Condensed Matter Physics
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Lecture - 13
Debye Theory of Specific Heat, Lattice Vibrations – Worked Examples

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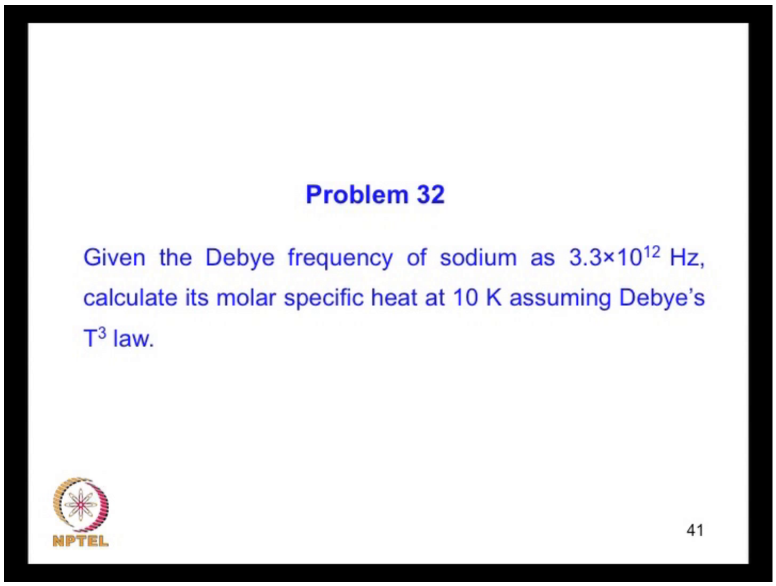

In the Debye theory of Debye frequency of sodium is given as 3.3 into 10 to the power 12 hertz.

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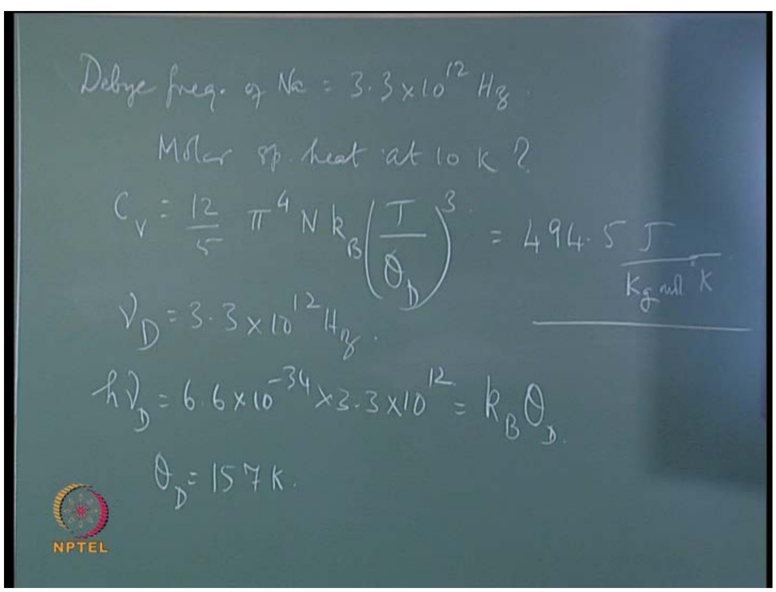
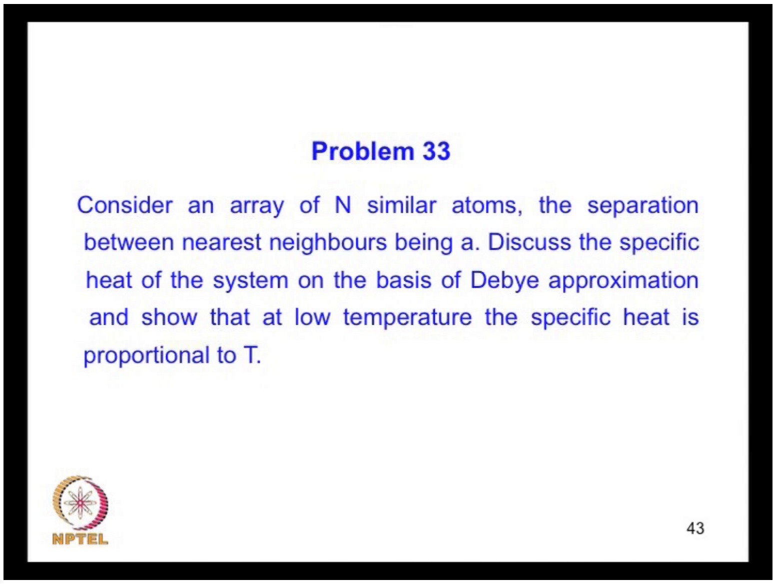


So we are ask to calculate the molar specific heat at ten k using Debye theory, law at low temperature. So basically, we have the expression C v according to Debye theory, 12 by 5 pi to the power 4 N k b T by theta d cube. And to calculate theta d, we are given nu d as three point three into ten to the power 12, so h nu d is 6.6 into 10 to the power minus 34 into Plank constant 3.3 into 10 to the power 12. And therefore, and this is equal to K B theta D where K B is the Boltzmann constant. So with this theta D works out to be 157 Kelvin, using this C v can be straight away find found out to using this expression as 494.5 joules per kg mol per degree k. (Refer Slide Time: 02:03)



We next proceed to consider an array of N.

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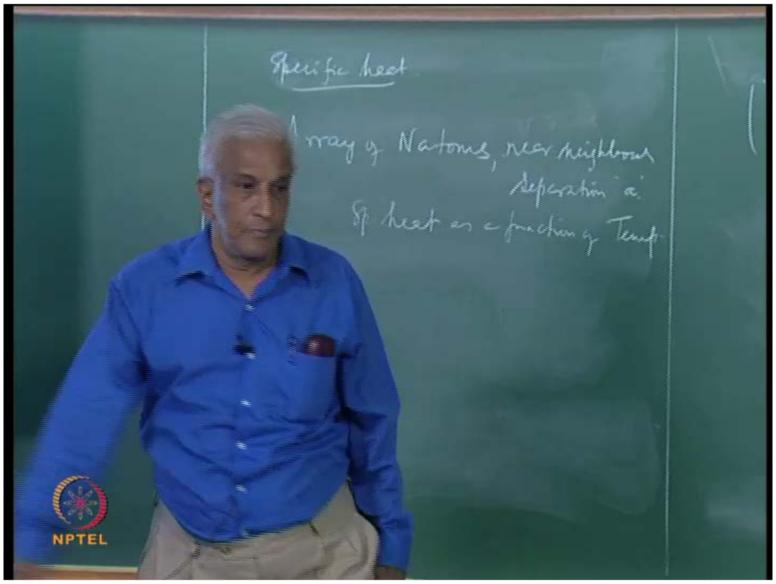

Atoms near neighbor separation of a, so we are ask to discuss specific heat as a function of temperature.

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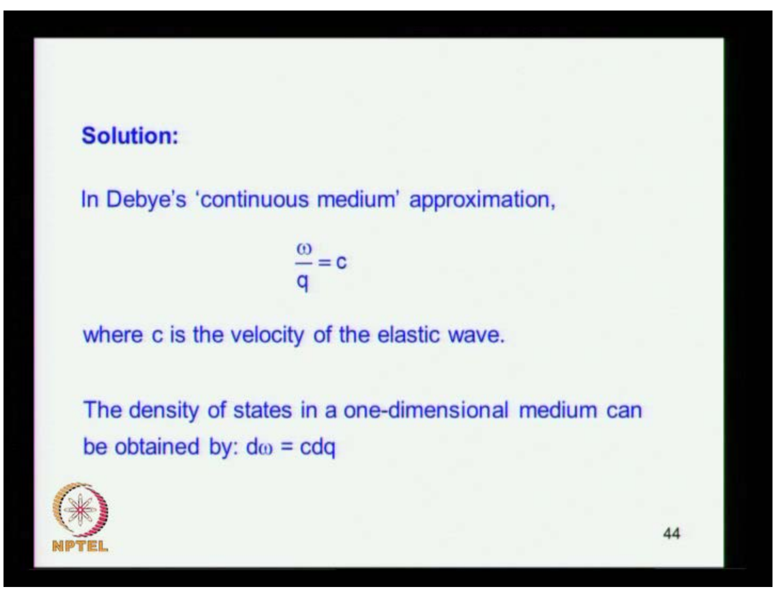

Now we have to do this, we go back to calculate the phonon dispersion relation, phonon in Debye's continuum, we can say this as dispersion relation for sound waves, omega equal c q, where or v q if you like where v is the speed of sound. So d omega is v d q.

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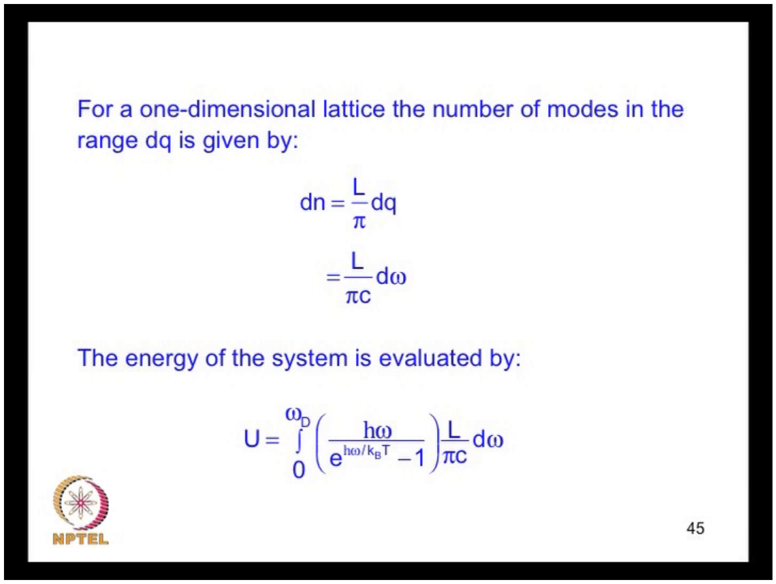

We have an array of N atoms in a linear passions, so we have a one-dimensional lattice with lattice constant a.

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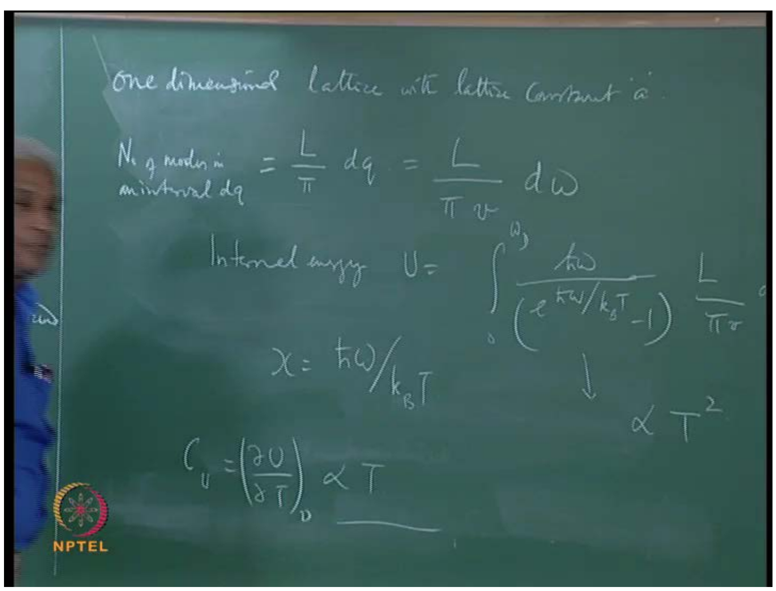

We have already discussed the vibration such crystal lattice, and the modes are given by L by pi dq that is the number of modes number of modes in an interval dq, so converting this into omega. And the energy internal energy is known to be say integral 0 to omega d of h cross omega by e to the power h cross omega by K B T minus one into L by pi v d omega.

So we make a change a variable by taking x equal to h cross omega by K B T, so that the integral goes as proportional to T square. And therefore, the specific heat, which is du by d t constant v is proportional to T. So the specific heat of such a system is a linear function of temperature.

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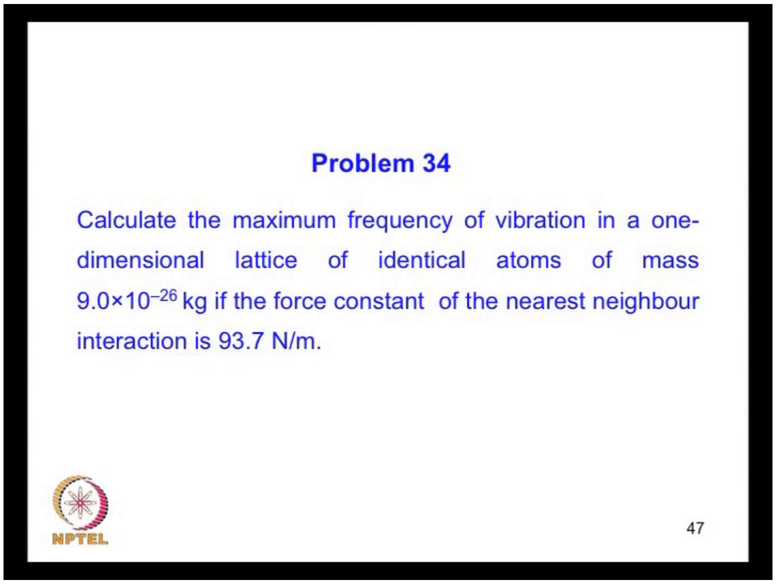

Next we are ask to calculate the maximum frequency of vibration of one dimensional lattice again a problem on lattice vibration.

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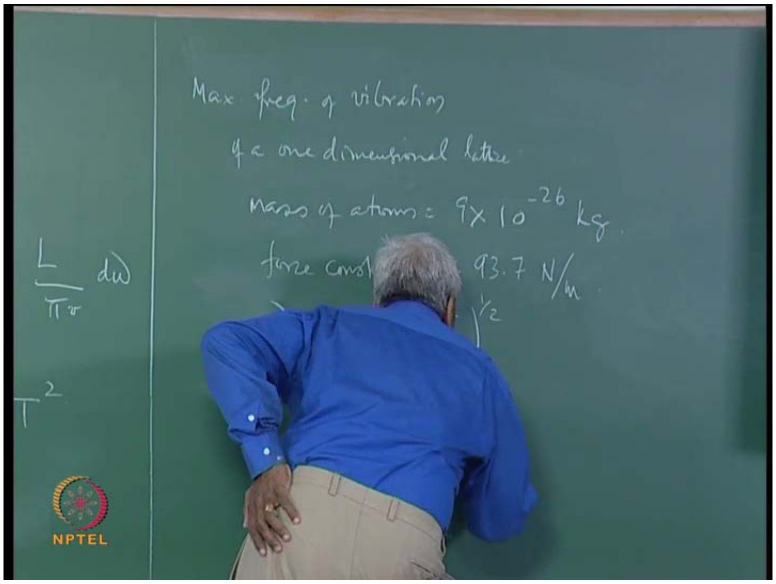

Where the mass of atom is given as 9 into 10 to the power minus 26 kilograms. The force constant between near neighbor is given as 93.7 Newton per meter. So the maximum nu max is 1 by pi into f by m to the power half, this is this. So substituting, we get 9.9 into 10 to the power twelve hertz.

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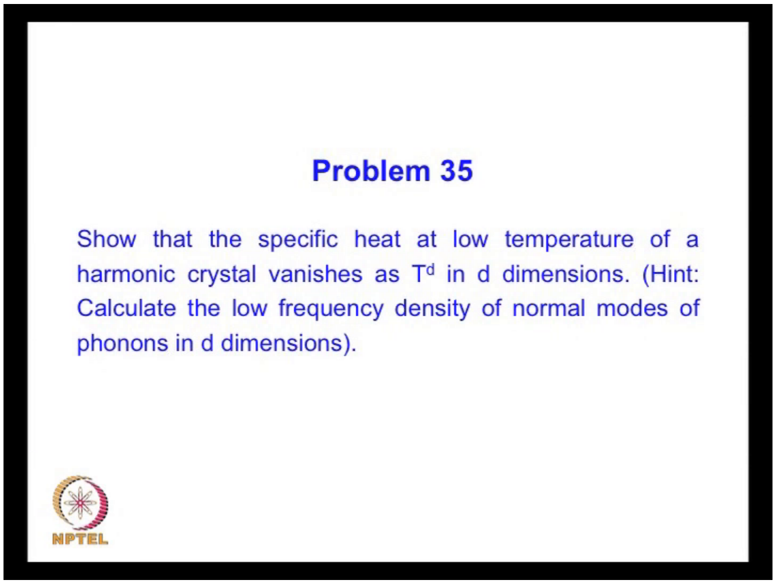

The next question is to show that the specific heat at low temperatures of a harmonic crystal in d dimensions.

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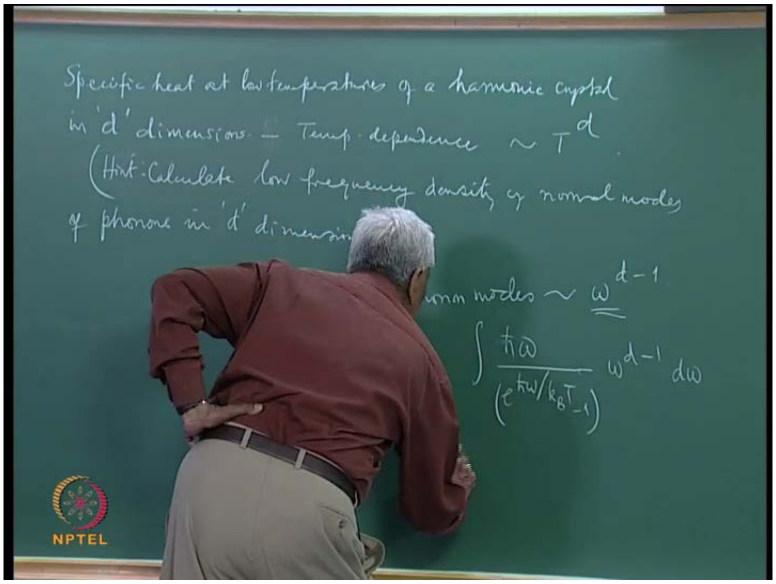

The dimensionality can be one dimension, two dimension, three dimension can be in general d dimensions. So we want to see what is it temperature dependents. We want to show that the specific heat goes as T to the power d, where T is the temperature. So we are given the hint that we had to calculate, this is the hint, the low frequency density of normal modes of phonons in d dimensions, that is the hint given.

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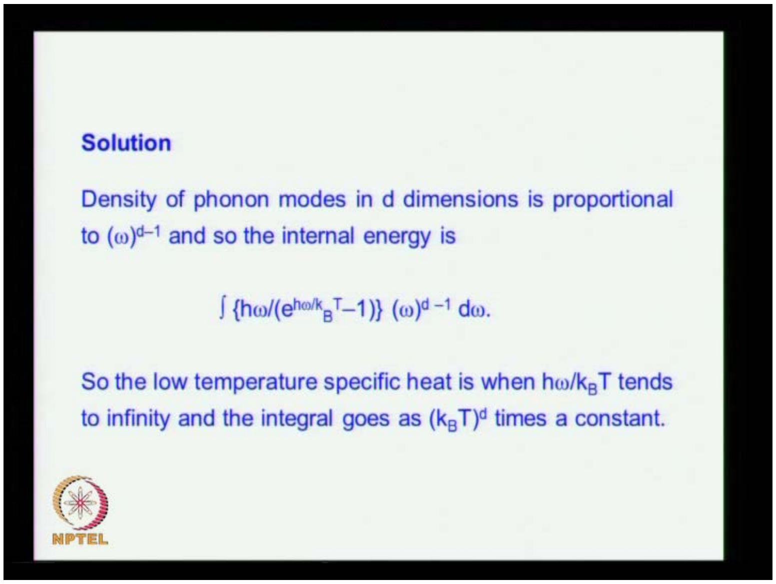

So if we take phonons in say, in general we have the density of phonon modes goes as omega to the power d minus 1. If d is the dimensionality then for example, if d is 3, if it is 3 dimensional, we know the density of phonon modes goes as omega square. If it is one dimensional, it doesn't depend on omega at all, so it is omega depend 0 and so on. So this is something that we have already considered.

Therefore, the density of phonon modes goes to the omega d minus one, therefore the internal energy which is the average energy u is the integral of h cross omega by e to the power h cross omega by K B T minus 1. This is the weighting function the average energy times omega to the power d minus one d omega. This is the energy and then this is the phonon density of states, so this is the integral that has to be calculated, so the U goes as this, so this will be in general goes as integral something times omega to the power d by e to the power h cross omega by K B T minus one d omega times h cross.

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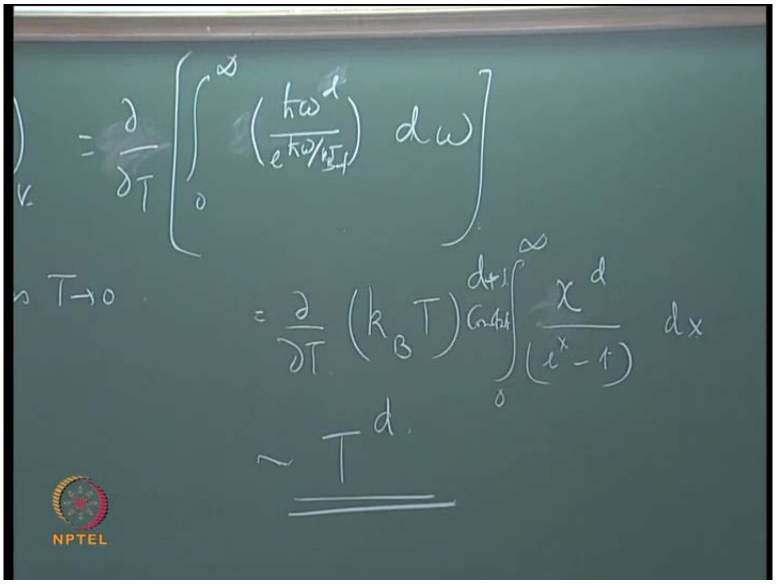

And therefore the specific heat is just dU by dT at constant volume, and therefore, the specific heat and in particular we are talking about low temperature limit, so we are considering the limit h cross omega by K B T, where T tending to 0 is tends to infinity as T tends to 0. So the integral will become integral 0 to infinity of this quantity the derivative with respect to temperature of this. So this will go as constant into omega.

So here we have the function h cross omega to the power h cross omega to the power d by e to the power h cross omega by K B T minus 1 d by dT d omega. And when I make a change in variable, this is going to give me d by dT of K B T to the power d times a definite integral x to the power d by e to the power x minus 1 dx ah into some constant. So this going to be a number because this is definite integral, this goes as T to the power d, this is going become d plus one and when I differentiate with respect to temperature, this is going to go as T to the power d in d dimensions is what we are required to prove.

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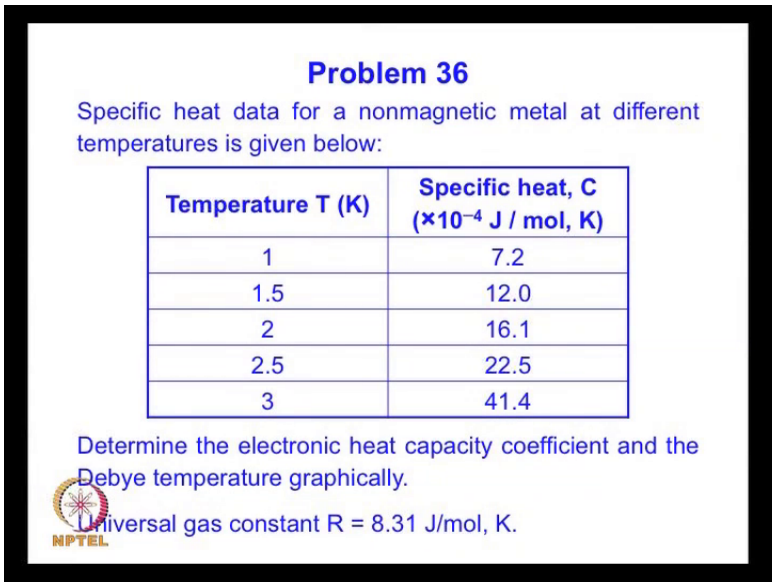

The next question we are given the set of specific heat values data, experimental measurements, the results of experimental measurements are tabulated specific heat versus temperature.

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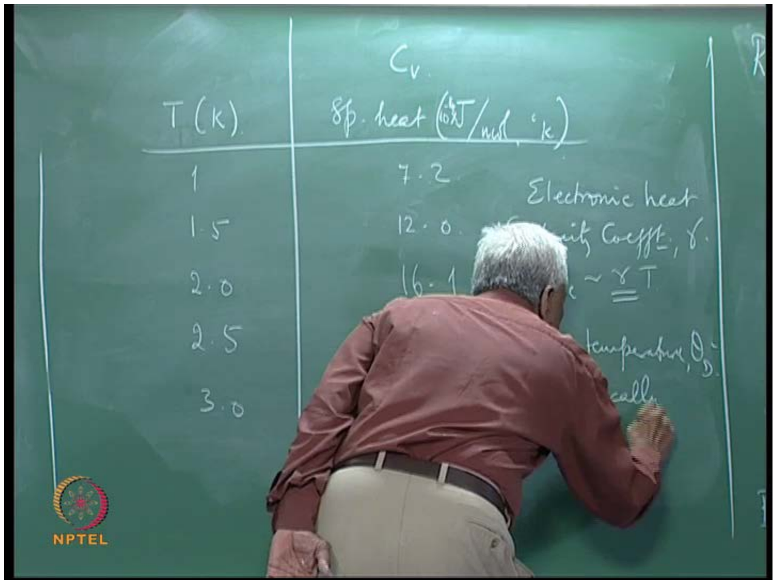

So T temperature and specific heat in joules per mol degree Kelvin into 10 to power minus 4. So at a temperature of 1 K the specific heat is seven point two into ten to the power minus four. At 1.5 K, this is 12; at 2 K, it turns out to have a value, 16.1; at 2.5 K, it is 22.5; and it three K, it is 41.4 into 10 to the power minus 4 joule per mol degree K. So this is the data that is given. So we are require to find the electronic heat capacity coefficient, remember we wrote this as C electronic goes as gamma T, so this is the gamma and the Debye's temperature by a graphical method. How do we do this, we already have consider this problem in a metal the heat capacity generally is given as sum of two terms in total heat capacity is the electronic heat capacity plus the lattice heat capacity.

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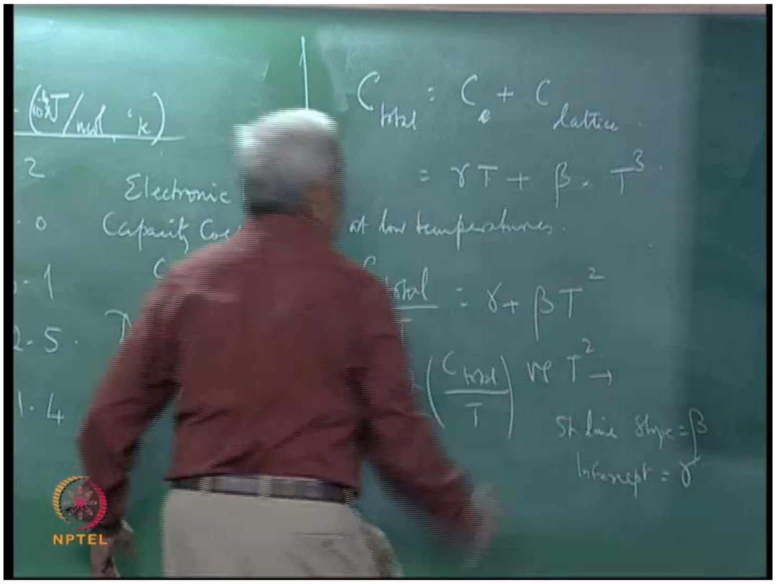

The electronic heat capacity go as goes as gamma T and the lattice heat capacity goes as constant times T cube at low temperature. Therefore, C total by T is gamma plus beta T square; constant I can write as beta. So this means if we plot the total specific heat divided by the temperature versus T square, this will be a straight line, whose slope is beta and the intercept on the y-axis gives you the electronic heat capacity coefficient.

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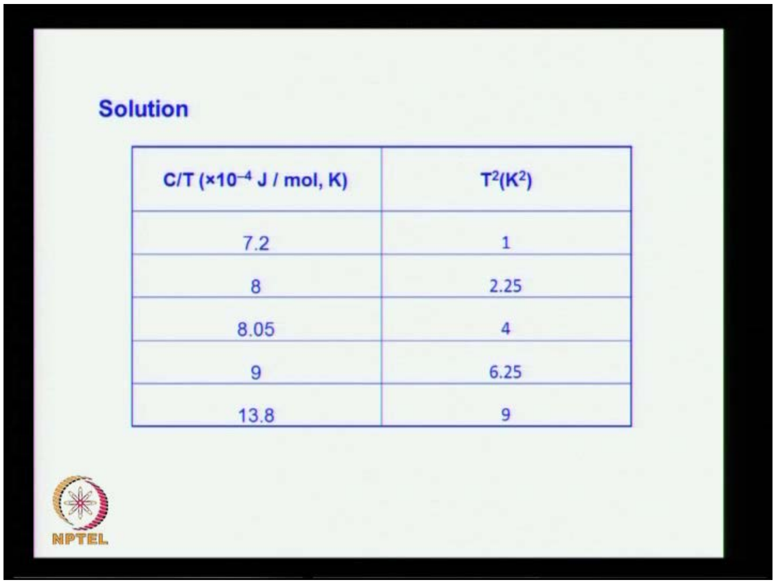

Solution

C/T ($\times 10^{-4}$ J / mol, K)	$T^2 (K^2)$
7.2	1
8	2.25
8.05	4
9	6.25
13.8	9

So using the given data, we arrive at the following table.

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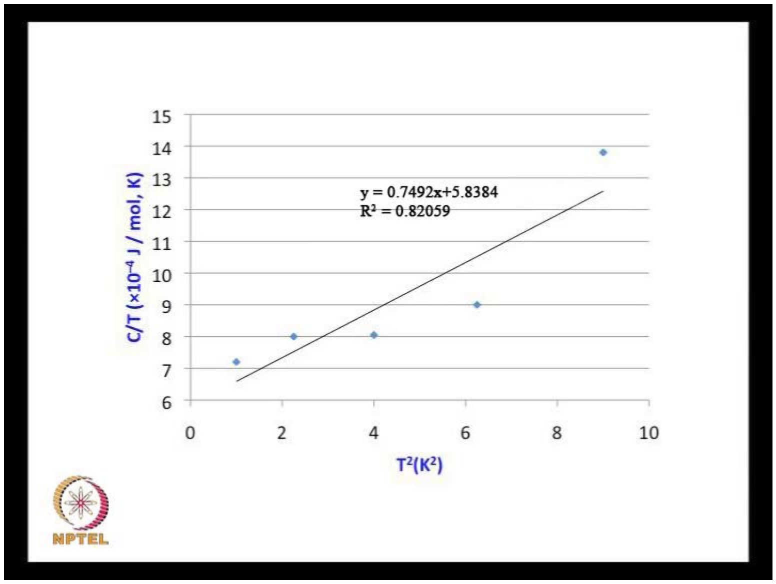

So these are the values, which are plotted graphically, and we find the plot of C by T versus T square turns out to be an approximate straight line at low temperature which diverges later.

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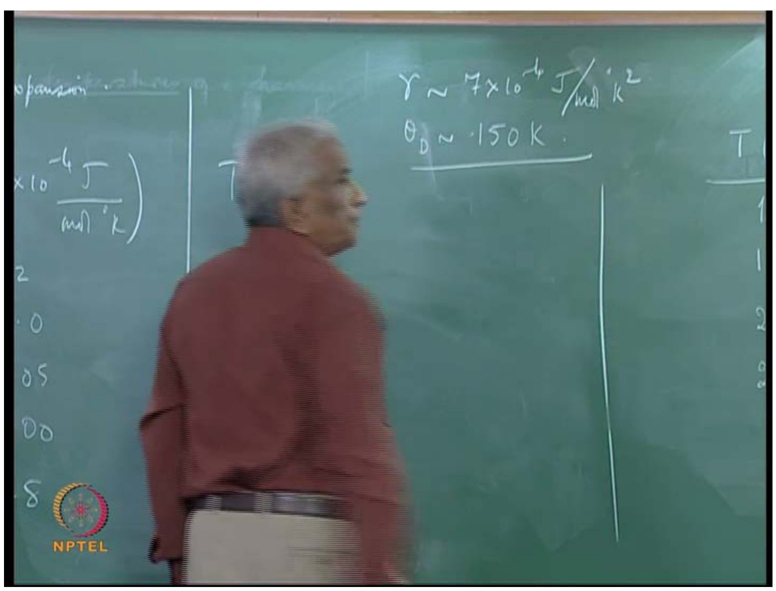

So passing a straight line, through these points, we get the slope and intercept which give the heat capacity coefficient gamma as approximately 7 into 10 to the power minus 4 joule mol K square. And Debye's temperature is approximately 150 Kelvin.