Condensed Matter Physics
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Lecture - 18
Dielectric Solids - Worked Examples

In this session, we are going to solve a number of examples, which are meant to illustrate the concepts explained in lectures on dielectric materials.

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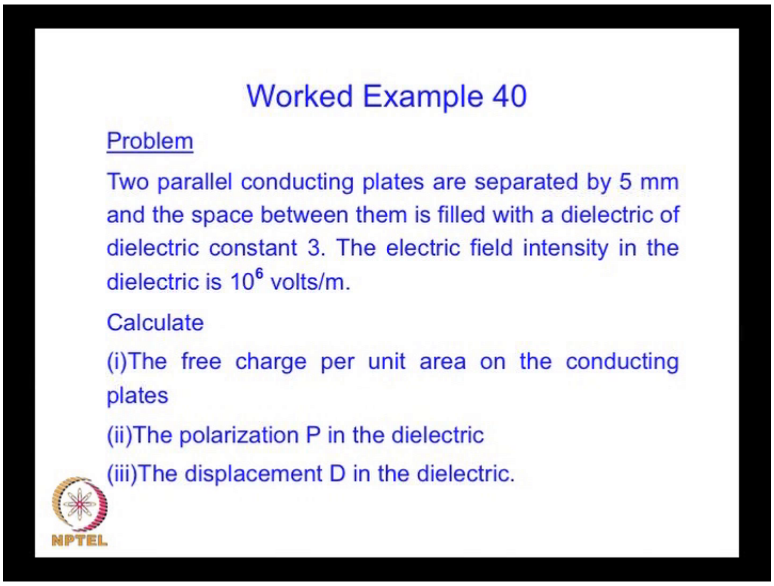

The first problem is a general one in which we are told that there are two parallel conducting plates, which as separated by a 5 millimeter, and this space between them is filled with a dielectric material who's dielectric constant is 3. The electric field intensity in the dielectric is given as 10 to the power 6 volts per meter, we are asked to calculate the number of quantities first the free charge per unit area on the conducting plates second the polarization P in the dielectric third the displacement vector d.

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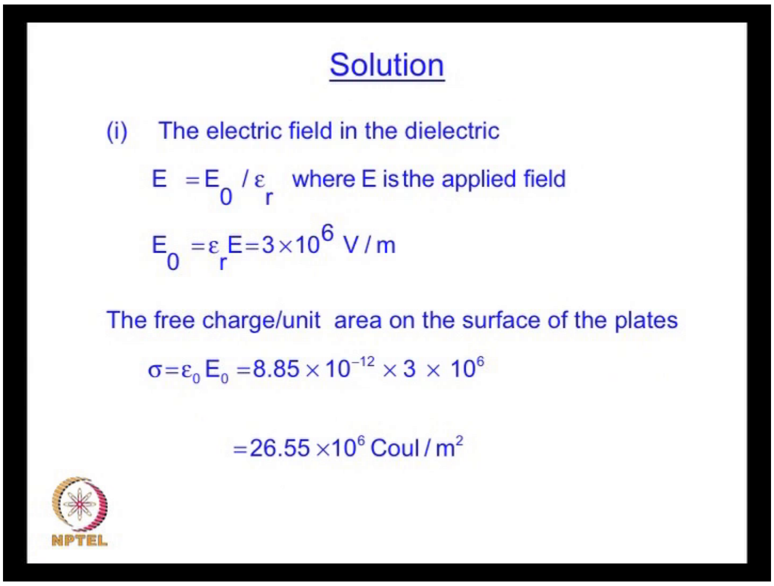

In the dielectric magnitude of it first in order to calculate the free charge we have to calculate.

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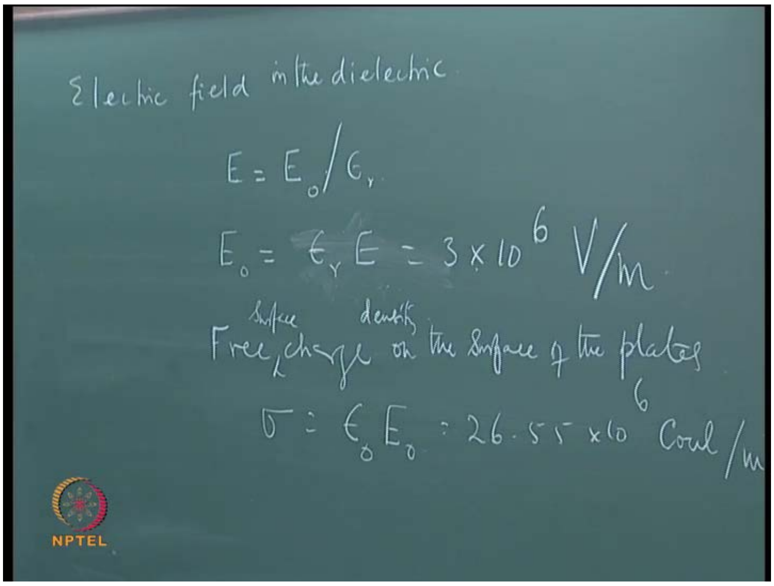

The electric field we have been given the electric field in the dielectric that is E magnitude is the electric field in free space times the relative dielectric constant epsilon r. So, this the field in the dielectric, this is the field in free space. So, E naught the field in free space is E by epsilon naught, and therefore E, which is three into 10 to the power 6 holes per meter from the given data. Because the relative dielectric constant is 3 and the field in the dielectric is 10 to the power holes square meter having calculated the free space a electric field intensity. The free charge on the surface of the plates is sigma is a free charge surface charge density is epsilon naught E naught in that would be 26.55 into 10 to the power 6 coulomb per meter square. So, this is the free surface charge density well that is what we are asked to calculate free charge by unit area having got this.

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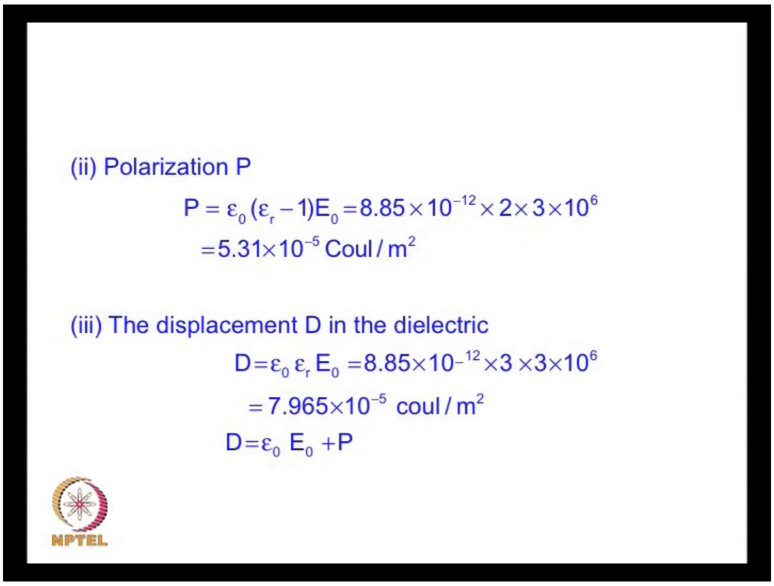

We can next calculate the polarization P, which is epsilon naught epsilon r minus 1 and E naught.

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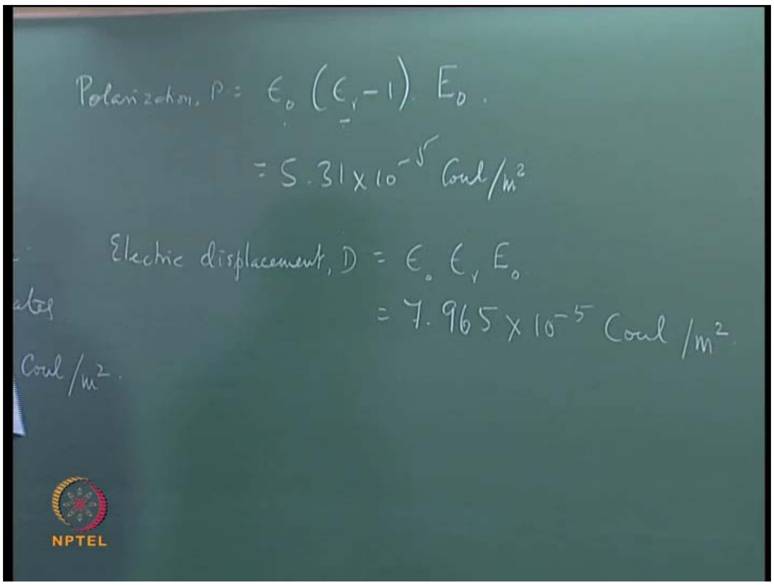

So, we know what is epsilon r and what is epsilon naught, therefore substituting values this turns out to be 5.31 into 10 to the power minus 5 per meter square I did not got P. And E naught it is a simple thing to calculate the electric displacement d has epsilon naught epsilon r E naught. Therefore, substituting the values this turns out to be 7.965 into 10 to the power minus 5 coulomb per meter square 1 can immediately readily verify that the relation d equal to epsilon naught E naught plus P form the values.

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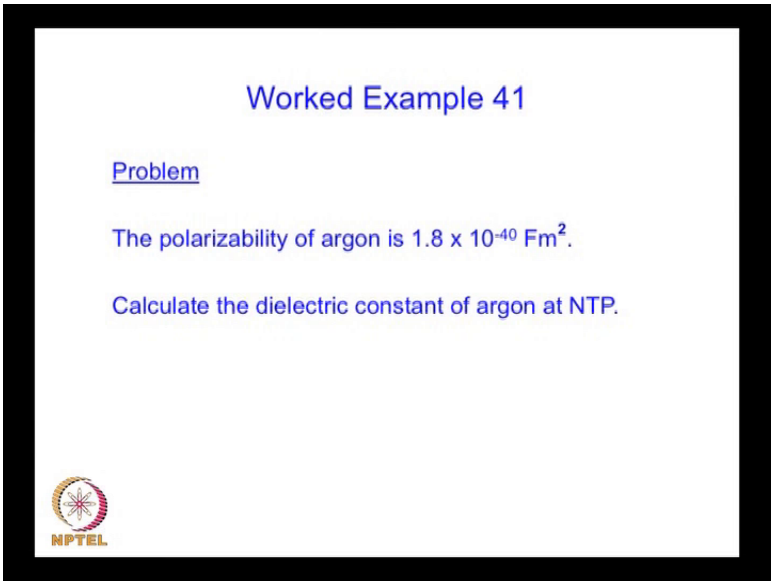

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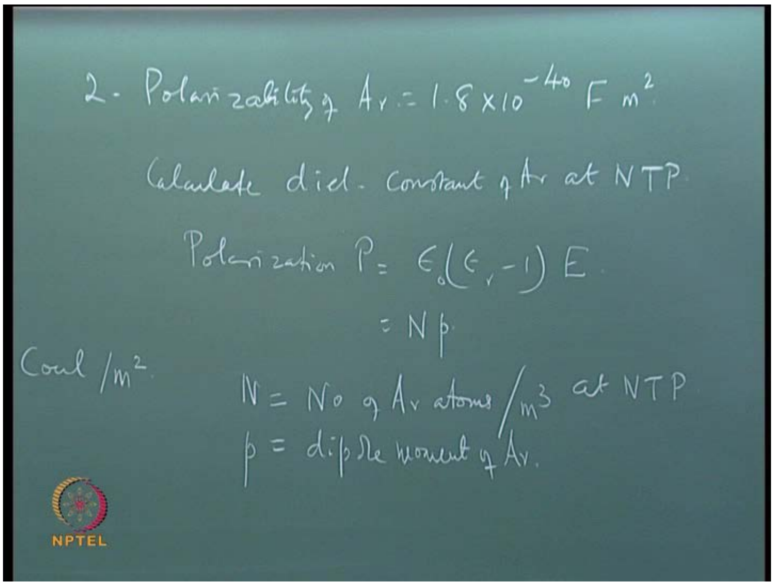

Next we are told the polarizability of organ is given to be one point eight into 10 to the power minus 40 for it meter square. So, we are asked to calculate dielectric constant of argon act normal temperature impression.

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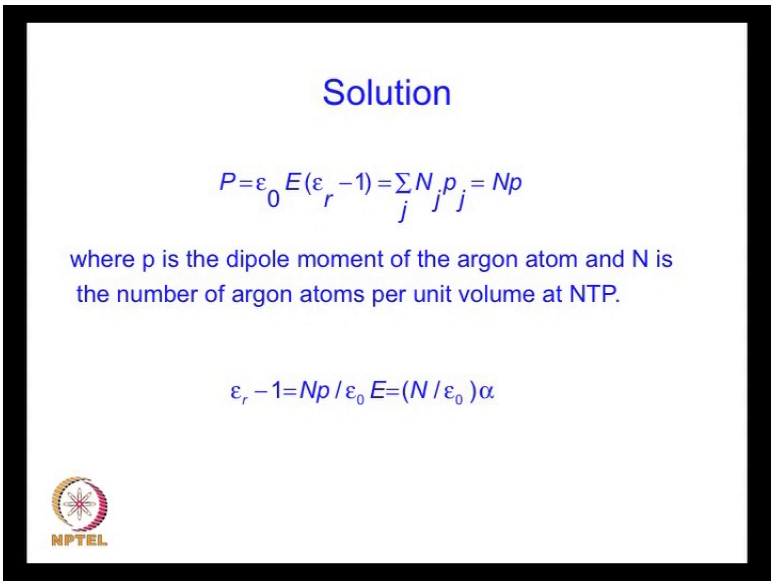

So, in order to do this we go from the fact that the polarization P is epsilon naught epsilon r minus one time E, and that is equal to N times P where P is a individual dipole moment. And N is the number of argon atoms in this case for unit volume at and NTP P is the dipole moment of argon, this is the dipole moment is related to the polarizability.

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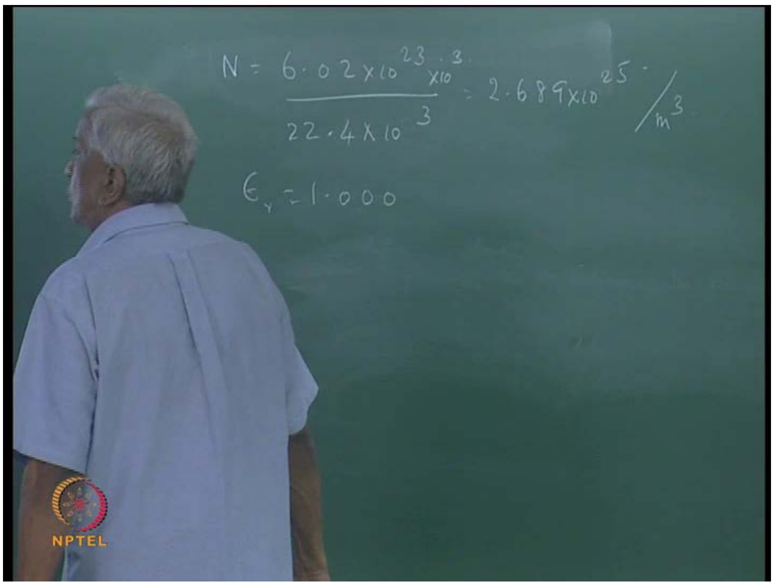

First let us calculate the number of atoms n.

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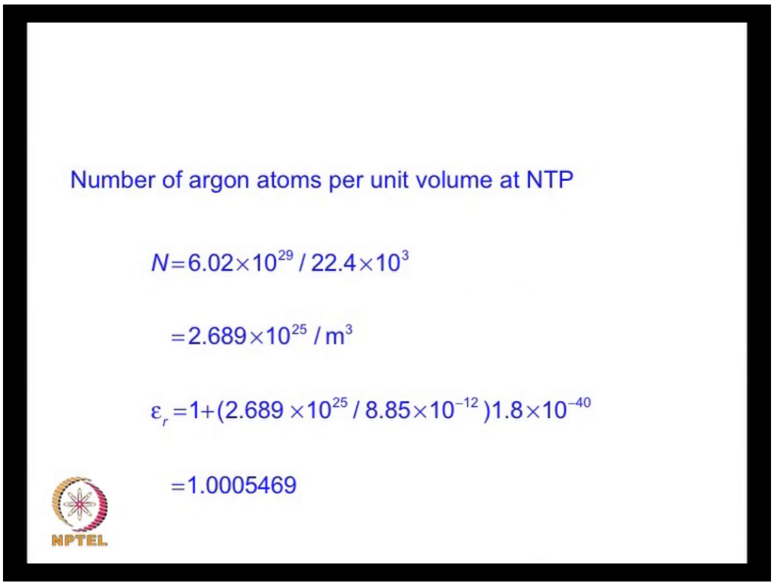

So, this would be the number by a the volume of one more which is twenty two point four liters. So, that would you be 2.689 into 10 to the power 25. So, and epsilon r is a therefore, substituting these values this will become 1.0005469, because we have taken the polarizability and multiply it by electric field.

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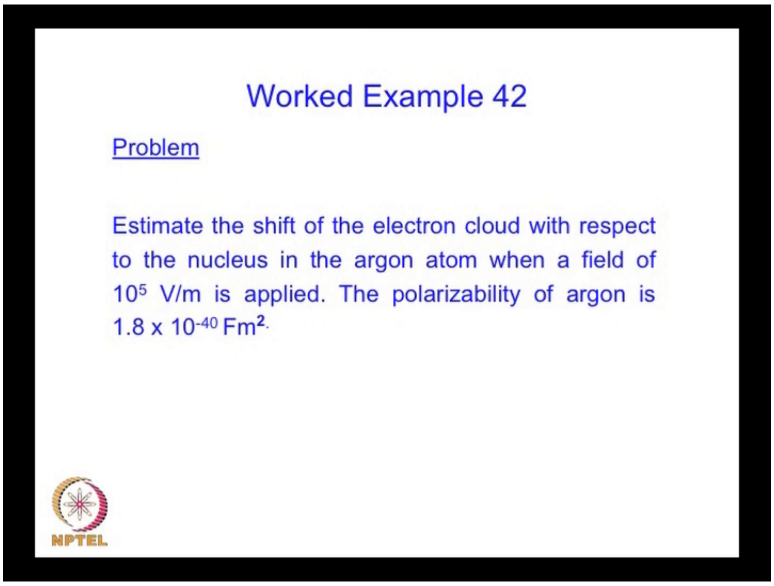

Next we are asked to estimate because this is not an accurate calculation, therefore we are simply asked to estimate the shift of the electron cloud with respect to the nucleus in the argon atom when I filled 10 to the power 5 volt per meter is applied again the polarizability of argon atom is given. So, in order to do this we go from the polarizability to the dipole moment.

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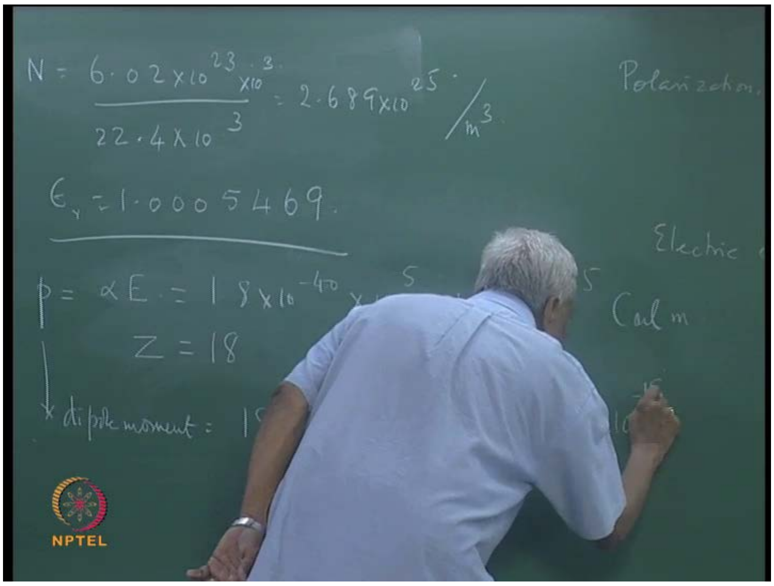

So, we are told that the polarizability is 1.8 into 10 to the power minus 40 parameters square times 10 to the power 5. So, that will be one point eight into 10 to the power minus thirty five coulomb meter in the case of argon we know that the atomic number is eighteen and the dipole moment is 18 E. That the nuclear charge times the delta the shift of the cloud therefore, from this it turns out that we have delta as 6.25 into 10 to the power minus 18 meter.

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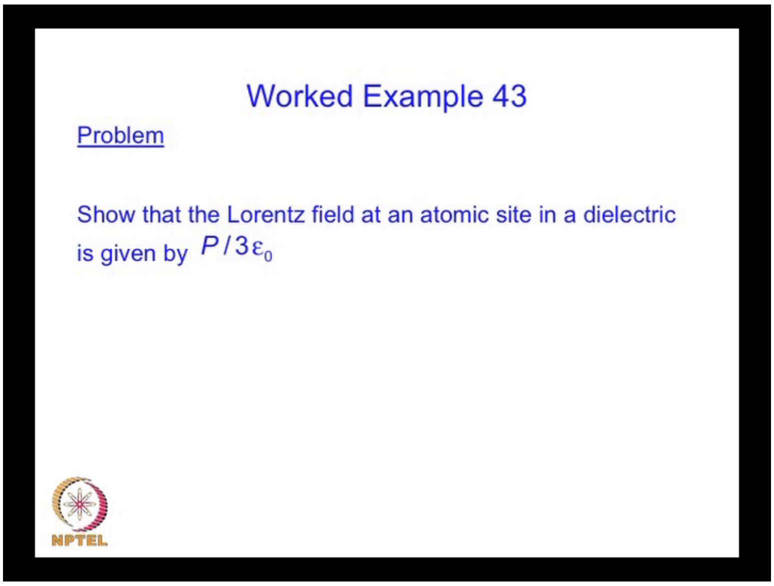

Next is a standard result we are asked to show that.

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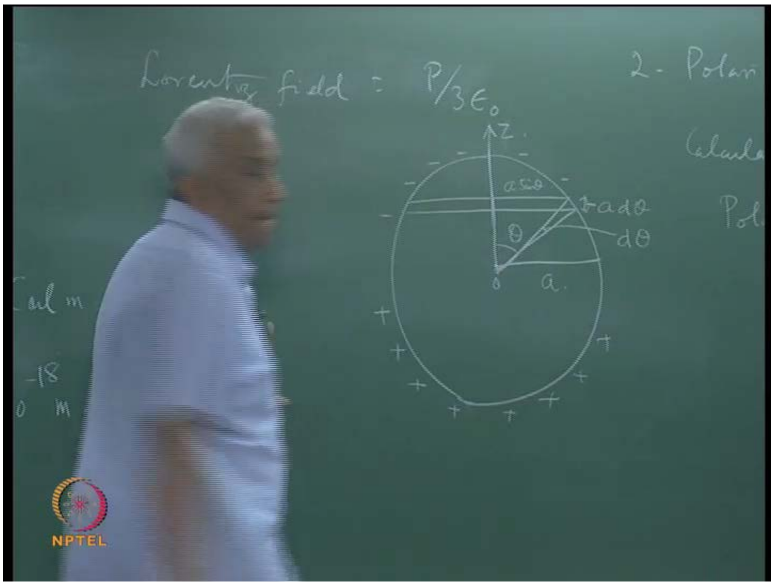

The Lorentz internal field at an atomic site inside a dielectric is given by P by three epsilon naught this was a standard result which was left without proof in the lecture.

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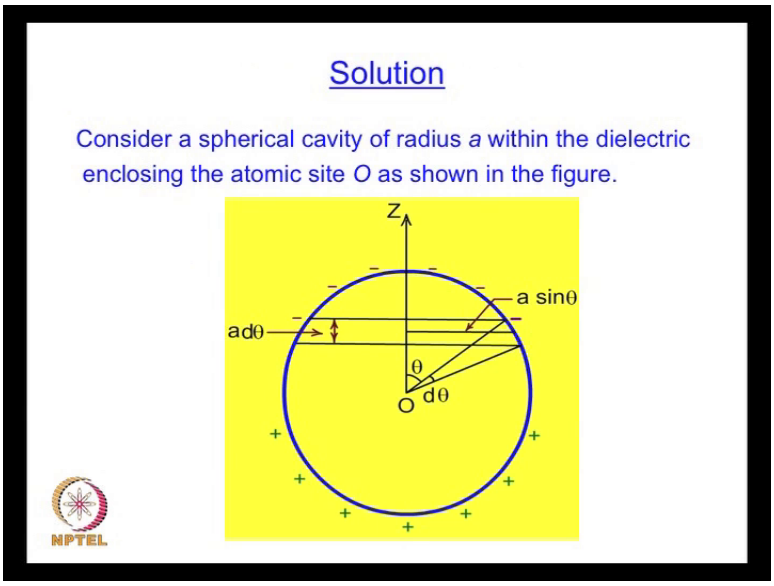

So, we are simply asked to due to this we have in the figure we consider a spherical cavity of radius a within the dielectric this is the cavity enclosing the particular atomic site o, which is kept at the origin as shown in the figure. So, that will be polarization charge s on the surface of the spherical cavity negative charge will be here for a polarization here along the z axis. So, this is taken has the z axis. So, there will be negative charges here and positive charges here. So, we consider the polarization charges and consider a elementary ring of charge the radius of this ring is this is theta. So, this is a sign theta and this is a d theta that is delta d theta.

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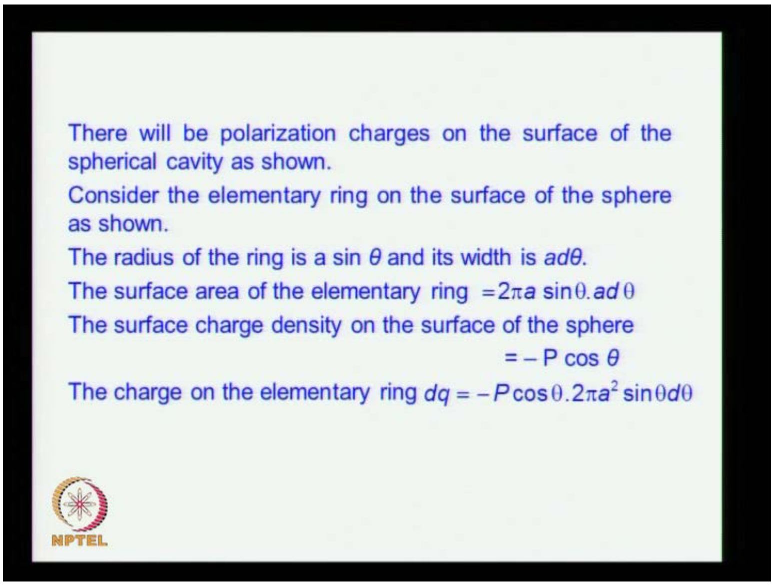

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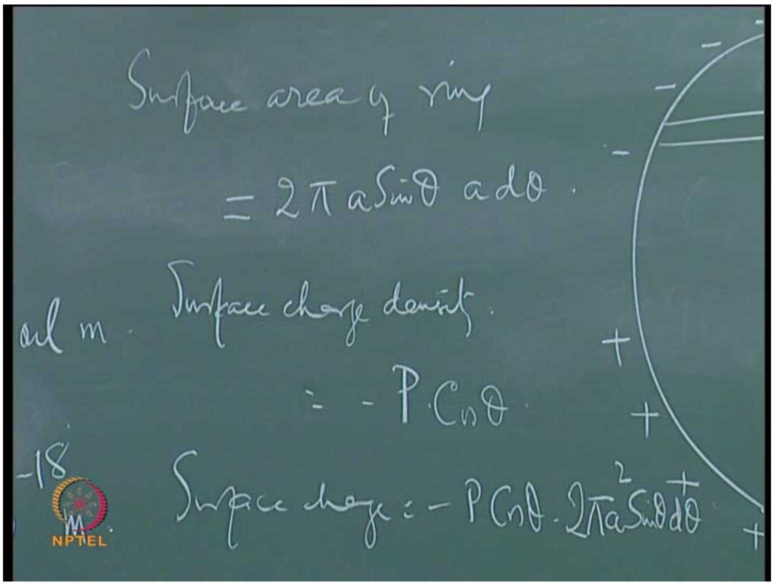

So, we know the surface area of ring is two phi a sign theta into a d theta. So, the surface charge density is just the surface area the charge density is of course, minus P cos theta. And therefore, the surface charge is just the product of these two. So, that is the charge carried by the ring of charge.

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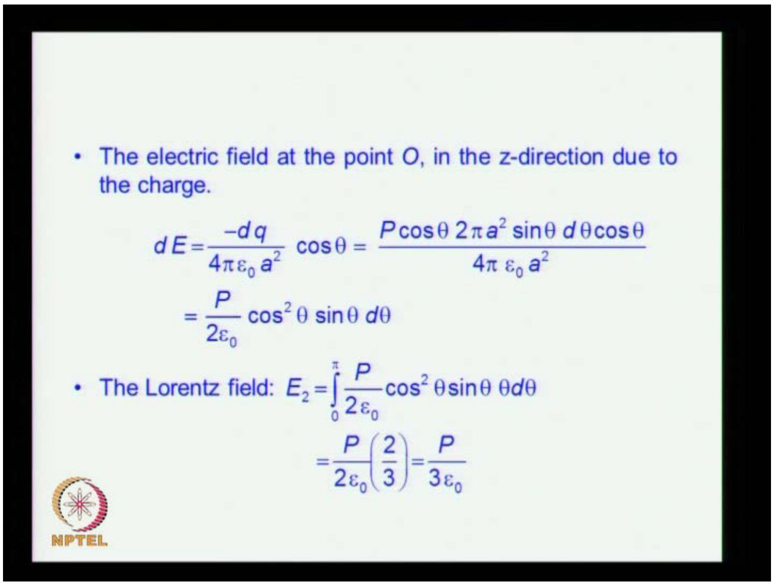

So, the a simple matter to calculate the electric field intensity at o due to this charge density by applying coulombs law.

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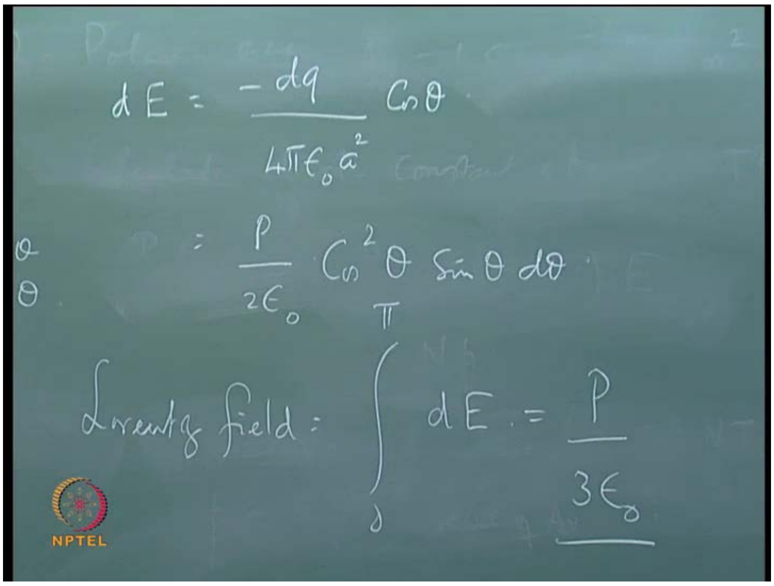

So, the differential electric field b-minus b q by four phi f epsilon naught a square times cos theta that would give me P by two epsilon naught cos square theta sign theta d theta, because the total Lorentz field d1ue to the entire charges fear will be delta zero to phi of the E.

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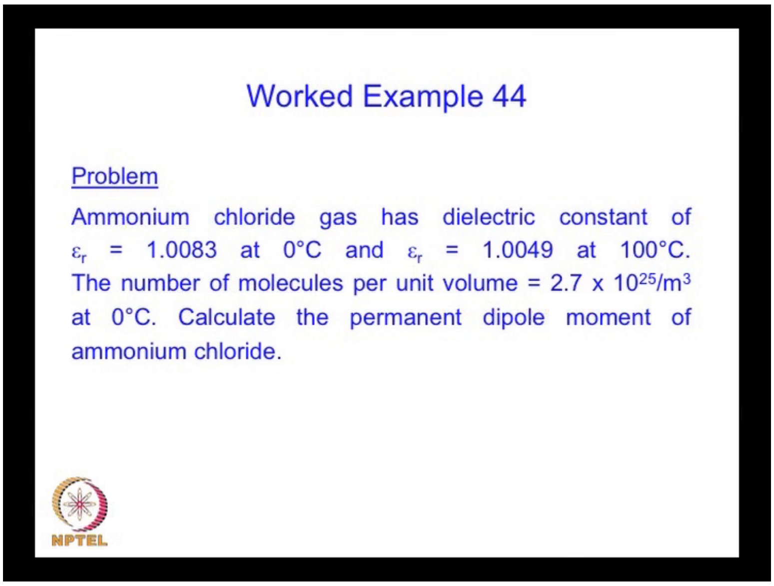

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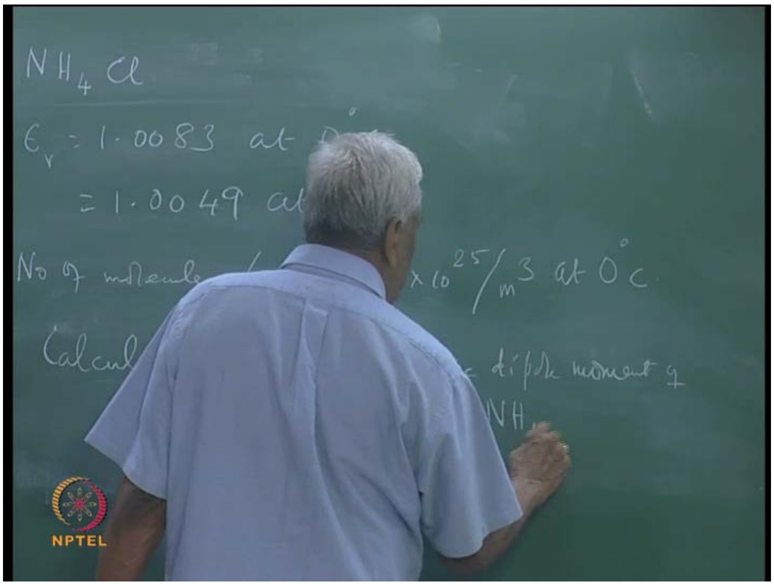

And that would can be readily shown to be P by three epsilon naught you pass on to the next question which is about ammonium chloride gas ammonia. Ammonium chloride it has a dielectric constant let you dielectric constant of 1.0083 at zero celsius and it is 1.0049 at 100 degree Celsius. We are also given the number of molecules of ammonium chloride per meter cube and 2.7 into 10 to the power 25 per meter cube at zero celsius. So, we are asked to calculate the permanent dipole moment of ammonium chloride.

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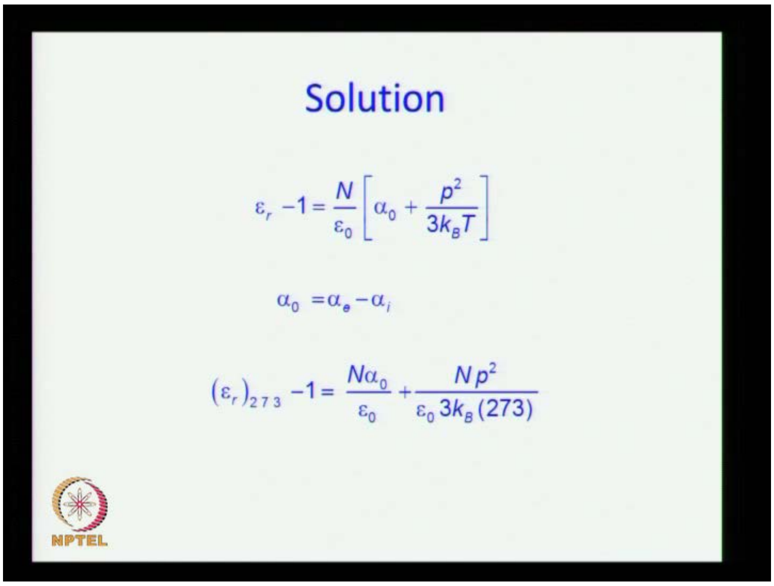

This is the polar molecule in order to do this, we have to go from the orientation expression.

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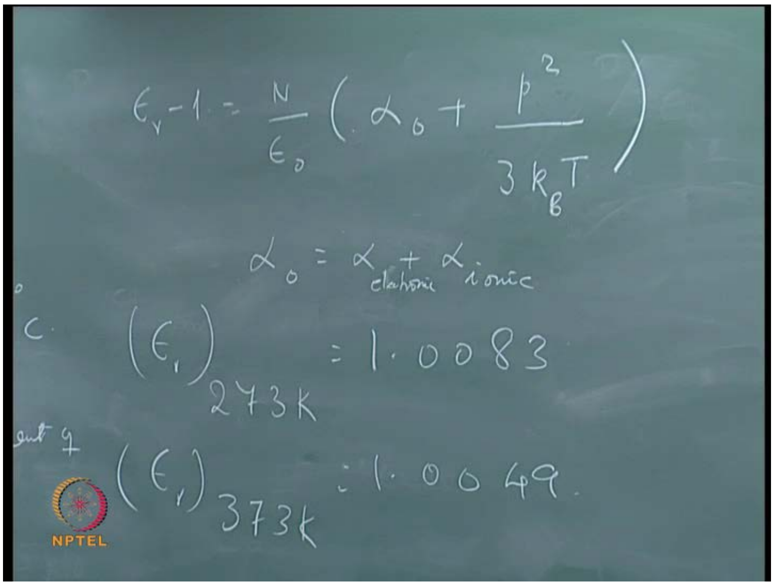

For the dielectric constant relative dielectric constant as N by epsilon naught into two contribution one is alpha zero, which is the temperature independent part of the polarizability plus P square by three kb t which is the orientation polarizability alpha zero contains the electronic plus ironic contributions. And we are given that epsilon r at zero degree celsius which is 273 kelvin as 1.0083. And similarly epsilon r at a three seventy three kelvin given as 1.0049. So, it is relatively straightforward to substitute these two values in these two corresponding to these two temperatures and therefore, eliminate everything.

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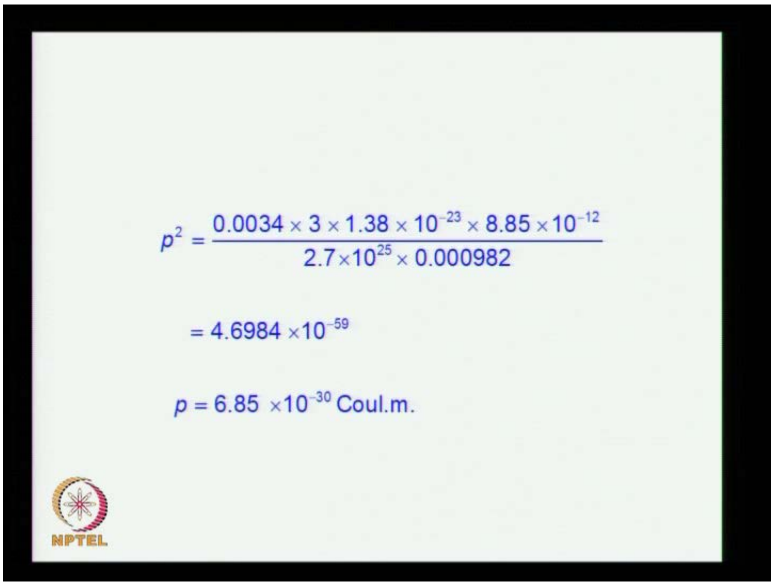

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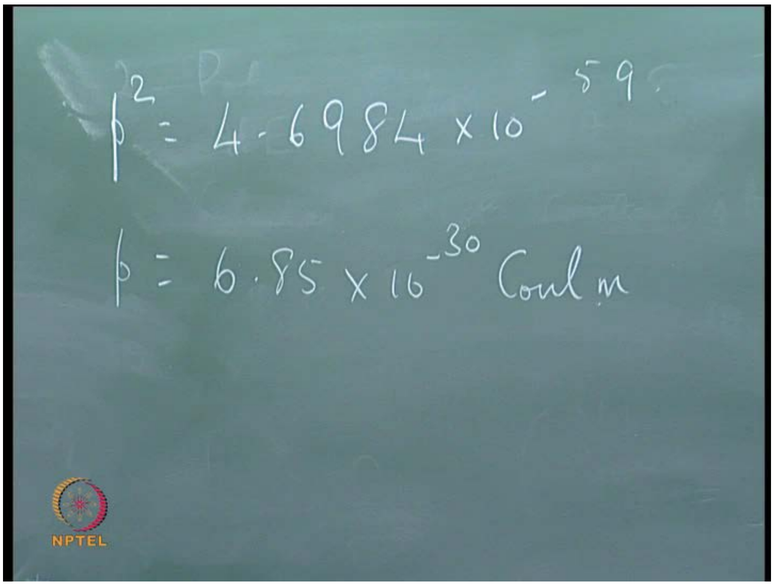

And find P square it is a simple substitution of problem P square 4.6984 into 10 to the power minus 59 giving P is 6.85 into 10 to the power minus 30 coulomb meter.

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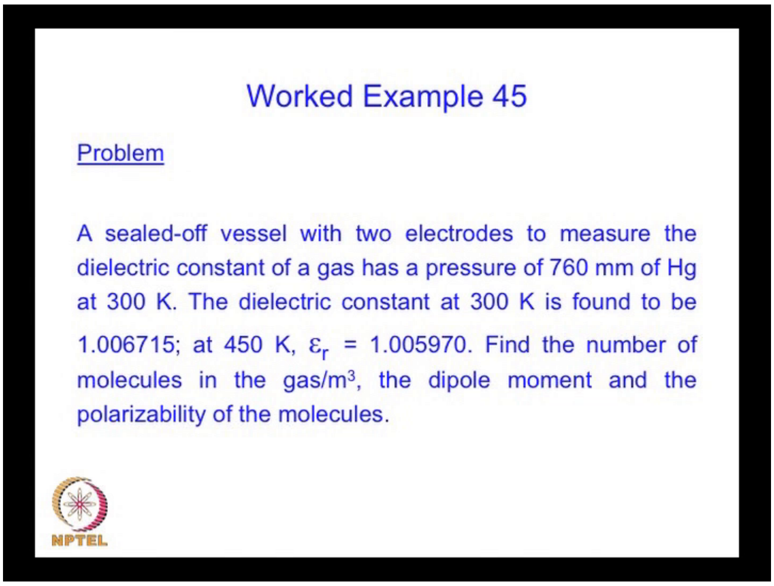

We next pass on to example six which states that we have a sealed off vessel the two electrodes to measure the dielectric constant of a gas.

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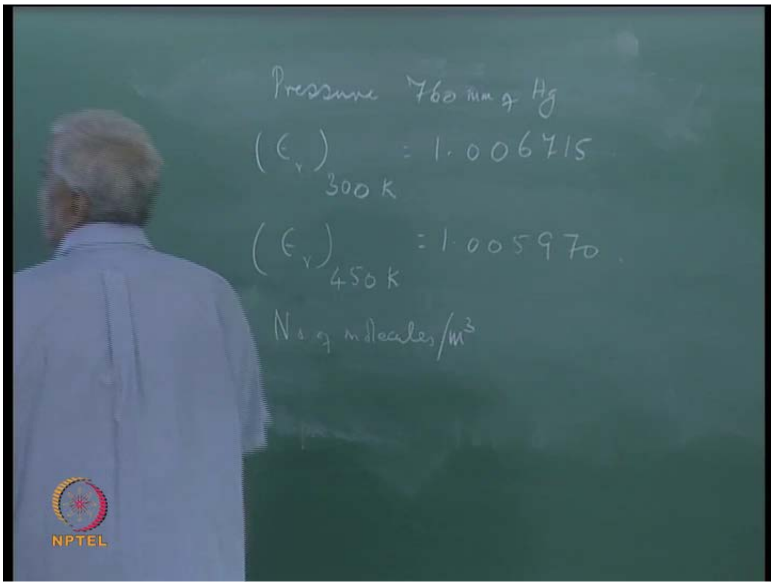

And the vessel has a pressure 760 millimeters the mckee pressure atmospheric pressure seven sixty millimeters of h g and the dielectric constant at three hundred k is found to be 1.006715. And dielectric constant at four fifty k is one point zero zero five nine seven zero we are asked to find the number of molecules of the gas and the dipole moment.

And therefore, the polarizability again very simple question which is based on the fact that there is a orientation polarizability, which is temperature-dependent.

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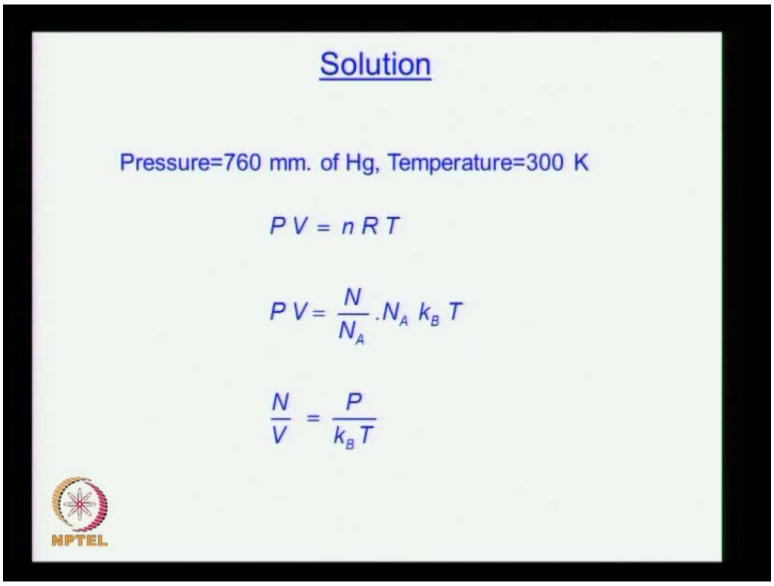

So, we are given the pressure is 760 millimeters of mercury.

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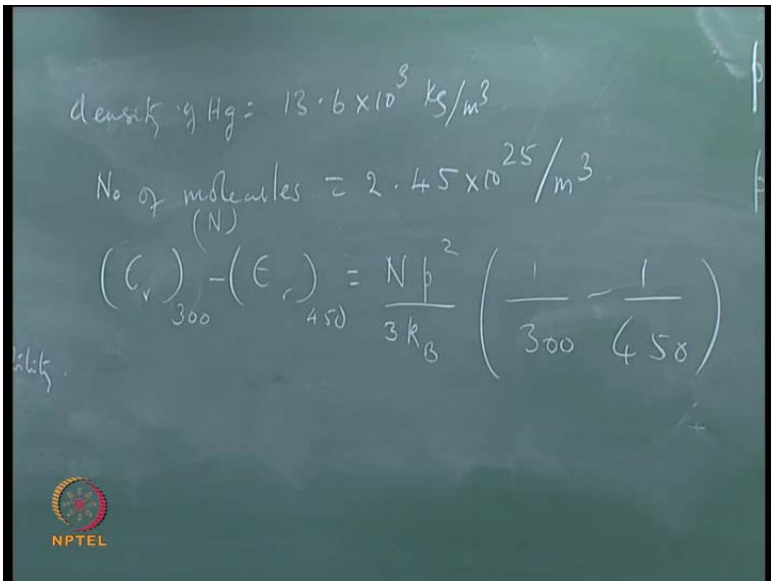

And density of mercury because we have mercury here the pressure is measured in units millimeters of mercury. So, given that we can convert this pressure and therefore, find the number of molecules, because they are given by pressure. And we are given the temperature, so it is a simple question of dividing the pressure by the k Bt. So, if we get the number of molecules has 2.45 into 10 to the power twenty five by per meters cube. And now again epsilon r 300 minus epsilon r 450 is N p square this is N by three k B t one by 300 minus 1 by 450 and from the value is given week.

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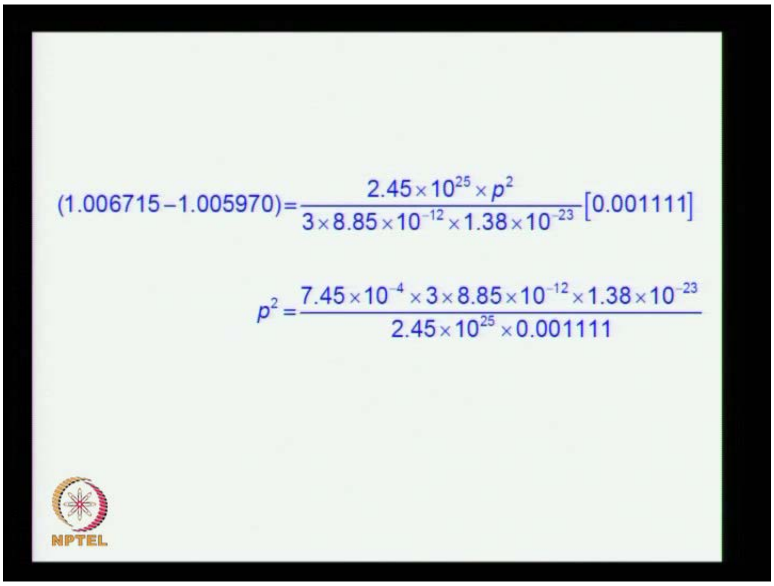

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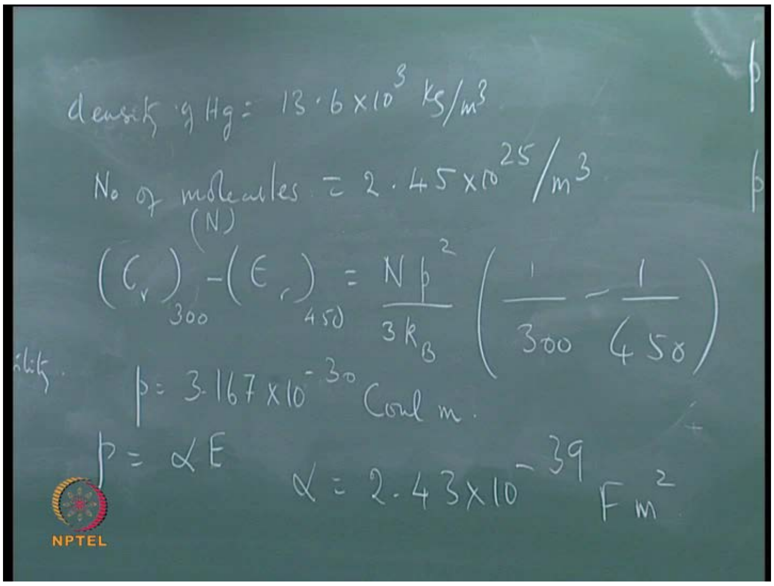

We can substitute and find P readily as a 3.167 into 10 to the power minus thirty coulomb per meter tool and epsilon r minus one is just the P by epsilon naught E therefore, we have we can find the alpha from P equal to alpha e.

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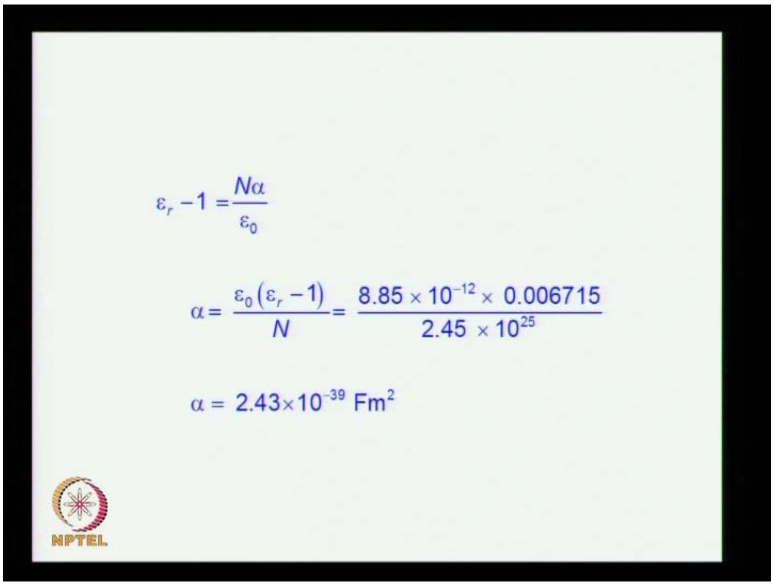

And we also have the relation from epsilon r and P and E therefore, substituting this we get alpha as 2.43 into 10 to the power minus 39 per meter square

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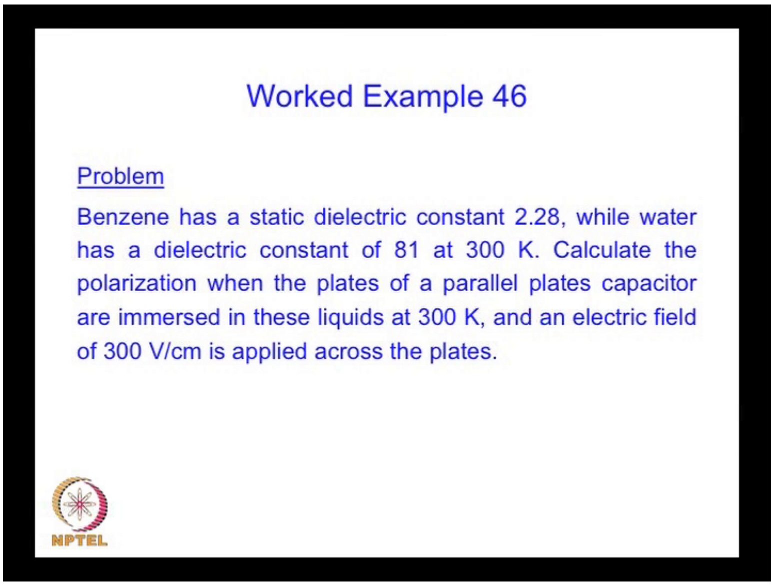

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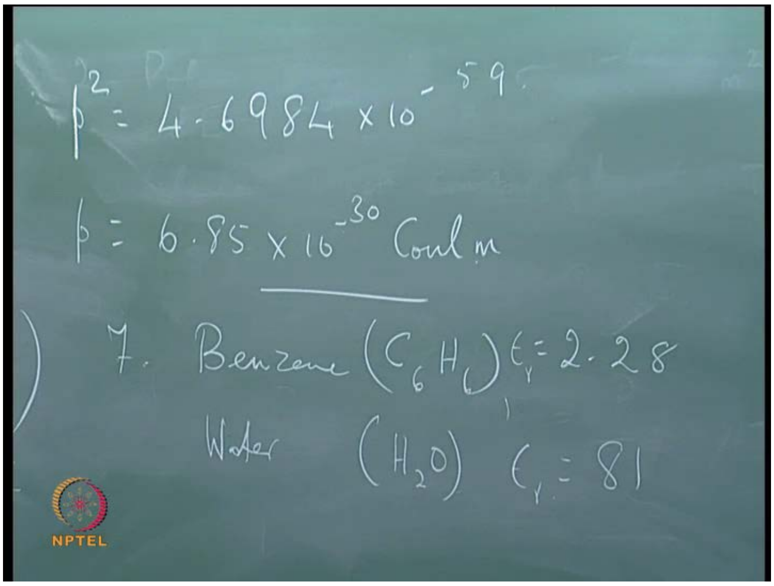

Next you are given a question about benzene which is polar molecule. So, we are told that as a dielectric constant of 2.28. We are also told that water has a corresponding value of 81. We are asked to calculate the polarization when the plates of a parallel plate capacitor are immersed in these two liquids at 300 k, and then an electric field of three hundred volts per meter is applied between the plates.

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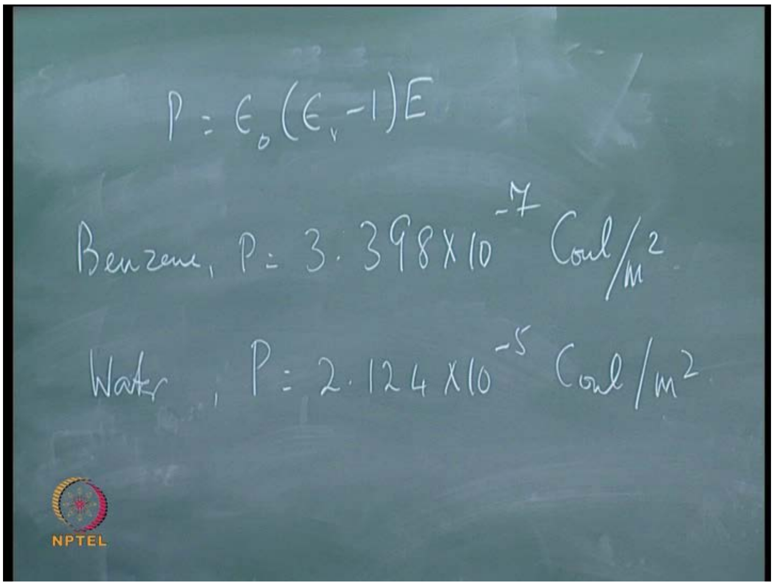

So, again we have to start from the polarization. P epsilon naught epsilon naught minus one E in the case of benzene begin the values.

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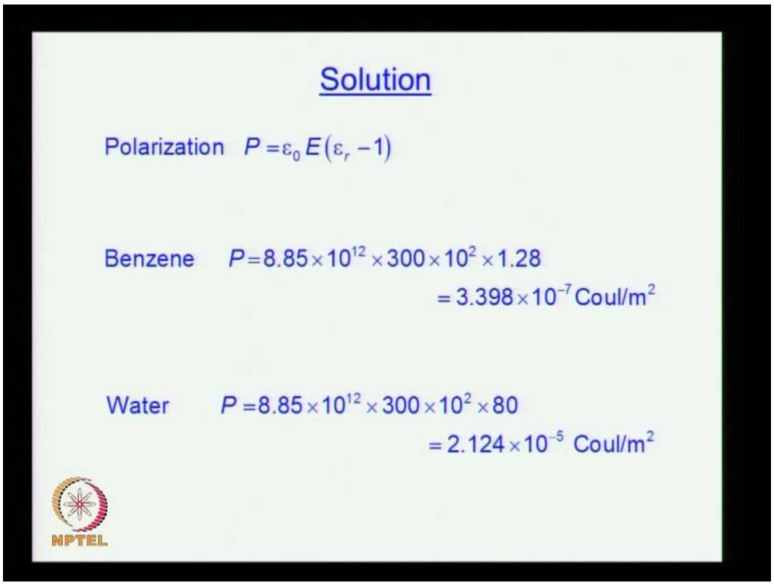

It turns out to be three point three nine eight into 10 to the power minus seven coulomb per meter square, whereas in the case of water same quantities lead to a polarization which is 2.124 into 10 to power minus five because of the high dielectric constant.

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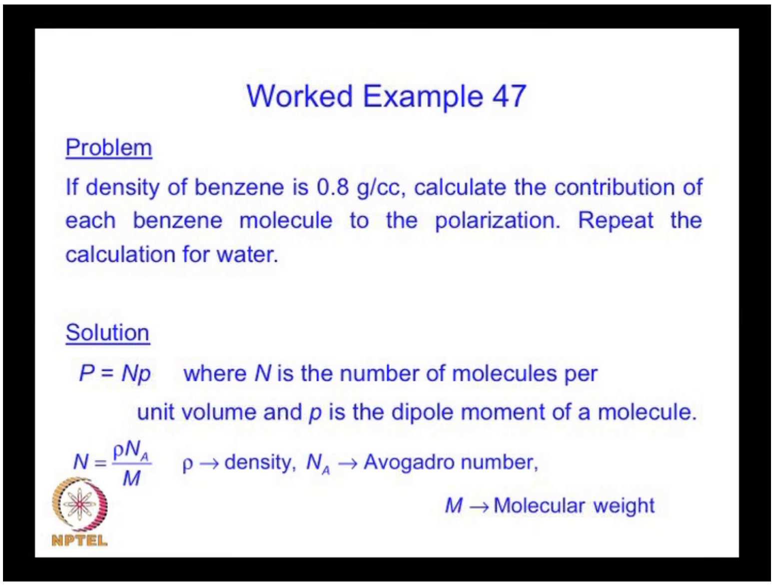

Having got this we can go on to find other quantities relating to benzene.

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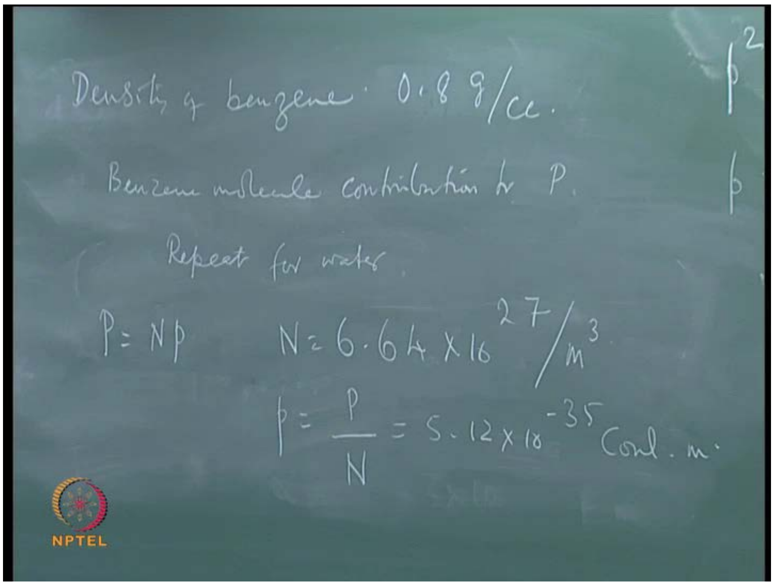

They are told that the density of benzene is is point eight grams per c c therefore, they are asked to calculate the contribution of each benzene molecule to the polarization contribution to P. And we are also asked to do this for repeat also for water since we know the polarization is is N times P where P is the individual dipole moment and we can calculate the N from alligators number and the density et cetera. So, N turns out to be six point six four into 10 to the power twenty seven per meter cube and therefore, P is just the polarization divided by N in polarization it is already calculated. So, this gives me five point one two into 10 to power minus thirty five coulomb meter.

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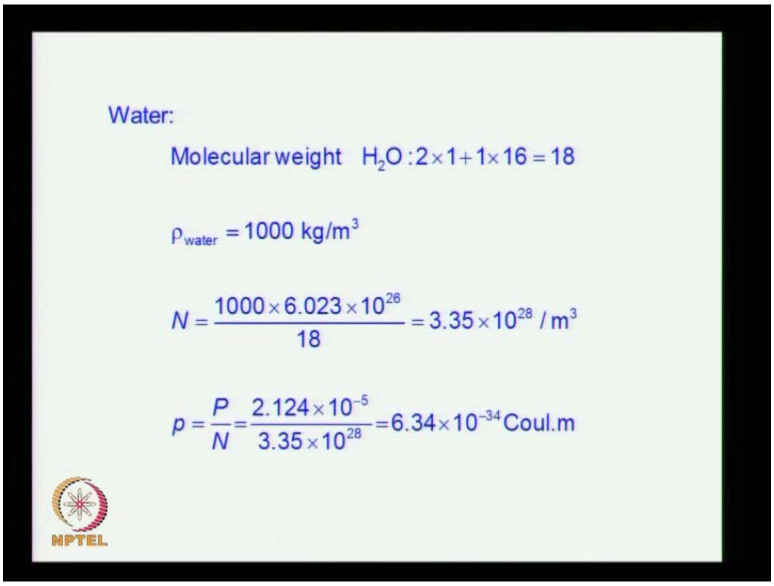

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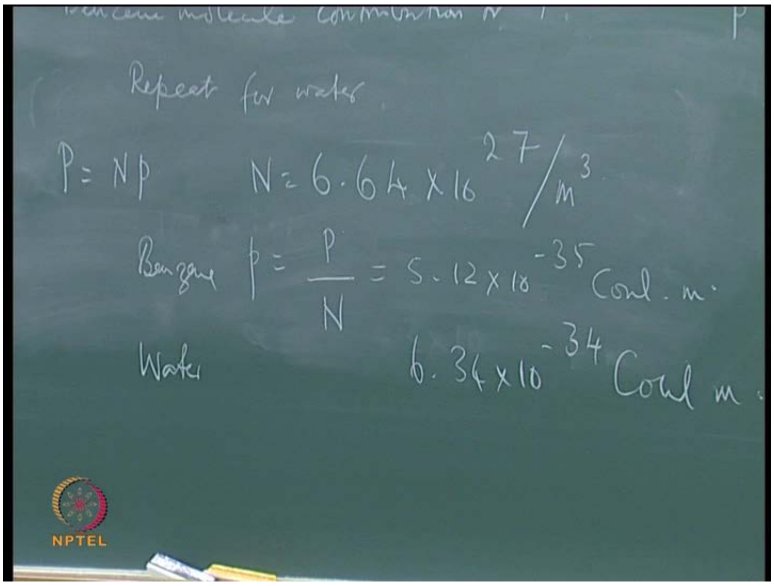

So, for water the same calculation for water is for benzene gives you a value 6.34 into 10 to the power minus 32, 34. We pursue the comparison between benzene.

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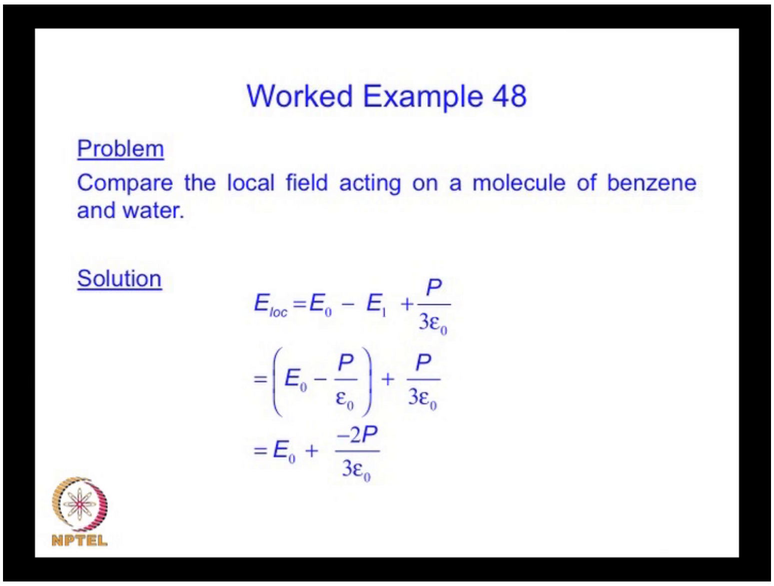

And water has polar substances by comparing the local.

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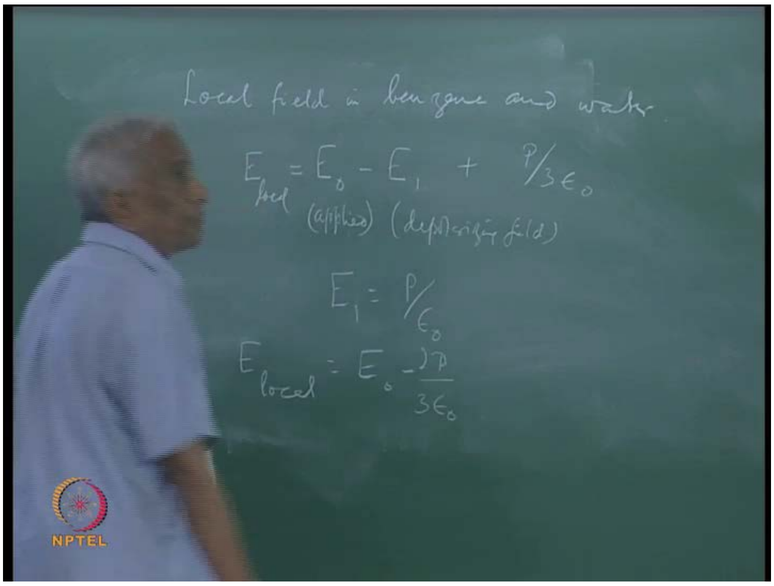

Field in benzene and water the local field is E naught he applied field minus the depolarizing field, this is the applied field is the depolarizing field plus the Lorentz interval. And he know that the depolarizing field P by epsilon naught therefore, the local is E naught minus P by 2 P by three epsilon naught substituting these values.

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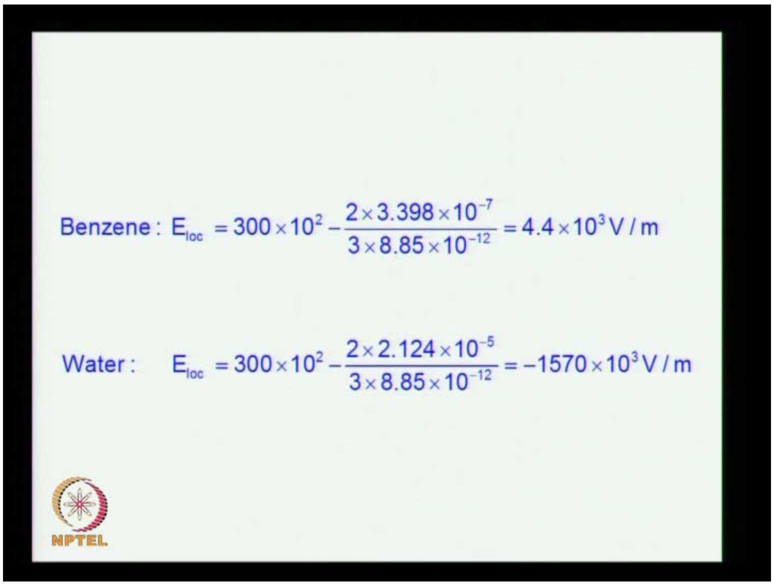

You find that for benzene the local field.

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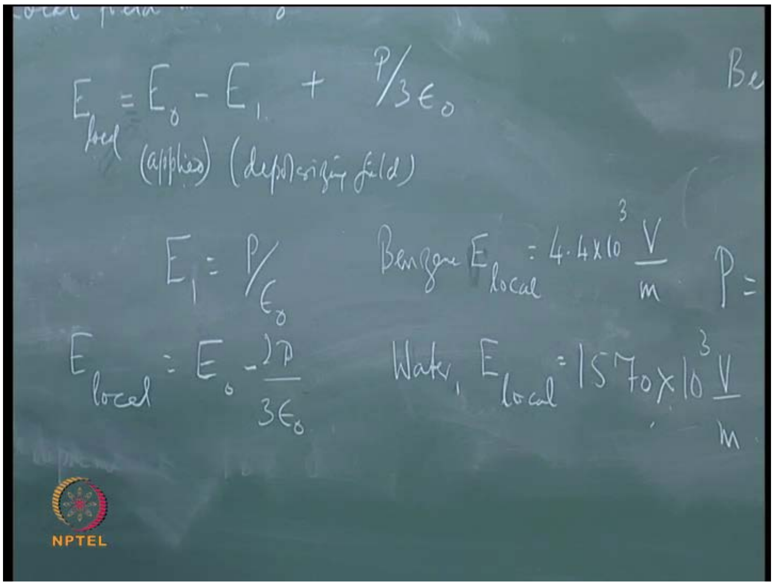

At a E local turns out to have the value 4.4 into 10 to the power three whole square meter whereas, water the same has the value of fifteen seventy into 10 to the power three. So, the internal field is much larger get the water.

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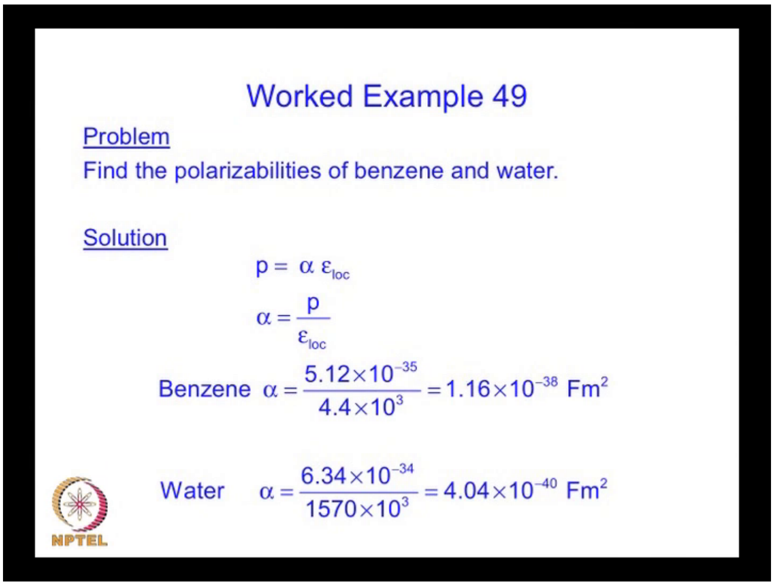

Therefore we can go further and calculate the polarizability a benzene and water.

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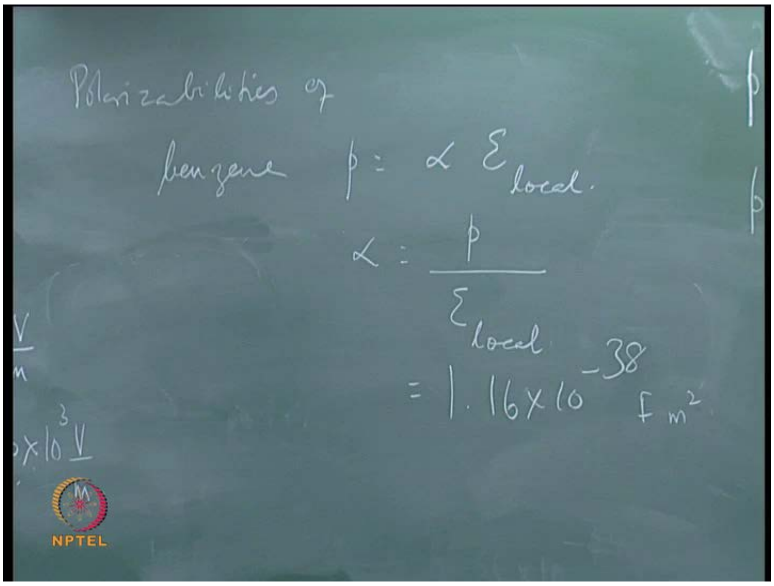

How do you review that you know that P is alpha E local. So, he has a local field therefore, the alpha is polarizability in dipole moment by the local electric field. So, this gives me the value. We have already calculated a dipole moment for it 3 1.16 10 to the power minus thirteen fare meter square minus situation yes were as for water this value is 4.04 10 to the power minus 40.

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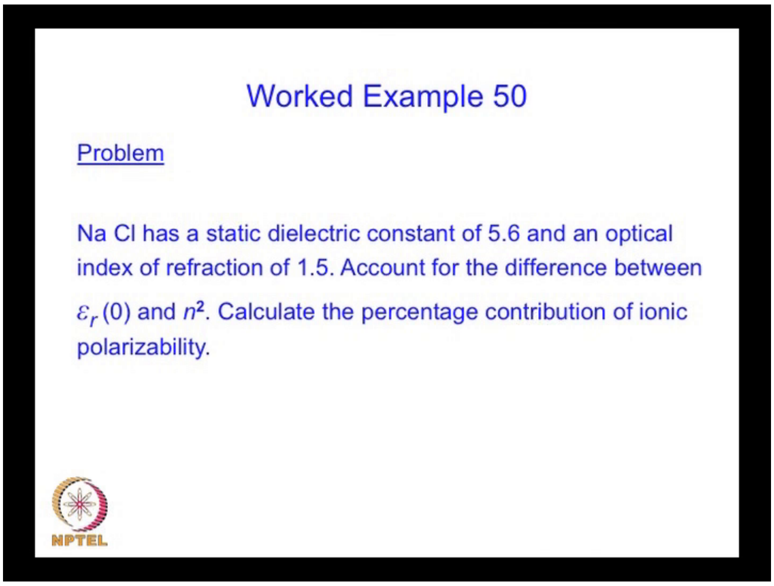

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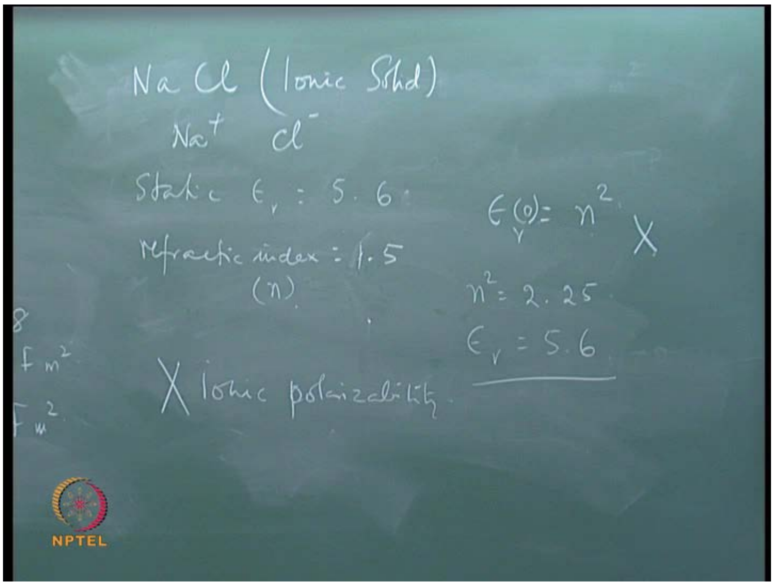

We now pass on to a different situation namely that of an ionic solid, which has sodium chloride is an ionic solid with N a-plus c l minus. So, it has static dielectric constant which is 5.6, but its optical refractive index is 1.5. We know that from electromagnetic theory epsilon r is N square square has a dielectric constant. So, if we take N n square in this case is 2.25 whereas, the static dielectric constant is much higher five point six. So, this is not satisfied this relationship if not satisfied. So, we are asked to account for the difference between epsilon r zero static dielectric constant and this square has a refractive index at optical frequencies.

Obviously, the difference is because the total polarizability has an ionic contribution due to the shift of the iron from their equilibrium position ironic polarizability, but at optical frequencies the time variations of the applied electric field in the light are too fast and the iron are unable to follow these variations. And therefore, the ironic contribution will fall out of the total contribution whereas, in the static case there is a full contribution which includes that of the ionic polarization.

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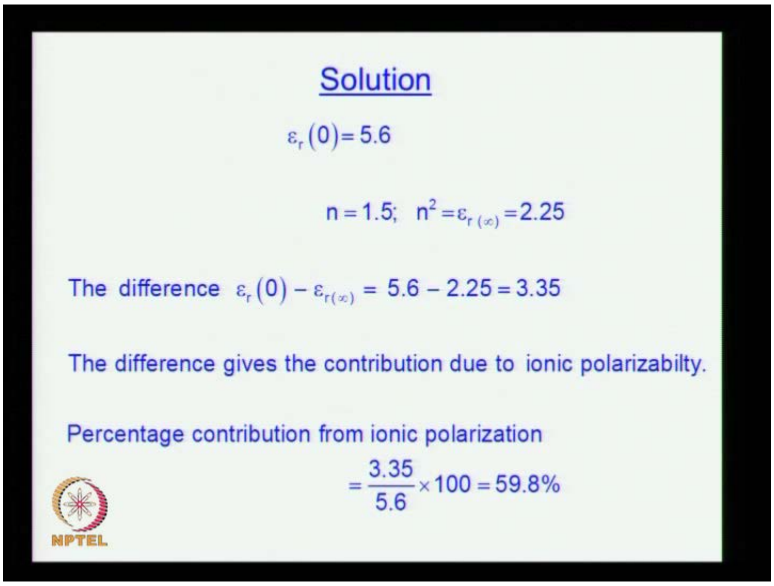

So, the difference is because of the missing ironic polarization.

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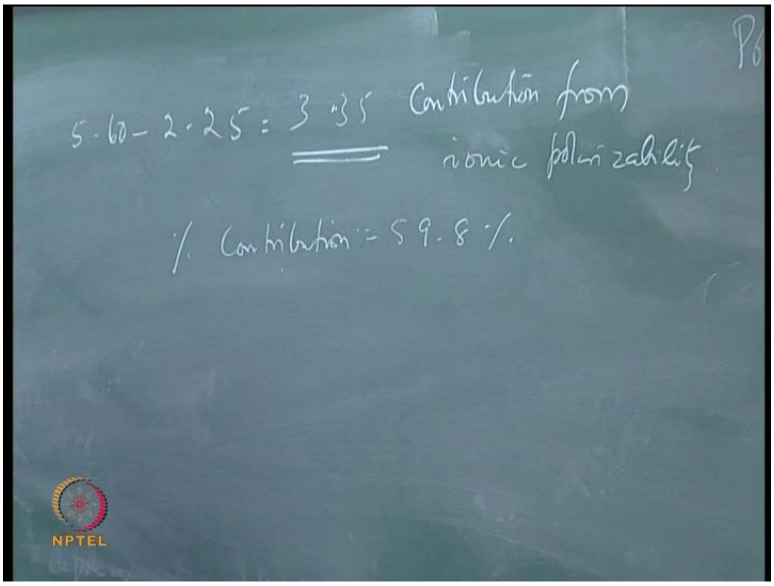

So, the difference between these two numbers a really 5.6 minus 2.25 3.35. So, that is the contribution from the ionic polarization, which is missing at the optical frequencies therefore, refractive dec falls from the square root of the static dielectric constant value. So, the percentage contribution turns out to be 59.8 percent.

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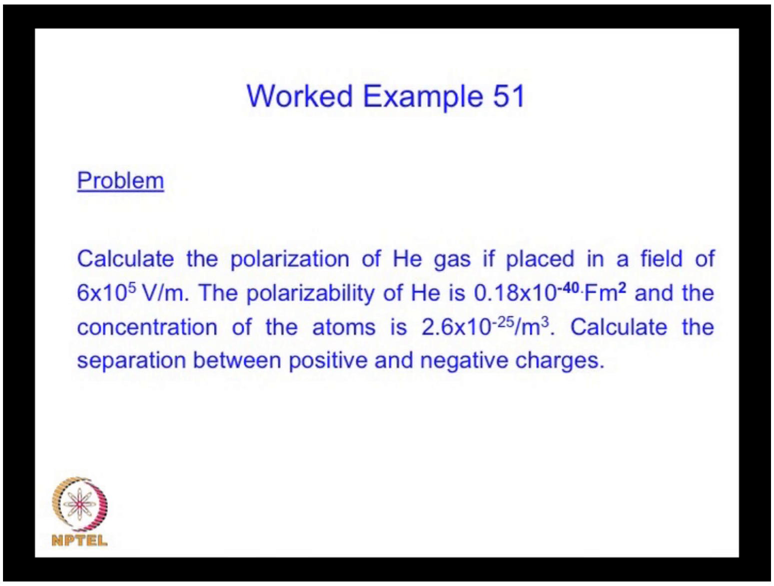

We go on to solve the example, which talks about helium gas helium as we all know is an enough gas.

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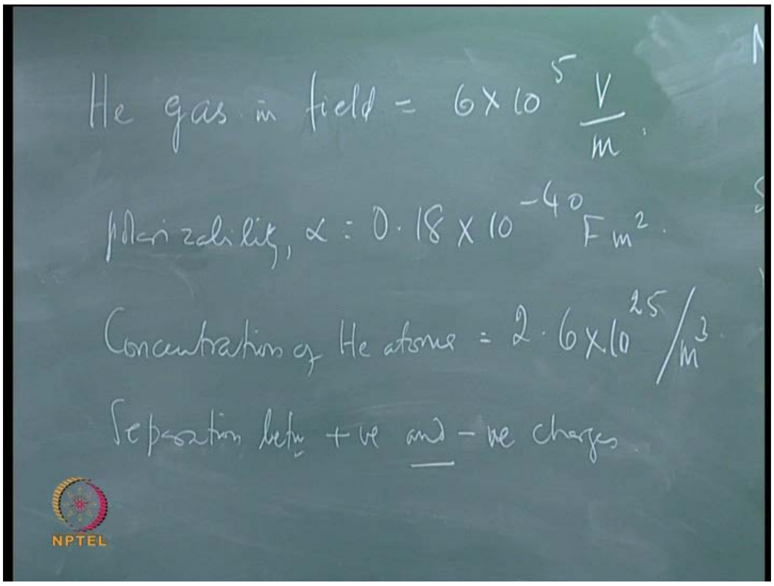

So, if it is placed in field of field is six into 10 to the power five whole square meter, we have told that the polarizability alpha is 0.18 into 10 to the power minus 40 meter square. And the concentration of the atoms, which is the number of atoms per unit volume helium atoms is two point six into 10 to the power twenty five per meter cube. So, we are asked to calculate the separation between positive. And negative charge namely the electrons and the nuclei in the helium atom in the presence of the applied electric field at least straightforward again.

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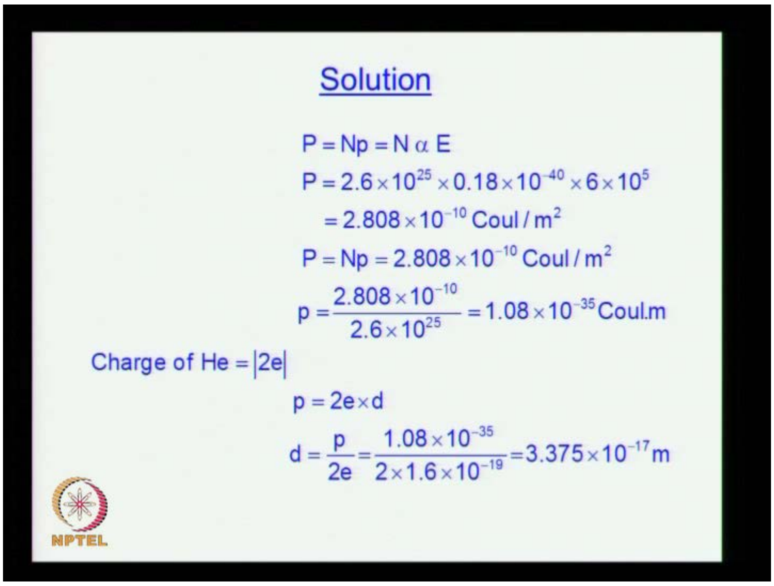

So, we are told that they have polarizability.

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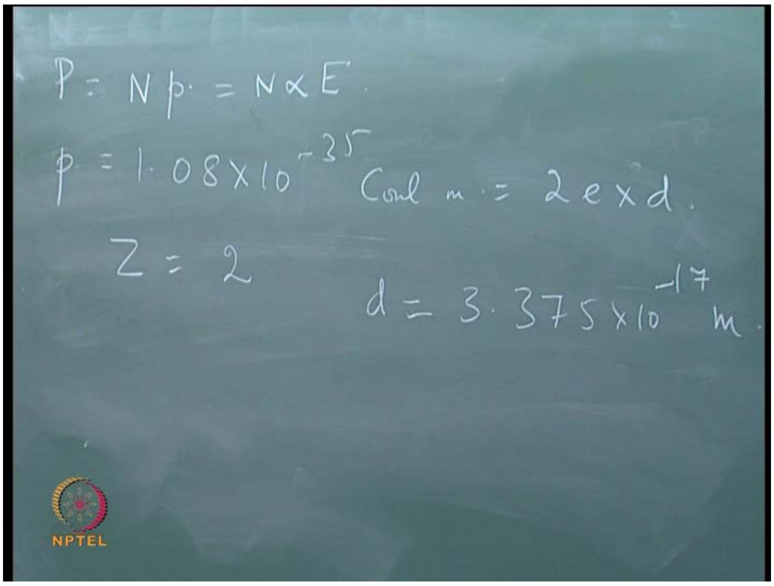

So, are given the number of atoms. So, P is N times P and this is N r by e. So, we are given alpha there are given e. So, it is of simple substitution problem. So, get P as one point eight 10 to the power minus 35 coulomb per meter and the charge of the helium atom is given by the atomic number of two. Therefore, this is two E to d where d is the separation. So, we know the electronic charge for substituting separation between positive and negative charge here turns out to be an extremely small quantity minus seventeen meters is something like 10 to the power minus four Fermi.

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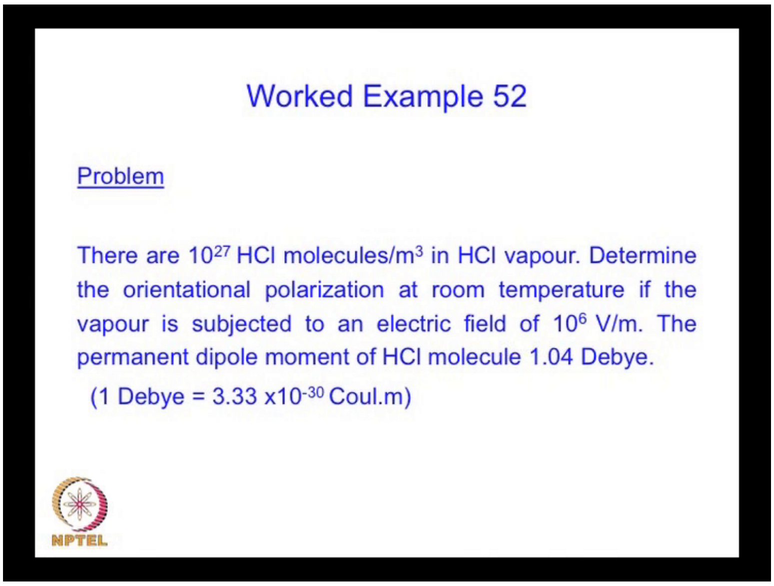

We go to another example of a dielectric namely hydrogen chloride H cl gas.

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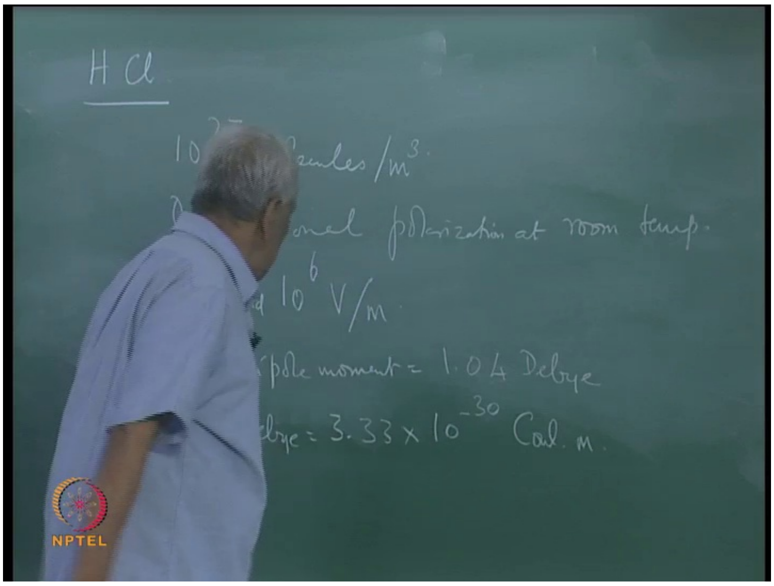

So, we are told that there are 10 to the power 27 H cl molecule per meter cube in h c l vapour, and we are asked to calculate the orientation and polarization at room temperature in an electric field of 10 to the to the power 6 holes per meter helium vapour a subject to this electric field. And they are also told the permanent electrical dipole moment of H cl is 1.04 debye one debye is one point three point three three into 10 to the power minus thirty coulomb meter is the conversion. So, rest if it is a situation problem in the standard relation for the orientation and polarizability.

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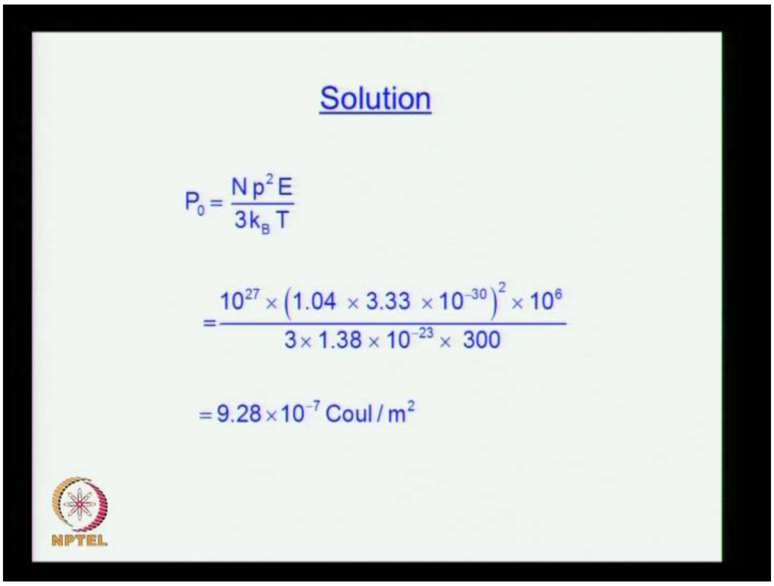

So, the orientation polarizability is given as N P square by 3 k B T times E. So, plugging in all these number.

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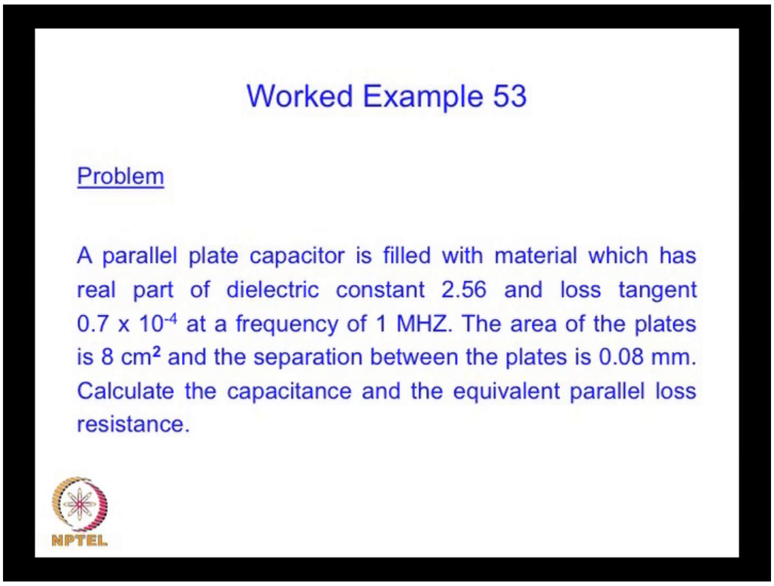

Next question concerns the frequency-dependent of dielectric constant, we are told that the there is parallel plate capacitor.

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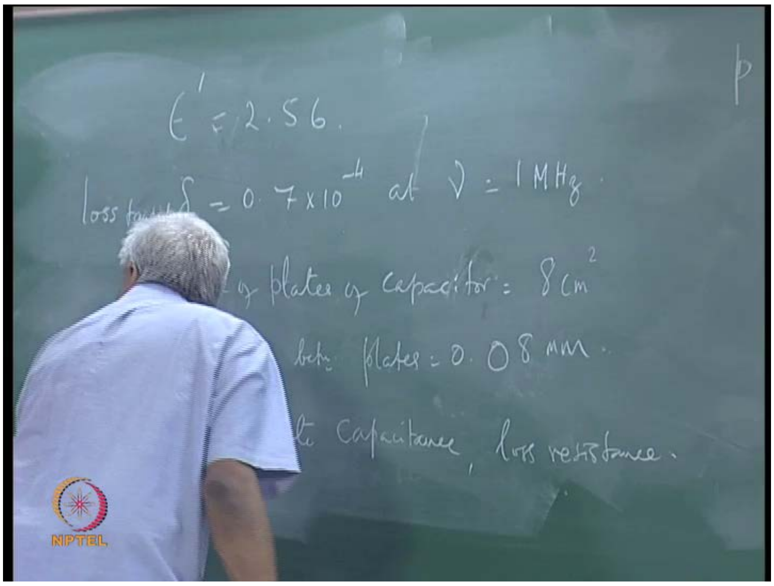

And it is filled with substance, which has the real part of the dielectric constant is two point five six and it has a last tangent delta of point seven into 10 to the power minus four the frequency of one megahertz. We are also told that the area of the plates of the capacitor is eight centimeter square in the separation between plates this point zero eight millimeter. So, we are asked to calculate the capacitors and is a lassie capacitor loss resistance.

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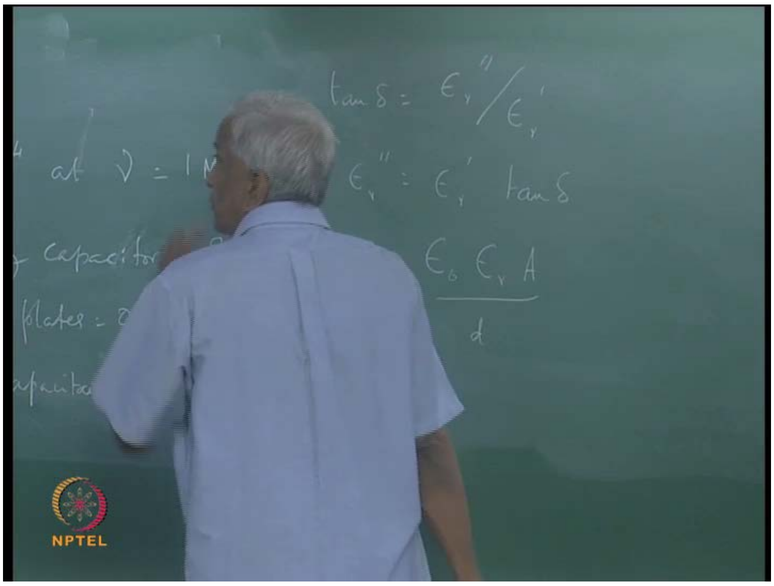

So, we have tan delta is epsilon r double prime divided by epsilon r prime in the definition of the last tangent therefore, from this we can substitute for epsilon r prime when find epsilon r double prime. So, that will be the tan delta is given as point seven into 10 to the power minus four, and the capacitance is epsilon naught epsilon r a by b. So, substituting all the values it turns out to be into 10 to the power minus 10 farrar and the last current can be readily shown to be omega epsilon r double prime.

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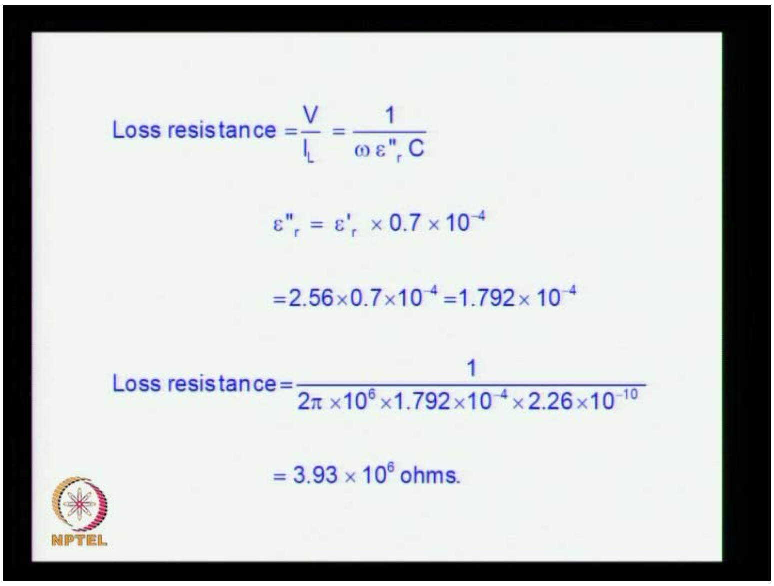

So, because a, this we can readily find the loss resistance v by I L this is i l.

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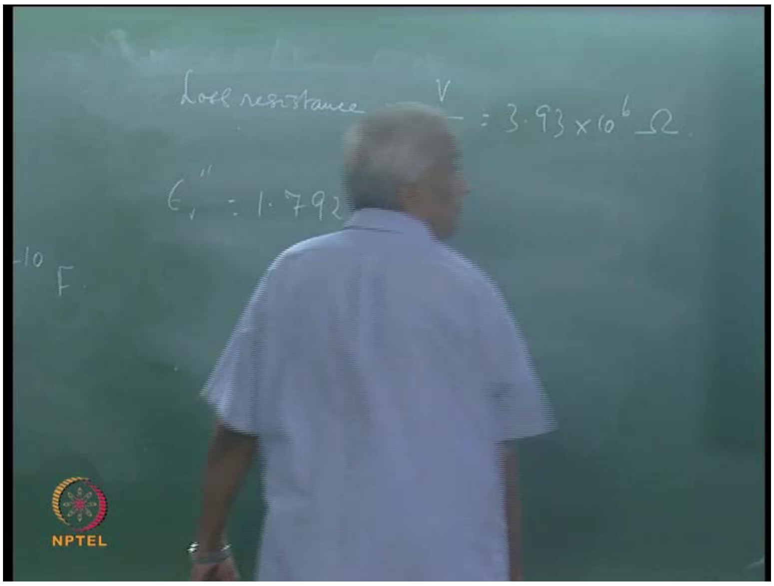

So, we get epsilon r double prime from this relation as 1.792 into 10 to the power minus 4, and therefore substituting these values we get loss resistance s 3.93 into 10 to the power 6 and capacitance is already calculated.