Condensed Matter Physics
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Lecture - 16
Dielectric (Insulating) Solids

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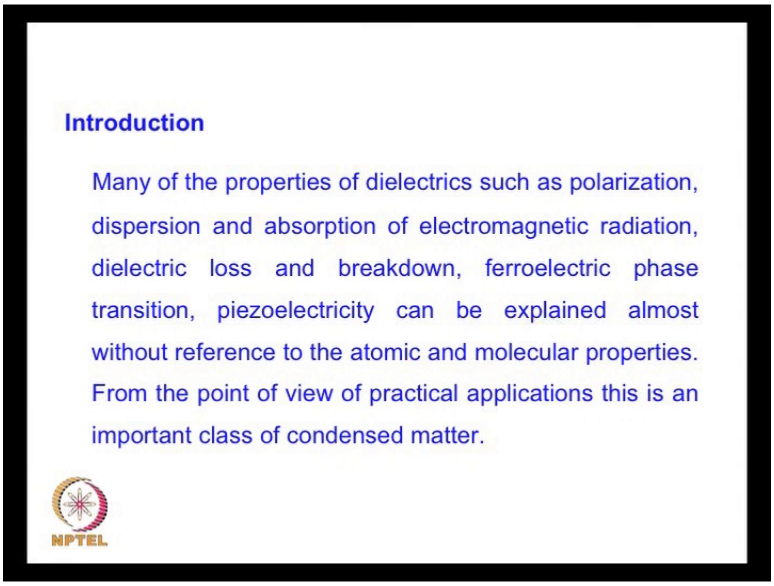

Introduction

Many of the properties of dielectrics such as polarization, dispersion and absorption of electromagnetic radiation, dielectric loss and breakdown, ferroelectric phase transition, piezoelectricity can be explained almost without reference to the atomic and molecular properties. From the point of view of practical applications this is an important class of condensed matter.

Today, we will to talk about insulators are dialectics now many of the properties have to dialectics such as electric polarization dispersion, and absorption of electromagnetic radiation by insulating materials dielectric loss dielectric break down, ferroelectric phase transition, piezoelectricity etcetera can be explained almost without reference to the atomic and molecular properties.

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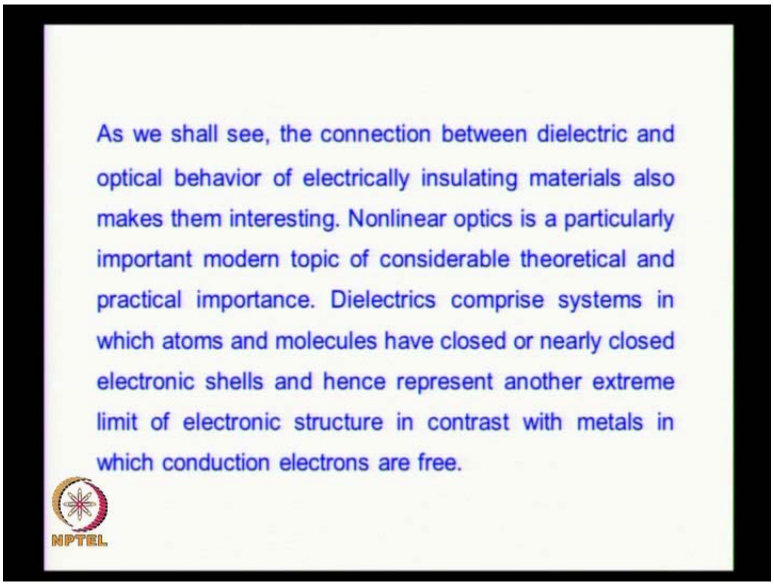

As we shall see, the connection between dielectric and optical behavior of electrically insulating materials also makes them interesting. Nonlinear optics is a particularly important modern topic of considerable theoretical and practical importance. Dielectrics comprise systems in which atoms and molecules have closed or nearly closed electronic shells and hence represent another extreme limit of electronic structure in contrast with metals in which conduction electrons are free.

And the point of view of practical applications, they insulators are dielectric constitute an important class of condensed matter. The connection between dielectric and optical behavior as we will see of such electrically insulating materials also makes them interesting from the point of view of modern areas such as non-linear optics which is a particularly important modern topic of considerable theoretical and practical importance dielectric comprise systems. In which atoms and molecules have closed on nearly closed shells structure for electrons. And therefore, they represent another extreme limit of electronics structure in contrast with metals in which conduction electrons are free.

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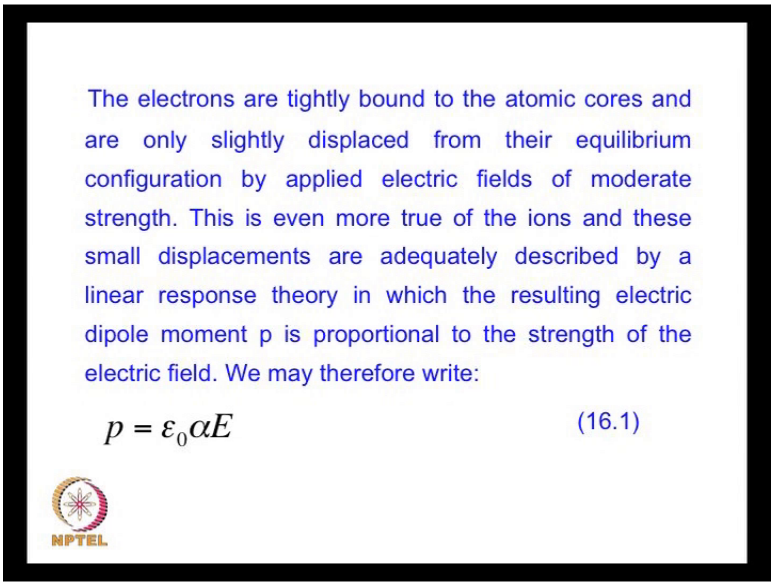

The electrons are tightly bound to the atomic cores and are only slightly displaced from their equilibrium configuration by applied electric fields of moderate strength. This is even more true of the ions and these small displacements are adequately described by a linear response theory in which the resulting electric dipole moment p is proportional to the strength of the electric field. We may therefore write:

$$p = \varepsilon_0 \alpha E \qquad (16.1)$$

The electrons in dielectric materials are tightly bound to the autonomic course and are only only slightly displays from that equilibrium configuration by applied electric fields of moderate strength, this is even more true of ions and these smaller displacement are adequately describe by a linear response theory.

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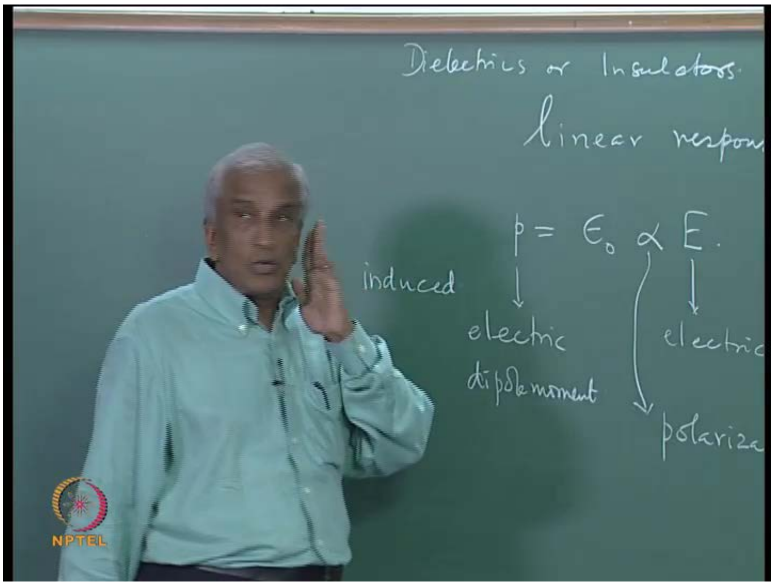

In which the linear response theory assumes that the electric de poll movement which results from the application of a in electric field of moderate strain it is proportional to this strength of the electric field therefore, if p is electric dipole moment. We can write p equals epsilon naught alpha e now here e is the electric field p is electric dipole moment and alpha is the constant of proportionality is known as polarizability. The atom are molecule that is a constant which is measured in units of epsilon naught the permittivity a free space. So, this p is the induced dipole e moment induced by he applied electric field. So, dielectric polarization this is the individual dipole moment induced per atom molecule.

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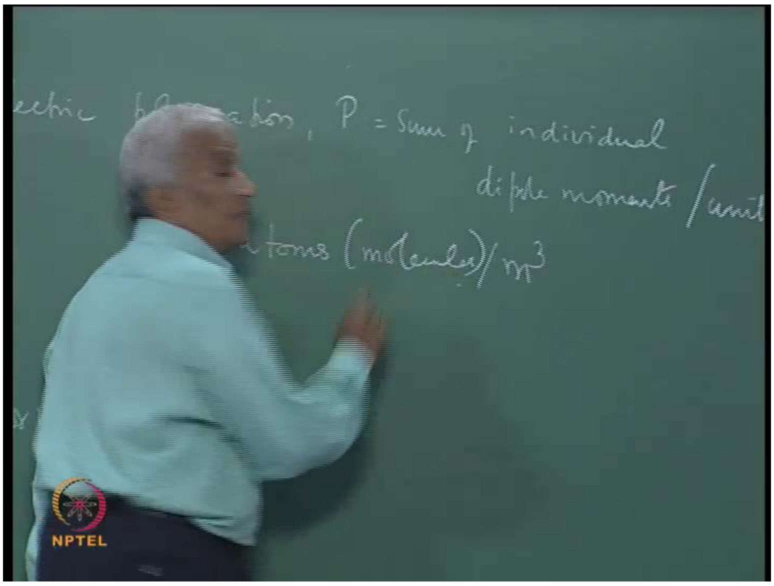

Dielectric polarization capital p is the vector sum of the individual dipole moments some of individual dipole moments and usually defined per unit volume of the material. So, if there are n such atoms are molecules n atoms or molecules per meter cube.

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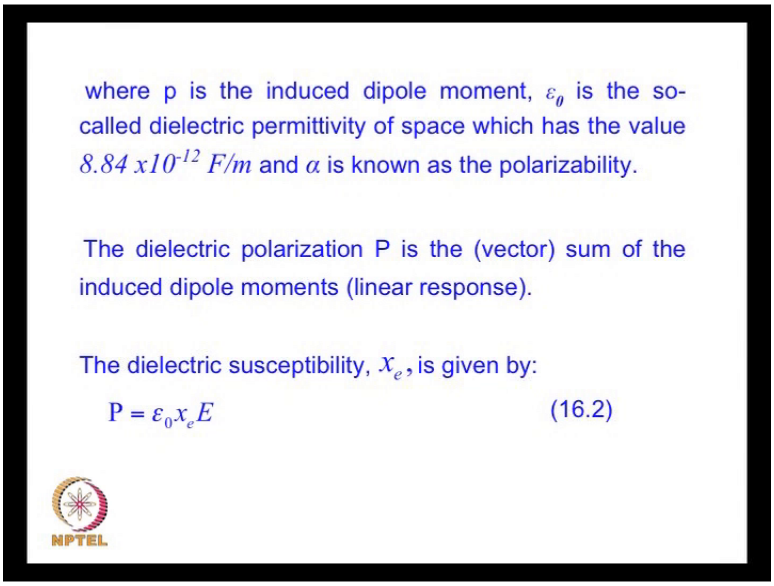

where p is the induced dipole moment, ε_0 is the so-called dielectric permittivity of space which has the value $8.84 \, x 10^{-12}$ F/m and α is known as the polarizability.

The dielectric polarization P is the (vector) sum of the induced dipole moments (linear response).

The dielectric susceptibility, x_e, is given by:

$$P = \varepsilon_0 x_e E \qquad (16.2)$$

So, and then polarization is also related to the applied electric field by the so-called electric susceptibility therefore, combining these relations we get immediately x i.

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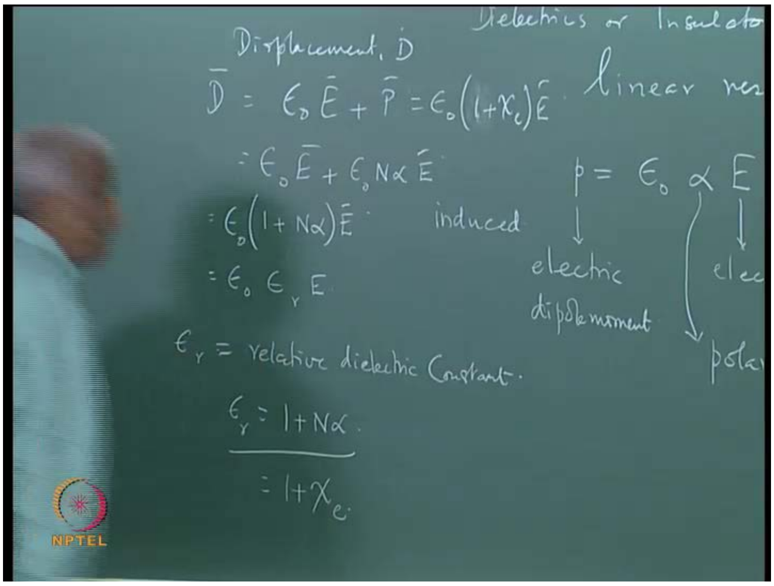

Dielectrics or Insulato

Displacement, D

$\bar{D} = \epsilon_0 \bar{E} + \bar{P} = \epsilon_0 (1+\chi_e)\bar{E}$

$= \epsilon_0 \bar{E} + \epsilon_0 N\alpha \bar{E}$

$= \epsilon_0 (1 + N\alpha)\bar{E}$ induced

$= \epsilon_0 \epsilon_r E$

ϵ_r = relative dielectric constant.

$\epsilon_r = 1 + N\alpha$

$= 1 + \chi_e$

linear res

$p = \epsilon_0 \alpha E$

↓ electric dipole moment

elec

pola

There is an important vector call the displacement D which is be defined by d equals epsilon naught e plus p. Therefore, plugging our epsilon naught e plus p is epsilon epsilon naught n alpha e therefore, I have epsilon naught times one plus n alpha times e and this is usually defined as epsilon naught epsilon naught e where epsilon r is the relating dielectric constant. So, that we arrive at epsilon r equal to one plus n alpha and and we also known that p is x I e epsilon naught x I e. Therefore, we can write here epsilon naught xi e one plus x I e times e. So, comparing this and this we also get one plus x I e.

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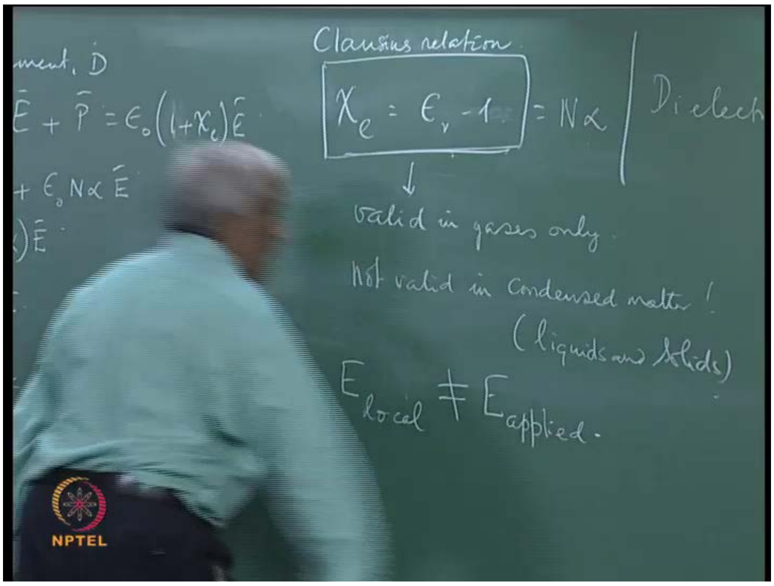

ment, D

$\bar{E} + \bar{P} = \epsilon_0 (1+\chi_e)\bar{E}$

$+ \epsilon_0 N\alpha \bar{E}$

$)\bar{E}$

Clausius relation

$\boxed{\chi_e = \epsilon_r - 1} = N\alpha$

Dielech

↓ valid in gases only.

Not valid in condensed matter! (liquids and solids)

$E_{local} \neq E_{applied}$.

Therefore we get x e is equal to epsilon r minus one of course, this is relation which is valued only in gases. In this case, we as assume that will that we are applying here is the same as the field is seen by the atom or molecule this assumption is not valid not valid in condensed matter such as liquids and solids. So, this relationship between the electric ability and the dielectric constant, why are these so, x e also equal to n alpha. So, this has been verified a great extent many gas, but in a condensed material like a solid dielectric this assumption this relation is not valid this is known as the Clausius relation this is not valid because they dipoles which are present the atomic or molecular dipole interact themselves. There is dipole interaction, which produce internal electric fields. So, that local electric field seen by a solid at or liquid atom or molecule is different from the field which is actually apply, so these local field defers. So, e local is not equal to e applied that is the reason why this relationship is not valid how does it differ?

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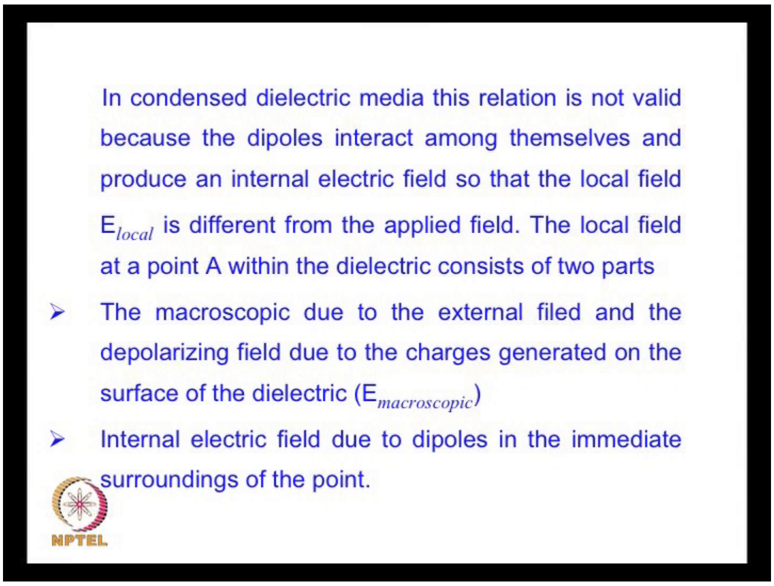

In condensed dielectric media this relation is not valid because the dipoles interact among themselves and produce an internal electric field so that the local field E_{local} is different from the applied field. The local field at a point A within the dielectric consists of two parts

- The macroscopic due to the external filed and the depolarizing field due to the charges generated on the surface of the dielectric ($E_{macroscopic}$)
- Internal electric field due to dipoles in the immediate surroundings of the point.

Now, the local the local electrical field as two contributions, let us look at the process of polarization carefully.

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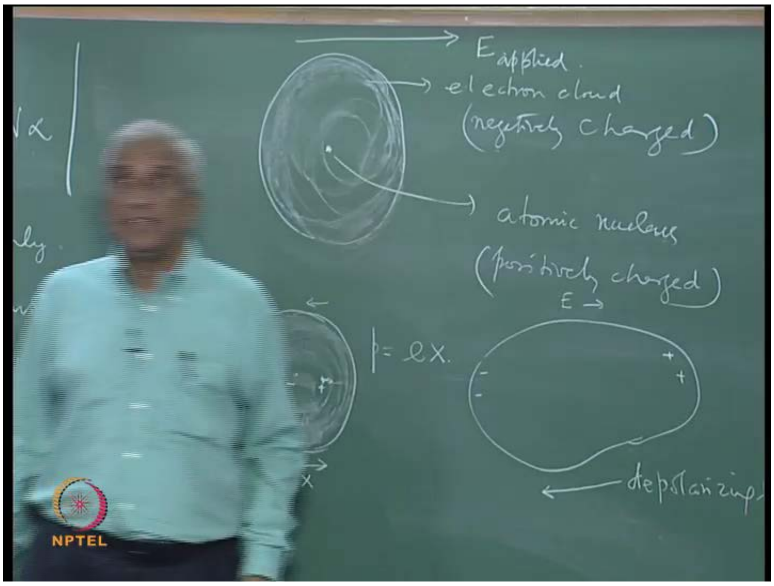

So, I have an atom in which there is an electronic clouds uniform charge density of electron and there is a central nucleus now this is a spiracle atom we are assuming it to spherical atom to keep the discussions simple such that centre's of positive and negative electric charge is the electron clouds negatively charged and this is the atomic nucleus which is positively charged and these two are equals wire. So, that is atom or molecule is electrically neutral to start with the centre of the negative charge distribution coincides with the central positive atomic charge.

Therefore, negative and positive charge that coincide and therefore, there is no dipole moment to start with, but when you apply electric field say this along this direction then the the nation electronic clouds moves away. So, this is the way the electronic clouds is displaced in this direction from the original position we assumed that it is a rigid displacement without any distortion. So, that this moves like this and the positive nucleus moves a little bit this is the nuclear charge.

So, this is displays to much smaller extent because the nucleus is much more massive therefore, it get displays to a much smaller extent, but still what is happening is the central of the negative charge and deposited charge here there is the relative displacement and this is what gives you a net dipole moment which is induced, and it is present only as long as the electric field this existing the moment you remove the electric field is dipole moment will also vanish because the everything will relax back to the original situation. So, this is really induced dipole moment. So, this is the mechanism of polarization. So, what really happened is that you have in a microscope dielectric you have positively charged going in one way and the negative charge is going in another way in the presence of an applied electric field. So, this is what gives you a net electric polarization, but because of this displacement there is also an electric field due to the presence or the positive and negative charges here. So, that gives you an internal electric field which depolarizes the original field because it is acting in the opposite direction from the positively the negative charge. So, this is the direction of the de polarizing field.

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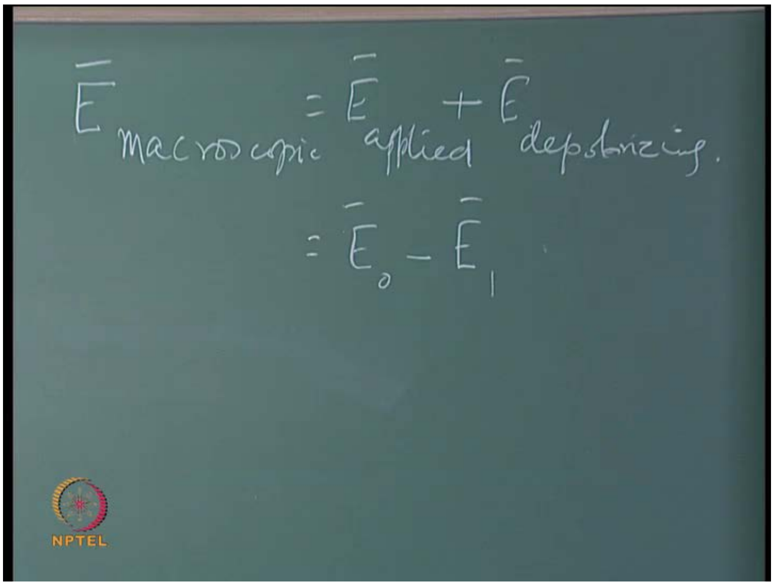

$$\bar{E}_{macroscopic} = \bar{E}_{applied} + \bar{E}_{depolarizing}.$$
$$= \bar{E}_0 - \bar{E}_1$$

So, the macroscopic electric field due to the applied field and de polarizing field duty the charges, which have been produced on the surface. And the dielectric that gives you a net macroscopic electric field which is the sum of the e applied vector sum and the de polarization, but in addition to that there is a interaction among the d I polls as I told you which produces another contribution to this internal local field. Now this therefore, let us called it e naught since this is opposing the applied field let us called it e 1. If you have an ellipse solid specimen, if you are dielectric is in the shape of an ellipse solid, this de polarizing field can be shown to vanish. But in all other geometries in the spicemen shape is different from a ellipse solid Debye de polarizing field is not vanishes, but even when it is not zero it can be determine for standard geometries such as medical specimen or a long thin brought and so on. So, we can find the de polarizing field and add iteratively to him apply a electric field to obtain the macro specific electric field.

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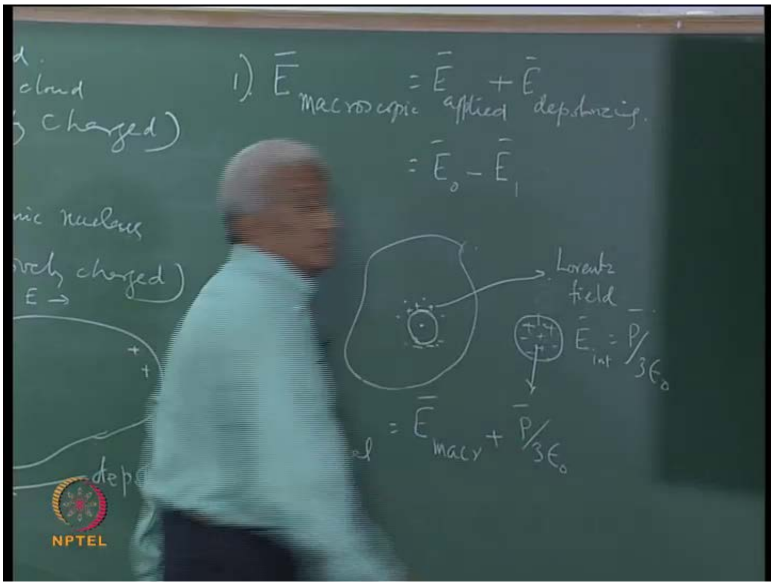

So, this is the first part the second part is due to the dipoles which are in the immediate neighborhood of suppose you have a dielectric and you have a given atomic dipoles sum were here it its immediate neighborhood there are other dipoles. So, these are all different dipoles. So, these dipoles interact with this dipoles and therefore, we have de pole de pole interaction which produce and internal electric fields this situation can be considered quantitatively by considering spherical cavity we assume that we scot out dielectric materially of course, a spherical plug and their four now you have charges on this surface this cavity.

So, these charges on the surfacewi9ll produce an electric field and then you have scooped out this material, but it within this cavity there are dipoles. So, these produced an internal electric field this electric field is known as the Lorentz field who calculated its value and it can be shown that this field is e internal is p by three epsilon naught for sphere and then this this is due to the surface charge. And in addition this, field due to the charges in this spherical cavity can be shown to be zero for a cubic material. So, we have all these various contribution and the macroscopic electric field e local the local electric field is e macroscopic p by three epsilon naught. So, it is this local electric field which is seen by the atom or molecule the dipole and therefore, this is what we have two here right here.

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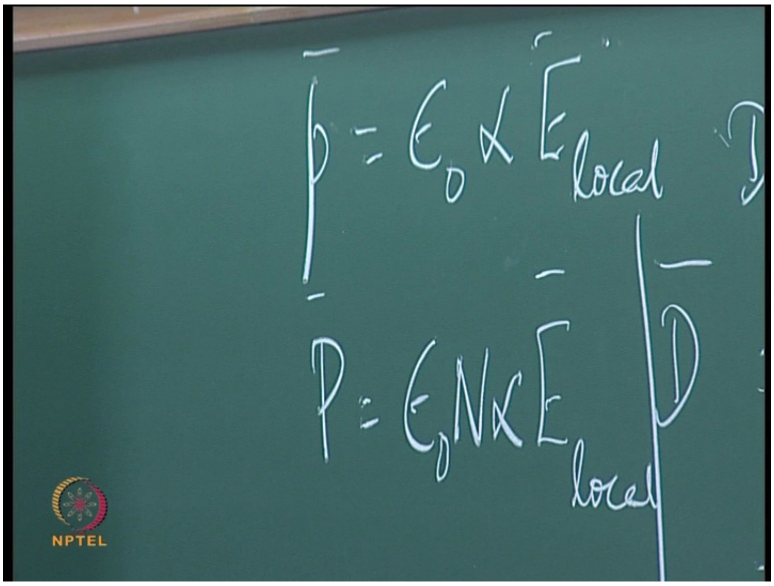

So, the individual dipole is now dipole moment is epsilon naught alpha e local where e local is given by this therefore, correspondingly p also will change accordingly. So, p is n n alpha e local. So, let the substitute this value of pin c what we get.

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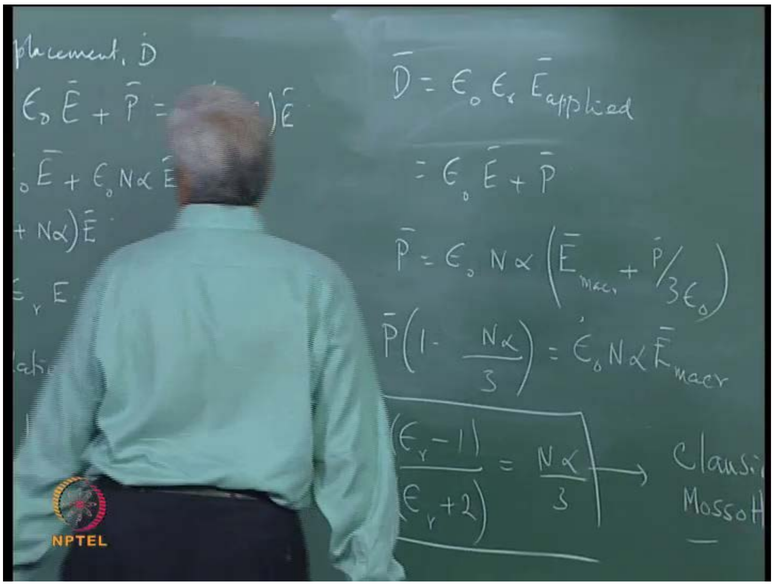

So, we will get d is epsilon naught epsilon r e applied and this is equal to epsilon naught e plus p. So, this will give me a relation p will be now epsilon naught n alpha into e local which is e macroscopic plus p three epsilon naught. So, collecting the p terms there is a together p times one minus epsilon naught n alpha by three epsilon naught cancels equals epsilon naught alpha e macroscopic. So, because of this we get a modification in the clauses relation between dielectric constant. All these are and the and this therefore, we can show straightforward to show that epsilon or minus one by epsilon on plus two equals n alpha by three this is the relation which replaces the Clausius relation for a gas the case of a solid or liquid dielectric this relation is known as the Clausius Mossotti equation. So, the presence of internal electric field produces this term in the denominator that is all this is change.

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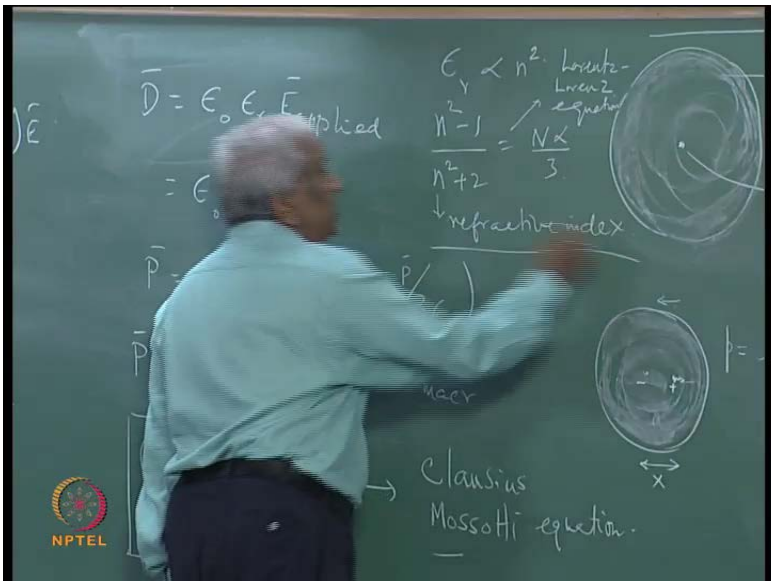

And since the relative dielectric constant is proportional to the squire or therefore, we can also write this relation the Clausius relation in this form where n is the refractive index say an optical frequencies. So, in this form this is known as the Lorentz equation. So, these are the basic equations which govern the relative dielectric constant in the refractive index in the case of condensed matter in the case of solids, so this situation.

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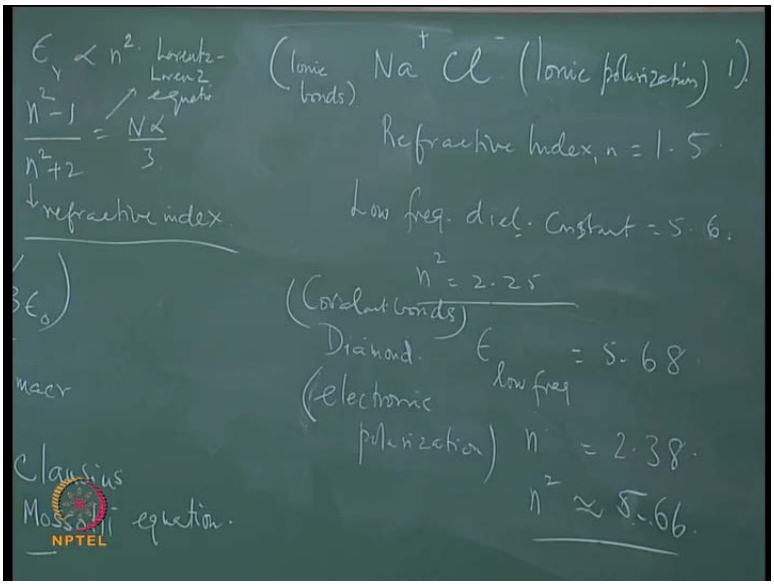

$\epsilon_r \propto n^2$ Lorentz-Lorenz equation

$\frac{n^2-1}{n^2+2} = \frac{N\alpha}{3}$

↓ refractive index

$3\epsilon_0$)

macr

Clausius Mossotti equation.

(Ionic bonds) $Na^+ Cl^-$ (Ionic polarization) 1)

Refractive Index, n = 1.5

Low freq. diel. Constant = 5.6.

$n^2 = 2.25$

(Covalent bonds) Diamond. $\epsilon_{low\ freq} = 5.68$

(electronic polarization) $n = 2.38$

$n^2 \approx 5.66$

So, for example, if you take a material like sodium fluoride common salt, so this as a refractive index n which is one point five whereas, this low frequency dielectric constant are starting dielectric constant is 5.5, 5.6. So, if you squire the refractive index n squire it is only 2.25, so in going from low frequency career-high to a frequency the value of dielectric constant as comedown. Whereas, the case of material diamond in which the bonds are stronger therefore, the low frequency dielectric constant low frequency dielectric constant is five point six eight the refractive index at optical frequencies two point two three eight. So, squire is almost close to the epsilon. So, there is not. So, much difference n squire is 5.66. So, you can see that in the case of diamond polarization mechanism is entirely electronic the electrons contribute polarization whereas, in the case of sodium chloride you have sodium ions and clarian ions forming alternate alternatively occupying a cubic lattice. So, these ions also have dipole moments. So, there is an ionic polarization in addition to the electronic polarization only.

So, these are all readily how the electrons this start to produce give rise to here dipole moment that is the electron is polarization mechanism. So, we have two different polarization mechanism one of electron due to the displacement and the electronic cloud and the other it is due to displacement at the positive and negative ion as in allowing. So, when both are simultaneously present then the ions being massive are not able follow the frequency variations such as high frequencies therefore, ionic polarization drops of become decreases as the frequency increase.

So, hard high-frequency the electronic contribution alone remains whereas, at low frequencies both the ionic and electronic contribution give rise to a large dielectric constants static dielectric constant in the case of diamond it is exclusively electronic polarization there are no ions there is a covalent solid covalent bonds in the case of diamond whereas, here these are ionic bonds. So, when there are covalent bond it is only electronic polarization which contributes to the polarization, dielectric polarization and hence it is the dielectric constant when you consider material like water water as in addition to these the water molecule as a very large dipole moment ordinary water.

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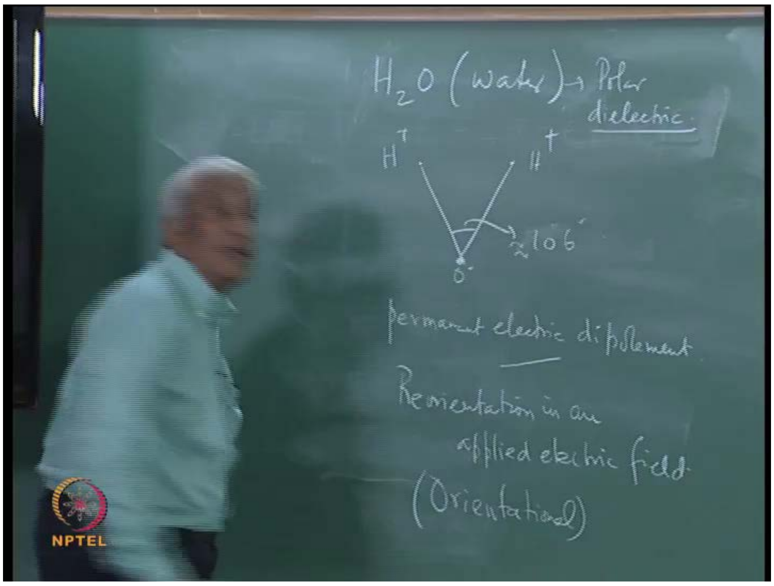

So, it has an oxygen and two hydrogen’s. So, this is the structure as the water molecule. So, the o h o bound angle is approximately tetrahedron something like hundred six degree approximately. So, this has a net permanent dipole moment it is because to have the positive and negative charges do not coincide and therefore, there is a net dipole moment even in the absence of an electric dipole electric field unlike the case of the spherical atoms which we saw in the case of electronic cloud.

So, this is known as the polar molecule poll dielectric which is an inherent non vanishing dipole moment and these dipoles due to the water molecules tend to reorient themselves in an applied field. So, reorientation in an applied electric field. So, this gives you in addition to the ionic and electronic contribution this gives us an additional orientation all contribution to the polarization and then and the dielectric constant.

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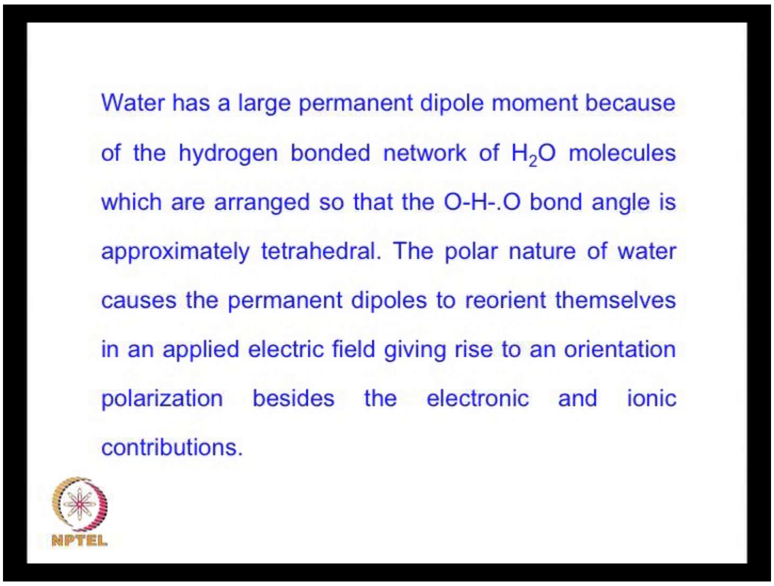

Water has a large permanent dipole moment because of the hydrogen bonded network of H_2O molecules which are arranged so that the O-H-.O bond angle is approximately tetrahedral. The polar nature of water causes the permanent dipoles to reorient themselves in an applied electric field giving rise to an orientation polarization besides the electronic and ionic contributions.

So, you have in all three contributions arising from different mechanism to the net direct polarization and Hensley dielectric constant.

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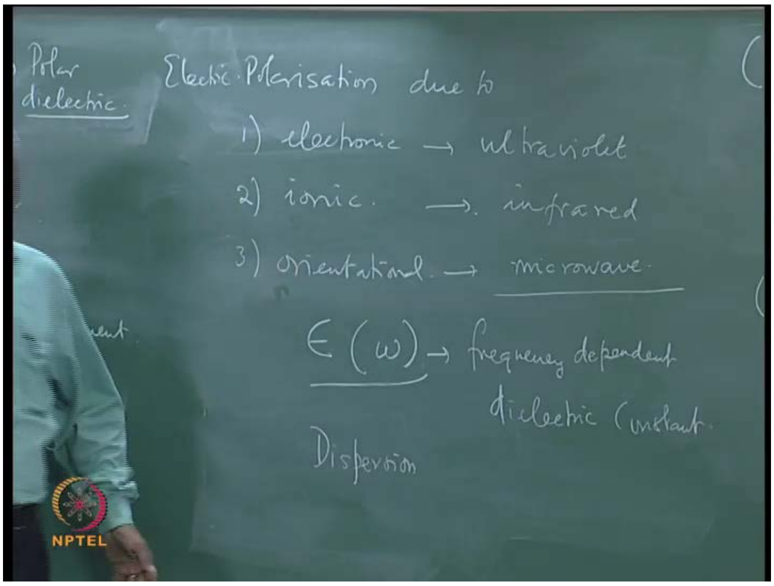

Polar dielectric.

Electric Polarisation due to

1) electronic → ultraviolet
2) ionic. → infrared
3) orientational → microwave

$\underline{\epsilon(\omega)}$ → frequency dependent dielectric constant.

Dispersion

What are the polarization electric polarization due to one an electronic polarization two and ionic contribution and three orientation all contribution in general all three will be present for example, in the case of the water molecule, but when there are no permanent dipole moments the electric field induces only electronic ionic contributions the ionic contribution is there is only when there are ions which get displays and therefore, give you a polarization otherwise when there are no ions the structure like that of diamond the electric polarization mechanism is completely electronic.

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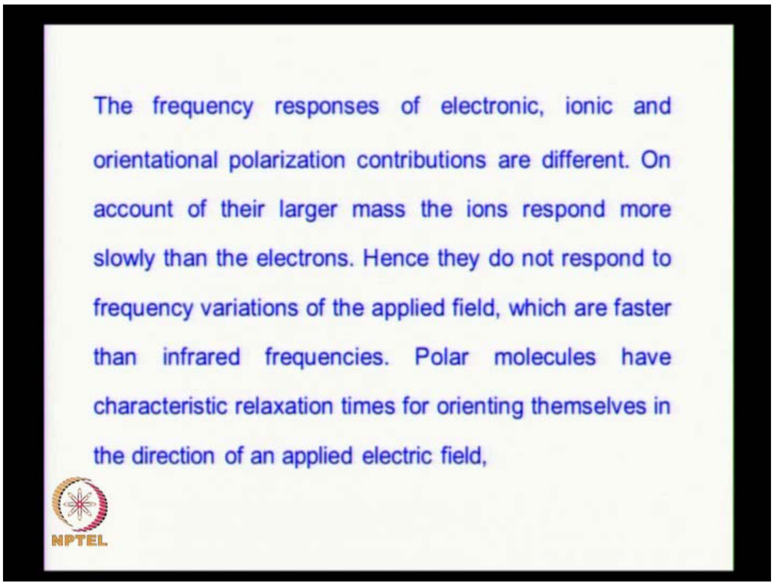

The frequency responses of electronic, ionic and orientational polarization contributions are different. On account of their larger mass the ions respond more slowly than the electrons. Hence they do not respond to frequency variations of the applied field, which are faster than infrared frequencies. Polar molecules have characteristic relaxation times for orienting themselves in the direction of an applied electric field,

So, these are even among dielectric there are different kinds of dielectric and since the electrons respond rapidly to even rapid frequency variation of the apply electric field the electronic contribution continues very high frequencies of the applied electric field. Therefore, ions being more massive they are unable to follow the rapid field variation of a certain frequency therefore, this ionic contributions are present only at will relatively low frequency similarly the orientation and the permanent dipole I gain happen at a relatively lower frequencies at the applied electric field.

So, the electronic contribution is usually the ultraviolet range ultraviolet frequencies the ionic contribution are in the infrared range and this is orientation contribution occurs the microwave frequency. So, as you change the frequency at the applied electric field and go through micro-wave infrared and ultraviolet ranges of frequencies these various contribution will give rise to polarization and n contribute to the dielectric constitute.

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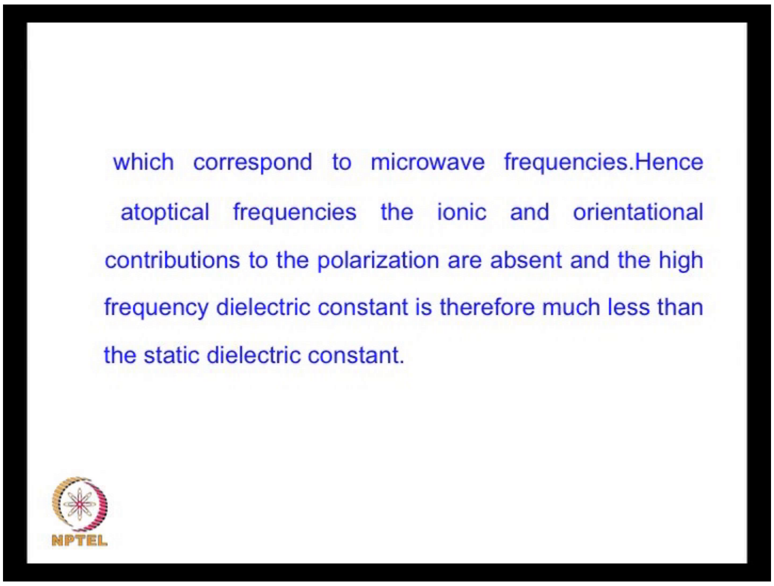

which correspond to microwave frequencies.Hence atoptical frequencies the ionic and orientational contributions to the polarization are absent and the high frequency dielectric constant is therefore much less than the static dielectric constant.

So, the dielectric constant is frequency dependent and it is frequency dependence which gave rise to a frequency dependent refractive index because epsilon disproportional to n squire and therefore, refractive index determines the phase velocity of electromagnetic wave in any given dielectric medium therefore, you can see that the phase velocity changes and this is what we know has dispersion. So, the phenomena of dispersion give rise to the arises from the frequency dependent of the dielectric constant.

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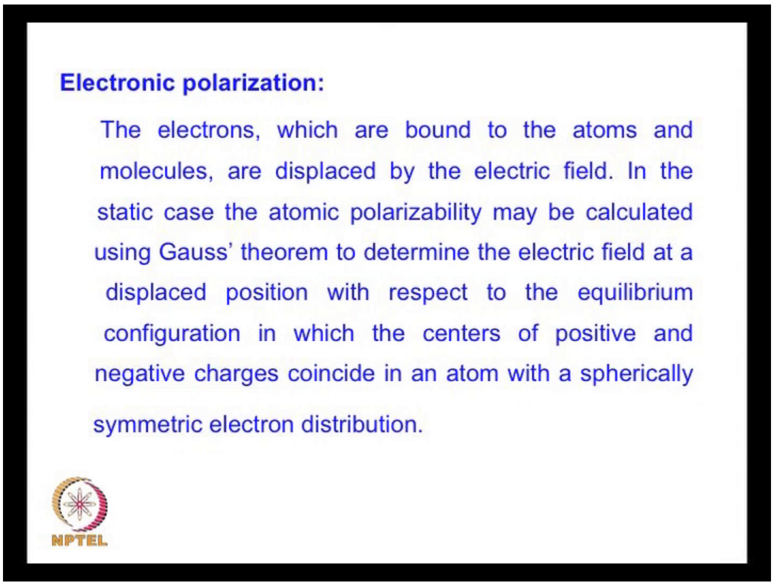

Electronic polarization:

The electrons, which are bound to the atoms and molecules, are displaced by the electric field. In the static case the atomic polarizability may be calculated using Gauss' theorem to determine the electric field at a displaced position with respect to the equilibrium configuration in which the centers of positive and negative charges coincide in an atom with a spherically symmetric electron distribution.

Now, let us consider this electronic polarization mechanism in a little detail when we already saw that the electrons in a dielectric material or bound to the parent atoms and molecules under displaced by the electric field. So, in the static limit when the electric field is a static electric field the atomic polarized ability can be calculated using cause theorem.

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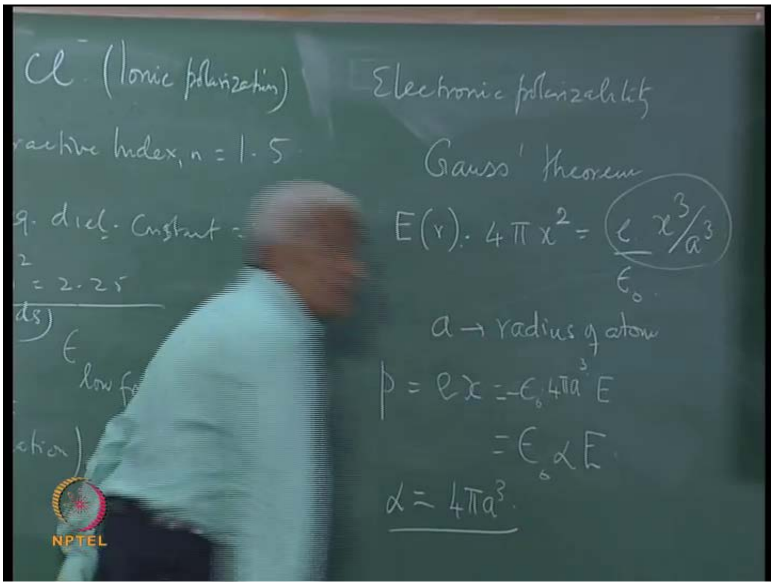

So, we assume a spherical atom and we apply cause theorem it is well-known in electro statics.

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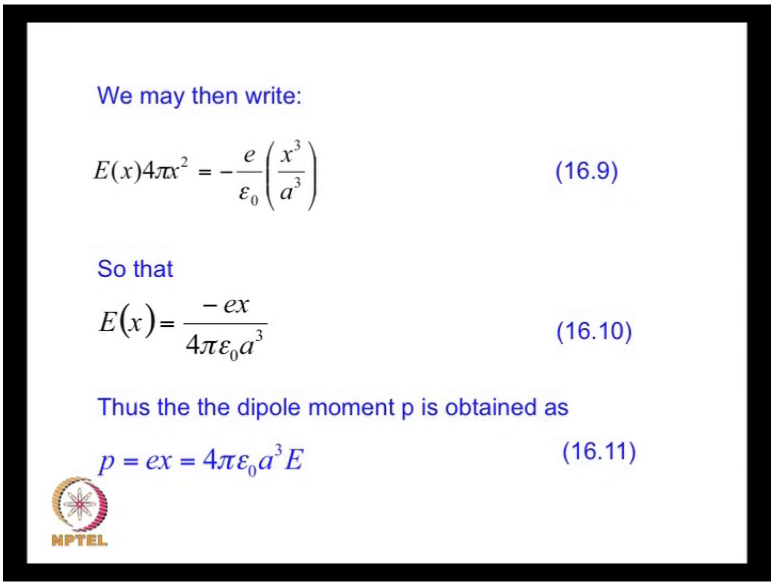

We may then write:

$$E(x)4\pi x^2 = -\frac{e}{\varepsilon_0}\left(\frac{x^3}{a^3}\right) \quad (16.9)$$

So that

$$E(x) = \frac{-ex}{4\pi\varepsilon_0 a^3} \quad (16.10)$$

Thus the the dipole moment p is obtained as

$$p = ex = 4\pi\varepsilon_0 a^3 E \quad (16.11)$$

So, if we consider gauss theorem we can write e f r for a spherical atom the electric field is radically directed that is why I am writing times four pi x cube where x is the relating displacement between the electron and the nuclear charges. Therefore, this will be this will be the electric field and this is gauss theorem giving you the flux duty the electric field and this is given by the charge divided by epsilon naught. Now if the charge electronic charges is distributed over and atom of radius a a is the radius at the atom. Then the electronic charge is sphere over a volume four third pi cube of which we want to calculate be fraction of this charge intersected by spherical region of radius x therefore, this will be fourth third pi x cube.

Therefore, that is a how you get the fourth third pi cancels leaving you this fraction this is the fractional as the electronic charge which is closed. And therefore, this gives me the e x as e e times epsilon naught into for pi a cube with a minus sign in order to show the moment as the negative sign the charges negative. So, the dipole moment this is the induced dipole moment. So, this gives me comparing it with epsilon naught alpha e where alpha is polarized ability we arrive at polarized ability as four pi a cube. So, the polarized ability at the atom a spherical atom is simply four pi times the cube of the atomic radius the polarizability is proportional to the atomic volume let us now consider we will consider in the next discussion what happens when we go from a static field to be a frequency dependency in applied alternating field we will consider this.