

**Condensed Matter Physics**  
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**Lecture - 11**  
**Thermal Conductivity of Metals – Worked Examples**

Today, we continue discussing problems relating to thermal conductivity of metals. The first problem that we are going to do today is rather elementary which is based on the definition of thermal conductivity, Fourier's equations, steady state equation.

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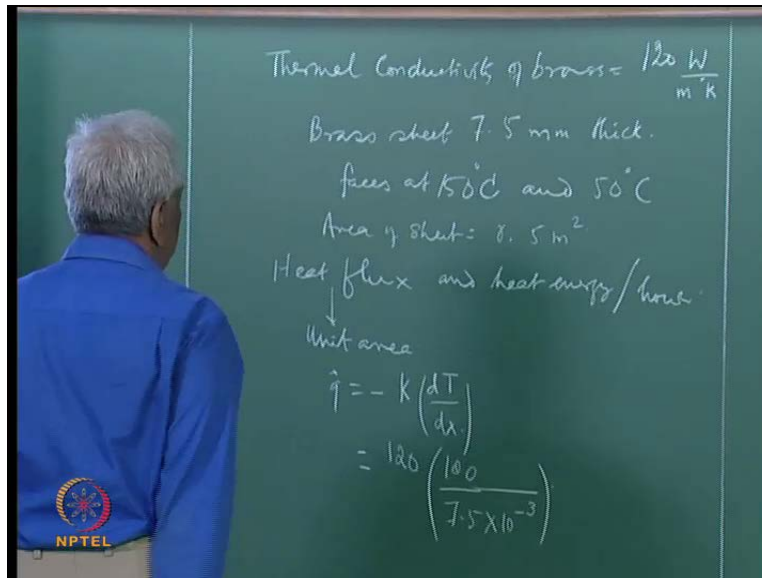
**Problem 30**

Calculate the heat flux through a sheet of brass 7.5 mm thick if the temperature at the two faces are 150 and 50 °C. (Thermal conductivity of brass is 120 W/mK). If the area of the sheet is 0.5m<sup>2</sup> calculate the total heat energy transmitted per hour.

 NPTEL 32

So, we are asked to calculate.

(Refer Slide Time: 00:49)



The heat flux through a sheet of brass sheet 7.5 millimeter thick, faces at 150 Celsius and 50 Celsius, thermal conductivity of brass is given to be 120 watts per meter Kelvin, area of the sheet 0.5 meters square. So we have the standard equation, we are ask to calculate heat energy heat flux and heat energy wherever. Heat flux is defined with respect to unit area, so it is given by  $\dot{q}$  is a minus  $K \frac{dT}{dx}$ , where this is the temperature gradient, so we are given we have thermal conductivity which I 120 and then  $\frac{dT}{dx}$  across 7.5 we have a gradient of millimeter, so into 10 to the power minus 3.

(Refer Slide Time: 03:12)

**Solution:**


Heat flux

$$J = K \times \frac{dT}{dx} = 120 \left( \frac{100}{7.5 \times 10^{-3}} \right) = 160 \times 10^4 \text{ W / m}^2$$

Heat conducted per second through the area of 0.5 m<sup>2</sup>

$$= 160 \times 10^4 \times (0.5) = 80 \times 10^4 \text{ Joules / sec}$$

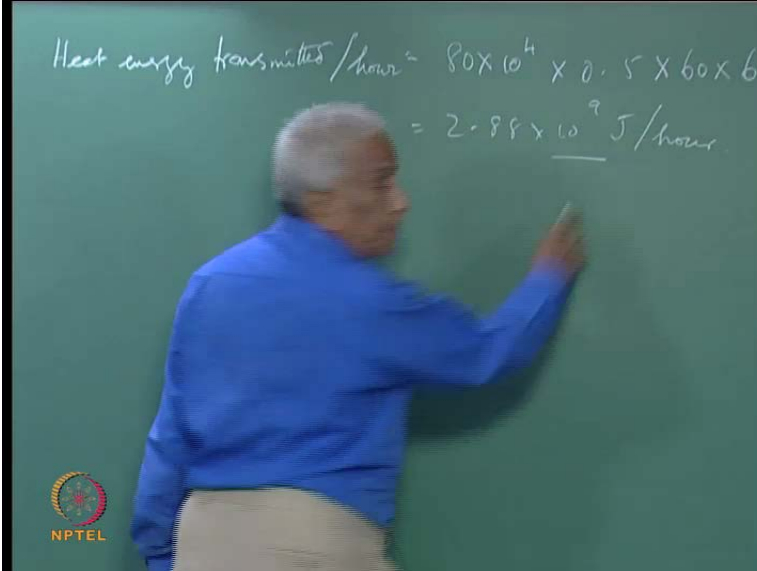
Heat conducted per hour  $80 \times 10^4 \times 60 \times 60 = 2.88 \times 10^9 \text{ J / hr}$




33

That would work out to be 8 into 10 to the power 4 joules per second.

(Refer Slide Time: 03:31)



Heat energy transmitted / hour =  $80 \times 10^4 \times 0.5 \times 60 \times 60$   
 $= 2.88 \times 10^9 \text{ J / hour}$



And taking area into consideration, the heat energy transmitted per hour is just a this quantity into the area which is 0.5, so that would give that is per second into 60 into 60 per hour, so that will be 2.88 into 10 power 9 joules per hour.

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**Problem 31**

Show that thermal conductivity of a perfect gas is given by

$$K = \frac{1}{3} C l \bar{v}$$

where

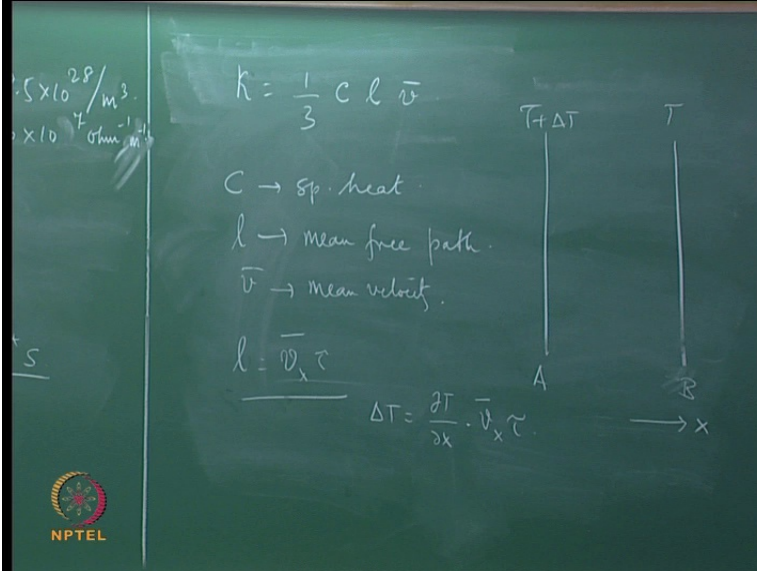
- C is specific heat
- l is mean free path of the molecules
- $\bar{v}$  average velocity of the molecules



27

The next question is on thermal conductivity of metals.

(Refer Slide Time: 04:31)




$k = \frac{1}{3} C l \bar{v}$

C → sp. heat  
l → mean free path  
 $\bar{v}$  → mean velocity

$l = \bar{\lambda} = \frac{1}{n \sigma}$

$\Delta T = \frac{\partial T}{\partial x} \cdot \bar{\lambda}$

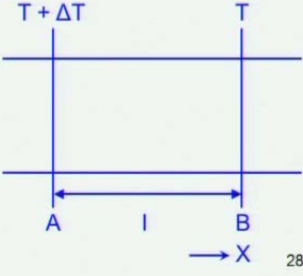



We ask to prove the thermal conductivity the kinetic theory expression of thermal conductivity for a perfect gas as one third  $C l \bar{v}$ ; where C is the specific heat, l is the mean free path, and  $\bar{v}$  is the average mean velocity of the gas molecules. So this is the expression that we will adapt for discussing thermal conductivity of metal as well.

(Refer Slide Time: 05:09)

**Solution:**

Consider two Sections A and B separated by distance equal to mean free path  $l$ . Let the temperature at A be  $T + dT$  and the temperature at B be  $T$ . The mean free path  $l = \bar{v}_x \tau$  where  $\tau$  is the time interval between collisions.

$$\Delta T = \frac{dT}{dx} \cdot l$$
$$= \frac{dT}{dx} \bar{v}_x \tau$$


 28

So we consider two sections at temperatures  $T$  and  $T$  plus  $\Delta T$ , so there is a temperature gradient along this direction, so let us call this  $x$  direction. So there is a temperature gradient and let us take these two sections call them A and B. And the mean free path  $l$  is  $\bar{v}_x \tau$ , where  $\tau$  is the relaxation time or collision time; and  $\bar{v}_x$  is the average  $x$  component of the speed. Therefore, if this section is separated by a mean free path, then  $\Delta T$  is  $d T$  by  $d x$  times  $\bar{v}_x \tau$ .

(Refer Slide Time: 06:22)


Number of molecules that cross-section A per second per unit area is  $\frac{1}{2}n\bar{v}_x$

where  $n$  is the number of molecules per unit volume.

Total heat energy given up in moving from A to B  $-\frac{1}{2}n\bar{v}_x c \Delta T$

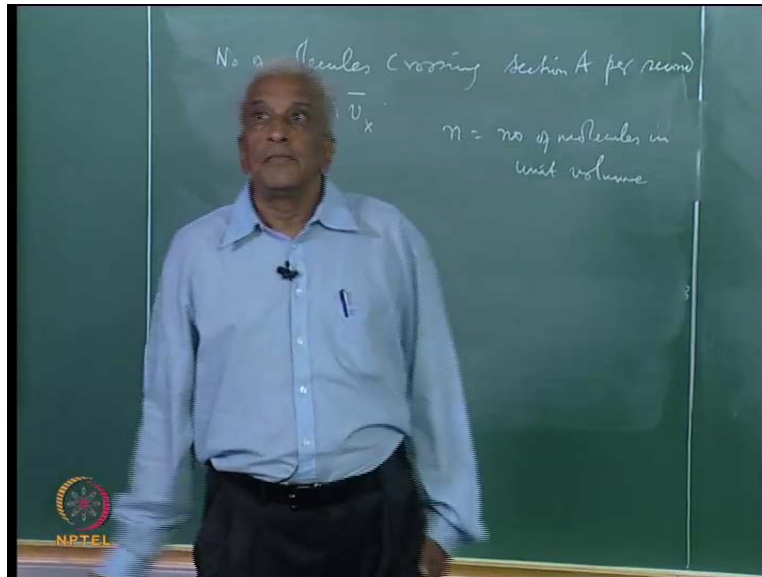
where  $c$  is the specific heat per molecule.

Total heat energy gained in moving from B to A  $+\frac{1}{2}n\bar{v}_x c \Delta T$

 29

Now we calculate the number of molecules which transport energy thermal energy to the two sections by moving by carrying heat along the x directions.

(Refer Slide Time: 06:37)



So number of molecule that across the section A crossing section A per second, this is we know that is half-n v x bar. And these where n is the number of molecules in unit volume. Now therefore the heat energy.


(Refer Slide Time: 07:30)

Since,

$$\bar{v}_x^2 = \bar{v}_y^2 = \bar{v}_z^2 = \frac{1}{3} \bar{v}^2$$

$$J = -nc \left( \frac{1}{3} \bar{v}^2 \right) \tau \left( \frac{dT}{dx} \right)$$

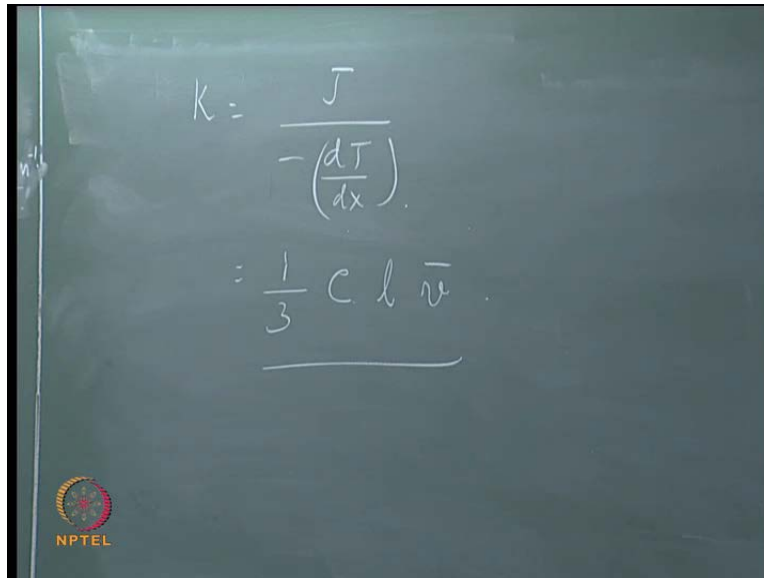
$nc = C_{\text{total}}$ , so  $J = -\frac{1}{3} C_{\text{total}} (\tau \bar{v}) \bar{v} \left( \frac{dT}{dx} \right)$

$$K = -\frac{J}{(dT/dx)} = \frac{1}{3} C_{\text{total}} \bar{v}$$


31

And then per unit temperature gradient  $K - J$  by since.

(Refer Slide Time: 07:34)


$$k = \frac{J}{-\left(\frac{dT}{dx}\right)}$$
$$= \frac{1}{3} C l \bar{v}$$

We are consider unit area already so this is just this and therefore, it just comes as. And  $v$  bar tau is 1, so it is  $\frac{1}{3} C l v$  bar which is what we are took true.