Indian Institute of Technology Madras
Present

NPTEL
NATIONAL PROGRAMME ON TECHNOLOGY ENHANCED LEARNING

TOPICS IN NONLINEAR DYNAMICS

Lecture 27
Quiz

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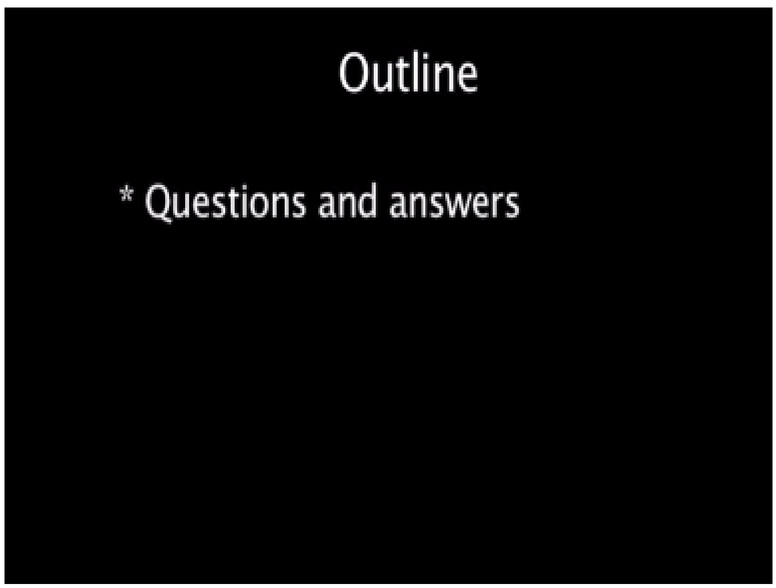

Let us discuss today the quiz paper that we had recently and then we will go on and see where this takes us and as I said last time we will switch to a different topic right after this so first the quiz first we had a number of statements which were supposed to be mark true or false and here they go any map of the unit interval that is non-invertible leads to dynamics that is chaotic true or false definitely false because the non invertibility is not enough you need much more than that in order to have clay also we have numerous examples of maps which are not invertible.

But which are not chaotic at all give me an example and non-invertible map that does not lead to chaos one dimensional map yeah the logistic map at parameter value < μ ∞ for example at the value of μ like 3.1 or something like that that is not chaotic definitely there is some stable some period cycle which is their planet so it is not care it is not invertible goes up and comes down like a parabola so that is certainly not true the Lyapunov exponent of the logistic map.

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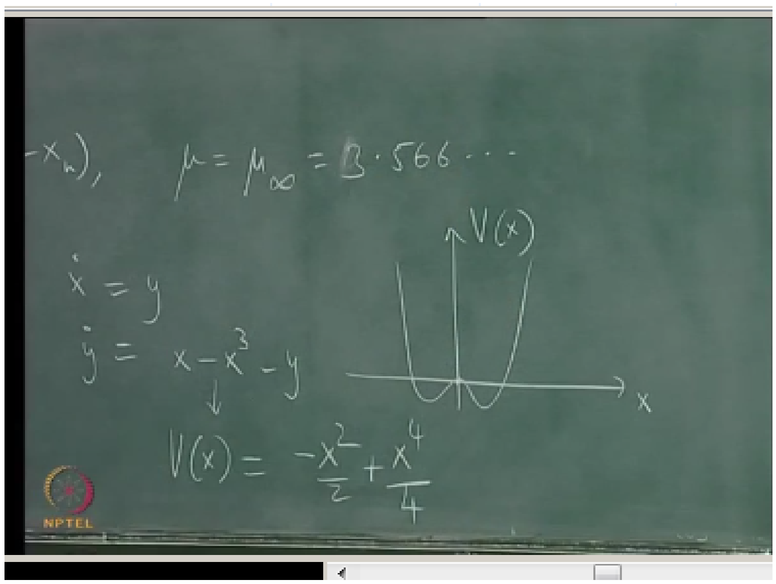

xn +1 μ times xn so xn + 1 = μ times xn 1 - XM at $\mu = \mu\ \infty$ = 0.5 6 6 etcetera this is the end of the period to cascade of doubling period doubling cascade of bifurcations this at this value 3 point sorry 3.5 66 etc so the statement is that the Lyapunov exponent at this value of μ is = log to its false it is 0 because it is the onset of chaos a little bit beyond this $\mu\ \infty$ you have chaos and as signified by a positively up now exponent in the logistic map this becomes to the Lyapunov reaches log2 only when μ hits 4 and the map leads to fully developed chaos till then it is less than log 2 for any chaotic attractor the generalized dimension.

D 0 is = the dimensionality of the phase space itself I repeat for any chaotic attractor the generalized dimension D 0 = the dimensionality of the phase space itself for the maps we looked at fully developed chaos in one dimension the tractor was the unit interval itself except four sets of meta 0 and then this statement was true but that is not true in higher dimensions certainly whenever you have chaos in a dissipative system in general there is shrinkage of volume and the system sits on some attractor whose dimensionality is less than that of the dimensionality of the system itself definitely even.

In the case of the logistic map it is clear that the dimensionality of an attractor could be a fractal dimension between 0 & 1 even though the phase space is one-dimensional so this is a false statement not necessarily true if the Lyapunov exponent of a one dimensional map is positive we may conclude that the dynamics is chaotic for all initial conditions false because we know that there are initial conditions which lie on periodic orbits unstable periodic orbits and then this is not true that set of initial conditions may have zero measure or may not but you certainly cannot conclude that the layup.

If the Lyapunov just because there is chaos the Lyapunov exponent is positive we cannot conclude that the dynamics is chaotic for all initial conditions not true stability analysis using a Lyapunov function enables us to decide on the stability of a critical point even in cases where linearization and the vicinity of the critical point is invalid this is true this is one of the great advantages of Lyapunov method now once you have a properly open off function.

You can make definite statements about the stability or asymptotic stability or instability without having to do linear analysis so that is the great advantage of a Lyapunov function the logistic map the same map undergoes a bifurcation at $\mu = 3$ a hoof bifurcation at $\mu = 3$ it undergoes a flip bifurcation from a period one fixed point to a period to cycle the period one cycle the fixed point becomes unstable the fixed point at $1 - 1 / \mu$ becomes unstable and bifurcates to a period to cycle which is stable for some range of μ hereafter and that is a flip bifurcation.

It is not even a pitch fork bifurcation period doubling bifurcation it's certainly not a hope bifurcation this system does not have any limit cycle at all hoof bifurcation cannot occur in a Hamiltonian system it leads a limit cycle which happens only in this repetitive systems not in a conservative system and therefore the statement is true the origin so the next question was the origin $X = 0$ $y = 0$ is a global attractor for the system given by X dot is y and y dot is $X - X^3 - y$ this statement is false

What sort of system is this what kind of system is this can we think of a physical system which has this set of equations any potential yeah this damping it is clear that if you regard this as the position in one dimension this is the velocity with appropriate units and then this is a damping proportional to the velocity in magnitude so it is a damped system provided this portion can be identified as a force arising due to some potential what would that potential be so if I plot x here versus V of X VFX would be - the integral of this since the force is - the derivative of the potential the potential is -the integral of the force.

So what would this give you this leads to a potential V of X which in these units is = what + or - $x^2/2$ - $x^2/2$ so it is - x 2 / + X 4 /4 fashion right and then what sort of potential is this it is a double well potential so definitely it is a potential looks like this in this fashion and the origin is a point of equilibrium it is a critical point but it is actually a saddle point in this case and you end up with two in the undammed case and then you end up here with two attractors there are actual attractors.

Here because this is a damped system they are not centers if the damping were absent this would be a center these two points would be centers and that would be a saddle point but now you actually have two critical points here which are as important what sort of critical points are they there is damping present in the system yeah these things would actually be asymptotically stable spiral points and then in between you have an unstable equilibrium point so this is not a global attractor nor is this nor is that there are actually two attractors.

In the system and depending on what your initial conditions are you would fall into one or the other there were two basins of attraction for the two attractors so this is just the Duffing oscillator the double well potential with unfor stuffing oscillated with linear damping it's the Duffing problem the winding number of the singularity at the origin of a planar vector field has been given to you and you are asked to show that asked whether the winding number is = - 2 or not let us check.

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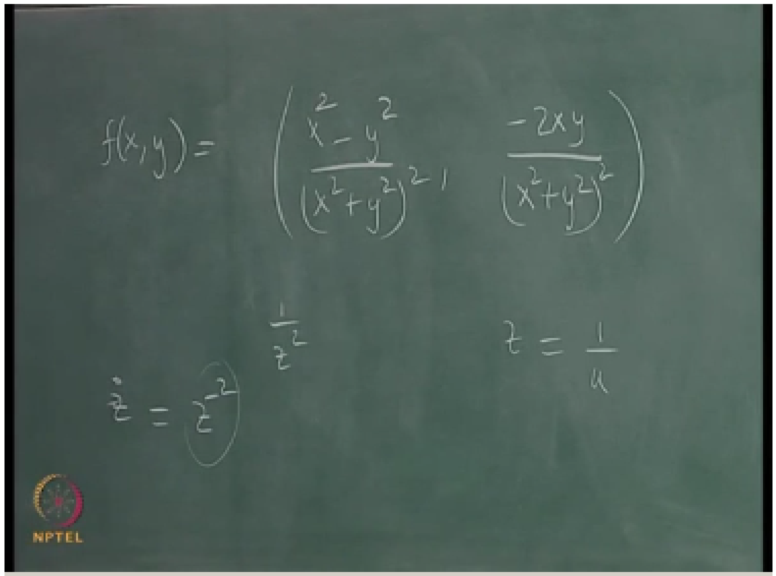

Let us check so (F)(X, Y = x ^{2} -y ^{2} /X $^{2+}$ y ^{2})^{2,}(- 2x y / x^{2} + y ^{2}) ^{2} it is a planar vector field which is singular at the origin as you can see probably blows up at the origin because of these denominators now what are these things suggest .I mean this x and y components of this vector field suggest that they are the real and imaginary parts of some function of complex variable z what would that that be that bar square so this these things are this thing here is one over set square that is the vector field so if i write this as (x + y I y) ^{2} I get an (X - I y)^{2} on top and below its (x^{2}+ y ^{2})^{2}.

So that is precisely the real and imaginary parts of this so you could write this in complex notation as Z dot if it is a dynamical system you would write this as Z dot = Z to the - 2 and the vector field is e to the mind that of - 2 so what is the winding number I go around once around the origin in the Z plane and what is the amount by which the argument of this vector field increases or decreases well if Z goes to z to the 2 πi I goes around once then how much does that increase by - fourth I I right.

So that is what the argument changes by so the argument changes by - 4 pi now as a multiple of 2 π - 2 times 2π therefore the winding number = - 2 yes up to the a ^{2} + y ^{2} a the dipole field would correspond to set squared + 2 yeah if you replace my way this way yeah and then it is equal to the type of it except for the factor below yes but that's all important that is absolutely all important you see whatever happens in the dipole field at the origin suppose I put z = to 1 /μ for example then clearly whatever happens in the origin in Z happens at ∞ in you and vice versa so because it is $1 / Z^2$.

What you should really do is to make a change of variables 1/ Z in which case this field is just u squared the winding numbers +-two but now you map back to 1 /μ and then it changes sign because whatever goes around counterclockwise once in this direction viewed from the point of view the point at ∞ it is going around in the opposite direction right if I imagine the point at ∞ to be the North Pole and the point or the origin.

To be the South Pole in stereographic projection and I go around once around the South Pole in specific direction viewed from the North Pole it is going to be in the opposite direction so that is precisely what is happening this - 2 is what appears here and there for the winding number is - 2 1 + 2 okay, so that is just a small change on the usual dipole field this field is singular at the origin it does not vanish at the origin it blows up at the origin becomes infinite energy the next question was the damped.

Unforced Duffing oscillator cannot have any limit cycles true or false it is true we prove this by the Poincare benison the bendix and criterion we showed that in this case in the damp ton for stuffing oscillator whose equation we wrote down a little while ago you cannot have any limit cycles at all simply because the vector fields divergence has a specific sign so we saw that as soon as that is the case you cannot have anymore any limit cycles we use greens theorem in the plane to establish this fact the brand Dixon criteria.

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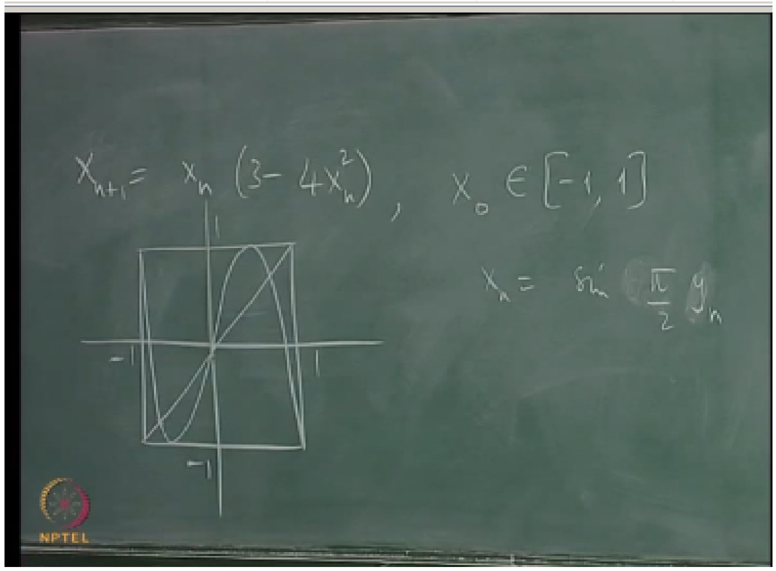

Then the next question was consider the map xn + 1 is xn x 3 3 - 4x ^{2} so let me write this background xn + 1 = X n X 3 - 4x ^{2} and X0 is an element of - 11 and the statement is this map has a stable period 3 cycle is that true or false it is false because we can see this immediately what does this map reduce to yeah it just reduces to like the Bernoulli shift except in ternary the slope 3 because we can see that directly let us plot this map let us plot it here is - 1 1 this is - 1 and 1 it is a non to map as you can see and when xn = 0 xn + 1 is also 0 when X n is + 1 then it is 3-4 it is - 1 and vice versa.

So this map does something like this it is a cubic map + this it does have fixed points there is a fixed point here the slope at the origin is three that is bigger than one so it is immediately unstable that is clear and is an easy matter to see that these slopes are also greater than one in magnitude and therefore all the fixed points are unstable if you iterated this map what would happen if I took f2 and f3 and so on and eat rates of this map what would happen they just go up and down a few more times

But the slope would again be much > 1 get increasingly increase as the number of iterations increases so it is clear that this map has no period cycles at all no stable periodic orbits at all so not only are the fixed points but all the higher periods periodic orbits of this map are completely unstable it is fully chaotic so you should not imagine that these three points form a period three cycle to start with they just fixed points of this map and it has no fixed points which has no periodic points which are stable now to see that this map is actually shift of some kind.

It is not very difficult all you have to imagine is to put xn = some trigonometric function sin or cosine or whatever it is so you put sine θ n and then you discover that you have a formula for sin of 3 θ as sin θ in terms of sin θ here and it says that 3 θ θ n + 1 is 3 times θ n - 1 θ N and therefore θ n is 3^n times 3 it or not and that is like the Bernoulli shift in this θ variable so since we have it between - 1 and 1 the obvious thing to do is to make it like this so let me call it Y N and then y n runs from - 1 to .

So with this change of variable this is immediately solved this map is immediately solved in closed form it is the analog for the cubic map of what the logistic map would have been at μ=4 we are all so we made a trigonometric change of variables and we got the Bernoulli shift the doubling shipment the next question was let x of to be a dichotomous Markova process in which X jumps randomly between two values x1 and x2 with mean residence times τ 1 and τ 2 in the two states we said let the mean be zero.

That is not necessary let the mean value be zero the statement is the autocorrelation function of the process is a decaying exponential function of T this is a true statement for a dichotomous Markova process no matter what this levels are and no matter what the rates of transition are the autocorrelation function is an exponential function the reason I said let the mean be zero was because I did not want X - the mean value at X of 0 - the mean value times x of t - the mean value that is the general auto correlation function. And I want to get rid of that mean so I define the mean to be 0 here.

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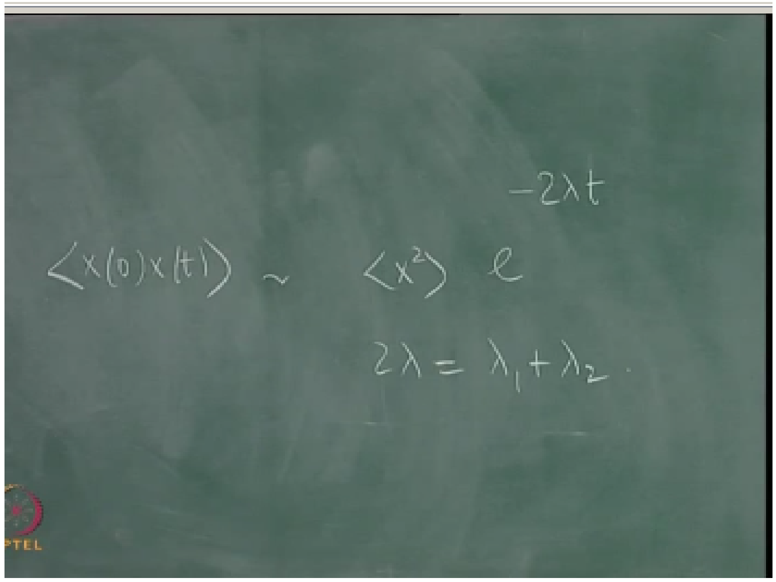

So in this case you have a process which goes up and down in this fashion between two values and we saw that if the value here is X 1 and the value here is X 2 and we arrange the rate so that the mean is 0 so that there and if this rate is λ 1 and this rate of transition is λ 2 then this autocorrelation function X of 0 X of T this goes like the mean square value whatever it is x^2 multiplied by e to the - 2 λ T and 2 λ is λ 1 + λ 2 that is a general statement not very hard to derive for a dichotomous process.

So it is an example of a very simple model of a Markov process which is exponentially correlated occurs in numerous applications and the characteristic time is 2 λ inverse which is the sum of the inverse of the sum of the two rates that is worth noting if the meantime of stay in this state is τ 2 which is λ 2 inverse and this is τ 1 which is λ 1 inverse the mean times a τ 1 and τ 2 then one should not come to the conclusion that it is a correlation time is τ 1 + τ 2 not true.

Let us in fact this so this guy here to λ inverse is the inverse of this which could be written in terms of τ 1 and τ 2 and what does the correlation time come out to be in terms of those 2 τ 1 τ 2 over τ 1 cross found so much for that then the next question give a set of numbers between 0 and 1 so let s be the set of numbers such that the decimal expansion of any X element of s is of the form X = 0 point a 1 a 2 etcetera.

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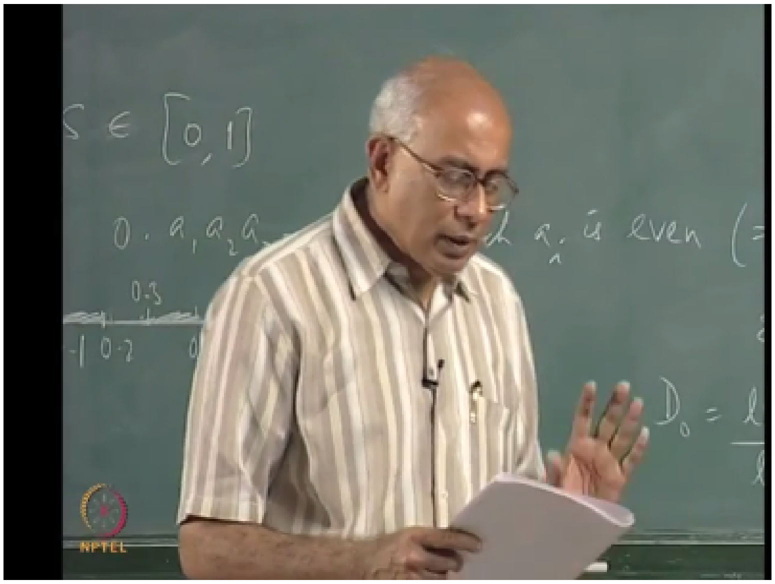

So we are starting with all the numbers between 0 and 1 this is the S is an element of 0 1 and then we have the set of numbers is of the form X is zero point a1,a2,a3 dot ,dot, dot each a n is odd namely they cannot have the sorry in other words the digits each of the digits is even cannot have the values each is even does not matter but (= 0, 2, 4 6, or 8)and you are asked to calculate the fractal dimension capacity dimension or the box counting dimension of the set s so what you do is simply say take the interval between 0 and 1 break it up into 10 = parts so 126 9 and 10 so 0 to 1 and this is 0.1 0.2.

Here etcetera right up to 0.9 so this is 0.3 0.4 0.5 0 60 70 point eight and one now a one can only be even it cannot be odd therefore it cannot be between 0 point 1 and point 2 because then a 1 would be pawn and even an odd number so this is forbidden this region is forbidden this is forbidden this is forbidden this is for that is forbidden therefore a one can only be in these intervals then you break up each of these into 10 parts and the second decimal would again be in one of the five intervals out of the 10 every alternate.

One is permitted and this process is self-similar at every stage it is exactly the same process therefore what is happening is that you have got a resolution add magnification factor ε which is 110 and at each stage you are breaking up a unit of the previous stage into n of ε parts where this is = 5 because the other five have been erased and therefore d 0 = log 5 divided by the log of 1 over ε which is locked tense that is the fractal dimension okay now of course you could make this = question a little more sophisticate by asking for various probabilities.

If I did not associate equal probability measures with all these things but I had biases to one side or the other then I would get a multiracial and I did get generalized dimensions DQ which are different from d0 but otherwise once I do this kind of coarse graining and I continue this then all the DQ's are the same as d0 nothing changes and it is just a regular fractal rather than a multi-front finally

We come to the last question pardon a multiracial is something where well many ways of defining it but if you have many dimensions generalized dimensions associated with a set I call it a multiracial in other words we have a whole spectrum of dimensions DQ as we defined a generalized dimensions.

Then it is a multi-factor but if all the DQ's collapse to some single value d 0 then it is just a fractal so these are in that sense regular simple fractals okay the next one was just a problem in matrix algebra and went free identical tall glasses ABC contain water to respective heights X naught Y 0t and Zenith the levels in A and B are first equalized by pouring water from A to B from or b 28 depending on which one has more the levels in B and C are then similarly equalized and then C and are equalized so that is a complete operation.

And then you repeat this operation over and over again and it is intuitively clear that eventually the levels in all the three would become equal if no water spilt and that level would be one-third of X 0 + y not + Z 0but the question is what is the rate at which this limit is approached in other words what is the actual value what are the values of the levels and the three glasses after n iterations of this step so this is done in a very straightforward way we could randomized this problem as well. But this is completely deterministic so it is just a set of three recursion relations.

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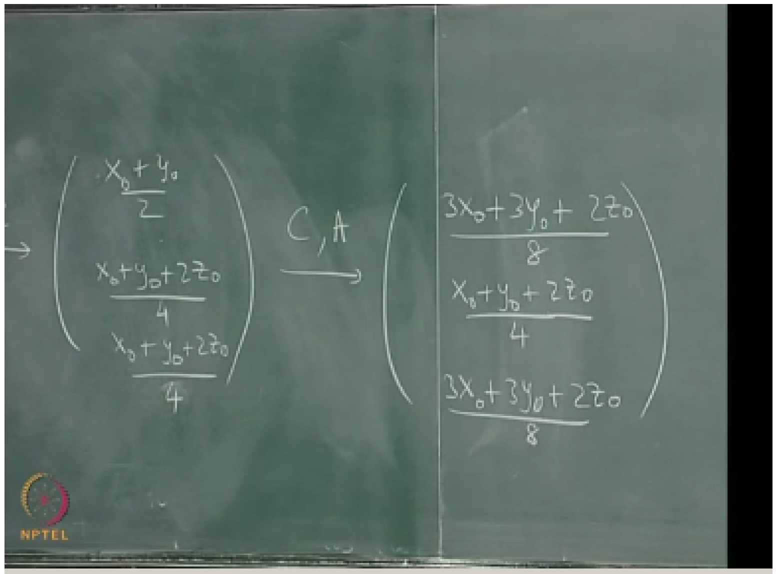

I start with the some three glasses which are and this one has some X 0 this one has maybe more we do not care why not and that one has some Z 0 you keep pouring from one to the other and you ask what happens to the levels finally well we start with X 0 let us put them down as a matrix Y 0 Z 0 and I take the two glasses A B and equalize so what I have done is to make this X 0 + y 0/to X 0 + y 0/ 2 and z 0 then I do the same for B and C so this remains as it is but these two guys get equalized

So this is this + that divided by 4 so it is X 0 + y 0 + 2 Z 0 / 4 and so is this in this fashion and then I change C and A I add I equalize these two levels which is equivalent to saying I take this and this and take the arithmetic average of the two which is a 2 X 0 + + X0 so that is3 X 0+ a 2 y 0+ or why not that is 3 y 0+ a to Z 0 the whole thing is divided this the common denominator already so it is equalized made in out there and that is exactly what you have here 3 X0 + 3 y 0+ 2 Z - / 8 and this remains as it is appointed by four.

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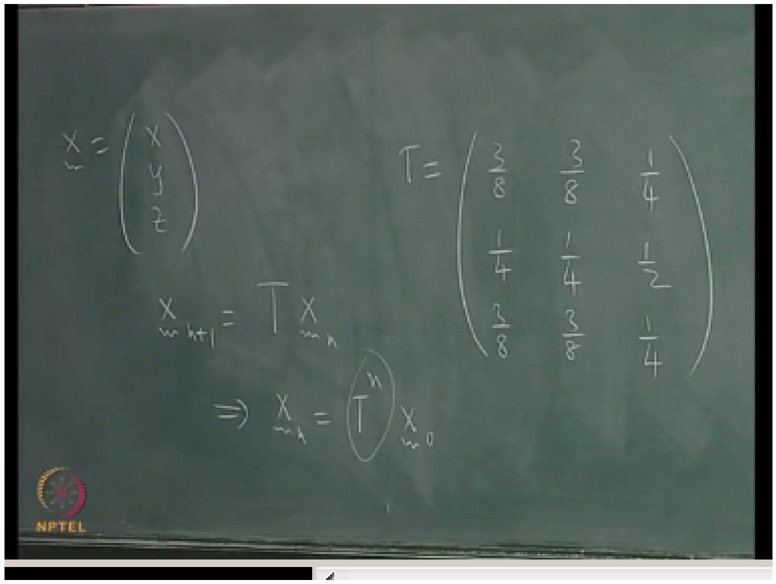

So what is essentially happened is that you have a column vector X which is X Y Z in this fashion and what we are doing is to say X at time n + 1 =some matrix let me call this matrix T X n time xn and the matrix t is written down there it is = 3838 and one-fourth on this side it is a ¼ a 1/4 and a 1/2and again it is 3/8 creates and one for that sector and this of course would imply that x ^{n} n x at time 0 and we need to find this we essentially need to take the end power of this matrix now what is interesting is to find what its limit is going to be.

We are guaranteed that what will survive finally for XYZ would be one-third of X 0 + y 0 + that not all the three would become equal in this case so the system would tend to the uniform distribution all three glasses have exactly the same amount of liquid now what is noticeable about this matrix it has zero determinant but it is a stochastic matrix because the material is not being lost so in some sense 0 basically a stochastic matrix in the sense that some of the rows is equal to one each row sums up to one each column also sums up to one this is doubly stochastic in this case is it a symmetric matrix not as it stands not as it stands.

So we really cannot immediately assert that the left and right eigenvectors are the same and I said that at all notice that because the rows are up to one you end up with the uniform eigenvector so 111 is an eigenvector and similarly 111 a row vector is also an eigenvector left eigenvector of this what are the eigenvalues of this matrix 0 has to be one because the determinant is 0 1 would be an eigenvalue exactly one would be an eigenvalue that is what would correspond to the equilibrium distribution finally.

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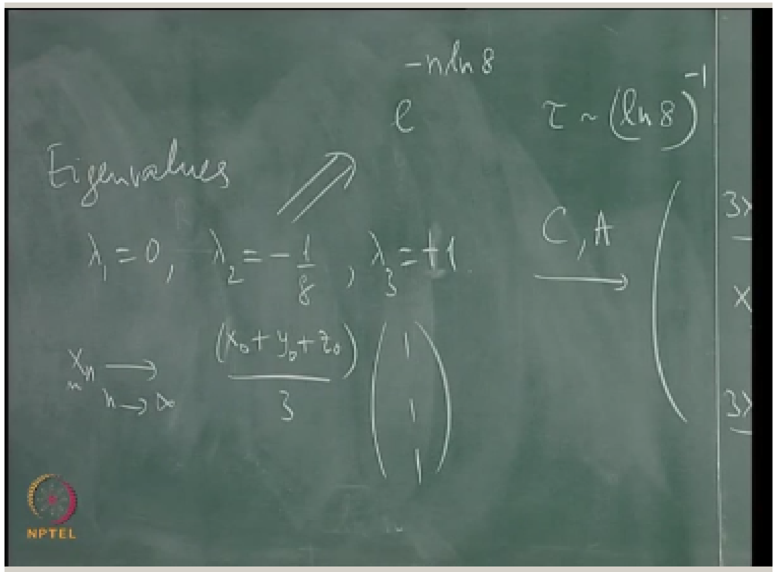

So Eigen values λ 1 = 0 λ 2 = something and λ 3 =1 + 1 1 would be an eigenvalue the third one in this case is not hard to discover it is - 18 so now we can diagonalise this matrix write down what the T ^{n} is and so on zero would just go remain zero and the one would remain one but then when you take the nth power this guy would give you - 18 ^{n} that gives you a time scale in the problem. I am not going to work that out is a matter of detail but we can see that eventually X will tend as n tends to ∞ X n this would tend to X0 + y0 + Z / 31 11 normalized.

To this and the question is what is the time scale how does it do so what is the characteristic time so you have an exponential here and if you raise λ 2 ^{n} we have 18 ^{n} which could be written as e to the - n log 8 so this would ⇒ that there is a terms of the form e to the - n log eight here so the characteristic time τ would be like log that is the interesting part so it is a deterministic process it is a map but it is very much like a Markov chain in this case it is a given by a certain transition matrix and this thing here is a stochastic matrix.

So it is exactly like the transition matrix for Markov chain and it has an equilibrium distribution finally as I said we can make this problem more interesting by putting in random elements into this some probabilities with which you do certain operations before others and so on and then the matter would become a little more intricate and interesting but this is completely deterministic you equalize a and B first B and C next C and A but you can do this in various orders with probably various probabilities and then you could ask similar questions.

What is the probability of finding the distribution of the levels at a certain stage this is also answerable okay, so I more or less done okay .I would like to open sort of conclude anything else any other questions and all that we have done so far I have to yeah the question is what is meant by a correlation function an autocorrelation function for example the simplest of these cases so let me explain that in brief again let me take the example of a random variable x scalar random variable.

And ask what can you say about this random variable it does not have to be a Markov process could be anything at all then clearly the following are relevant questions.

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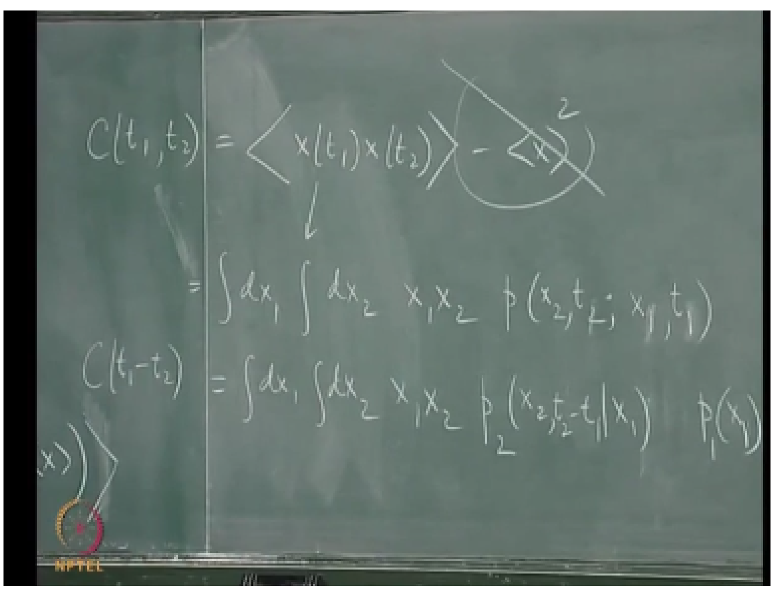

To ask to relevant weight relevant physical quantities to analyze for a random process one of them would be to say if you have this what is the average value at any instant of time it does not have to be a stationary random process if it is then the average is independent of time because you would evaluate this over a distribution which is independent of time you can generalize this and ask what is the nth moment of this object this too if it is stationary would be independent of time now you could have stationary.

At the level of the mean at the level of the autocorrelation and so on and so forth but let us assume we take the definition of a stationary process in the strict sense of the word namely all the distributions all joint probability distributions single time ones are independent of time to time distributions have depend only in the time difference and so on in other words the origin of time is irrelevant the next question you could ask is all right what is the average value of the product of the value of the variable at some time t1 and the same variable at a later time T 2 what is this equal to this would in some sense characterize the amount of memory present.

In this variable in this random variable if it is a Δ function into t1 - t2 then I would say it has no memory whatsoever like a noise a white noise for instance but in physical problems this would always depend on t1 and t2 if the variable is a stationary random variable then. I would expect that this quantity would become actually a function only of the time difference between t1 and t2 and not of the two absolute times t1 and t2 separately in fact I should be a little more careful here.

I should really define X (t1) - the average value at that time X (t2) - the average value at that time and then take the average value of this and this is what I would call the correlation function which in general is a function of t1 and t2 this is what I'd call the autocorrelation function of this random variable if this is a stationary random variable then certain simplifications occur so stationary $\Rightarrow$ immediately that this thing becomes the average value of (x (t1)- the average value of x average value of x and that is independent of T.

So just this average but we can separate this out it is x at t 1 times x average - average of x times average of x here - another of those same things because these quantity is no longer get averaged because they are constants and then + the average of x the whole squared so you could also write this as C (t 1)x (t 2) = - X^2 so it is like the generalization of the variance except that you have time arguments here and now the statement is if this is a stationary random variable what is meant by this thing here.

Let us try to write it out suppose it is a continuous variable let us try to write out what you explicitly what we mean by this explicitly it is an integral over all values let me call X 1 the value at time T 1 and X 2 to denote the value of the variable at time t2 so it is an integral over all x1 it is an integral over all X 2 and then x1 x2 because that is what you are averaging this product is what you are averaging multiplied by the joint probability density that you have (X 1 t 1, X 2, t2) but this quantity by definition can also be written.

So this quantity is equal to that let us forget this for the moment apart from that this thing is equal to this and then this is = DX 1 integral DX 2 x1 x2 this joint probability if for instance t 2 > t1 t1 is the earlier time I use slightly different notation i use the later times I move to the left so let me leave that same note let me write this as x 2 t 2 x 1 t 1 pardon me yeah I can shift the origin but let us do it slowly so this is x 2 p 2 given x 1 at time T 1 multiplied by P of X 1 T 1 this is the single time probability this is the two-time probability densities.

But since it is a stationary random variable we assumed that this is independent of T this is this quantity here and then once again since its stationary I write this as t2 - t1 and I could write this as 0 but let us just remove that or together it stands for some origin of time so we come to the conclusion that C is in fact a function of t1 - t2 so this thing here is a function of t1 - t2 to start with let us forget about this smell let us set that =0 or shift the whole origin of this variable so that the mean is zero.

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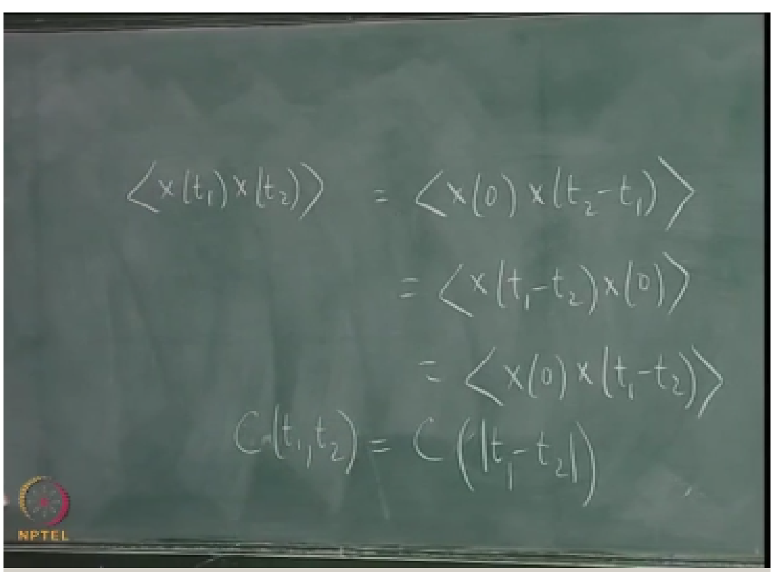

But you can say a little more than that notice that if it is stationary you could actually write this C (t1) (t2) so this quantity X (t1) X (t2) if I subtract from both sides t1 and I said t 1 = 0 or a shift the origin to p 1 you could also write this as x at 0 x of t 2 - t 1 but I could also have shifted t2 so you could also write this as x of t 1 - t to x of 0 but if these are classical variables there is no problem about commuting them this is also equal to x of 0 x of t 1 - t so in fact it says that this correlation function is a symmetric function of the time difference the two-time arguments.

So you actually come to the conclusion that c of t 1 t 2 = C of the modulus of t1 - t2 it is a function of the magnitude of the time difference and it gives you some idea of how rapidly the system loses memory in the case of a dichotomous Markov process this was a single exponential but it does not have to be so it could be much more complicated than that but what we do know is that you end up with something which is a function of this modulus alone now physically.

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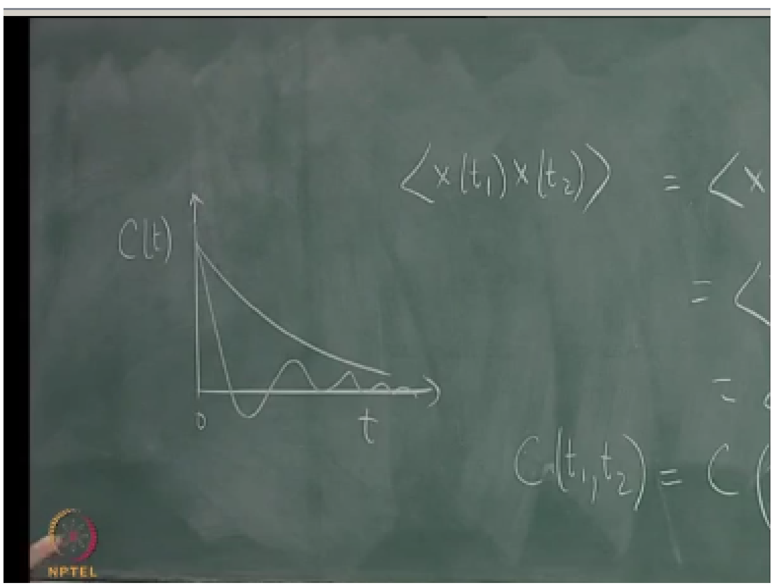

I would expect if the average value was 0 for example I would expect the behavior of the C so let us call it C (t) as a function of T and let me just plot the positive side of it the negative side would be symmetric in this case it would start here and perhaps go to 0 it would DK may be exponentially maybe slowly like a power law the memory would gradually go down that would be decided by how fast the limit of this quantity as t2 - t1 goes to ∞ goes to p1 because I know that in the limit this forgets.

The initial condition and the limit of this quantity is in fact just p 1 of X 2 so it would depend on that it does not have to do this it could do this to do crazy things physically I would expect it to be a non increasing function. I want to expect the correlation to increase but of course it could be purely periodic suppose this variable is not a random variable at all but it is the position of a simple harmonic oscillator and I take averages over time what would you expect this correlation to be suppose it is a simple harmonic oscillator of frequency ω what would I expect this correlation function to be I know there is no averaging I average over the actual dynamics I know if I start with an X 0 I know what the system does it would just be a periodic function

Once again it would not go down at all it will just start at unity and then perhaps do this but in real random variables in more realistic situations I did expect this correlation to die out and it is a measure of how much memory there is in this in fact you can find a time scale from this C of T because notice that you could integrate this C of T so I could do the following in the case of a stationary variable I could take C of T.

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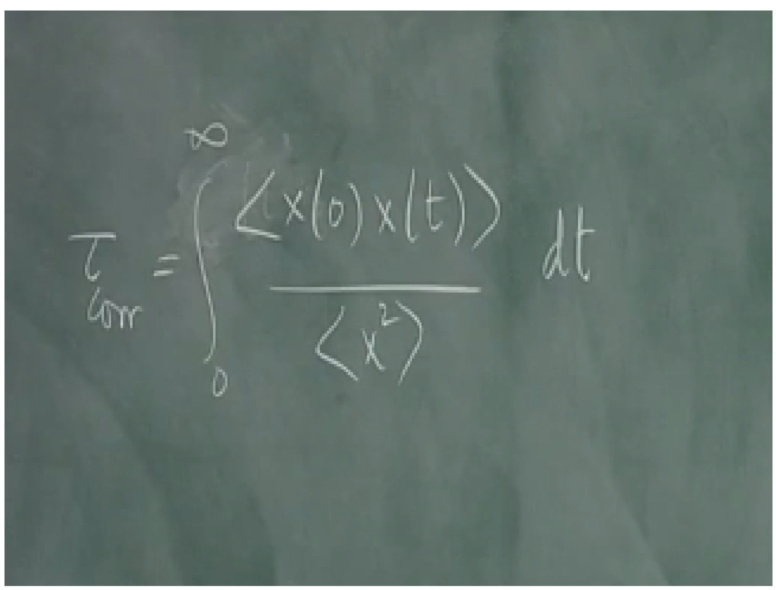

I could take the quantity X (0) X(T) / x ^{2} so as to make a dimensionless and integrate this over T from 0 to ∞ all right call this the effective correlation time because it has physical dimensions for tiny as you can see if this is mean value of x^2 e to the - λ T and I do this integral then the correlation time is just 1 / λ for a single exponential but otherwise I get some effective correlation time there is another thing you do with the correlation function and that is the following you could also take its Fourier transform so you could also take let me call C of T the correlation function.

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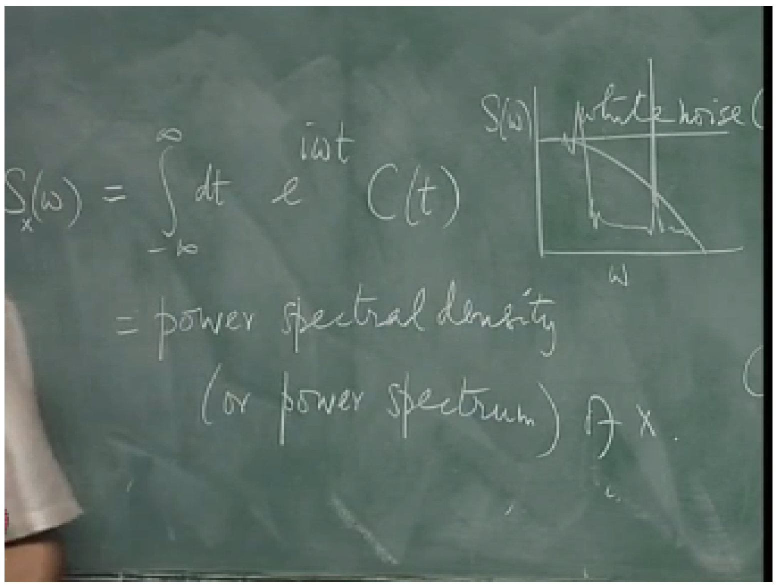

You could also take integral - ∞ to ∞ d t e I ω T C of T that is a function of ω here so you could ask what is this equal to call it something else so let me call it s to show that it is over the function f for the random variable X let me define it in this fashion and this is called the power spectrum the power spectral density of a short or power spectrum of the random variable X if X is a noise then this thing here tells you in rough terms the intensity or the strength of this noise in a frequency window between ω and ω + D ω and if it is white noise this is a Δ function and then it goes to a constant so that is that is the reason.

Why you call it white noise because the power spectrum is flat physically that will never happen of course we know that things will always go down so the power spectrum if you plot this s of ω versus ω in the ideal case for white noise it would look like this ideal white noise but in practice it would come down this fashion and a great deal of information is obtained by looking at the power spectrum so you could in fact start with any time series for any random variable even a chaotic time series and look at the power spectrum.

If the system has hidden periodicities in it then they will be detected here so anything that becomes periodic here you notice if I have a single period here of period ω 0 what would this become it will give me a pulse it give me a Δ function at ω0 so what happens in practice is that you have a power spectrum it looks like this maybe there are some spikes of this kind and at those spikes you know that there are periodicities in the system so in a complicated time series the power spectrum which is a Fourier transform of the autocorrelation function helps.

You to detect hidden periodicities if the system is completely noisy white noise that which have absolutely no such structure at all even the way it falls off tells you something about the underlying random process whether it goes like a $1/x^2$ 1 / F or whatever where F is a frequency there is a whole class of processes called 1 over F noise which corresponds to a power spectrum which dies down for large ω like one over on ω to a power which is roughly between say point eight and 1.2 and this is called 1 / F noise.

It is very ubiquitous it appears everywhere if this were purely brown in a Brownian motion kind of noise then it would go like $1/\omega^2$ so the asymptotic fall off of s (ω) also gives you physical information if it is a chaotic the power series as opposed to pure noise true randomness or noise then the power spectrum behaves very differently and is very broad band there are no periodicities so it is sort of spread out very irregular very broad band whereas if it is complete knows we like white noise or falls off like a 1 / ω to some power and if it is periodic.

Then it ends up with spikes if it is quasi periodic it ends up with a large number of spikes so the power spectrum the physical way a very practical way of seeing something about the underlying dynamics regardless of whether it is deterministic or noisy or chaotic or a mixture of all these okay should we stop you.

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