Condensed Matter Physics
Prof. G. Rangarajan
Department of Physics
Indian Institute of Technology, Madras
Lecture - 1
Principles of Condensed Matter Physics

So, we shall be starting these lectures on condensed matter physics. At the outset, we would like to answer the question, what is condensed matter. So, in this lecture, we will try to answer this question, and basically try to explain the relative interplay of thermal and potential energies of atoms and molecules in condense matter. The onset of short and long-range order, the concept of broken symmetry, soft modes and topological defects that would be the scope of today lecture. In particular, we know the matter is in the gaseous state at relatively high temperatures, but as the temperature is lower, it liquefies below the boiling point of the liquid; and then as you cool down further, it freezes into a solid at much lower temperatures in particular below the freezing point. A familiar example is that of water – H 2 O.

At high temperatures, we all know that water exist as stream or water vapor. It is a gas, which has a uniform density in any enclosure inside which it is kept. And point of view of various physical properties, we may characterized the water vapor has been isotropic and homogeneous. What do you mean by this? At the microscopic level, this means that if try to find a water molecule, in any small region within a close space the container, the probability of finding a water molecule at anyone point of space within this region is equal to the probability of finding it at any other point. In other words, there is an equal probability of finding a water molecule in any region within the enclosure. Therefore, there are no density variations over a distance equal to the wavelength of the light. So if we shine light on a container of steam or water vapor, there are no density variations over a wavelength of light, and therefore there is no scattering, very roughly. This is an explanation, why light is not scattered by a gas, and if light is not scattered, we do not see in the vapor. So that is the reason why a gas is invisible.

On lower in the temperature of course, at the molecular level, what we know is a temperature is the measure of the kinetic energy of the molecules, atoms and molecules. Therefore, when we lower the temperature, the kinetic energy decreases.

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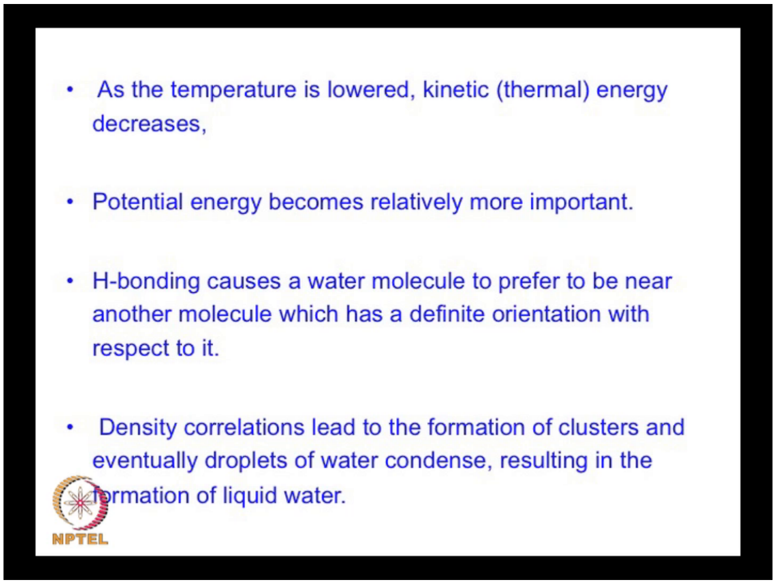

- As the temperature is lowered, kinetic (thermal) energy decreases,
- Potential energy becomes relatively more important.
- H-bonding causes a water molecule to prefer to be near another molecule which has a definite orientation with respect to it.
- Density correlations lead to the formation of clusters and eventually droplets of water condense, resulting in the formation of liquid water.

And therefore, the potential energy gains important relative to the kinetic energy. It progressively becomes much more and more important. In the case of water, this potential energy is mainly because the water molecule is a good strong electric dipole; so each water molecules interacts with the neighboring water molecules through dipole, dipole interactions. In addition, the hydrogen atoms in the water molecule provide an additional mechanism for bonding, which is known as hydrogen bonding, and this is a highly directional bonding. Therefore, a given water molecule prefers to spend time near another molecule which comes in a particular orientation. In other words, inter molecular correlations start becoming important.

So as we lower the temperature, the density fluctuation grow in amplitude, because the molecules the water molecule stay preferentially together, because of these intermolecular interactions. So, the density fluctuation grow in a amplitude and also persist for longer time. So, in comparison to the gaseous space, the density is still uniform, but only when average row, large regions of space or over longer intervals of time. Because of the density fluctuation, critical opalescence takes place, when clusters of water molecules grow in size, and can scatter light that is the reason for the phenomenon of critical opalescence. Finally, a point is reached, when the dense region of water molecules covalence at the bottom of the vessel because of gravity. And there is a meniscus starting appearing which separates the liquid from the gas that's one we recognize that a phase transition has taken place from the gaseous state to the liquid state.

Now there is a certain set of thermodynamic variables like temperature, pressure, volume etcetera, which are characterizing this phase transition. A particular point is reached, which is known as the critical point. Just like everywhere else the word critical, we use the word critical in the case of patients, who are terminally ill and who are who one cannot say whether they are going to be alive or whether they are going to die, so we say that the person is in a critical state, when it is difficult to say whether a person is going to be alive or will be dead.

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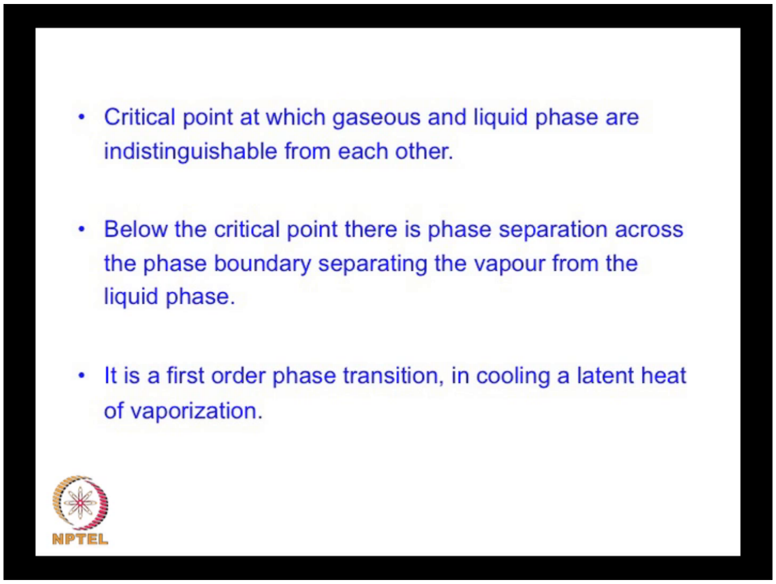

- Critical point at which gaseous and liquid phase are indistinguishable from each other.
- Below the critical point there is phase separation across the phase boundary separating the vapour from the liquid phase.
- It is a first order phase transition, in cooling a latent heat of vaporization.

Similarly here the critical point is define as the thermodynamic state corresponding to which it is difficult to distinguish between the gaseous and liquid state. So there is continuous change in the density via what is known as a second order phase transition. But as you go down in temperature below the critical point, this transition this change in density becomes discontinuous and then this discontinuous change takes place across the gas, liquid phase boundary. It becomes what is known as a first order phase transition. This is through this takes place to the nucleation of droplets, say on a dust particle inside the medium, and migration of these droplets to form larger aggregates of the liquid. In the case of water, the critical density is zero point three two two grams per centimeter cube as you can see in figure one.

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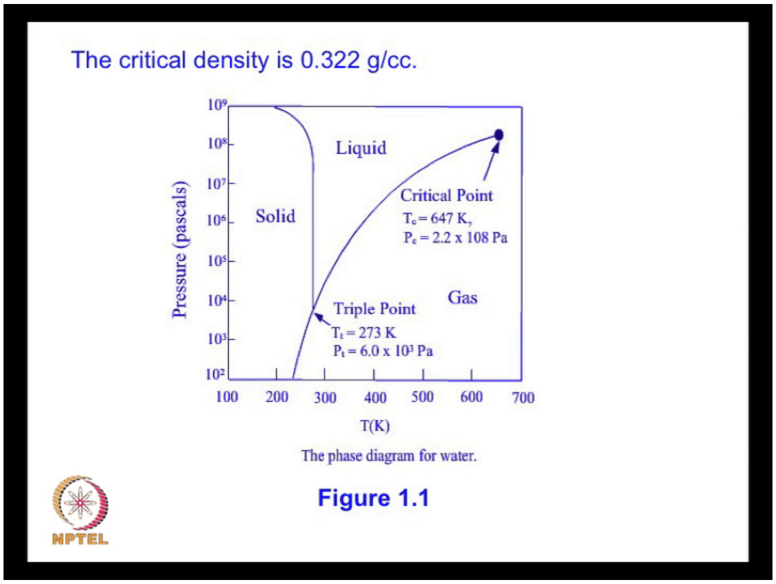

The critical density is 0.322 g/cc.

The phase diagram for water.

Figure 1.1

So you have the critical point which is given in terms of the Kelvin scale of temperature as 647 Kelvin; and below the critical point, you go further down and eventually you reach a point known as triple point where the three phases gas, liquid and solid meet together. So you have the co-existence of the gaseous phase, the liquid phase and the solid phase at different thermodynamic conditions, so that is the so called phase diagram of the water. So as we already said at the critical point, it is possible to go from the gaseous to the liquid phase with I do extremely small changes in pressure. If you change the pressure slightly, you can go into it. In other word, a very small change in pressure causes the large change in the density, because it becomes liquid. Therefore, they so called isothermal compressibility, is relates the pressure change to the change in density, the isothermal compressibility diverges.

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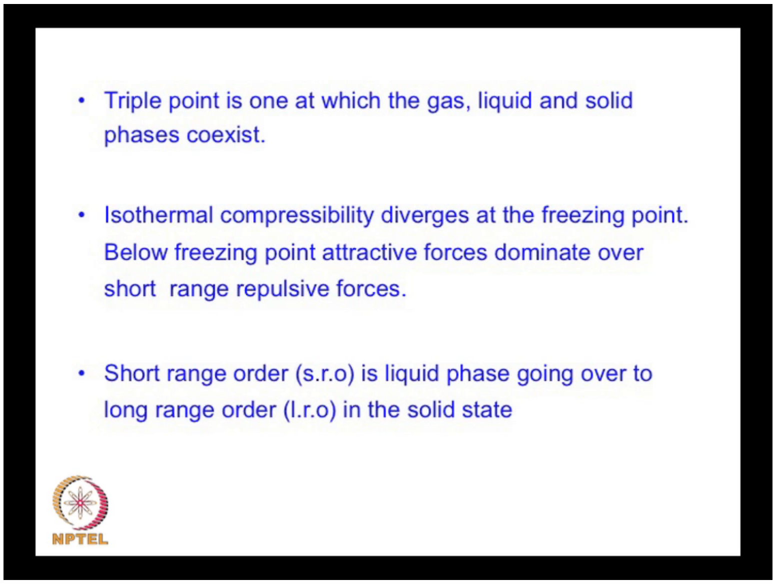

- Triple point is one at which the gas, liquid and solid phases coexist.
- Isothermal compressibility diverges at the freezing point. Below freezing point attractive forces dominate over short range repulsive forces.
- Short range order (s.r.o) is liquid phase going over to long range order (l.r.o) in the solid state

Now when you lower the temperature further, as I already remark, there is a competition between the attractive interactions which cause condensation and the hardcore repulsive interactions such as coulomb repulsion or exclusion principle, which prevent atoms from occupying the same place. For example, in liquid argon, this results in triangular and tetrahedral arrangements of spherical atoms. In water, these leads to tetrahedral coordination, because the so called loan paired electrons in the water molecules or directly towards the hydrogen atoms. So there is a short range order in the immediate neighborhood of a given atom or molecule, there is a local order which is known as short-range order. And this is reveal by x-ray diffraction as we can see figure two.

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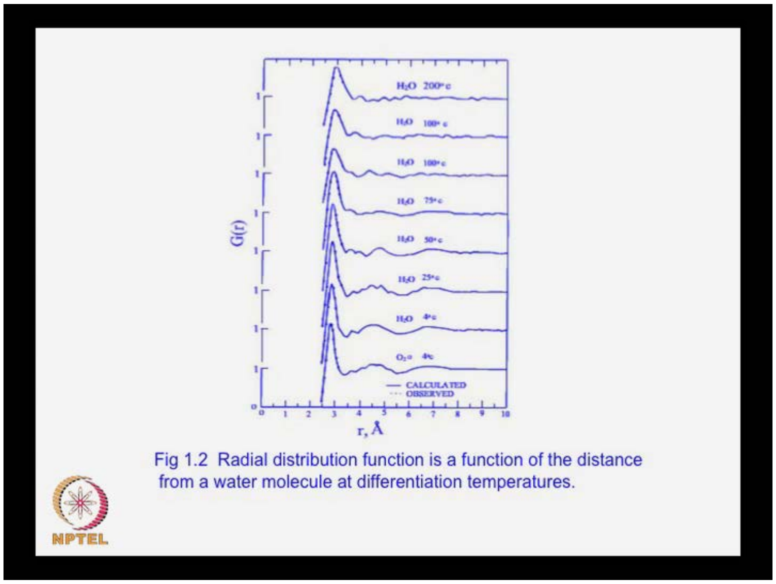

Fig 1.2 Radial distribution function is a function of the distance from a water molecule at differentiation temperatures.

In this figure, we are showing the radial distribution function in the x-ray diffraction, and how it reveals the existence of short-range order in the water liquid – water. So you can see that there are peaks in this radial distribution function of function corresponding to the different coordination shells, surrounding a given atom or molecule. As I already remarked, when you approach the triple point, the liquid becomes a solid and ice crystals are formed in the case of water. Therefore, we have yet another phase transition from the liquid to the solid phase; in which the water molecules are frozen at definite lattice sides as in ice forming hexagonal layers which are responsible for the faceting of ice crystals as shown in the next figure.

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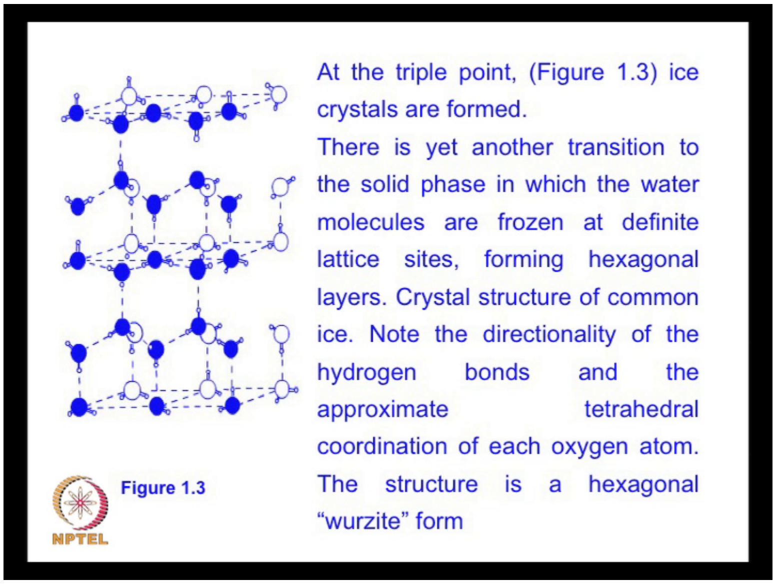

At the triple point, (Figure 1.3) ice crystals are formed.

There is yet another transition to the solid phase in which the water molecules are frozen at definite lattice sites, forming hexagonal layers. Crystal structure of common ice. Note the directionality of the hydrogen bonds and the approximate tetrahedral coordination of each oxygen atom. The structure is a hexagonal "wurzite" form

Figure 1.3

So you can see, the so called long-range order – L R O in ice crystals. In this case, the water molecules are frozen at definite lattice sides, and therefore all translational and rotational degrees of freedom are frozen in. Therefore, the isotropy of the gas phase is completely destroyed.

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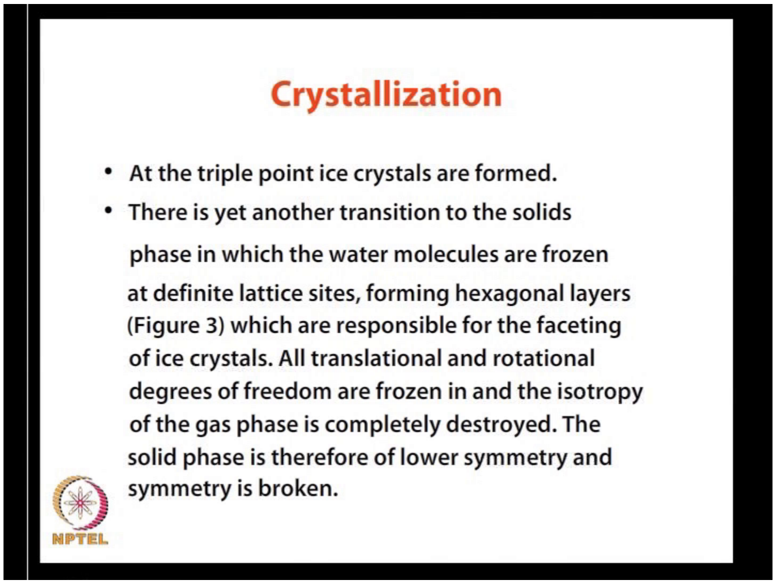

Crystallization

- At the triple point ice crystals are formed.
- There is yet another transition to the solids phase in which the water molecules are frozen at definite lattice sites, forming hexagonal layers (Figure 3) which are responsible for the faceting of ice crystals. All translational and rotational degrees of freedom are frozen in and the isotropy of the gas phase is completely destroyed. The solid phase is therefore of lower symmetry and symmetry is broken.

The solid phase is definitely of much lower symmetry, it is anisotropic and the symmetry of the higher temperature phase is broken, we speak of a broken symmetry. Associated with this broken symmetry is appearance of rigidity to shear which prevents the solid from flowing unlike a liquid. Now how this rigidity to shear appears, this is because of the creation of topological defects known as dislocation which are needed to produce a shear strains. Now this periodic arrangement of atoms in a crystal lattice, such are use of the Fourier transform of the scattering potential for x-rays, neutrons or electrons. And this is what gives raise to Brag peaks in the diffraction patterns revealing the long-range order in the solid.

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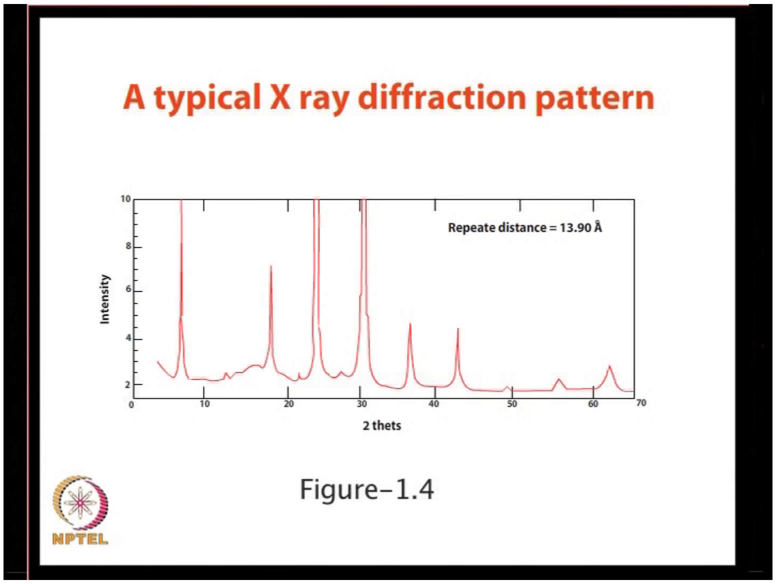

A typical X ray diffraction pattern

Figure–1.4

This is what we see here in the next figure a typical x ray diffraction pattern showing the appearance of Brag peaks at definite angles of scattering of the incident x ray. So the existence of these Brag peaks is a definitive indications of the on set of long range order in a condensed solid phase. Now we spoke about the divergence of the shear modulus or the disappearance of the shear mod, there is a appearance of the rigidity to shear, in the solid phase. We can understand the reason for this as follows.

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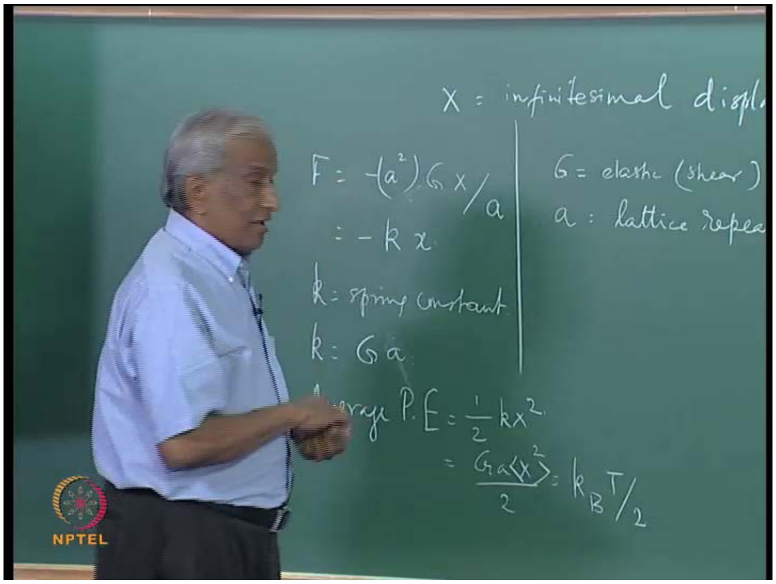

When you have small atomic displacement x from the equilibrium position, the force on a unit cell is given by the stress which is G x by a, where G is the elastic modulus for the shear and a is the, G is the elastic shear modulus, and a is the lattice – crystal lattice repeat distance or periodicity. X is a infinitesimal displacement of an atom or molecule from its equilibrium position. So G x by a is the force, no this is the stress and which should multiply it by the square of the, sorry so that would be force. So this is equal to in the case of a totally harmonic, purely harmonic solid, where the vibrations are simple harmonic, then this is equal to minus K x for equilibrium. Where K is the spring constant, which is due to the inter atomic intermolecular forces. This means K is equal to G x, G a sorry. And the average potential energy which is what we started with we said that the equilibrium relative importance of the potential and kinetic energy is what determines the nature of the equilibrium in condense phase. Therefore, we have to find the average potential energy, which is half K x square for a harmonic solid.

And therefore, this will be substituting for K and this is equal to this is the kinetic energy is half K B T by Boltzmann ((Refer Time: 18:23)) partition theorem. Therefore, we get the mean square displacement x square becomes the shown that this is inversely proportional to the G. So if the latter, the elastic modulus is zero, the former the mean square displacement will diverge, that means that the displacement will go to infinite. So there will be a sliding of the atom with respect to layer zones, and then this is the solid starts melting and start flowing. So that is what you understand as the difference, main difference between a liquid and a solid. So when these random displacement becomes comparable to a lattice constant periodic order and the discrete Brag piece are all destroyed. Rigidity is therefore seen to be a necessity condition for the existence of periodicity, associated with every phase transition, where symmetry is broken as we already discussed. There will be a new rigidity or elastic constant preventing thermal fluctuations from destroying the new state. So that is the key to understanding of broken symmetry.

Now this distortion of periodicity in a solid depends on the relative displacement of neighboring atoms or molecules. So the dynamical modes are associated with these relative displacement or elastic waves, but we know that in water compressional sound modes which are associated with these elastic displacement exist both in liquid water and in solidest; whereas, shear modes exist only in ice, but not in water. So in the long wavelength limit, the frequency or energy of the shear modes tends to zero. And there is no restoring force against the long wavelength displacement, which is what we call the flow. The appearance of such a hydrodynamics or long wavelength mode with zero frequency, this is known as the phenomenon of modes ((Refer Time: 20:55)) is yet another general features of every phase transition which yields a broken continuous symmetry.

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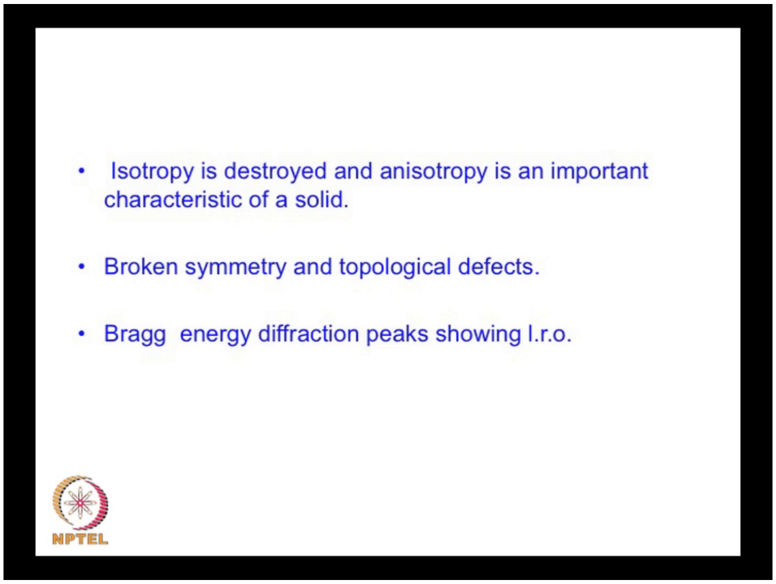

- Isotropy is destroyed and anisotropy is an important characteristic of a solid.
- Broken symmetry and topological defects.
- Bragg energy diffraction peaks showing l.r.o.

In order to make ice flow however it is not necessary to break all the bonds between the hexagonal planes, so that the atoms could slide over one another. Suppose, we imagine as in the next figure, figure five that we cut half the bonds in a plane move them over one lattice to the left and reattach them as shown here.

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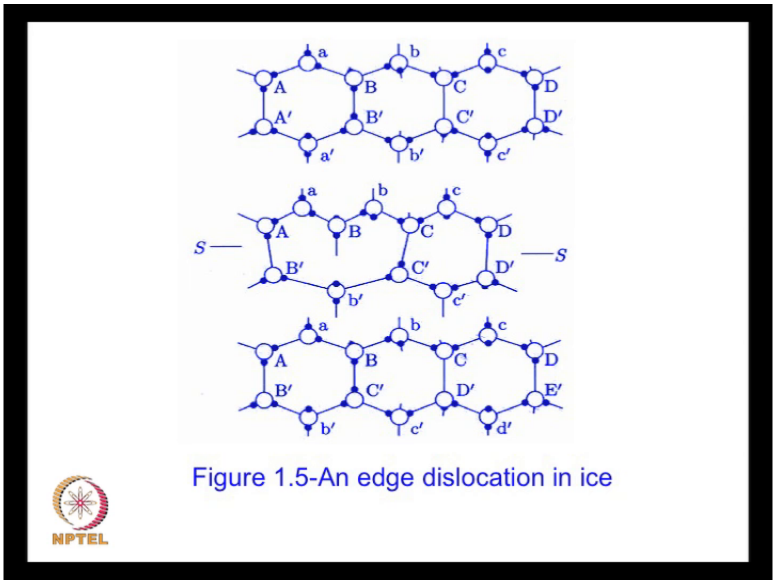

Figure 1.5-An edge dislocation in ice

In this figure then we get effectively a relative displacement. So you have a line defect or dislocation which can be moved readily, since it means breaking only a line of bonds and remaking them one site away, energetically this is much lower, the energy cost of this is much lower. The edge dislocation is set to glide easily in the plane and this allows the ice to shear above and below the plane. This is what gives raise to the so called phenomenon of creep in a solid.

Spinning of dislocation and dissipation associated with a motion of dislocation is responsible for most of the mechanical properties of crystalline solids. The dislocation is the topological defects, which like rigidity is a general features of all broken continuous symmetry. Summing up we can make the following overall picture, at continuous phase transition, characteristic length, relevant susceptibility or compressibility and relaxation times diverge; whereas, at discontinuous transition there is nucleation at sufficiently low temperature. When the potential energy dominates over kinetic energy, equilibrium states will generally have lower symmetry than the high temperature phase, this is what we known as broken symmetry. If the broken symmetry is continuous, the lower temperature phase is characterized by a rigidity and the presence of topological defects. At high temperature, however the kinetic energy dominates over the potential energy and equilibrium phases in a gas for example, or isotropic and homogenous. However the temperature is lower, there occur phase transition to more strongly correlated phases, these phase transition can be continuous or discontinuous.

Phase transition are described by thermodynamics theory. The microscopic mechanism of ordering processes can be probed by a variety of techniques as we discussed for example through the phenomenon of critical opulence, through x ray scattering and observation of Brag peaks etcetera. While the kinetic theory of gases gives an adequate description of gases, symmetry provides the key to understanding the behavior of perfectly periodic crystalline solids. Of course, liquids are in between they are harder to describe theoretical. So in the next table, I have shown the properties of some representative broken-symmetry phases.

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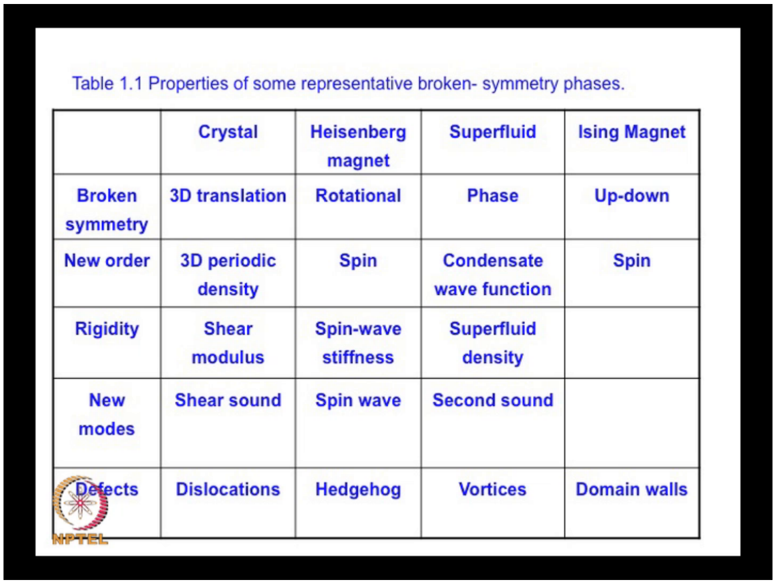

Table 1.1 Properties of some representative broken- symmetry phases.

	Crystal	Heisenberg magnet	Superfluid	Ising Magnet
Broken symmetry	3D translation	Rotational	Phase	Up-down
New order	3D periodic density	Spin	Condensate wave function	Spin
Rigidity	Shear modulus	Spin-wave stiffness	Superfluid density	
New modes	Shear sound	Spin wave	Second sound	
Defects	Dislocations	Hedgehog	Vortices	Domain walls

For example, Heisenberg ferro magnet or in a ordinary normal fluid to what is known as a super fluid or the transition - a magnetic phase transition into an ising ferro magnet, these are all standard phases ordered phases in a three dimensional crystal. It is the translational in all three-dimensional which are the broken symmetry at the three-dimensional periodic density is the new order. In the case of a Heisenberg ferro magnet, the broken symmetry is rotational and the new order is a spin ordering. In the case of a super fluid phase, this new order is what is known as the condensate wave function, we will discuss it later. In the case of an using magnet, you have an up to down broken symmetry due to spin ordering again. The associated rigidities for all these transition are associated with it. A shear modulus, in the case of crystal a three-dimensional solids; spin-wave stiffness, in the case of a Heisenberg magnet; and the super fluid density, in the case of a super fluid.

The new modes which appear are undulation in the second sound, we will have to discuss all this separately. Then the shear sound wave modes are the new modes which appear in a solid – crystalline solid. Spin wave modes appear in the case of Heisenberg magnet; second sound appears in the case of liquid helium, which is the super fluid. The associated topological defects are, in the case of a Heisenberg magnet the topological defects is a hedgehog; and in the case of super fluid the associated topological defects are what are known as vortices. (refer time: 27:00) In the case of an ising ferro magnet, it is the domain walls which are the defects. Well, I think, we covered basic principles governing the condensation of disordered phases into progressively ordered phases which exhibit short or long range order, as in the case of liquid or a solid respectively.

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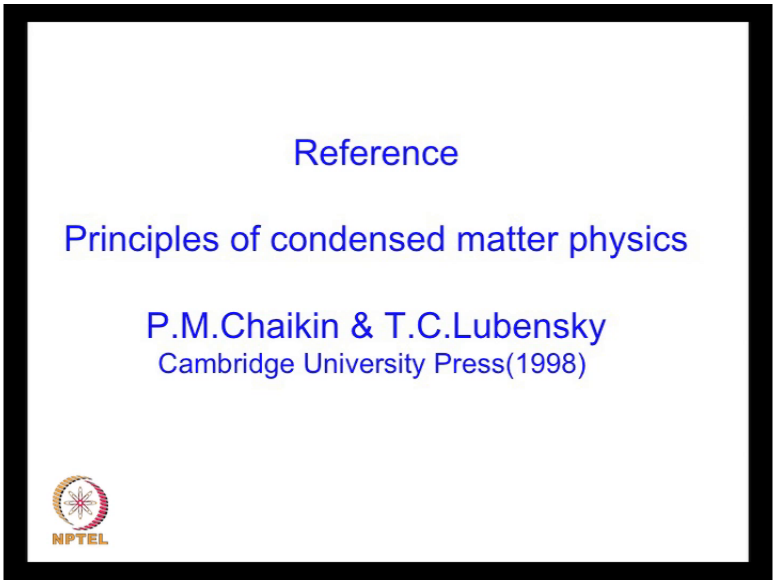

Reference

Principles of condensed matter physics

P.M.Chaikin & T.C.Lubensky
Cambridge University Press(1998)

Now the idea is that we have discussed in this lecture or discussed in much greater detail in the book by Chaikin and Lubensky the Principles of condensed matter physics by Chaikin and Lubensky published by Cambridge University Press. So you can refer to it for greater details. I have try to present unified brief account of what happen at when there is an on set of phase transition from a disordered into a condensed phase. So this is the key to the entire ((Refer Time: 28:10)) condensed matter. In the next lecture, we will be talking about the role of symmetry and learn how to define and describe symmetry are different kinds in a crystalline, purely perfect crystalline solid.