Indian Institute of Technology Madras
Present

NPTEL
NATIONAL PROGRAMME ON TECHNOLOGY ENHANCED LEARNING

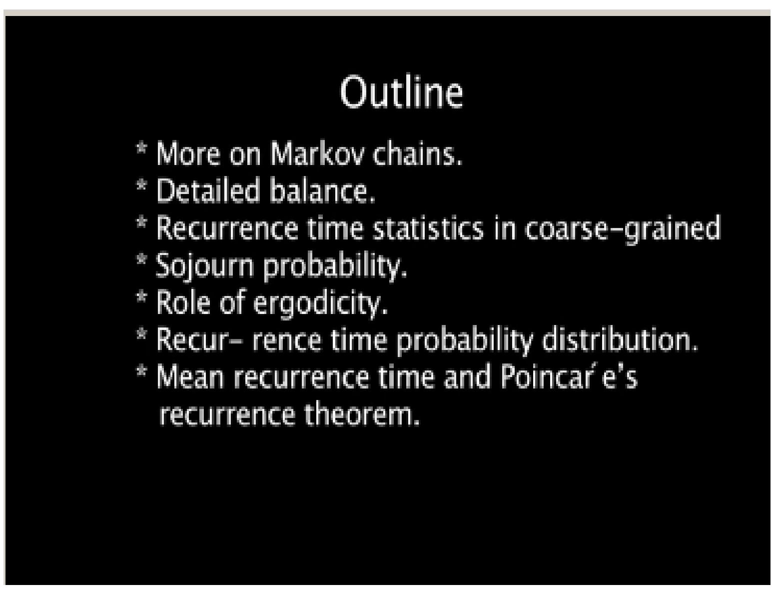
TOPICS IN NONLINEAR DYNAMICS

Lecture 24
Stochastic dynamics(part iii)
Coarse-grained dynamics in
Phase space (part iii)

Prof. V. Balakrishnan

Department of Physics
Indian Institute of Technology Madras

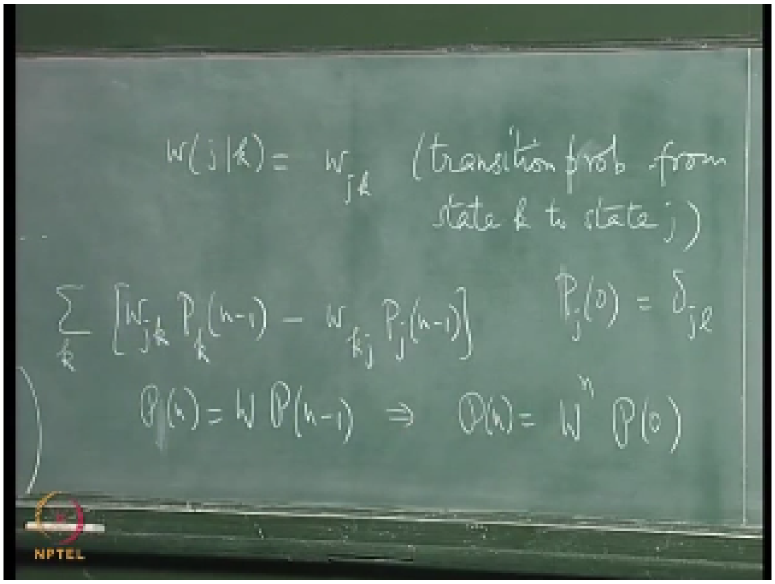
(Refer Slide Time: 00:11)



So we were talking about coarse graining dynamical systems such as Maps and we saw how in the case of a simple tent map an asymmetric tent map we saw that the coarse graining into two cells a left cell and a right cell led to a matrix W of transition probabilities and I left it to you as an exercise to check that this really was the transition matrix for a Markov chain so I hope you have done it and convinced yourselves that indeed the dynamics reduces to that of a Markov chain let me spend a few minutes now and explain a little more in detail what we mean by a

Markov chain so that in general this concept becomes familiar to you because it is extremely useful this is the discrete-time analog of a continuous-time process Markov process which we talked about a little bit.

(Refer Slide Time: 01:04)



So let me explain again what we mean by Markov chains so we imagine there is a random variable which takes on a discrete set of values suppose you take these values to be X 1 X 2 X 3 etcetera and we work in discrete time just as in the case of maps and let me label the values of these of this random variable X sub j j = 1 2 3 and so on so I label these values by just J and when the random variable takes the value XJ I say that the system is in the state J so from now on we do not worry about the x's itself.

We just talk about the state of the system and that is labeled by an integer this set of integers could be finite or infinite so the number of possible states of the system could be finite could be infinite but it is discreet and countable in this fashion and now we ask what is the probability P that at some specific instant of time indiscrete time the state is Jay that is the probability that the system is in the state J at the time n and we ask what is the sort of rate equation you write for this and this becomes on the right hand side.

Because we have now discrete set of states you have summation rather than an integration and this probability depends on the preceding step and nothing more than that so it is equal to P a summation over all the possibilities k PK of the probability that the system is in the state k and then makes a transition from the state k to the state J at a certain rate a transition rate which we wrote as W so this is equal to W from the state K to the state J so W I am not sure how I wrote this the last time.

We wrote this as W j/k I wrote this as w JK or KJ it is a matter of KJ yeah this is the initial state and that is the final state so what did I write this as JK oh I just wrote it in the same order transition probability from state K to state that was my definition so this becomes w WJ PK at time n -1 having reached the state K at time n - 1 it then jumps fromk2j so that it contributes to PJ of n at time n minus there is a loss rate which is KJ p j at time n- 1 this is the chain equation this in fact is the master equation not the chain equations.

The master equation for a Markova chain but of course you have to specify some initial condition you have to say in what system state the system is in could be in a distribution of states or could be in a specific state if it is in a given state then of course you would say p j at time 0 if this thing here is in a specific state let us call, call that s 8l for instance at time 0 then this thing becomes equal to Δj l.

So it is one if j is l and 0 otherwise so the task is to solve this setter this equation here with those initial conditions and of course exactly what we wrote down earlier place comes through and that is if i define a vector column vector P of n which consists of P 1 of nP 2 of n etcetera then this whole business could be written as PJ of n is w times P P of n WP n -1 where there is a certain matrix whose elements are given can be read off from this equation and this of course immediately implies that p of n = $w^{n p}$ of 0.

And that is the formal solution to the Markova chain now there is a way of classifying these Markova chains is a systematic classification depending on what kind of transition matrix or transition matrix of transition probabilities w that you have if it is possible to go to any state in the Markova chain from any initial state then we say that this chain is irreducible because there is a connection between any state and any other state maybe not in one step but given enough time if this is true then this chain is irreducible if it turns out that there is a flip-flop between a few states for instance if from state three you go to five to seven.

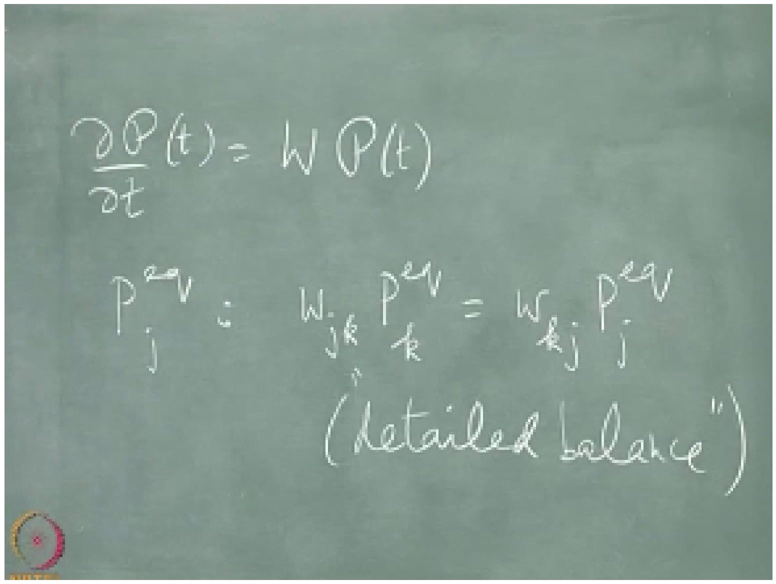
And back two three and so on then you can have these periodic cycles these Markov chains in between but if no such periodic cycles exist then you say the chain is a periodic and in general the non trivial cases of those where it is not a periodic as well as irreducible so that all states are connected up this fashion so everything depends on the nth power of this matrix and we know that this matrix can be put in Jordan canonical form by a similarity transformation after which rise raising it to the nth power is a matter of algebra.

So in principle this Markov chains can be solved some technical difficulties arise if this summation goes on till ∞ so from one if it goes up to ∞ then you have to worry about convergence questions of convergence of these matrices of what you mean by then the power for arbitrarily large n and soon arise some technicalities arise but otherwise in principle this is all that a Markov chain does and the whole thing is guided by what this w does a very important question which is important for us.

To in the context of dynamical systems is does this thing reach a steady state at all is there any stationary distribution associated with it is there a quote-unquote analog of an equilibrium state an analog of critical points in the case of dynamical systems or fixed points in the case of maps is there an unlock of that in Markov chains and the answer is the item that is very important we would like to know if after a long time if there is any invariant probability distribution just like an invariant measure is there something that does not change under further time iteration at all.

So is there a limiting form to this as n tends to ∞ is there some kind of equilibrium distribution if so it says that the distribution at time n does not change at all from that a 10-1 as n tends to ∞and that would happen when you equate this to that and you solve for the invariant distribution on both sides if you recall in the case of continuous time Markova processes in continuous time we ended up with an equation.

(Refer Slide Time: 09:17)



Which looked like Δ P over ΔT was equal to some W on P of T and I pointed out that. The stationary distribution would correspond to a right Eigen vector of W with 0 is the Eigen value because you want Δ P over ΔT to be 0 so you would like to have a non-trivial Eigen vector such that when W acts on it from the left you get 0 on the right hand side and the right-hand side was pretty much something like this except that it was in continuous time and what did we say then well if this vanished this quantity vanishes.

Then you have a steady distribution the many ways in which this sum can vanish you would have to solve a set of homogenous simultaneous equations but one important way which happens in many physical problems is if each term in this bracket vanishes if term by term this vanishes then you have a steady distribution which obeys a principle called detailed balance in other words if the steady or equilibrium distribution let me label it as P equilibrium J it is independent of time this thing here one possible solution is that this solution is such that w JK p k equilibrium is equal to w KJ.

And this is called detailed balance it is the detailed balance condition which is sufficient to produce for you an equilibrium solution because it just says that the equilibrium distribution is just the ratio of these rates w JK /WK g gives the ratio of pj equilibrium over p k equilibrium of course you could ask but that only gives a ratio of these probabilities how do I find the numbers themselves what would you say I would normalize the whole thing has to be normalized to unity and that gives the overall constant.

So this is a very, very important subclass of possibilities this is an important class of possibilities where you have detailed balance and of course whether it obtains or not depends on the physics of the situation mention this because we will come back to this we will see that there are reaction reactive diffusive equations where we are going to impose the detailed balance condition based on physical considerations yeah if it has a non-trivial equilibrium distribution yes certainly yeah if it is an a periodic irreducible chain then.

And it has in equilibrium distribution it will certainly be recording yes it will jump from one to the other it is not saying that things are going to stop it is just like an invariant measure so the dynamics continues it is just that under time evolution that distribution does not change thank you for example the gas particles in this room if I assume that these collisions have sufficiently short term memory that the processes are all describable by Markov processes then look at the velocity distribution of the particles of gas in this room.

It is a max William distribution under equilibrium conditions the whole idea is in spite of the collisions going on in spite of the dynamics going on the maxwellian distribution does not change so it is an invariant distribution it is an equilibrium distribution so that is exactly the point it is exactly like saying we went back a few steps saying that the density phase space density always an equation like H with, with row and the whole point of equilibrium statistical mechanics was that in equilibrium thermal equilibrium this is zero.

So we discovered that those distributions the phase space distributions had to be functions of the Hamiltonian what functions they were dependent on the external conditions if you kept a system in isolation a closed system in thermal equilibrium then that density is uniform on the energy hyper surface that is the micro canonical ensemble if you kept it in contact with the heat path such that it could exchange energy with the surroundings.

But not matter but not mass then this row was some specific thing called the Gibbs distribution it was a to the -β H where β is one over KT Boltzmann's constant times the temperature if you kept it as an open system which could exchange matter as well as energy with the surroundings then you worked in the grand canonical sample and this row involved not just the Hamiltonian a to the -β H but there was an extra factor which depended on the chemical potential of the system and the number of particles in it.

So this thing here is just this is this is precisely this is an example of the kind of invariant distribution that you deal with in physical situations the point I am making is that the distribution itself the probability distribution itself is invariant under the time evolution but it is not that transitions are not occurring they are occurring all the time but keeps the entire distribution unchanged so that does not answer the question okay I was point out that detail balance does not have to obtain.

Always there are special physical reasons in certain systems why obtains and then you have a very particularly simple as particularly simple solution to the problem of finding the invariant distribution but that is our equilibrium distribution it does not have to happen always you can however in principle ask is that does yeah, yeah this was for the discrete case so if I want to convert this now to a time equal like Δ P over Δ T yeah.

Then I would subtract from this pj of n - 1 and then go to the limit in which the time becomes the time step becomes 0 and then I end up with the master equation so this equation here is for a change it just says at time n- 1 you reach this at every at unit intervals I am making transitions and I reach this state k @ n - 1 this is the probability and the next step is to flip to J so this gives me the probability but having reached the state K at time n -1 in the next step I am at state J and this tells me how much flows out in this stage I have already am in the state J a 10-1.

And in the next step I move out her eyes exactly I have assumed throughout that these things are independent of time these are constants some given constants what would you say is this mark what would is what sort of chain would you say it is Markov chain would you say it is if these were themselves independent would still be Mark off Markov just means one step memory.

But what sort of chain would that be if these were independent this would be like the non autonomous systems we looked at in the case of dynamical systems so what would you say what kind of chain would this be the statistical property is change with time and it would therefore be a non stationary random process still be Mark off.

But non stationary then of course there is no question of a stationary distribution in that case so I have assumed that this, this set of numbers is independent of n itself and the independence comes here completely yeah yes it is given to you yes think how do you justify time step so that you no no, no yeah this is a good question the question is the time step arbitrary or not okay there are problems where if the time step is given to me and it is discreet and the state space is discrete it is a Markov chain if the Markov conditions are satisfied okay.

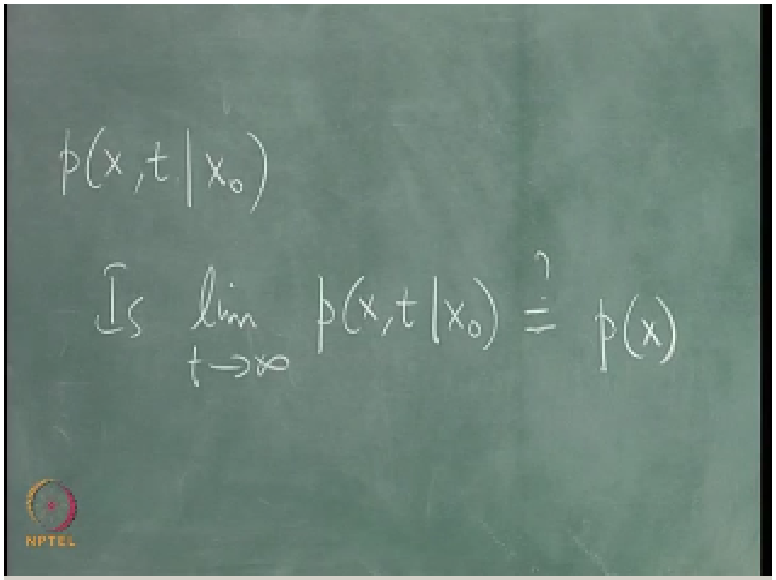
However there could be other problem which I could start by modeling in terms of a discrete-time dynamics but where the dynamics is actually running in continuous time and then to derive those differential equations I could start by writing down difference equations and then moving to the continuum limit that is the way you derive differential equations in any case in most physical problems you start by asking what happens at finite increments and then go to the limit in which ΔT goes to 0.

And it would lead in general to differential equations yeah no not at all I mean not at all I mean this is nothing to do with whether it is discrete or continuous or anything like that no I do not have to set that I would say this is equal to I mean if I if I say this is independent of n yeah exactly I take the limit in which n goes to ∞ and ask is there a non-trivial limit right if there is a non-trivial limit that is my stationary distribution is not it yeah.

So all I have to do is to ask does this have a finite limit does this does this thing have a limit as n goes to ∞ and the idea is that in most cases if the chain is erotic that limit will be independent of what P of 0 is as long as everything is connected to everything else just like the invariant measure was independent of what the initial distribution was in the case of the dynamical systems we looked at so in the same spirit it is if it is now.

That is a subtle question is this true or not is the question let me let me explain this is a good point let us look at it in continuous time we saw that for stationary Markov processes everything was decided by two quantities one of us a probability distribution of the variable itself the probability density function for continuous variables and the other one was a conditional density which looked like this so we assume stationary we assume time is continuous we assume that the state pace is also continuous.

(Refer Slide Time: 19:47)



And these are probability density functions this one is the one time density function but that is independent of the origin of time so there is no T sitting there this one was a function of X T X 0, T 0 but by stationary I subtract the t 0 out it is a function of just one time here okay now the interesting question is, is limit T tends to ∞ p of x t x 0 is this such that it loses memory of its initial condition and becomes equal to P of X or not this is the question one asks always.

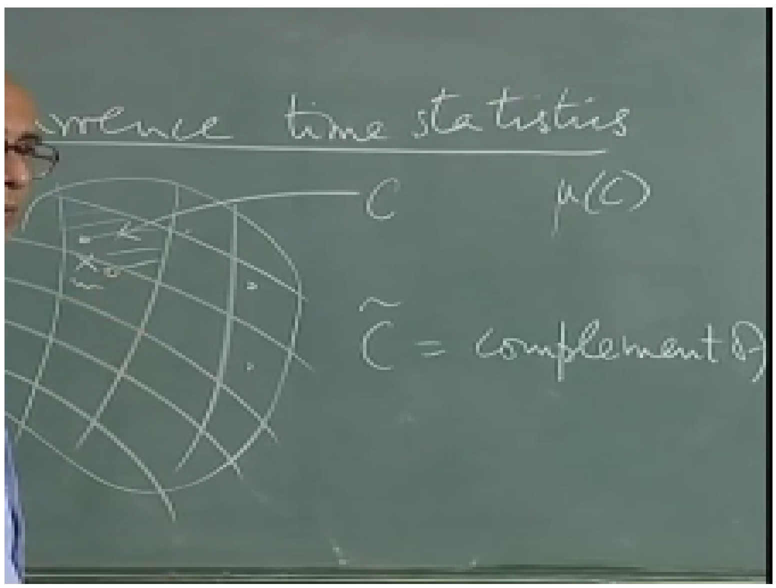
And the statement I made was that if the system has a sufficient degree of mixing then this is true when this happens so this is essentially what happens in the case of these Markov processes the kind of Markov process we are talking about this is what happens the fact that it is in continuous time is irrelevant if mean I could rewrite this in discrete time it does not matter yeah not necessary it is not very clear why that should be so why should it be so not every Moses yeah it actually implies yeah it implies is not necessarily true it implies that this system.

It is not a question of the memory being shot that has been taken care of already in saying that everything all the joint probabilities are decided by the Justice two-time probability but now if the correlation between the variable with the autocorrelation function of this variable dies slowly enough as T goes to ∞ there is actually no reason why this should happen by this limit shooting go to this it is an assumption that is made in general it is a sort of consistency condition.

But to prove it rigorously is another story altogether so it is not I do not believe that this is so that as soon as you say the process is mark off that this is true but I am NOT hundred percent sure I will check this out but the strikes me that this is independent this is an independent statement further input that has been put into this that is got to be added but I will check this out all right let me go back now and talk about the idea of a recurrence and this is something which we will deal with when we do coarse-grained dynamical systems we looked at a to sell dynamics if you like for the 10th asymmetric 10thmap in which you went from the left to the right.

And back again and so on but let us do this in a slightly more general setting higher dimensions in general and see that there is an extremely simple formula for the mean recurrence time which is important to understand it is called the Poincare recurrence formula is valid for all our garlic systems does not have to be chaotic or anything like that and it goes as follows the derivation is simple so let me do that and goes as follows so we look at a recurrence.

(Refer Slide Time: 23:24)



Let me call it recurrence time statistics we start with a phase space of this kind arbitrary dimensionality we will work it out indiscrete time in terms of maps and we could make it continuous later there are some subtleties involved but for ease of illustration let us look at this free time dynamics and I have a point an initial point X not out here and at time1 it jumps somewhere else and then somewhere else and somewhere else and an orbit is formed by this point X 0.

And I ask the following question I divided up my phase space into sense in this fashion and I focus on one particular cell let me call this LC I assume there is an invariant measure and I assume that the dynamics is a gothic in other words any set of initial conditions that little volume element would visit all of this available phase space given enough time I do not assume anything else just as God is it nothing more than that and now I asked if I start with the initial point in C then what is the first time that I come back to see what is the probability.

That I come back to see at the end time step that is one question the second question is what is the mean time in discrete time steps of sometime step tau indiscreet multiples of this time stepped out what is the mean time to come back then I could ask what is the variance what is the statistics in general so I would like to discover what the statistics of recurrences to this LC looked like that is the target and I assume that then I invariant measures there is an invariant measure on this and the measure of this cell C let me call it me of c and let us assume that the whole phase space is no its measure invariant measures normalized to unity.

So I do not have to keep dividing but me be computed and we say well we said that this is equal to the time step divided by mu I would like to derive this formula I would like to derive this from first principles but in principle in general I would like to derive the statistics itself not just the meantime maybe the variance I would like to find out exactly what it is so this is the target now how do we go about this well it turns out is a very elegant.

And a simple formalism to do this which goes as follows first let us ask what do I mean by the probability of a recurrence to this cell what I mean by it is the joint probability that and now let me in keeping with our notation right earlier times to the right and later times to the left I want the joint probability that if I start at the cell C at time zero I leave this cell and I come back at time n so I want the joint probability that having started here I am in the complement of this cell so let me call the rest of it that means the rest of this other than the shaded portion.

So I am in C tilde at time 1 and let me call this time step down let me set it equal to 1 for the moment see tilde 2 and so on till I hit C at time n see tilde at time n- 1so this conditional probability is what i would like to compute what is that equal to well we know that this conditional probability is a certain joint probability / the absolute probability this thing here so this is equal to p c and c tilde n -1 dot C till 1 c 0 divided by P of C 0 but this P of C 0 since I am assuming that the system has an invariant distribution measure.

And everything is stationary this p c ,0 is independent of the time origin it is independent of this time argument and it is just P of C and that is nothing but the measure of this cell itself so this is the same as mu of see I use P of C and mu of C interchangeably they are exactly the same thing remember that I have normalized the total volume or the total measure invariant measure of the space, space to unity so then I can talk I can replace P of C by just move see and it is this quantity I would like to compute.

But what is this quantity since the variable that I have is this point X0 which moves around here so let us do the following let us write this as a multiple integral over the phase space x what now what should I write here it is d μthat is the measure of x the moon this is ρ of X DX if you like the μ of x over this measure right times what is the first point that I should write down here I am going to start at time zero inside the cell.

And let me define the so-called indicator function π of X = + 1 if X is an element of the cell equal to 0 if X is an element of seated it is like a theta function so it is equal to one if the point is inside and zero if it is outside since I am going to start there so this is chi of X definitely and how does this X evolve we have already assume that this X evolves X n is equal to some F let me not use this cumber some notation it is some operator T acting on X n - 1 this is the map function.

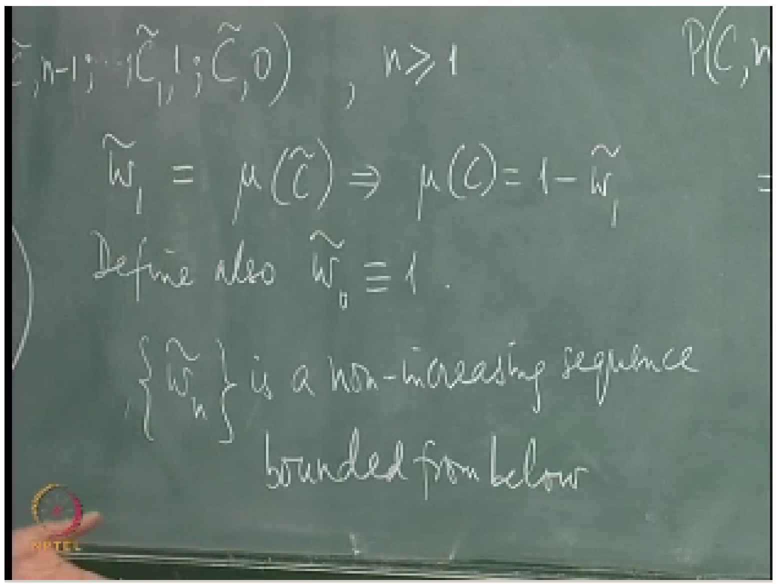
If you like but I have written it here are some operator which acts on xn –one introduces X of n just for ease of notation this is the time development operator which takes me from time n -1 to time it so what happens next at time 1 it should be outside therefore you should multiply this by 1- the indicator function of T on X because a time one this guy should be outside and the same thing should happen for time 2 and 3 up to n – 1.

So let me write at E^{K} it says this is the same map iterated k fold on X a product from k =1 up to n - 1 and then so you start here you jump out and you stay out and comeback at time step n so the last factor is Chi at time n so it is T n X so formally this is this probability the whole thing normalized by 1 over U of C so although the notation looks elaborate the reason for it is its incomplete generality.

So very complicated time evolution is taken care of here by writing this abstract operator T if this is the set of very complicated very complicated nonlinear map it is still taken care of by writing this and this TK stands for the Kate iterate of this map just as T to the N is the end literate of this map and you have to do this integral in principle.

And that gives you this conditional probability which is the probability of a recurrence to the cell at time n so this is the probability with respect to which I start taking averages but first I have to compute this number in some simple fashion now let me introduce the following auxiliary quantity.

(Refer Slide Time: 33:35)



Let me define w n tilde to be equal to the probability so for that I just write P from ability but if I start in the complement of this cell at time 0 I remain there till time n - 1 so this stands for C compliment 0 is a joint probability you for starting the ir remaining they are all these do exactly the same thing and the last one this is a definition p of being in this cell c tilde at time n this one n > 1.

So I define the measure of those events where I start with the representative point in the complement of this cell and I do not leave this at all I do not leave the compliment I do not enter the cell C but at time n - 1 I am still in the compliment but me all of them are see tilde every one of them is c villa so I do not enter the cell c at all so here is c and the rest is seated I start somewhere here in the orbit goes on but never enters this the measure of that set of points let me call that w n tilde it is clear that w 1 tilde is equal to reset n = 0 n = 1 in this.

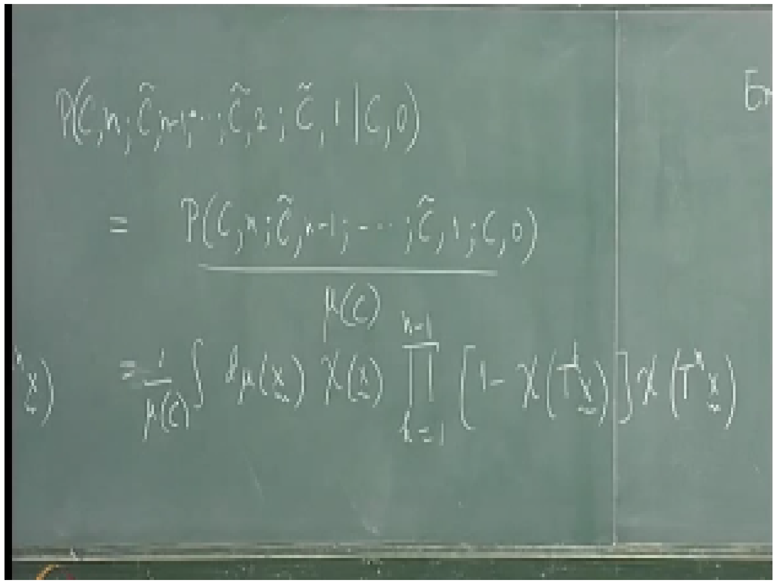
This is just the invariant measure of c tilde so this is μ of see Taylor because it is just P of C tilde 0 the origin of time does not matter so it is just μ of C tilde therefore this implies a very useful relation which is μ of C = 1 - w1because remember the total measure is one so mew of c +μ of c tilde is equal to 1 by definition therefore μ of c is 1- w1 Taylor let us also further define it is become useful w0 tilde to be identically equal to 1itself we will see why that is necessary and useful what can we say about this sequence w n tilde this is a sequence of numbers now starting with 1 w 1 tilde is some number less than 1 between 0 & 1and.

So on what can we say about the set of numbers is it an increasing sequence or a decreasing sequence as n increases it should decrease because this is the probability that the system does not move enter see at all so it is a sequence which is bounded from below because these have got to be non-negative numbers being probabilities so the sequence is bounded by 0 from below starts at one and is a non increasing sequence bounded from below their fourth is a theorem.

And analysis which say such a sequence has a limit point in other words just the statement that this sequence w n is a non is a decreasing and non increasing sequence bounded from below by zero because it is clear that this set of numbers can become negative implies limit WN limit n tends to ∞ W and Tilde exists that is a rigorous theorem in analysis there is no reason why this limit should exist it could just awe.

So lit but this is guaranteed that such a limit point such a sequence has a limit now what is a gad a City have to say about this sequence what would you say is implied by a god dimity for this limit we know that given enough time any set of initial conditions has to visit the entire phase space including see therefore it is quite clear that as n increases and n tends to ∞ what is the limit of W and tilde should be zero by a goddess city so that is the assumption that is the rigorous assumption it says by a goddess City.

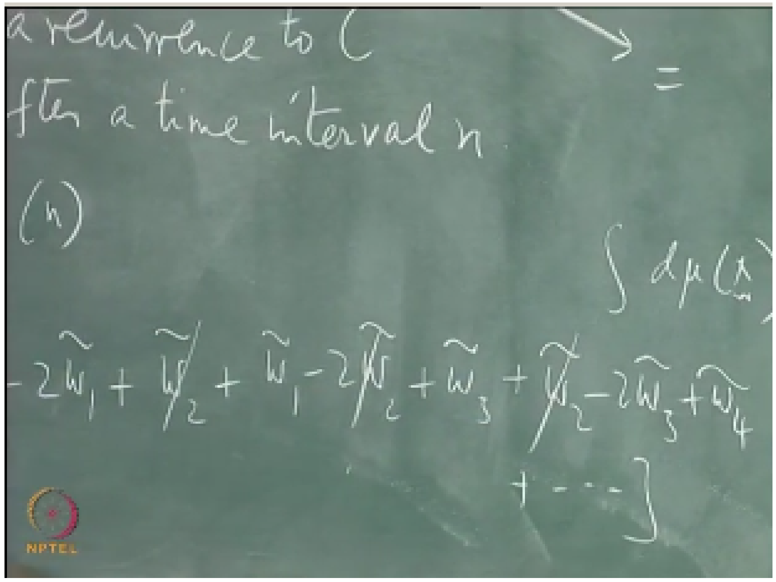
(Refer Slide Time: 39:15)



And let me write that here this is where the input goes in is strictly zero this limit exists and is in fact zero now let us try and simplify this a little bit so what is the trick one would use well if you had only these factors and nothing more just a set of these factors then you can easily see that is related to this sequence because this precisely the probably joint probability that you start with C tilde and you remain in C tilde if you had this thing here.

But unfortunately you cannot do that because you have this factor and you have this factor what would you suggest we somehow have to convert them to these factors so what would you suggest add and subtract one right you add and subtract one this is all you would do so if you did that then the following happens so let us do that and then a remarkable formulae merges.

(Refer Slide Time: 40:48)



So this P of C n see tilde n - 1 1so for the moment let me just look at the numerator. So this too is unconditional which is this multiple integral could be written as equal to integral d mu x times instead of chi of x let me call this 1 – π of X and then subtract the one should be careful about minus signs though so let me let us do that slowly so let me write π of x KY of T n X let me write this, this quantity and subtract the rest of it out so this minus well the first term that I have to subtract out is this one clearly.

And then what, what else do I have to subtract from not what else do I have to subtract +π of X +Chi of T to the power right -1- write it in this fashion now if I plug this in here I put that inhere then what would this integral become it would it says you are going to start at Z so it would correspond to starting it would correspond starting in the cells see tilde at time zero and then continuing all the way up to time in here.

So what would that become it would be w tilde of n +1 because we define w tilde of n as staying in this compliment from 0 up to n - 1 so you would immediately get a w tilde of n + 1 right and then you have this term and this term right so let us write this once again by adding and subtracting they write this as equal to so this portion let me rewrite this as - 1-Chi of X in this fashion so that takes care of this portion and then I am left with this in this term let me write it as so s=- 1 -x of TN x this faction.

And what have I done now I have subtracted this so i write it s + 1 so i put them all together and what do I get I end up with this probability is equal to we already saw that you had a w n + 1 tilde and this term here corresponding to just the product pie from k =1 to n - 1 right so that would give me w tilde of n - 1 + n -1 that takes care of this term and it takes care of the original term this term this product and then I have minus this minus that now what is this term.

So this says you are going to some with this product what does that say what does that give you it gives you a cub little dog n with a minus sign what is this given the urchin what does that give you that is where you need a little bit of subtle tea because this term here is really saying it is integral D mu X let us write it out product from k =1 to n - 1 1- Chi of T and X TK x and then it is x 1- chi of t n x it is this term agreed.

So I could make this N and get rid of this what is that equal to what is that equal to unfortunately I cannot write it as a w immediately because the integration is over X but the characteristic functions here are over t x + k runs from 1 upwards unfortunately what use is that going to be it is inside the integral it is inside the integral unfortunately so what can I do about this you are integrating over X which is the time zero like but.

Then everything else is happening all the integrand involves whatever happens at time 1 2 3 etcetera so this is giving you the probability some kind of probability it is trying to give you but it is over TX T^2 X and so on but you see this is an invariant measure that is the whole point of invariant measure that if you apply the T operator to X the invariant measure does not change at all so this is true that is the meaning of invariant measure.

So it does not change at all which implies I can change variables from x2TTX and nothing happens once I do that then this becomes WN immediately so that gives you another factor of w n which is twice this divided by mu of c to get the actual recurrence probability so we collect all these results and let us write our final result which is that this conditional probability that we are interested in this thing here.

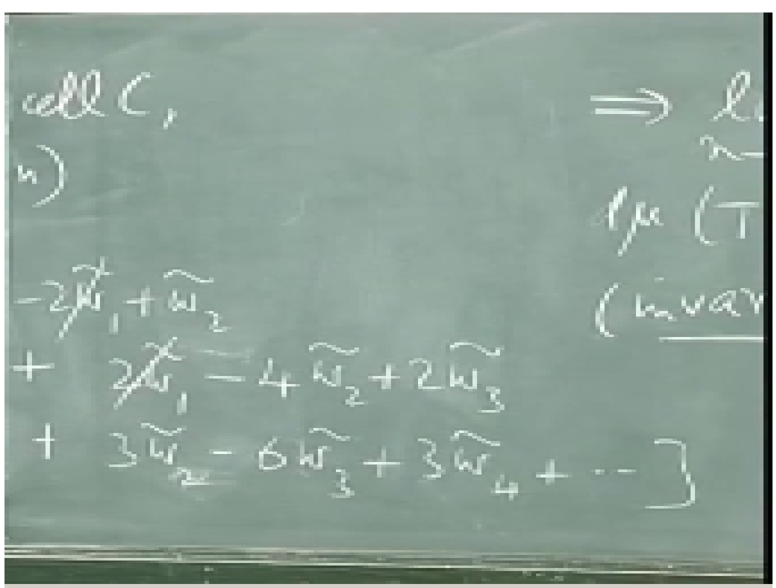
Now been rigorously proven to be equal to Wn - 1 -2 w n + w n +1 filter / M U of C but mew of c was 1-μof c tilde but see tilde was w1 tilde but we define this to be w 0 so that gives you the actual recurrence time probability this is the probability after time interval you are guaranteed we have to check normalization but you actually guarantee that this is a positive number because there is a decreasing sequence tending to zero in the limit and therefore this is like the second derivative of this object.

And you can see that it is actually saying that this is a sequence if you plot is a continuous function is concave upwards because it is got a lower limit which is finite so this quantity is non-negative and is in fact the exact probability distribution but we can now ask what is the mean time to recurrence therefore if I call that mean time to recurrence to the cell c p sub C this is equal to first of all I need to show that this is normalized you have to make sure that this guy is normalized.

So let us check that out first what is let us call give it some number less some name so let me call this guy are for recurrence cr0 we have to make sure that ∑ r c of n over all allowed values of n should be one otherwise there is no guarantee that recurrence is a certain event we know it is a gad da can better be equal to 1 we going to make sure of this so what is this equal to what is the least value of n for which this is valid from 11effect.

So what would you get so this is a common factor we call it one then this is w0 till -2w 1 tilde + 2 + W 1 tilde - 2 + it is what happens now so it is quite evident the w 2 tilde for example will appear twice and get canceled so we will w3 w-4 etc and you are left with just finally so everything gets canceled except w not tilde -W10 +2+ single times w 1tilde so this whole thing collapses to just this that is equal to one. So it is evident that if you this thing is normalized it is a normalized probability distribution now we can find the average now what is the average distribution what is the average recurrence time.

(Refer Slide Time: 54:35)



So since we are using n at this P time n c is the mean4-cell see what does this give you this is equal to a ∑ n = 1 to ∞ n times r n r c of n now what is this equal to so again is simple to see if one over w not tilde -w 1 tilde x 1 x this sow not tilde -2w 1 tilde + W tilde plus twice when n equal to 2 so I is 2w 1 tilde - 4 times W to kill de +2 w 3 to them Plus.

Now thrice the next one so fries w 2 tilde - 6 w 3 tilde plus etcetera so this so blazingly cancels out and what are we left with so this guy cancels out so twice this +4- X cancels out and so on everything cancels out and you are left with this is equal to W not tilde over na tilde - 1tilde but W not tilde is 1 by definition and this guy is 1 -W 1 tilde which is the measure of the compliment therefore it is 1 over μC so this whole thing finally is equal to 1 over in time steps of 1 had we used a time step tau it would be tau over this is the punker a recurrence theorem.

So proves rigorously that when you have coarse grained dynamics of this kind if the system is erotic and has an invariant measure then you are guaranteed that the mean time of recurrence to any cell is inversely proportional is just the reciprocal of the measure of that same variant measure of that cell a useful piece of information and a very general statement we went through a little bit of formalism but it is a very general statement the assumptions were minimal and completely regress we just assumed their goddess city.

And the existence of an invariant measure and this statement follows at once if you have time steps which then go to 0continuously that becomes a little more subtle because it is not very clear if this formula can be just translated directly because if i put a Tau instead of a 1 and let her go to 0 then formally any finite matter itself will have a zero recurrence time which is absurd and that is because of a flaw in the argument which does not take this possibility into account we counted as a recurrence something where the system stays in the cell itself.

Because we started at n = 1 so really you should start by saying it goes out and comes back and then taking the continuous time limit is a little trickier you have to subtract the measure of all those points all those events where the system starts at time zero in this cell and remains there at time one that is a fake recurrence with a recurrence time of one so you should subtract that and if you improve the formula for which there exists such a formula then you get an improved formula for this we change is this slightly.

But in general apart from that technicality this is a very general result is useful in many cases so it gives you a quick order of magnitude estimate of how long it takes to recur now this is used in principle even in statistical mechanics in large systems it turns out that this the time for coming back the pond a recurrence time the mean time and when you have a very large system then this thing here can be shown the time can be shown to grow like the exponential.

Some exponential of the number of degrees of freedom which is why the macroscopic world appears irreversible to us because even if the statistical properties did not change and everything went on as before without aging even then the time for the system to recur would become exponentially large in the number of degrees of freedom and if you have 10 to the 23 degrees of freedom then this is more than astronomically large eat ooh the 10 to the 23.

So large that it does not matter whether I measure units in time in units of seconds or micro seconds or ages of the universe it does not matter at all it is exactly the same impossibly large number so that is the reason why microscopically things appear to be irreversible even though in principle if you did not have any dissipation you had dynamical systems which for even if you it said that the system was conservative.

And did not have any irreversibility built into it the recurrence times would become impossibly large unrealistically launched that is the reason you do not see it in practice okay I do not want to get into the details of macroscopic irreversibility here it is a subject by itself but this result is used in reducing these orders of magnitude okay so let me stop here and we will continue next time slightly different topic.

Online Video Editing / Post Production

K. R. Mahendra Babu
Soju Francis
S. Pradeepa
S. Subash

Camera

Selvam
Robert Joseph
Karthikeyan
Ramkumar
Ramganesh

Sathiaraj

Studio Assistance

Krishnakumar
Linuselvan
Saranraj

Animations

Anushree Santhosh
Pradeep Valan. S. L

NPTEL Web & Faculty Assistance Team

Allen Jacob Dinesh
Bharathi Balaji
Deepa Venkatraman
Dianis Bertin
Gayathri
Gurumoorthi
Jason Prasad
Jayanthi
Kamala Ramakrishanan
Lakshmi Priya
Malarvizhi
Manikandasivam
Mohana Sundari
Muthu Kumaran
Naveen Kumar
Palani
Salomi
Senthil
Sridharan
Suriyakumari

Administrative Assistant

Janakiraman. K. S

Video Producers

K. R. Ravindranath
Kannan Krishnamurty

IIT Madras Production

Funded by
Department of Higher Education
Ministry of Human Resource Development
Government of India

www.nptel.ac.in