Indian Institute of Technology Madras
Present

NPTEL
NATIONAL PROGRAMME ON TECHNOLOGY ENHANCED LEARNING

TOPICS IN NONLINEAR DYNAMICS

Lecture 23
Stochastic dynamics (Part II)

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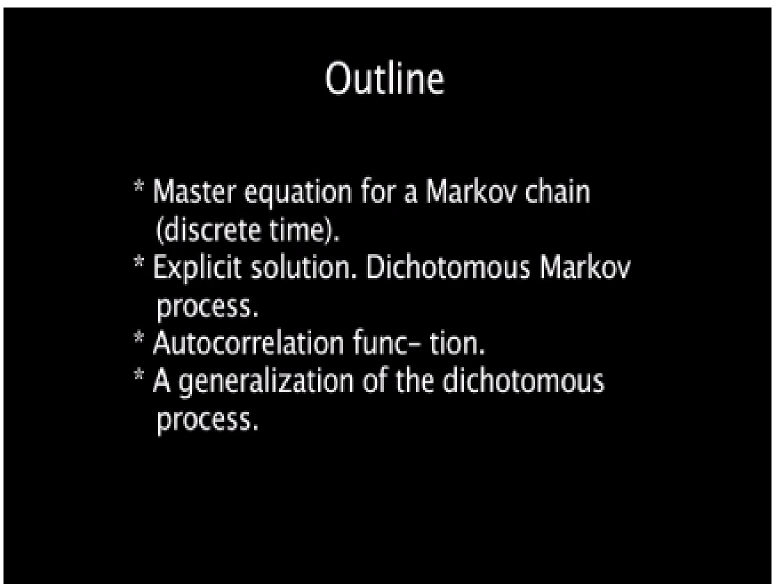

Yeah we were examining the elementary properties of Markov processes and let me continue with that give a little more bit of an introduction to some elementary Markova processes and then we go on to applying whatever we have learnt to dynamical systems, specifically to coarse dynamics. If you recall i define the Markova process as one in which the conditional probability for an event to occur at a certain instant of time is dependent only upon the preceding instant of time.

Whatever happened at the preceding instant of time and this short term memory led to the consequence had the consequence that the entire family of probability densities or probabilities for a Markova process are in fact expressible in terms of a single conditional probability and this means that the system is completely known all statistical averages can be found. Once I give you an equation for this conditional probability or probability density in the case of processes in continuous time this probability density obeys a certain master equation.

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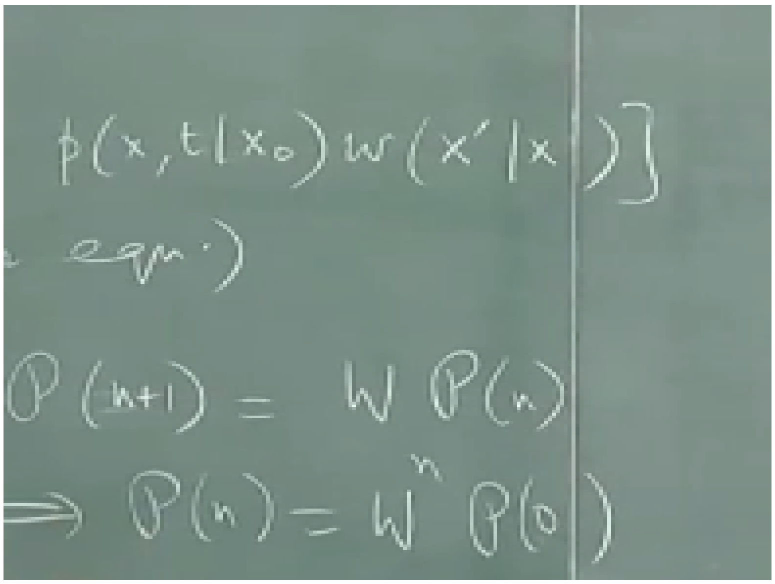

And the way we wrote this equation was ∂ over ∂ T P of X and T given an initial value X_0 was written on the right hand side as = an integral over all intermediate States X' P of X' T X_0 multiplied by the transition probability to go from X' to X the transition rate to go from X' 2x - P of X T X_0 multiplied by the transition rate to go from X out to X' and this was like a gain term and that was like a loss term and this is what the master equation read this thing is called the master equation.

And I further pointed out that if you took these quantities and wrote this out as some kind of column vector then you ended up with a matrix equation in the case in which the value of x the values of x or discrete values discrete set of values, otherwise it is an integration or replaced by a summation in the case of a discrete set of values. This kind of equation is not easy to solve it is a linear equation in P but it is an integral differential equation on this side and it is not altogether trivial to solve it although there are well-established techniques.

Now for handling this sort of the equation and making considerable progress of course the input information has to be this you have to know what these transition rates are to go from one state to another. The same thing if I wrote it out in discrete time for a discrete set of states and let us say the variable X takes on only a discrete set of values X 1 X 2 X 3 etcetera let us label that by some integer J and let me say the system is in state J, if the variable random variable has a value X sub J.

Then I can ask what is the probability that it is in the state J at time n or time n + 1 what is this = this is = the probability that at time n the system has reached some state k for example this case let us put the time index inside because we are going to write this as a matrix equation, so p j at time n + 1 is p k at time n multiplied by the transition rate of making jumps from the state k to the state J per unit time, so in one more step it jumps to this point and in our notation this thing is a summation over K and in our notation this is a transition from k2j.

We have written it from X ' 2x I write this as from k2j this is the gain term, so this stands for the transition probability from k to j - pj of n w kj. So again this is now the form of a rate equation it really is telling you that here is the gain term which contributes to the probability in state J and this is the loss the rate of loss and if you write these p_js together as a column matrix then of course this is a matrix equation of the form P physique p at time n + 1 is = some w p at time n.

And since this is a constant matrix which are given once and for all the transition rates between the different states it is immediately obvious that p at time n is w to the N P at time 0, so this at once implies that p at time n is w to the N P at time 0 and all you have to do is to calculate this matrix here. Now the way to calculate the nth power of a matrix for large n if this state space has got some large number of dimensions is not altogether trivial what you do is to try to diagonalizable this matrix or at the very least if you cannot diagnose this matrix at least bring it to Jordan canonical form after which you can take its nth power without too much difficulty.
So this is a problem now in matrix algebra to find out what P n is given any initial distribution p 0, let us look at a few examples let us look at an extremely simple example in continuous time just to get used to the idea of these things in continuous time and let us look at an example where the state space just contains it is two dimensional just two possible values. So we have a variable which switches between two values say x1 and x2 at random instants of time and what would this look like if i protect the graph.

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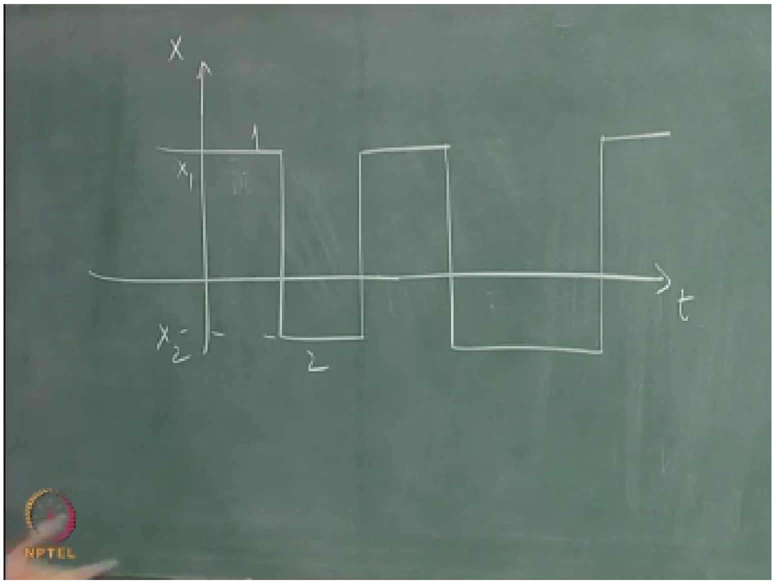

So let us say there are two values x1 and x2, so as a function of T I have the variable X and suppose this is the value x1 and this is the value X 2 just for illustration sake let me take it to be negative it does not matter. So this is the value X 2 it continuous for a while and then flips back to x1 goes on for a random instant of time back to x2 for a random instant of time and so on. So here is state 1 and here state too and there is a certain transition probability or rate per unit time of switching from state 1 to state 2 let us call that λ 1.

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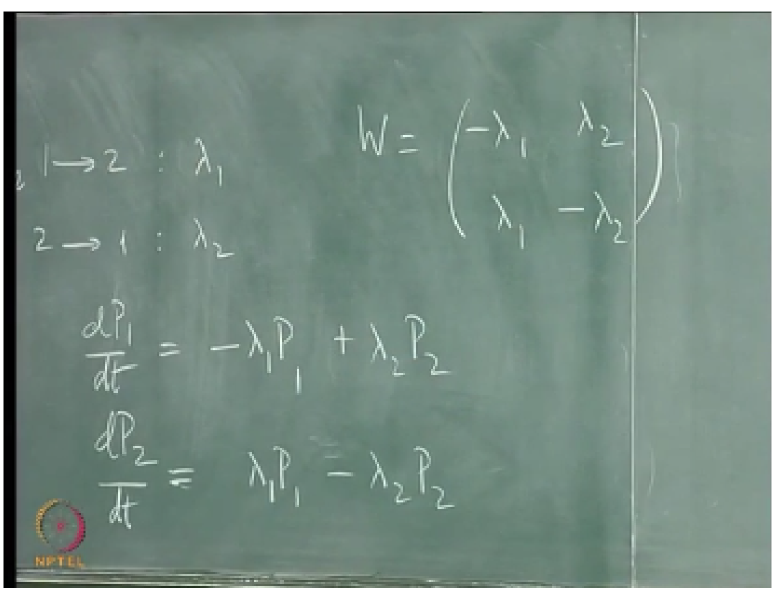

So λ 1 so 1 2 rate of switching this is λ 1 and 2 to 1 back again this is λ 2 in general and suppose this process goes on for a very long time and now I ask what are these equations what is dP 1 over dt =and what is dp 2 over dt = it is evident that at any instant of time p 1 of t + p 2 of T must be =1 the system must be in one of the two states and it flips back and forth between these two states now what would this equation be =, if the system has reached the state one already it makes a transition to two with a rate λ 1 and that is a loss term for p1.

So that is - λ 1 P 1 because they are switching out of 1 into 2 like this and then the gain would be if you switch from to back to one and that would be + λ 2 p2. On the other hand here for p2 what would it be here obviously λ 1 p 1 - λ 2 p 2 and this is as it should be because as you see the transition matrix if i write it down is in fact, so w in this case this transition matrix is - λ 1 λ 2 λ 1 - λ 2 and as i promised the sum of elements of each column is 0 as you expect.

And it has the immediate consequence that if i added up p 1 + p 2 I immediately get as you can see d over dt p1 + p2 =0 which implies that p 1 + p 2 =constant =1 if you normalize it initially it remains at one for all kind. The job of course is to now try to find out what are the steady state properties of the system by this process by the way that it very flips by Markova process where it flips between two possible values back and forth is called a dichotomous or deco Tonic Markov process very often abbreviated as d NP very popular and modeling in a very large number of situations.

And it is evident immediately that I can eliminate either p1 or p2 and i am going to get a second order differential equation for either p1 or p2 the same equation, in fact and then the solution depends on the initial conditions. So it would the solution would in fact be the sum of two Exponentials depending on the Eigen values of this matrix which are fairly straightforward right. Now we can fact computer e to the WT write down the full answer can do this in some simple form but first let me ask this.

What is the average value of x in this case this is an ongoing process for a very long time so you could ask what would be the equilibrium value, what would be the equilibrium you.

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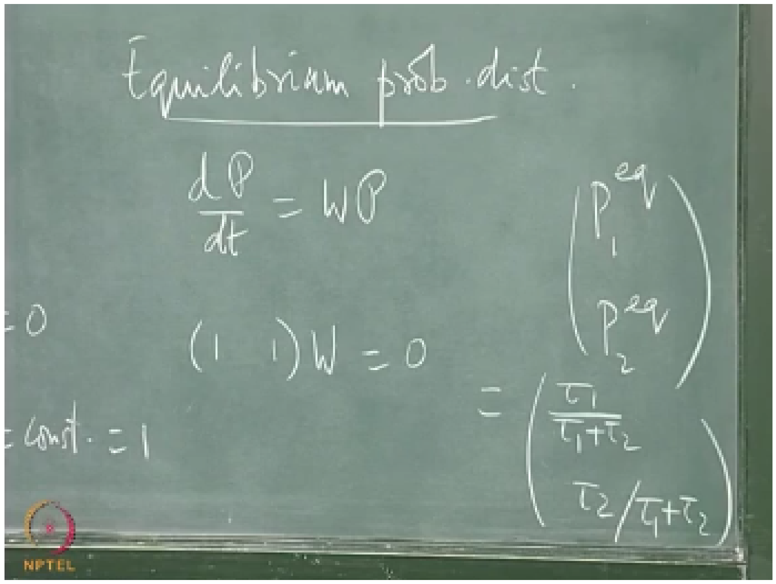

So that is or a stationary value equilibrium probability distribution what would that be well exactly as in the case of dynamical systems, we essentially have a system which looks like this and in equilibrium this should be 0 in the stationary state, so you want WP to be 0 and if i call that let us call this p1 equilibrium and p2 equilibrium this column vector tells me what the equilibrium distribution is between one and two what would that be for this process.

I would have to set this = 0 in other words I have to find a column vector P which is annihilated by w you act with W on the left and you get a null vector because the Ps the sum of each column is = 0 it is evident that the uniform vector is a left eigenvector with eigenvalue zero. So it is evident that if I took this one and operate it with W on this side i get 0 that is because the sum of each column is =0, so what I want is a corresponding vector right.

Eigenvector corresponding to this 0 and that will give me the equilibrium values out there but you can actually guess on physical grounds you can guess what happens here. What is the average time that the system spends instead state to, what would this be let us call this τ to beam mean residence time in state to let us call that τ, to what would this be what would the mean residence time here be and similarly τ the meantime here would be τ 1 what would this be.

Well it is clear the faster it gets out of that state the less there is none Stein in that state and if it goes on for a very long time then what you could do is to add up all these intervals and divide by the total time and take the limit in which the total time elapsed goes to infinity and similarly for state one and this would give you the relative fractions, the fraction of time that the system spends in either state one or state to over a long interval of time that would be in fact directly proportional to your equilibrium distribution.

So what would that be what is τ 2 going to be exactly it is just what exactly absolutely so this thing here is λ 2 inverse and this is λ 1 inverse λ 1 is a rate at which it switches out of one right, so λ 1 inverse is the meantime expends in state 1 therefore this p1 equilibrium would be =τ 1 over τ 1 + τ 2 and the other guy is τ 2 over τ 1, therefore this is λ 2 over λ 1 + λ 2 and these are the guys λ 1 over λ 1 + λ 2 that is physically obvious it is easy to trivial to check that if you took this and wrote down here those column vectors.

This column vector here with τ 1 is λ 1 inverse in town 2 λ 2 inverse you get zero on the other side what would the average value of this process p.

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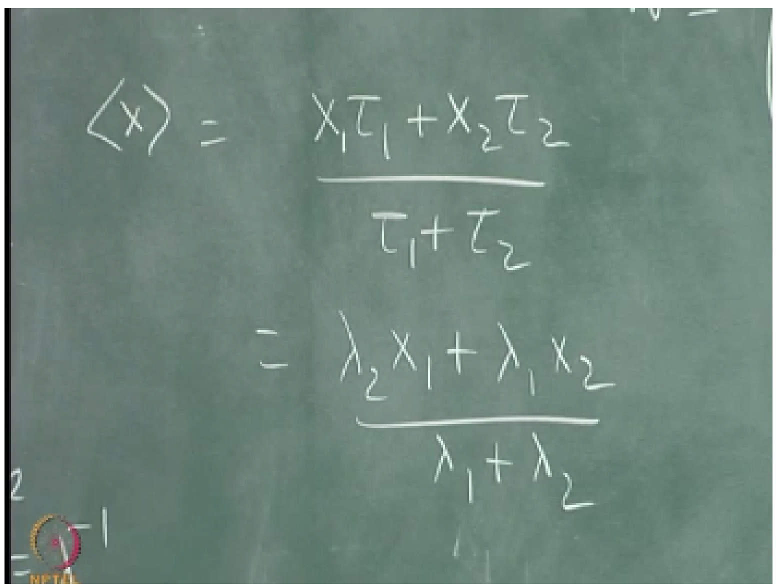

What is X average =well exactly so I would expect this to be X 1 to 1 + X 2 τ 2 over τ 1 + now this is what I would expect that would be =λ 2 X 1 + λ 1 X 2 over expect this to be the average so it is sitting somewhere here it depends on whether it is closer to this or this would depend on what the transition rates are in general. And similarly for the mean by mean ^{2} that would just be x1 ^{2} + x2 ^{2} here that is it and you can find the correlation function that is the crucial thing we would like to find out what do you think that is going to be that's the most important thing of all so in particular.

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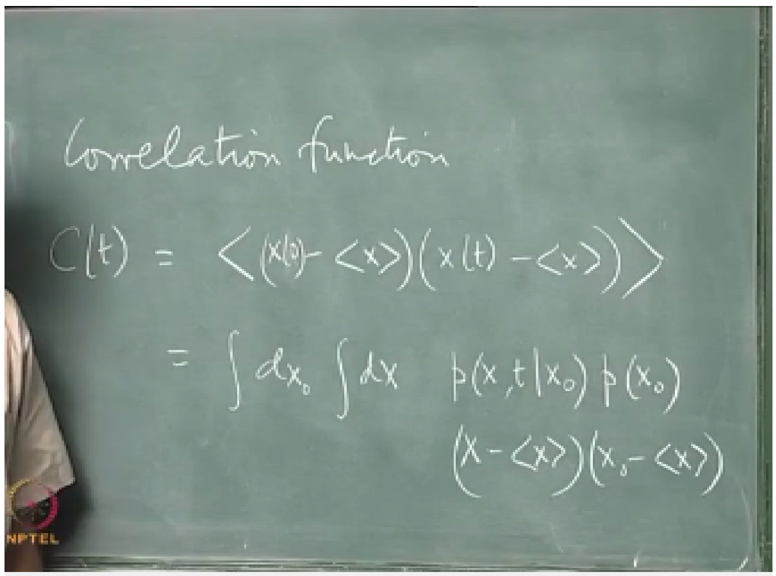

I would like to find the correlation function and let me call this C of T in equilibrium this would be = the average value of X - X average X of 0 - X average x of t - X and the average value of that that is my definition of the correlation function what do you think this is going to be in this process? You can compute this without too much difficulty but let us see what the exact expression for this quantity is what the actual expression for this is the formal expression.

If I say that the, so this is =if it is a continuous process for example I would say this is =an integral over the initial value the X_0 and integral over the value at time T let me just call it x times the probability that I have X at time T given X_0 at time 0 multiplied by the probability that i have x_0 at time 0, since it is a stationary process there is no time argument there x x - x average times x_0 - x act.

That is the quantity which I want to average over and this is what I have read over this probability distribution this is the conditional probability density and that is the probability density stationary density of the process. Now of course I have discrete values 1 & 2 so each of these is summed over X x1 and x2 and instead of these densities I write down the probabilities themselves. So it is straightforward calculations you can come complete this fairly straightforwardly.

Let us look at what these probabilities themselves are in the simplest instance so to make life a little simple let me do the following, let us take x2 to be - x1, so that it is completely symmetrical let us call this value some constant value C.

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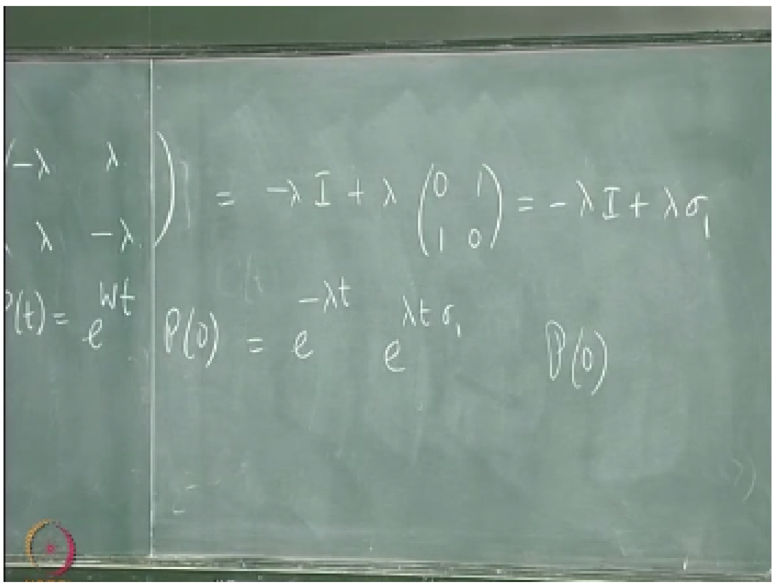

And the other value is - C etcetera – C, so it flips back and forth between + C and - C and for algebraic simplicity, let us supposes the transition rate is just a constant λ. So this is some τ =λ inverse and this average value is again τ =λ English average on either side the average duration in each state is some τ which is a reciprocal of a single common rate of λ of switching from one to two and two back to one, so this just becomes this quantity here then it is easy to compute.

So I write P of T once again is =e to the wp p of 0 whatever be my initial distribution and what see to the WT it is this but this is =i need to exponentiation, this find it ^{2}and it is cube and so on and so forth. So let me write this as - λ times the unit matrix + λ t a matrix which is essentially zero one zero so this becomes e to the - λ t because i have e to the power a matrix + another matrix and e to the a + b is not e to the a times e to the B unless I and we commute with each other.

But in this case a happens to be the unit matrix which commutes with everything else, so that is the reason I chose the special case so it is e to the - λ T and then it is the exponential e to the λ T times this matrix 0 1 1 0, the whole thing axon p of 0 what is the ^{2}of this matrix well let me call this matrix it is got a name - λ i + λ let me call this matrix σ 1 and then notice that when i exponentiation the σ 1 things are very simple.

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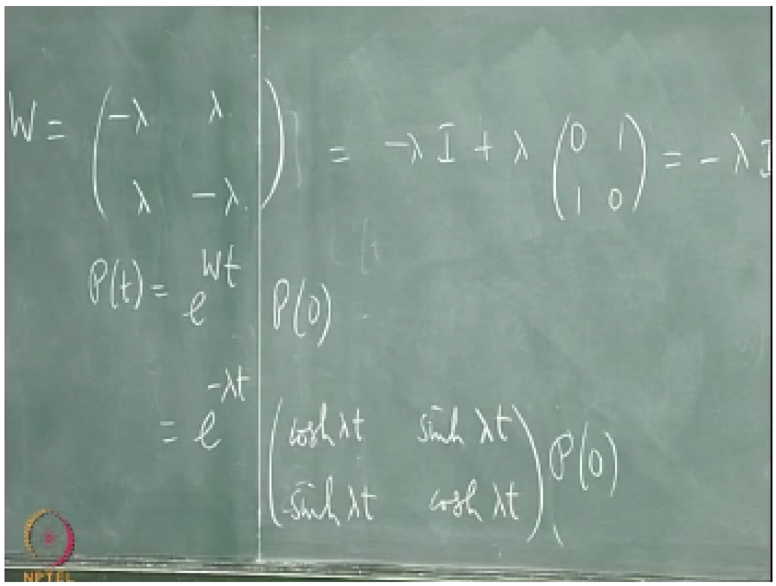

Because what σ 1^{2} it is the unit matrix right, so this becomes I once again right what happens next three factorial σ 1 cubed but σ ^{2} is the unit matrix so it is just σ 1 + . and I gather all the I is together and i have 1 + λ ^{2}t ^{2}by 2 factorial + λ t ^{4} over 4 factorial etcetera ad infinitum and what is that =+ I have another matrix which is λ T + λ Q of T^{3} over 3 factorial + λ π t 505 factorial + 1 and σ 1.

So what is this =yeah it is k is =cos hyperbolic λ T times I + and this is sine hyperbolic λ T I am σ 1 that is it. Therefore this thing here is now exponential it is finished it is =e to the - λ T times this matrix that matrix is just this cos hyperbolic λ T, sine hyperbolic λ T on this side sine hyperbolic λ T cos hyperbolic λ T is that matrix acting on p 1 of 0. So let us write that explicitly p of 0 and you begin to see immediately that it will do exactly what you would intuitively expect directly.

And that is the exact solution to this differential coupled differential equations which we circumvented by simply writing the matrix solution down right away.

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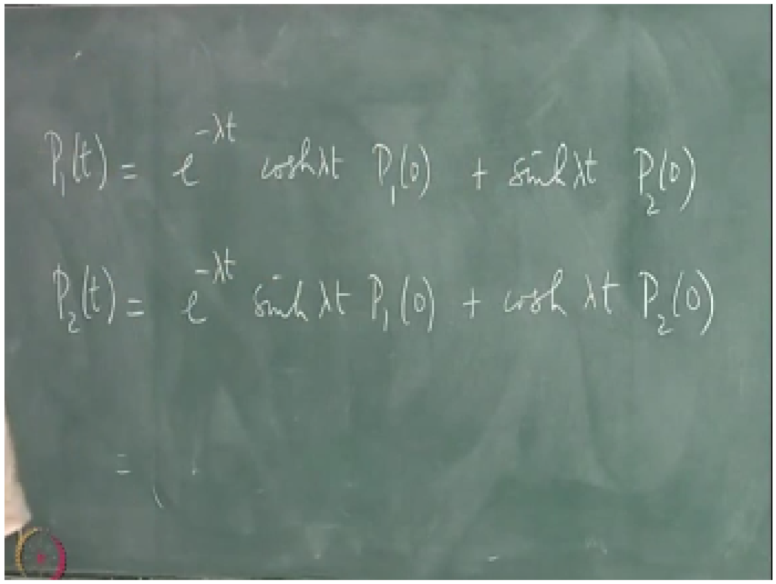

And therefore P 1 of T P 2 of T write it as a column vector this is =e to the, so let us say let us write this out explicitly solution p 1 of T is =and p 2 of T is =there is an e to the - λ T cos hyperbolic λ T times P of 0, so this gives you e - $^{\lambda T}$ V 1 of 0 + sine hyperbolic λ T P 2 of 0 and similarly that is it, so it gives you explicit solutions, if i start with the system in say the upper level then p 1 of 0 is 1 and p 2 of 0 what happens as you can see the probability is flip back and forth.

So if p of 0 is =10 then this quantity here becomes = this is 0 here, so it is just this and that is =1/2 1 + e to the - 2 λ T and that is it in that same case this becomes = 1/2 is an e to the - λ T. So this becomes 1 – engine, so at t =0 this is 0 and that is 1 and as T increases this fellow drops down from a number from 1 towards half as emphatically and this builds up from 0 towards a half pardon me yeah there is a bracket over here right.

And what is the correlation function become what do you think the correlation function would be and what are the steady-state probabilities, what is this equilibrium distribution in this case just half and half it is clear that the number the average durations are equal and it is half and half but that is precisely what to expect this to become as T tends to ∞ that is a check that there is sufficient mixing in this system that the conditional probability tends to the stationary probability as T tends to ∞.

The memory of the initial condition is gone and that is exactly what has happened here, it is not hard to show that the correlation in this case C of T the average value is 0, so we do not even have to subtract the average this quantity is = what if the value of the random process is + C or - C when I take averages multiply the two together and I ^{2} it and I want the correlation function it becomes clearly there is a C squared which appears here C of zero must be =one.

And then it is x if I compute those averages it just turns out to be e to the - twice λ T it is exponentially correlated with the correlation time, which is 1 over 2 λ in which is 2 λ inverse that is the time roughly on which this that is the time scale on which the correlation this memory is in the memory acts is not surprising because there is just one time scale in the problem. So you end up with something which is proportional to λ inverse there are many processes which are exponentially correlated Markov processes.

There could be in general more time scales there could be a sum of Exponentials but when processes are exponentially correlated with a single correlation time something very special happens and the DMP is a classic instance of such a two-level system to a two-state process you add a symmetry to it you can change the rates and so on and so forth, but the essential physics does not change very common in modeling very popular in modeling.

It is clear that it models something which is either on or off a two-state system either on or off you can make it three state system where it is on or off and then there are choirs and states there are active States and so on those are generalizations of these two state processes okay. We could make things a little more complicated you could say well and this is a very popular process so let me mention that, there is a class of jump processes where you end up with the following situation the variable X takes on a continuous set of values.

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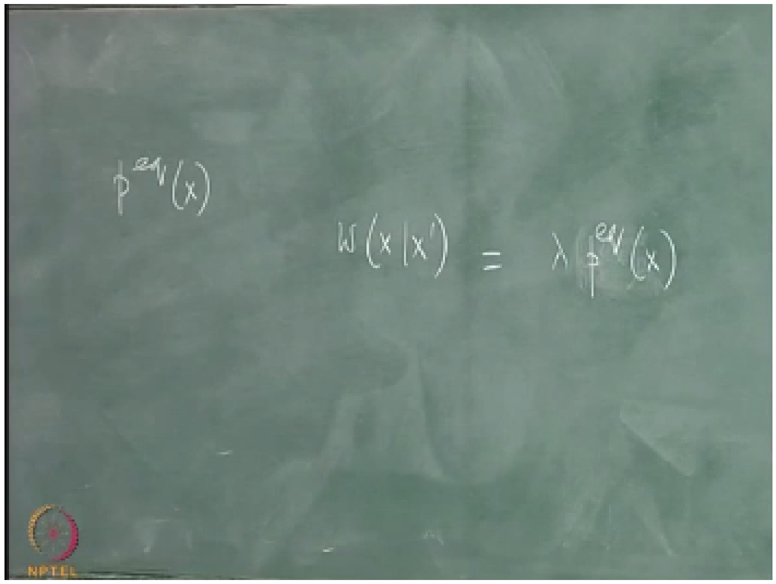

And there is some equilibrium state there is some equilibrium distribution P equilibrium of X some equilibrium probability density, then in the Markov process for which the transition rate is of this kind this is the transition probability or per unit time to go from X' to X we could model this in the following way we could say well this thing is a rate, so there is a time scale λ inverse sitting there x something which depends only on the equilibrium or final state distribution.

So you could say does not depend on where you are but it is instantly jumps and with something which is proportional to P equilibrium of X this to occurs in many physical situations. So the jump rate is independent of the initial state depends only on the final state and that two manner which is directly proportional to the stationary distribution itself. I urge you to solve the master equation with this assumption.

This is a normalized density so if you integrate this overall X assume it to be =1 and then it turns out you can actually solve for the conditional probability density p of x comma T given X_0 you can actually solve this exactly and show that the process is exponentially correlated. So it generalizes the idea of this to state process to a continued of states in a specific direction. So once again I leave you to work this out and find show that the correlation is exponential is an exponentially decaying correlation.

We will again talk about Markov processes we will again talk about similar considerations but let us go now change horses and go completely back to dynamical systems and see how a specific coarse-grained dynamics is in fact a Markov process this is what I would like to do and I would like to show you that the dynamics is exactly that of a Markov chain which is the sort of thing we examined here.

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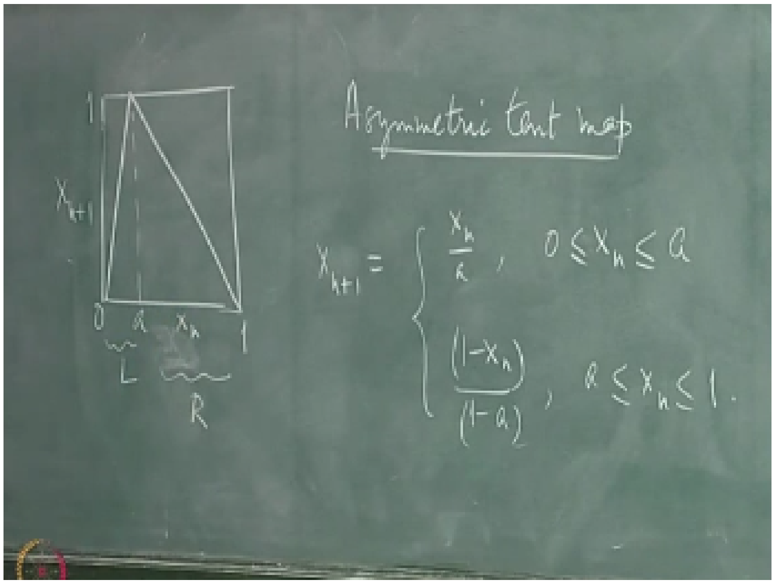

And now just to make things a little less accidentally degenerate let us look at the 10th map once again of the unit interval but we will make it an asymmetric 10th map just, so that we do not have artificial degeneracy and there is a specific reason why I do so let us take instead of a tent map which goes up and comes down to 1/2 let us say it goes up in this fashion and comes down there for the rest of it and let us suppose this is some number a which is < a half, so 0 and 1 here and this is unity here and this is my map function.

So this is how xn + 1 is given xn I call this the asymmetric tant and the map function is the following xn + 1 is =it is clear, its xn divided by a 40 or less than =e the slope is 1 over a because it reaches 1 when x n is a and what is the slope here it starts at one and goes to 0 therefore it must be 1 - X and the slope is 1 - a because if a is a half it is also a half it is also too, so a < =X n <= unity.

Now this map is fully chaotic and what I am going to do is to partition it into two cells a left and a right and the natural thing to do is to partition it from zero to a as one cell and a 2-1 as another, so let me call this left and let us call this the right cell and similarly for xn + 1 what is the invariant density of this system of this map so we write the Fresenius Peron equation once again.

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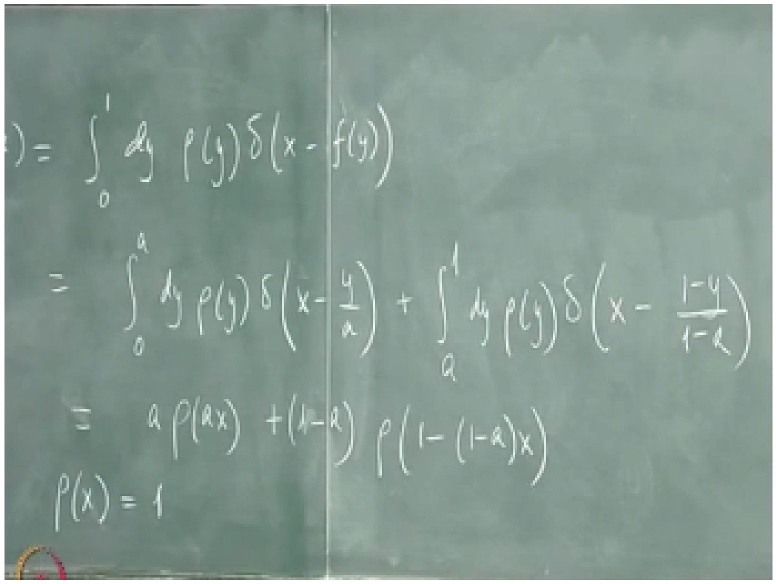

I have ρ of X =integral 0 to 1 dy ρ of why a ∂ function of X - F of Y in this fashion that is = well since I am breaking this map up there is a 0 to a which is one piece and an a to this which is another piece, so this is =0 to a dy ρ of why a ∂ function of X - y over a because that is what the map function is this region and then a piece which is a 2 1 dy ρ of why a ∂ function of X - 1 - y over 1 - a that is the Fresenius Peron equation and in the standard procedure we convert this to a functional equation and see if you can guess a solution to this equation.

So I pull out this a y becomes a X so it is ρ of X but if I pull out this a from the denominator it goes up here in the numerator there and similarly + 1 - a this factor comes up and then a ρ off now why here is =so 1 - y becomes =1 - a times X, so Y becomes =1 - 1 - 8x, so this argument is one - one - x times X that is the problem is power equation what is the guest solution yeah the fact that this and that seemed to cancel each other suggest that you put ρ =a constant again.

So yes indeed this is to ρ of X =1 is the unique normalize able non-negative solution to this equation, so it is a uniform density one second it does not matter whether it is a symmetric 10th map or asymmetric it is uniformly spread out. Now I am going to do the following I am going to ask what is the transition matrix to go from one cell to another we write this w down completely for this chain and then we test if this is a Markov process or not by finding out whether the two-step probabilities is just the ^{2}of the same matrix which you get in the one-step if it is we know that it is a Markov process.

Now the end step probability is just some it transition matrix raised to the end, if so if that is computable on both sides and you verify that this is so then it is as good as a Markov process but we need several steps to do this let us do that carefully. First let us ask what is the so my cells are here and this by the way is a also so this is cell left here and that is right here and this is left here and right here for the one step transition.

First I ask what is the measure of the left cell what is this =that is clear clearly an integral over the left cell which is zero to a DX ρ of X that is obviously that is the definition of the invariant measure just the integral of the invariant density over the cell and since this is constant = one this is a and you write =one - a that is certainly true I need to calculate the transition probabilities. So I need to calculate quantities like the probability.

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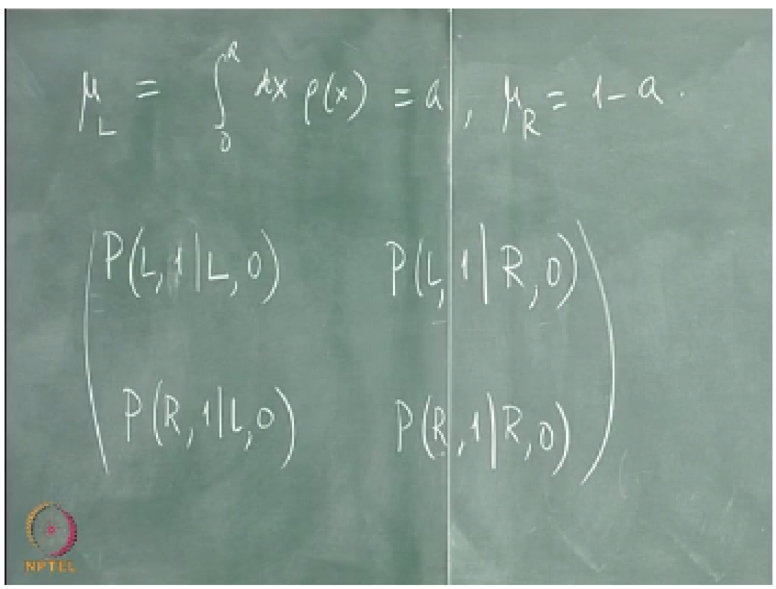

That if I start at 0 at time 0 in the left cell i am at so L at time 0 I remain in the cell l at time 0 I need to compute this number that is a probability upon me a time one at time one at the end of one time step. So let us put the time steps here similarly I need to calculate pl1 given that I started are on the right hand side or pr1 given that I started l 0 and t are one given that I started R. I need to calculate these probabilities and then I am going to put that in the form of a matrix call that my transition matrix at each step.

Now how do you define these quantities what would you do remember that the X itself is continuous but I have broken up the whole thing into cells and I am now looking at the symbolic dynamics of the cells, so the states of my system are either l and r l and r and that is what I have written here these are the probabilities I am computing. But how do i write this down for example how would I write this down what would be the procedure to write this down.

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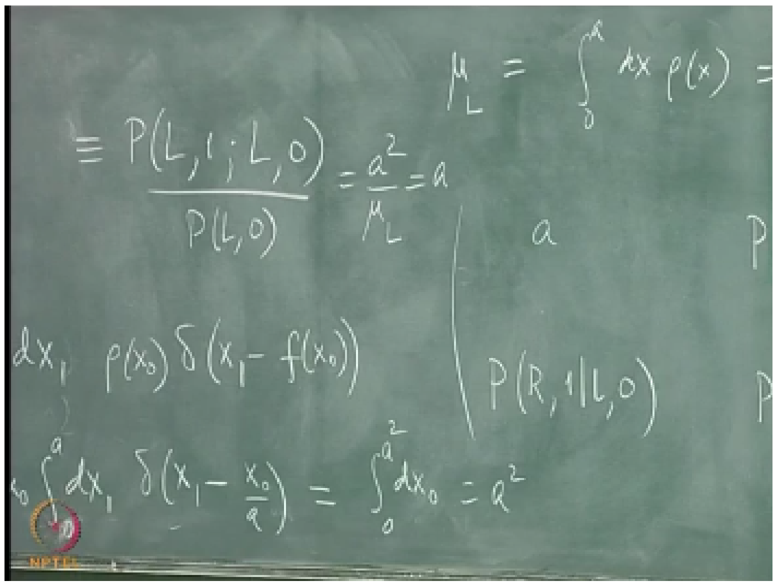

From first principles I would like to know what is P at l1 at l0 I have to be a little careful, now so I ask I say well what is the joint probability that I am at in the left cell at time 1 given that I am in the left cell at time 0 so I start with this quantity here and ask what is this joint probability and this as you will see intuitively is =an integral over DX not suppose X as the value of x_0 at 0 time, so it is an integral over D X_0 over the el cell and integrals over X 1 over the left cell x x what since x is continuous we are talking about probability densities right.

So this is multiplied by the probability invariant probability that you are in the cell l it you have the invariant probability for this variable X_0 ρ of X not integrated over L that gives you the probability of starting in L multiplied by the probability density for starting at X_0 and moving to an X 1 and that we know is a ∂ function kernel and we know that that is = this is a conditional probability density.

It says if you give me an X_0 I am guaranteed to go to an ex one whose density is given by this and that multiplied by ρ of X_0 gives me the joint probability density for X_0 and X 1 X_0 at time 0 and X 1 at time 1 and it is integrated over X_0 and X 1, over the regions that you are interested in. So this is in fact the definition of this joint probability right but what is that = what is this = well X not L the cells L is just 0 to a the X_0 and it is just 0 to a once again over DX one on this side and ρ of X_0 for this map is one.

So you do not need to put that in but you do need to put in ∂ of x 1 - f of X_0 this function and this function was what x over 80 is also its X_0 over a because that is what the branch on the left is so ∂ of X 1 of X_0 over 8. I need to compute this integral so the range of integration the region of integration you can do this geometrically in the following way one way would be to convert this to an integral ∂ function over X not finish that and then do the x1 integration but I can do it as it is because I also have to integrate over x 1 I am as well do it directly provided the ∂ function fires.

When will that happen here is the region of integration this ^{2}and here is the ∂ function constraint which tells you that x1 is f of a f of X_0 so what is the value of this integral. Now what is the value of this integral of course it does for all X not between 0 to this point wherever this intercept is for every X 1 in this region of integration between 0 to a that ∂ function fires. So I can do the x1 integration provided i restrict the x_0 integration to this region otherwise the ∂ function does not contribute.

And what is this point this value here it is a squared it is a squared because a^2 over a will give you this value a so this integral becomes =0 2 a^2 DX not and now the ∂ function has taken care of as become one because this integral is finished here and the answer is a^2 therefore what is this = that is the conditional then say probability for this L to L exactly. So it must be this quantity is this quantity x p of l0 is by definition =P of l1 l0 the joint probability is the conditional probability x the absolute probability here.

Therefore this quantity alone is this / p of l0 but that is =a^2 divided by what is this = P of l0 it is the invariant measure because it says what is the actual stationary probability of being in L I close my eyes and put my finger on this on this interval and what is the probability I am here or there and just mu l which is = e, so we have an a here what is this going to be well let us do that let us see what that thing is.

So again it is over l DX not but this becomes an R and now I am trying to compute P of r1 so this is finished this is over R I have to do this integral and what does that become I use the same strategy as before in this case since the invariant measure this unity it is actually very simple I have to integrate over this quantity, here the X_0 over L but the x1 i integrate over R. So really it is this range of integration that I am talking about and this range of integration for every value of x1 I have a contribution from the ∂ function provided X_0 runs from here to there.

And therefore this is = integral and that runs from a squared to a from here to there the X_0 and then the ∂ function contribution over X 1 gives you unity for the x1 integration. So it is a times 1 – a, so this guy gives you a times 1 - a but then I must divide in order to get this conditional probability m is divided by the measure of this cell here ufl, which is an a and therefore this just gives me 1 – e.

Similarly for the other one p of l1 are 0 I have to integrate over this rectangle and the contribution that is known that is non-trivial the only contribution comes from this region, what is this point remember this function is 1 - X_0. So this is X_0 here and this is X 1 it is 1 - X_0 over 1 - a and that is =a because that is the value here, so tells you 1 - X not =a - a squared or x_0 is =1 - a + a^2so this point is 1 - a + a^2.

Therefore if I integrate we are now doing L 1 and r 0 so this is over R this is over l and that integral is this interval and it is 1 - this guy so it is a - a^2 is a times 1 - a so this quantity here is a times 1 – a, but then i need to divide by the measure of this cell to get the conditional probability and that is a 1-8 and therefore it is just a and finally we need to know this length and / the measure of the right cell because we are now integrating over this region and that length is 1 - a + a squared - a it is 1 - a the whole ^{2}divided by 1 - a to get conditional probability just gives me a 1 - a so this here is an a and a 1 - a on this side.

The sum over each column is 1therefore it is a stochastic matrix the matrix with non negative elements such that each column adds up to one or each ρ adds up to one is called a stochastic matrix because it is connected to these probabilities and such a matrix would have a uniform left eigenvector had some ρ sums been = 1, it would have a uniform column vector as the eigenvector.

So it clearly has one as an eigenvalue and a uniform left eigenvector and the corresponding right eigenvector is not hard to find it would be related to some equilibrium distribution, in this case it will be related to a and 1 - a itself. So this gives me my transitions the transition at one time step and the question is can the transition matrix at two time steps be written as the ^{2} of the transition matrix over wine one time step.

So if i call this some transition matrix t it is related to the w that I wrote down earlier but the addition of a unit matrix, then the question is what is T^2 and is that the same as so the question we ask is the following is and I leave you to verify this is the matrix P of L at time 2 given L at 0 is this matrix $=T^2$ is the question, if it is then you have reason to believe that this is a Markov chain because all you are doing is to take this transition matrix one step transition matrix and multiplying it raising it to higher and higher powers.

Now what would you do to compute this yeah what I do is to first compute this L to L 0 and I write this as over L the X not let us just call it all right X_0 over L the x2 times ρ of X_0, which is unit unity in this case x a ∂ function of X 2 - f2 of X_0 so I take the second iterate of this map and play the same game and compute these numbers which are very easy to do in this case and check out whether that is =the ^{2} of this guy or not of this matrix here, if so then by iteration by induction I know that this is true in the end step as well.

I leave you to verify that this is indeed so this is a Markov chain which is equivalent to saying that this partitioning that we have done of the phase space in this map is a mark of partition because the symbolic dynamics of the system has been reduced to a Markov chain in discrete time. And therefore I can use the entire machinery of Markov chains to solve various problems here for instance.

I could ask questions like if i start with the cell l then what is the mean time for me to come back to the cell l because it undergoes excursions from l to r it stays in l for a while etcetera, so if it leaves l after a time step what is the mean time for it to come back what the statistics of these recurrence time and so on. So the entire machinery of Markov chains for which well-defined answers exist for such questions can be used in order to study the dynamics of the system.

You do not have to go back to the map anymore because that dynamics has been, now transferred to the properties of this Markov chain, it carries all the information that you need this transition matrix already carries everything. So we will study this little better because I would like to do two things I would like to show you that there is a uniform there is a very specific behavior of the recurrent statistics namely well when the system comes back.

Because it is ergodic you know that the system will come back to whichever cell you leave given enough time then a question of interest is how long does it take to come back what is the statistics of the recurrences are the successive recurrences independent of each other or not and so on, well defined answers to these exist and we will use this as a sort of case history to study this little deeper and I will do that next time.

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