, we will just read up back to the case of a metal which we have been discussing in some detail earlier and the incidentally want to see how this behavior this dispersive and absorptive nature is reflected in the properties of metals, metals as you know are highly conducting media. So, the question is, whether there will be any change, any difference in the optical behavior of metals in the response to light, because these are all know it is a matter of common experience that metals are highly reflective in contrast to insulate existences which are not so. We would like to see whether this treatment of dispersion and absorption can lead to a detailed understanding of the optical behavior of metals. Now one thing we know is that the metals have a very large concentration of conduction electrons.

(Refer Slide Time: 03:46)

Collective hetrovoir of conduction electrons
Skin effect
Skin depth: $S = \sqrt{\frac{2}{\mu \sigma \omega}}$.

And therefore, the main properties of metals are usually understood in terms of the collective behavior of conduction electrons. In particular, one remembers there is a phenomenon called skin effect, which is characteristic of metals and this result in the electrons being able to penetrate the metal only up to a short distance named our skin layer or skin depth. And this depth is usually given by the root of 2 by mu sigma omega; mu is the permeability, sigma is the conductivity and omega is the angular frequency.

(Refer Slide Time: 04:57)

So, this is one thing you will keep in mind before discussing this, and going back to our discussion of the response of a free electron unlike the response of a bound electron in an insulator.

(Refer Slide Time: 05:14)

The response of a free electron, in what way is it different from that of the bound electron insulator. We will see by writing the equation of motion as before as d square x by dt square minus gamma dx by dt, this is the dissipative or resistive term. So, there was in addition in the case of a bound electron, there was a natural frequency of oscillation which is in the form a restoring force here in the equation of motion. Now in the case of a frequency electron, there is no restoring force, there is no natural frequency, it free to move under the influence of an applied electric field. So, this term we seen in the equation of motion and we have. So, this is the equation of motion in case of a free electron in a metal this gamma is the damping constant per unit mass and be seek a solution to this differential equation in the usual way by saying that.

(Refer Slide Time: 07:02)

The response is also of the firm of x naught e to the power I omega p the probably a phase shift and as plugging this back we get x naught equal to and this for a small then gamma is very small compared to one we can just write this as e naught by m omega square.

(Refer Slide Time: 07:52)

This will lead to a polarization usual way once we have the displacement we can write the polarization and that take this farm and… So, this leads to the frequency dependent dielectric constant if the case of a metal following the same procedure we get now this

dump n e square by m x let me rewrite this. So, that this can be written let n e square by m epsilon naught d call this is a term which has the concentration of the electrons the electronic charge mass and the permutative of this space. So, this is the constant which is characteristic in the metal.

Now, that is take as omega naught is known as last month frequency once you have that substitution is simply arrive at a simple relation like this and. So, what is this say. So, it the dielectric function of as metal as simply form one minus omega naught square by omega square. So, this can be again related to the real and to the imaginary part of the dielectric constant this has the form which is shown in the figure. At very low frequencies, the epsilon one is a there negative and then it rises to 0 then omega approaches the frequency and then you goes up to the value one at high frequencies. So, that the behavior which is shown in the figure.

(Refer Slide Time: 10:38)

Next figures show the imaginary part of the dielectric constant which is related to the reflectivity. So, in this figure the reflectivity is blocked as a function of a frequency. So, this shows that the reflectivity is one at low frequencies which means that at low frequency the metal reflects all the incidental electromagnetic radiation except when the frequency approaches the plasma frequency.

(Refer Slide Time: 11:17)

Then this behavior is did explain for omega very small compared to omega naught there is a negative dielectric constant and a correspondingly the refractive index which is the square of the dielectric constant is purely imaginary and I imaginary effective index means that the electromagnetic wave is attenuated. So, it does not propagate inside the medium inside the metal and it is perfectly reflected which is the reason why a metallic surface highly reflective shines. When the frequency approaches the plasma frequency, the dielectric constant approaches zero and you have a longitudinal plasma wave which is propagated in the metal. And for higher frequencies, higher than the plasma frequency, the dielectric function change the varies from 0 to 1. And this is the usual behavior in which they are electromagnetic real propagates to the phase velocity which is given c by root epsilon.

(Refer Slide Time: 12:28)

So that is the overall behavior and in the case copper corresponding wave length is about 120 nanometers. This is in the ultra violet rate that is way copper reflects visible light and appears shine. The collective oscillations also take place in the case of the past two iomans a diatomic solid that at the which also give rise to plasma executions which are known plasmas, but these plasmas occur at much lower frequency because the ion masses much larger it is shifted with the infrared range.

(Refer Slide Time: 13:10)

Now, we have in this treatment taken gamma as very small compared to one and neglected the damping term. Now if the damping term is not neglected in the frequency dependent direct constant then epsilon omega turns out to the complex, and this results again in an imaginary ineffective index this corresponds to absorption and hence attenuation like the media, because this light is attenuated. There is an energy loss and this energy loss may equally well be described in terms of a frequency dependent conductivity. So, this is known as the a c is the electrical connectivity then the when the incident radiation incident wave has a time dependence.

(Refer Slide Time: 14:09)

So, you have an a c response and therefore, this known as the a c electrical conductivity in contrast to the d c connected which we have already discussed. If a metal now this can be discussed again the same formalism that which we discussed the conductivity in terms of the current density which is n e v and this is equal to the sigma e by owns law. And the for we the velocity is a I omega minus gamma and v equals e on to take it account to sign of the charge is the. So, this means that v is and the e gamma minus i omega me. So, this can be a rewritten because we know that gamma is by 1 by tau we remember tau is the relaxation time for connecting trans. So, this can be written in terms of using this and this we can write the sigma the frequency dependent conductivity as sigma naught by one minus I omega now. So, that is the expression for the frequency dependent a c conductivity of a metal.

(Refer Slide Time: 15:53)

And this again displayed in the next figure where the real and imaginary parts of the electrical conductivity of a metal you shown again as a function of frequency are very not well and you can see the hole behavior is the different from that the d c conductivity.

(Refer Slide Time: 16:18)

Now, we turned to a discussion of the ionic polarization we discuss. So, for the electronic polarization and be now turned to a discussion of the ionic polarization.

(Refer Slide Time: 16:34)

So, in simple alkali halides an example sodium chloride we have sodium plus chlorine ions sitting alternately in a diatomic. Now the ionic polarization is again given by and z e into x one minus x two where that is the relative net displacement of the ion from the equilibrium position are we can write plus and again following the same procedure.

(Refer Slide Time: 17:39)

We arrived a situation where x naught to the equation motion and solid salt it and law where this omega naught this k by mu and mu is reduce mass of the two ions given by from this once we know the displacement.

(Refer Slide Time: 18:34)

We can write the ionic polarizability in the ionic polarizability due to the relative lead to displacements of the two ions of the form following the same procedure which we followed up to now.

(Refer Slide Time: 18:58)

Of course, we have neglected here the individual atomic polarizabilities, which are alpha plus and alpha minus therefore, alpha total polarizability this plus this plus this term and this lead to the clausius mossotti equation we discussed already for the dielectric function an where alpha is given by this and therefore, going to the different limits define.

(Refer Slide Time: 20:18)

For example, the static dielectric constant static omega equal to zero there that will be and we have in this omega becomes zero.

(Refer Slide Time: 21:07)

So, I have whereas, the high frequency dielectric function which we write as epsilon infinity is is one is got in the same way by setting omega equal to infinity there. So, the frequency dependent term goes off. So, using these two limit we can write the epsilon omega the dielectric constant at any frequencies of omega.

(Refer Slide Time: 21:59)

This take the form that will be plus minus. So, solving for epsilon omega we get? Where omega t square, this is something which we one can figure out rather straight forward algebra.

(Refer Slide Time: 23:11)

So, what is the meaning of all this we have now returned the frequency-dependent dielectric constant at any frequency in terms the high-frequency dielectric constant and then the static dielectric constant and have a frequency the defined in this way. So, this

helps us to visualize what is going on in terms of what I have known in the longitudinal and transverse of pick more.

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tinal and transver $f(a)$

So, we now go on discuss the longitudinal and transfers optic modes and optic on usually they go by the contraction at the abbreviation l o and t o for us. So, in an optic mode of an ionic crystal, there is an non varnishing polarization p and associated with this is the electric displacement d and when there are no free charger. We have the general relation divergent to be vanishes now the apply an alternating field the e equal to e naught exponential I minus k or k dot r in general. So, using this, you can see d gives rise to k dot d equal to zero, because the divergence is going to taking the divergence means it would be replacing this by I k div d equal to zero goes to the equation k dot d equal to zero. And what the consequences of this this means that I either d itself is zero or d need is perpendicular to k if d is perpendicular to k take care e also is and p also that is the significance of this condition div d equal to zero.

(Refer Slide Time: 26:34)

Now, in addition we also have for the electric case the additional relation donation going equal to zero.

(Refer Slide Time: 26:45)

Now, curly cord is you know foreign hostility electric field again give rise to the relation k cross e equal to zero this means either e itself is zero or e is perpendicular to keep no parallel if is all. So, is the and p. So, that these are the two possibilities.

(Refer Slide Time: 27:42)

Now, we can discuss an l o mode a longitudinal in an optic mode in an l o mode what happens it is defined by the polarization being parallel to keep. So, if p is parallel to k naturally this condition is ruled out there for only we have the d equal to zero and therefore, e is minus p value where as for the t o mode p is perpendicular to k which means that p s parallel to k therefore, e zero. So, this is the main difference between l o mode and the t o mode.

(Refer Slide Time: 29:02)

In a longitudinal optic mode P is parallel to k and since $D = 0$, $E = -P/\varepsilon_0$ (18.23) For a transverse mode in which P is perpendicular to k **k** . $D \neq 0$ while $E = 0$. The dielectric permittivity, $\varepsilon = \infty$. Hence $\omega^2 = \omega_T^2$. (18.24) The frequency of the LO mode corresponds to $\varepsilon = 0$. This leads to: $\omega_L^2 = (\varepsilon_0/\varepsilon_0) \omega_T^2$. (18.25) Fhis is known as Lyddane -Sachs- Teller relation. * **MPTEL**

Therefore you have the frequency a the L O mode corresponds to epsilon equal to zero in; that means, omega l square plugging back into the expression for the along L O mode omega l square is epsilon naught by epsilon in finite a into omega q square. So, this very interesting relation between a frequency of a L O mode and the frequency of a t o mode goes by a special name lezyne sects tailor this can be experimentally verify because no one can measure the static dielectric constant the high frequency the frequency angular frequency that the L O and T O modes. Therefore, one can check this relations now this in general gives back omega is usually greater than omega t. So, the frequencydependent dielectric constant its variation for an ionic crystal is shown ionic the next figure.

(Refer Slide Time: 30:32)

And one can see that indeed frequency range between omega l and omega t there is a again and negative dielectric constant and on the ionic crystal is highly reflective.

(Refer Slide Time: 30:49)

So, no radiation can propagate in the crystal this frequency region laying between the transverse and longitudinal optical modes. Now we can also solve for the dispersion relationship for the transfers electromagnetic wave and modes and this gives you dispersion relationship and solved gives you a behavior like this. So, the dispersion relationship has to branches one laying below omega t and another laying above omega l. So, you have in the region there are linear regions one mode is clearly found like and one is clearly optical.

(Refer Slide Time: 31:49)

So, this excitation is known as polarity dance at at very low frequency is we see omega goes as linear and it is levels of the omega t and at highly frequencies about the other branch.

(Refer Slide Time: 32:12)

You have again in linear region at very low frequency is it levels off to omega t and again you have at high frequencies you have a linear region again it levels of up a syntactically to the longitudinal optical.

(Refer Slide Time: 32:28)

So, in the region between omega on in this frequency range you have highly reflected behavior and the e m waves are reflected from the surface at the custom. So, you can after many reflections only the component of radiation frequencies very close to omega t will survive. So, you can have a surviving radiation which is known as a rest trial or in English is a German-term which means residual unease. So, you can have a very precise to measure the transfers of the pick mode frequency and also produced monochromatic radiation gross e to omega t which lies in the intranet. So, these are some vacations of this behavior.

(Refer Slide Time: 33:37)

We know all closed is that their discussions brief discussion of very remarkable you kind a substances known as b. So, electric's.

(Refer Slide Time: 33:47)

In general all the dielectric material when you subject it to an external electric field have sufficiently large magnitude undergoing a small chain in a dimension. So, there is a strain generated which is proportional this cure of the electric feed at high enough fields.

(Refer Slide Time: 34:32)

So, this effect is known as an electron structure and this is observed in all the dielectric materials at high enough electric fields, but piezo electrics is different. This is a one special class electric where there is a significant change in dimension in and subjected to an electric field backed in a piezoelectric strain is proportionately to the field. So, they

strain depending on whether what is the polarity the field you can have a compressive if you are extensive string.

(Refer Slide Time: 35:10)

So, this is piezoelectric, now usually the electric effect mass the electrostrictive effect. Now you can have not only a direct electric effect best, but also in direct reverse effect in in which you have a field resulting and their an a electric field resulting from and applied strain jacks on the acuity first observed this effect in eighteen eighty and all means, but if that. So, that is the reason for the terminology.

(Refer Slide Time: 35:57)

Now, this is interests comes because of the applications apply it well generation all ultra sonic it is because it is trained needs used in sauna and then you can also have pressure are strain since we I based on director reverse piezoelectric effect you can also have actuators these electric actually. So, usually materials belonging to crystal which classes these processes and center of symmetry do not exit it piezoelectric effect because the linearity. So, it is only the non centro symmetric crystals which produce the piezoelectric effect.

(Refer Slide Time: 37:24)

Now, that shown in next couple of figure direct and the indirect is addicting effect I and that usually we have already discussed faro electrics all faro electrics display piezoelectric. Now in the case of a faro electric material the important characteristic is that there exists a spontaneous polarization which can be the reverse by applying the external electric field. So, a faro electric material exhibits a piezo effect only when it is polarized in this way that is when it is made to process ramnant and polarization the polarizing processes is call only the error that.

(Refer Slide Time: 38:15)

What are the material typical examples one as the material which is very well known a piezoelectric material is quarts.

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Quat the silicon dioxide usually piezoelectric, but not faro can used in the form of a single crystal and when have quat crystal.

(Refer Slide Time: 38:44)

This is use very often in frequency stabilization of as electives appeaser piezoelectric quad crystal gives you frequency stabilization.

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This figure shows a single crystal of quad's in the different cuts type which are implied for production of x piezoelectric slices with the different frequencies are shown in the figure.

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Now, they evident frequencies can lay in the kilohertz are mega hertz in depending on the direction of correct and the thickness of the custom.

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So, that is the oscillator the next material is barium title it we already discussed this material it becomes a fero electric head phone under on twenty d b celsius and as I a high dielectric constant exhibits a very sensitive piezoelectric.

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Another one is lead zirconate well it is lead zirconate titanate is it is that on. So, this is the widely used a piezoelectric it looks at me, but plans to use application now it is also used in the polycrystalline farm.

(Refer Slide Time: 40:35)

So, it as an advantage of easy fabrication that is the crystal structure of let us open it paid and this there the most widely used piezoelectric material.