Classical Field Theory
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Lecture - 38

For you to sort of see what we learnt in this course, carry forward to current day research and things like that...but the steps will be fairly slow and there are some beautiful ideas, which need quantum field theory; but I think, I can tell you the words and the pictures, which go with it so that you get an idea of where these things fit with the quantum field theory, quantum mechanics so on and so forth, and even current day research.

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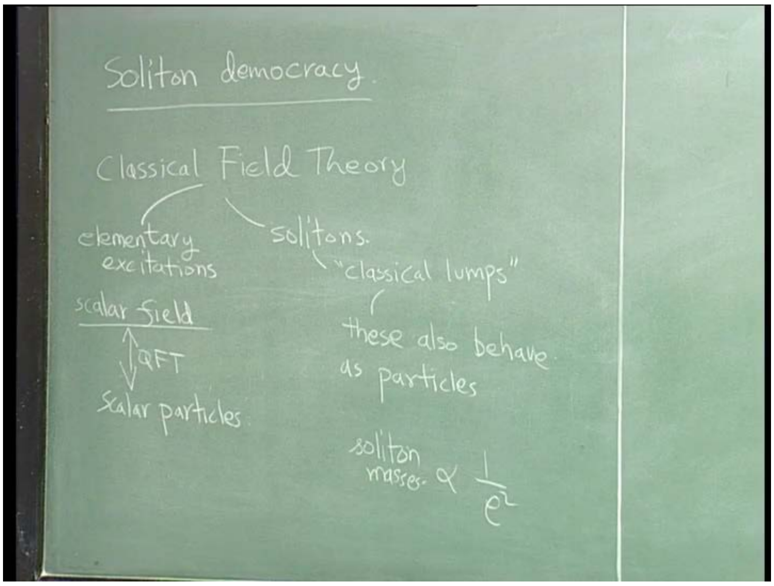

So, one title for this lecture could be, you know soliton democracy. This is just an adaptation of title of a paper by Thompson, which is titled p branes democracy, and that talks about democracy among p branes. Similarly, we will talk about democracy among solitons. So, the thing is, what we have seen is that, in classical field theory, so there are solitons and there are, what we will call elementary excitations. So, the idea of elementary excitations is that, suppose I have a scalar field. We are kind of told that, if we go ahead and go to quantum field theory, the normal elementary excitations or the low energy excitations would be excitations of the scalar field, which would be related to scalar particles.

So, this is something which we need quantum field theory, but at least we can go ahead and look at these things. But, solitons do not seem to come into that sort of a thing. They look like, they are lumps as Coleman likes to call it. They are classical lumps; lumps of energy. We have seen for instance, the kink soliton, it could be fairly localized. So, but these behave, these also behave as particles and you can do things like, can I try to quantize this thing like you quantize particles in quantum mechanics? You can talk of, talking about, you can talk, I mean quantum mechanics of such particles. So, there is a sense in which you could do this, but typically these things are massive, very very thing and they are not low energy excitations.

So, really whenever you talk of excitations of a field theory from a classical field theory to the quantum field theory limit, you actually thinking of these scalar particles. So, the democracy, which I am going to discuss today, is going to tell you this picture of separating these two things out is very (( )). There is a sense, in which these also have to be taken into account. For instance, in the path integral approach, you will see there are reasons you should look at configurations, which are related to solitons. But, I am saying something more dramatic or different. What I am saying is that, there could be an alternate description, where these solitons are elementary excitations of some other theory, while these scalar particles are solitons of that theory. That could happen and we are going to see an example of that.

So, in other words, there can be new variables or new fields, for which the solitons are the elementary excitations. But still, there is a problem. The problem is that, like I said, these things tend to be very heavy. So, there must be a sense in which they can become light. So, if you look at typically, what we see is that, typically soliton masses, which is the same as the energy tend to be proportional to some coupling. We go like 1 by e square or 1 by e and we are told that quantum field theory requires this e to be small. That is where perturbative quantum field theory works.

So, this is some book keeping parameter. You are doing some Taylor expansion or whatever, where e square or e square h bar or alpha prime, it is a small number and then you can see that 1 by e square is a fairly large number. That is why it is massive. But, if e is a parameter in your theory, I can ask, what happens if e becomes very very large? Then clearly you can see that, if this formula does not change, if you naively look at this formula and let us say this formula does not change for large e. You can see that the soliton mass becomes lighter and lighter. So, there can be regime, where your perturb, where e of course, your perturbative methods in quantum field theory die down, but there could be, I mean, you can see that these become, it becomes very cheap to create the solitons and destroy the solitons.

That is, so really I should say soliton elementary particle democracy in some sense. So, what we will do today is, to see three different examples. Time, I mean, I may get to the third, otherwise I will do to it next lecture. So, the idea here is that, suppose we have two different theories, which have the following property that the solitons of, say, let us call this A, solitons of theory A, become the elementary particles of some other theory and vice versa. We will call that a duality.

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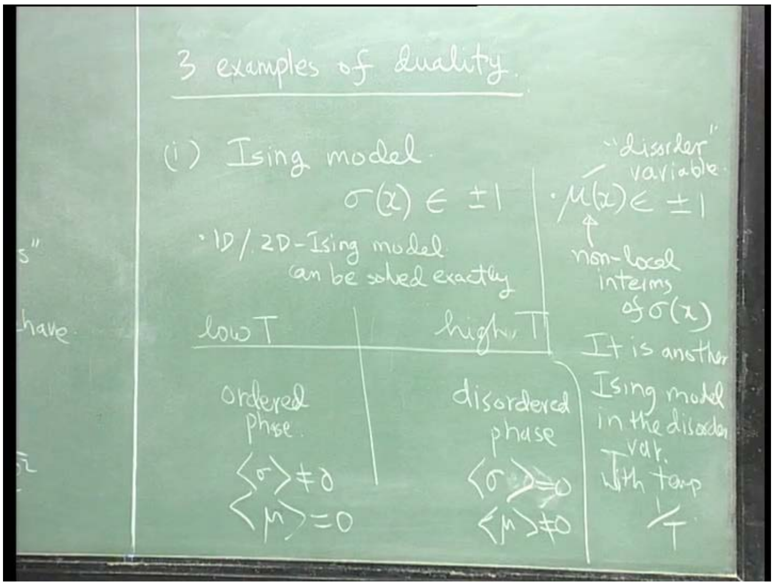

So, we will see three examples of what I will call duality and they are all, what are called strong weak duality, s duality. So, this is the kind of thing. One of them, so you can see what we are saying here is that, e becoming large; a strong coupling. So, this is a coupling constant of your theory. So, the first example is actually the Ising model. So, you are told that this is a field theory with variable sigma and let us say, I will just deliberately use continuum index, but you could think of it on a lattice, where the x takes value in some lattice, which belongs to say plus or minus 1.

So, we know that, for instance, the Tour de Force, which is the 2 d Ising model can be solved exactly you are told, 1 d and 2 d. Then you are told that the 3 d Ising model is the computation of the ground state energy is something like n p hard or whatever. So, but the, what I want to tell you is that, this model at low temperatures, it has the following behavior at low temperature. So now, the analogue of the parameter e is going to be temperature. So, it is something which I can control. I can change the temperature. At low temperature, we have an ordered phase and at high temperature, all the up and down, everything is this thing. So, you have a disordered phase.

So, in the ordered phase, sigma, the expectation value of the average, so this is an example, by the way, not from field theory, but from statistical mechanics. But, it does not matter. It is also a field theory and in this case, sigma equal to 0. The argument for a phase transition is done in the following way, is that, there is another set of variables you can construct, which I will call mu of x. These are non local in this theory non local in terms of sigma of x. I am not going to give the constructions. This you can go and look at it in books, in Statmac books, may be some of them will have it.

So, this variable, I will call this the disorder variable. For whatever, for want of better reason and you can show that, you can rewrite the whole hygiene model in terms of these variables mu and mu again takes only values plus or minus 1 and it is complicated to write it in terms of sigma's. It is non local in the sense that, to describe it, you need, there will be a tail if you wish, starting from some infinite point down to this thing. You have to write that. So, it will be, it is not some localized function of sigma of x. It is non local in that sense. But, nevertheless, I mean, that is ok. But, this also takes values plus or minus 1. The nice thing is that, you can ask what happens. You can show, you can rewrite the Hamiltonian if you wish of the Ising model in terms of the mu variables and what you will discover is that the temperature gets flipped. So, mu variables, it is another Ising model in disorder variables and roughly speaking, t goes to with temperature 1 by t.

So, now can see that large T, this is high T becomes low t for the disorder model, Ising model and it will be in the disordered phase. Will be when? When the disordered parameter is ordered. It is kind of confusing. So, I will just write something here. Obviously, this is like high temperature for this. In fact, anybody who has solved the Ising model, will see that you get the expression, there are 2 betas which come, beta and a beta tilde, which are related by precisely this kind of thing. So, the thing is, the point, so that there exists a phase transition, you can see is that, what is happening is that these variables go to the other variables. So, this is an example of a model, which is strong weak dual. I mean, strong here means, high and low temperature expansions get exchanged and it is self dual because it match to a model of the same kind.

So, I this is an example of what I said, theory a, theory b and I also said the variables are not going to be so simple to write. The solitons are not, I mean, they are not localized objects. They have some spread etcetera. So, there in terms of these variables, it will be pretty complicated.

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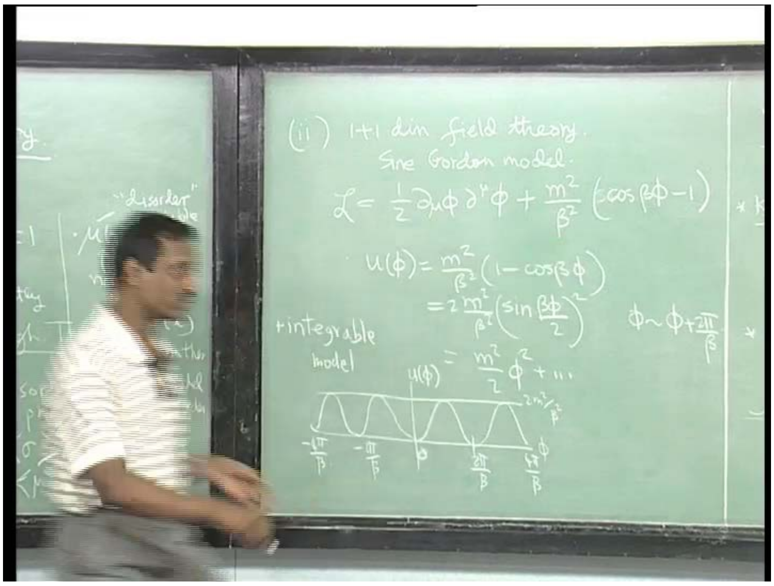

So, this is the first example and the second example is of a proper field theory. So, it is a 1 plus 1 dimensional field theory and it is called the Sine Gordon model. I guess, this name was given by Finkelstein in casual conversation with a student Rubenstein, but he never expected Rubenstein to go ahead and use that name in the paper. But, he said that, he was too late to take Rubenstein to task because Rubenstein had graduated. So, it is a, I will write a whole bunch of things. So, hopefully there are no errors here. I think this is fine, I will check for some factors I need to put. That sounds correct.

So, the reason this is called as Sine, so now, you can see, we can ask how this looks when you expand it for small pi. So, you can see that, so u of phi is equal to m square by beta square into 1 minus cosine beta of phi, which is equal to 2 m square by beta square sin beta phi by 2 whole square. So now, you can see for small phi, this whole thing will become, this is equal to m square by 2 pi square plus non-linear terms. So now, hopefully you can understand the joke. It is like saying, taking the Sine Gordon equation and replacing the mass term with the sin phi, with sin sin phi. So, that is why it is called the, this thing. So, is a model which is actually, exactly solvable in many ways. So, it is also a model which is called an Integrable model. In the strict classical mechanic sense, where you find that, that it has an infinite number of conserved quantities in invalution etcetera, which is related to the fact that it is exactly solvable.

So, in this model, now if you plot the potential, you can see that the potential u of phi; one more thing you can see here is that is a periodicity, which is phi goes to phi plus 2 pi over beta. So, let me plot u of phi. So, at phi equal to 0, it goes to 0 and then it sort of goes. So, this would be at 0 and first one would be 2 pi over beta, 4 pi over beta and the maximum value will be when this would be some 2 m square upon beta square. So now, this is a case, where you have many non degenerate, sorry, degenerate vacua you have. You have infinite degeneracy.

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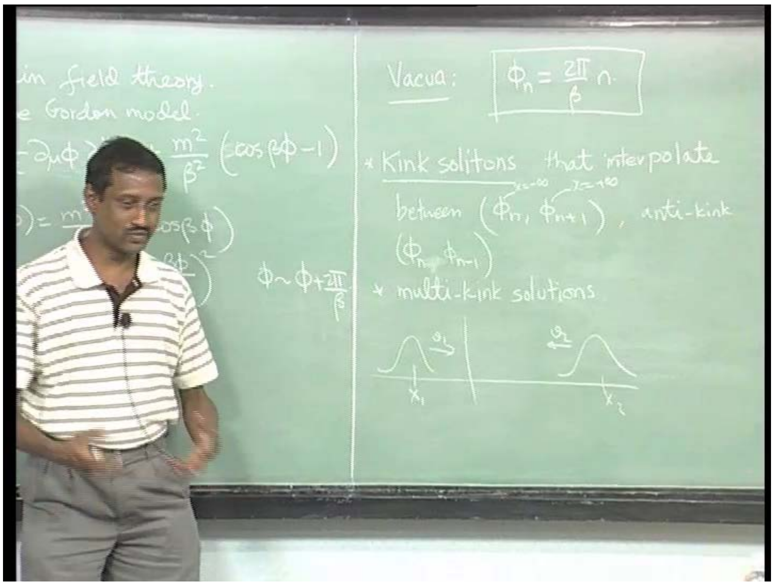

So, the vacuum solution, let us call them pi n equal to whatever values it take. When it becomes 0, that is just 2 pi over beta. So, for any of these things, you are at the minima of the potential and obviously, this has kink solutions, which interpolate. So clearly, so this has kink solitons that interpolate between say phi n and phi n plus 1.

So, this will be a kink and anti kinks will go opposite. So, this is the boundary condition at x equal to minus infinity and it goes to this, at x equal to plus infinity. So, that is what I am using in this notation. So, it can go from 1 phi n to pi n minus 1. Any one of these things you could do. It interpolates between them. But, unlike, so in the phi 4 theory, there was a kink and an anti kink. But now, I can have a situation, whereas a solution, which interpolates between 0 and 4 phi beta, so that, we can, so you can think of that, at least as, so for instance, in phi 4 theory, you could not put superpose to kink solutions because it will violate the boundary condition. But here you can.

If you put two kink's together, it will take you from, 0 to first kink will take you from 0 to 2 phi over beta and the next kink will take you from there. So, there are, so this has the usual kink solitons or has multi kink solutions. So, you could write a solution, where the two kink's are fairly, widely separated out. So, you could write something like this. So, just linear combination would work.

So, I am just plotting the energy density, so that you can see it localized. But, the point here is that, for instance, this is a two ink solution. You can write an exact two kink solution. You can solve it. More importantly, you could even say, you could solve it for, you know, this is localized here. So, there is, it is at x 1; this is localized at x 2. Let us say it has velocity v 1 and this thing has velocity v 2. You could write solutions, which actually do all these things and this is related to the fact that, it is an integrable model. That is some nice structure which comes out.

So, you can have quite a bit of control. So, you can ask, what is the kink kink reaction because you know all these things. So, you take the kinks, you separate them and ask is there some energy what is the energy between them. You can work out lots of stuff. Could do the dynamics of these things and write an effective potential between these things. So, it is a, so in that sense, it is a very very malleable and ductile model. You can do all sorts of things to it.

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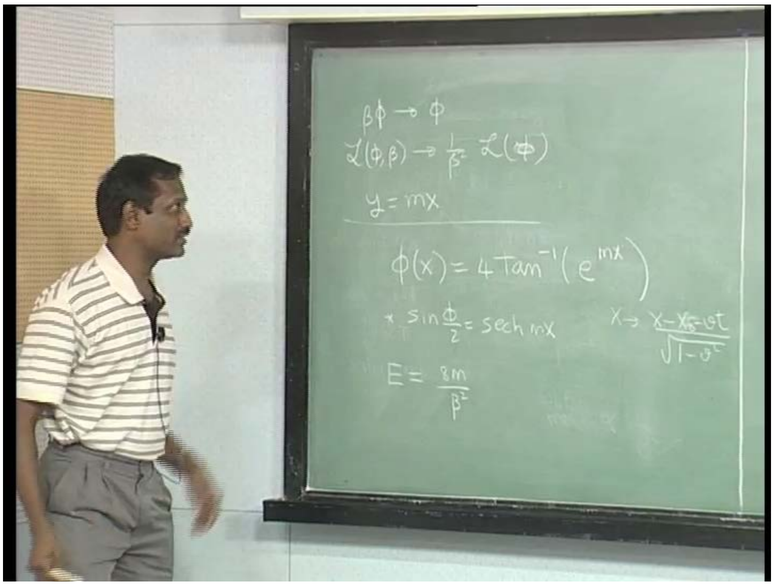

So, just for completeness, I will just give you the kink solution and let us do some little bit more things. You can see that, classically at least, I can do the following thing. I can take phi beta phi and just call it phi. What I mean by that is, if you are confused, you just say phi beta pi equal to pi tilde. You define it and remove the tilde. That is what I mean by that. So, what will happen here is that, this beta phi will become a phi. Nothing happens here. But here, we have to write, this will become a phi upon beta.

So, you get a 1 by beta square. So, the Lagrangian, which originally had just phi and beta, just goes over to 1 by beta square times, which just depends on phi and the beta square dependence just disappears. I mean, goes away in some sense. So, but it is useful to remember that the Lagrangian has a 1 upon, if you want to put back things, just remember that you put a 1 by beta square in. We will also be looking at time independent solutions. So, we could also; then one more thing you could see is that, there is a m square floating around. So, you can go to dimensionless. You can define y to be m x.

So, in terms of these new phi and y, beta and m completely disappear from the theory. So, if you want time dependent solutions also, you could do that. You could just say the time also scales that way. Then you perfectly find the m goes away. So, if you are looking at solutions on these books, you may in several books, you will find that some people would have said beta equal to 1. Few others would have said beta and m equal to

1. So, it is a little bit of a mess, if you are looking for the solution. But, you can see from this and you can actually put things going back.

So, let me just write out the one soliton solution. So, I am just giving you a soliton, which is localized this at is the. So, for this solution, you can actually check that. I think sin phi by 2 becomes sech and hyperbola m x shows this. This is a useful identity, which you need to and cost phi by 2 will be, I think un hyperbolic m x. You can go ahead and replace x by x minus v x minus x naught minus v t divided by square root of 1 minus v square, like we did for the other kink soliton. It will still solve the equation. The energy of this thing turns out to be 8 pi square upon beta square into m. So, dimensionally again, is it 8 pi square or 8 pi? No, it is just 8. Not even 8. No phi's.

So, this is again a simple calculation will tell you this. So far it looks like, you know, one more model we are looking at and for once, I have gone away from my, the only model I knew the potential, which is pi fourth. So, looks like yet another thing. But, there we will get, so we will see pretty soon that this model is equal to sigh equivalent to dual to something completely different. So, this is what we have. The energy is this thing.

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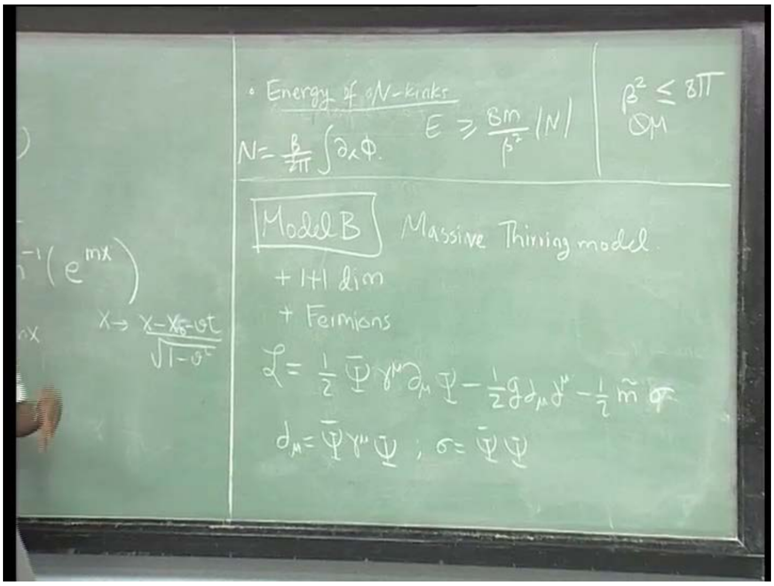

But, you can also show that the energy of n kinks generally greater than or equal to; I will put mod n with negative n representing anti kinks and explicit solutions are known. So, this is with respect to the kinks. But here, is the, so this is, will call this, we will call this model A. So, model B, I have to give you a lagrangian etcetera. It is called the

Massive Thirring model. Just one thing I want, one last thing I want to mention about this model is that, this model, classically like I said beta is not very important. But, quantum mechanically it is. So, then there is a value of beta square and it should be less than or equal to 8 pi for the quantum theory to have a ground state. Otherwise, you can show that by some Variational methods, for instance, Coleman shows that the ground state energy is unbounded. So, in sense, which you require, so this is a quantum mechanical statement. This is not something, which you can show classically. Classically we saw right. It just goes off. It goes for the right.

So, the Massive Thirring model is a model, which has fermions. It is a model in 1 plus 1 dimensions and it is a model of fermions and the lagrangian has the following form. So, some two dimensional Pauly matrices analog of, so this would be, this would give you a massless Dirac equation. We have not discussed fermions. But, it does not solve our purpose. It is just to show you what is going on and then it has coupling, which I will write out for you. It has two kinds of couplings, where j mu; so again you would have seen this in your lativestic quantum mechanics course. This is what would have coupled to electromagnetism for instance and this is like a cha. This thing is just, sigma is just psi bar psi. So, psi bar would correspond to psi dagger with some charge conjugation matrix. We usually, you take it to be, depending on your convention, it is typically gamma 0. So, this if you wish in components, like in 4 dimensions Dirac equation, Dirac fermions had 4, had 4 components and here, there will be 2 and roughly you can think of them as left and right modes.

So, here is the remarkable claim. So now, you can see that, this is the interactions. It has 4 psi bar j mu. j mu, which is psi bar power psi power 4. So, it is like 4 form interaction. This is a mass term, because it is quadratic in this thing, in the field. So, it is exactly like scalar field, except, there is little bit more stuff going with it. So, this is the kind of what we will call a mass term, but actually, this partic this theory apparently does not have one particle excitation. So, this you cannot think of, this as mass of that thing. You just think of it as some parameter. So, it has 2 parameters, g and m tilde. You can see that, when g equal to 0, it is just a theory of a Dirac massive Dirac particle in 1 plus 1.

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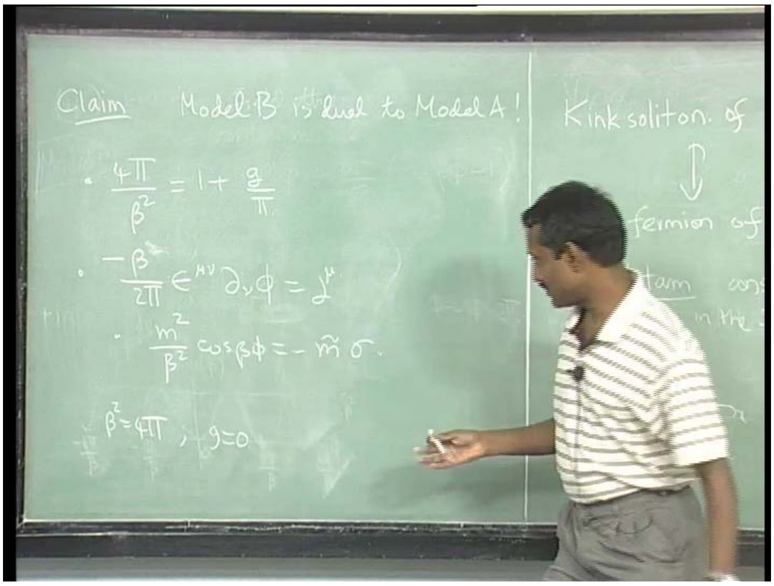

So, this is the model and claim. Model B is dual to model A. Kind of confusing, right? One is a theory, which is a scalar theory, model A. This is a theory which fundamental excitations are fermions and here is a mapping. The mapping is a following. I will explain all these things one by one. But, the key point here is to look at, first let us go look at the first term in this thing, which says that, the coupling beta goes. It is like an inverse of this. So, small beta implies large g and vice versa. So, this is exactly like that strong weak kind of stuff. Then this is kind of, this is also, this sort of gives you a hint about things. What it is telling you is that, this is your, this is the current, which is usually conserved. But, this is, like I said the electromagnetic current. So, it is know the current corresponding to electromagnetic invariant. U 1 global, that is the know the current for that in this theory.

So, on this side, but in this, what is this one? In the Sine Gordan side, it is a topological current. So, what is happening is that, a topological current on one side, you are getting mapped to a know the current in the new variables. In fact, you can see, you can show this n here. It is just beta. Is there a beta square here or a beta? Let me check that. It is just beta. This is just the topological charge and the kink number. It is really that. So, this gives you relationship between the 2 m's, if you wish. It also tells you that the interaction, so the interactions of the Sine Gordan theory get this. In some sense, this term gets mapped to this. That is about all I can say. But, you can also see something interesting happen when beta square is 4 pi, which is below this limit. What happens? g becomes 0. So, there is a point, which is the, we will say the free Fermion point or the massive Fermion point, which is beta square equal to 4 pi. So, let us try to understand how this model actually captures this soliton. What I call, the soliton and elementary particle democracy.

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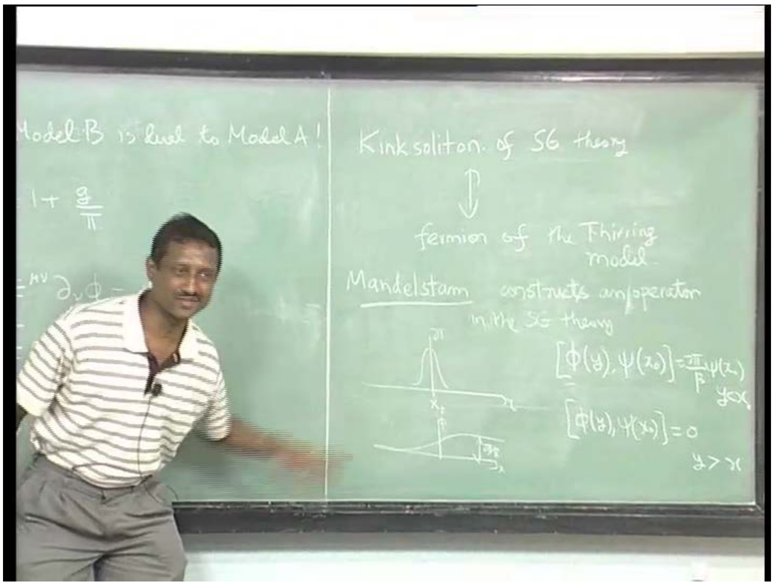

So, the idea here, so the thing is that, the kink soliton of the Sine Gordan theory actually is the fermions of the Thirring model. The way it goes, the way, so this was shown by, in a very nice paper by Mandelstam, so whose idea was the following. Suppose I have a kink localized out here at some point x naught. So, what is, so let us say that, what this is doing is that, it goes from 0 to; so it is a kink. In the sense of it is soliton number 1 and so it just does this. So, let us say it goes. So, the configuration looks like this. It does something like this. So, this is the energy density versus x and this is x. So, x naught is somewhere the location, roughly of this thing. So, suppose you want to construct. So, you want to construct an operator. So, what Mandelstam does, he constructs. So, if you want to destroy this kink, right, what you should do is, you should at this point, you should have something which will bring, roughly, it should undo this. So, he constructs an operator. This is non local in the Sine Gordan theory, which has the following property. So, let me just write that.

So, phi is just a field. This is when y is less than x and then; so what it is doing here is, on this side, so this one is telling you, is it? Yeah, psi. So, this just tells you that this phi jumps, this commutation relation just tells you that this jumps by your 2 pi by beta, which is what was really, so this had to be 2 pi over beta, right, for a single kink. So, it creates a jump. So, this is undoing this or if there was nothing, if it was acting on vacuum, it is changing it. So, in that sense, it has the same role as what the kink soliton was doing. He has to construct such an operator and he does and it is non local as you will see. Just as the disorder variable was non local. So, keep in mind that this psi, which I am writing, is completely written in the Sine Gordan theory.

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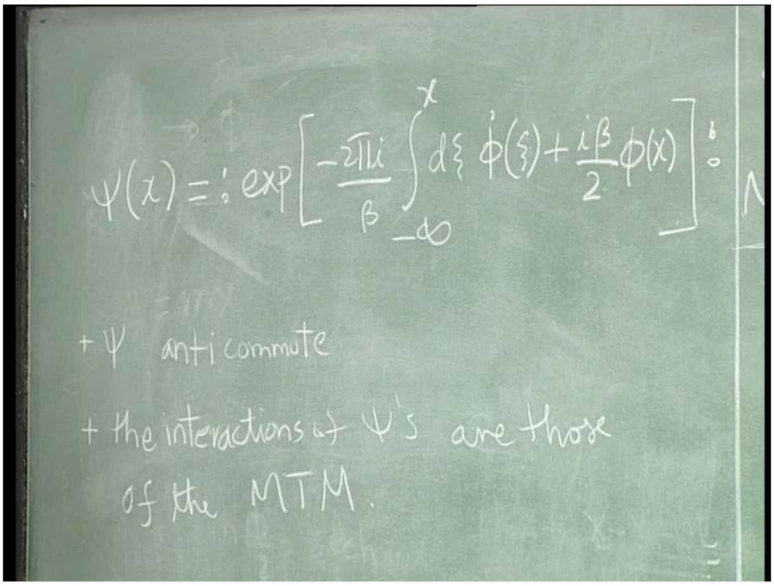

So, it is a quantum operator. It requires normal ordering. Just give me a second, i beta over 2 phi. So, you can see it is a non local object. First thing, this is exponential and it is an integral from minus infinity to x. So, it is going from minus infinity to this point x. This became x naught here. So, he constructs this and shows that it satisfies that. So, you need, I mean, little bit of quantum field theory is needed, but you can, what I want you to get from here is that, this is a non local object. Now, once he has this thing, he needs to show that this, so among other things, you would like to show is that, this psi behaves like a Fermion. So, psi is anti-commute and you should also show that it can recover the interactions of the psi's are those of the Thirring model, the Massive Thirring model. So, he goes ahead and shows this thing.

So, this is very very remarkable and this kind of shows you, what was the soliton in one theory. So, for instance, I would think that if beta square is 4 pi, it is like in some sense, strong coupling. There I would say that, it is probably better to think of describing that equation by Massive Thirring model, with g equal to 0. Probably, a nicer, simpler one to do it. So, this is my second example, where I want to show you, how solitons in a theory can become the fundamental excitations of another theory. So, are there any questions? phi in terms of psi?

How do you write that? So, the easiest way is to use this. So, this gives you psi bar psi is d nu of psi. So, this is the way. So, in fact, you said g equal to 0 and m tilde equal to 0. It is a free fermion. Free massless fermion. There is a method called Bozonization, which takes you from here to there and then this is the starting point. So, you can see that, this roughly tells you, how to go the other way. So, you might say, hey, this is toy model and this may not represent reality. Yes and no. We will see a slightly. So, let us look at a model in 3 plus 1 dimensions.

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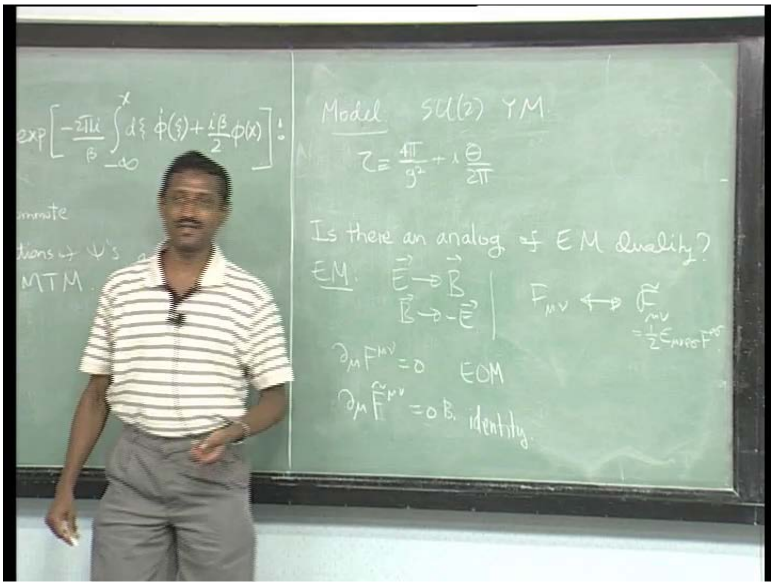

So, we will look at, so the model will be, say let us consider some non, yeah, let us consider for starters, let us just take s u 2. So, we have seen that that this thing has a nice, this thing has a coupling 4 pi upon 1 g square. So, I define something called; I combined things like this. I wrote, the coupling in the theory is given by this tau. So, the question is, can we understand is there an analog of electric magnetic duality and if so what does that look like? So, let us go back to electromagnetism EM and we know that there is a duality which does the following thing. It takes electric field to magnetic field and magnetic field; I do not know, somewhere there has to be a minus sign. So, I am putting it out here. So, electrical and magnetic fields get exchanged and we wrote this in terms of, by saying that the role of F mu nu and F tilde mu nu are exchanged, which is the dual, are exchanged.

So, but now, if you remember, there was exactly the following thing is that, the equation of motion, this was equal to 0. This was the equation of motion. But, this was just a trigger identity. This was the one which let you solve for F mu nu in terms of vector fields. So, this is an identity. So, this is without sources. So now, the thing is, suppose we had, under electric and magnetic duality, you can see that the role of F and F tilde get exchanged and what is happening is that, you have, so this is a Bianchi identity. So, the bianchi identity and the equations of motions, roles get exchanged. In some sense, that is similar to what happened here. Know the current is becoming, this is a trivial identity. This did not require equations of motion to solve it, but this required. Same thing is out here. This is the equations of motion and this is does not require anything. It is just an identity. So, topological current; so in that sense, it is similar to that. So, obviously, I can just do the same thing, except, if you have some non abilean gauge theory, I can always go ahead and exchange. Now, instead of working with these things, these would be adjoint valued objects. But, both of them are adjoint valued. So, I can exchange them. So, it is very clear.

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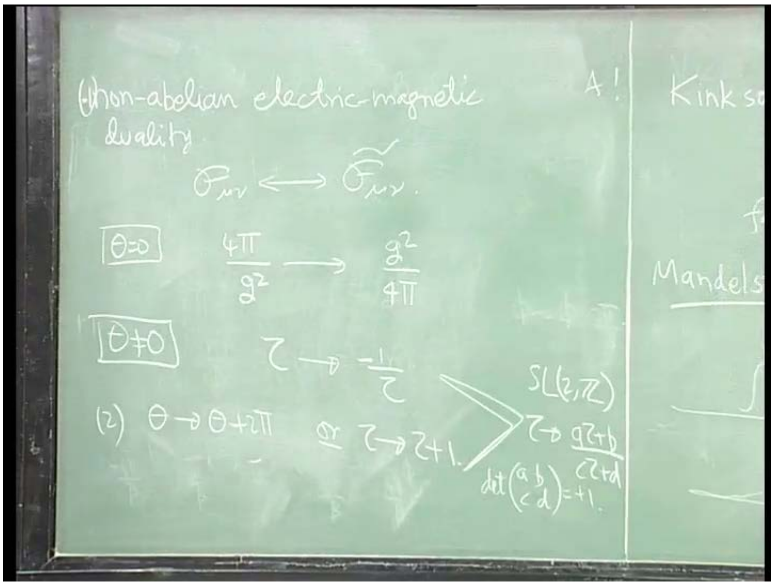

We can state, so the non abilian electric magnetic duality, it is just a tilde mu nu. F being a rather F, but actually it is not just that. It also corresponds to taking, so let us say that theta, when theta is 0, at least let me write, let us put theta equal to 0. It takes, it corresponds to taking 4 pi g square to the inverse of that. Sorry, let me redefine my tau in a slightly different manner. I think it is usually the real part. I exchanged the real (( )).

So, this is, so this actually has to be combined with this thing. So now, you can see that, electric magnetic duality, if you just look at this thing, small g is where perturbative things become this thing. But, so the coupling constant being small in one side, becomes coupling being larger on the other side. So, it is again a strong weak thing. The other way of seeing it, if you go back and look at the example of we had worked out the monopole for s u 2 cross u 1 and if you remember the mass of the monopole was very, it was heavy, right. It was. It went by and it was divided by g square. So, again it is exactly like the kink. The kink had a 1 by beta square. So, it is the same thing which is happening out here. So, but there is a nicer way of writing this thing.

So, when theta is not equal to 0, this is slightly more involved. I am not giving any proofs or anything. I am just writing these things out for you. tau goes to minus 1 upon tau and you can see that, that will work out because when theta is 0, the minus sign will get cancelled by the i going below and then again it will work out. It will be exactly what I wrote. But, there is another; so this is what we will call normal electric magnetic duality. But, because we have a theta term, we have actually, so now, theta comes. If you remember, the theta term comes out like this, right, times the instanton number and quantum mechanically, there is an equivalence theta goes to theta plus over to theta plus 2 pi.

So, theta is a period. It is an angle variable, not a, this thing. So, theta is angle. So, there is a second, so this is the first one. The second transformation is theta goes to theta plus 2 pi. But, that is easier stated. If you look here, that is r tau goes to tau plus 1. So, this looks like, it does not change anything in the quantum mechanical story. So, but this is a symmetry of this theory. These two actually combine. These two symmetries combine and generate a group called s l 2 z, which tau goes to, a tau plus b over c tau plus d and where, this a b c d are interior elements, such that, determinant of this is equal to plus 1. There is a yes.

So, there is a trivial equivalence of all a b c d e are minus of themselves. So, that is z 2. So, it is called technically p s l 2 v. So, one thing you can see here is that, again just going back to plain electric magnetic duality, what it tells you is that, if you are going to the strong coupling limit, then probably what you thought was, I mean, there could be monopoles, which become very light and your electrically charged objects become heavy.

So, this tells you that, what you thought was your electron in this sys, in this thing, in this set up would become a soliton in that theory and vice versa. So, there is a, this thing, but it also tells you, suppose you have a situation, where both electric and magnetic, there are things which dyons become the lightest. Then you could be in a situation, where actually there exist no local description and there is a class of, so in these theories, this is just I started out with normal s u 2 yang mills, but we could, we know little bit more when you add super symmetry to this theory.

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So, there, so whatever this means, N equal to 2 super symmetric yang mills, so this will have this. So, this again will have this parameter tau and the same things go through. But because there is super symmetry in the problem, we find that we have lot more control and many of the, so you can you can, so for instance here, the theory is determined uniquely by 1 function. So, at some, so we will just say it is some function of something of some fields and if you give me this function, I can write out for you the full

Lagrangian for the theory. So, it is determined completely because super symmetry relates various terms. It relates Bozonic terms to Fermionic terms and when there is lot of super symmetry, you will find that you can actually; so this pre potential it is called the pre potential. It determines the Lagrangian. So, if you take this kind of symmetry seriously, you can go ahead and you can ask, can we figure out this function which has this? It should have some properties. In fact, duality plus super symmetry plus a little bit more, but determine.

So, this F, you will start out writing your normal classical thing. But, what you want is a quantum corrected this thing. So, you can actually determine the quantum pre potential and this is Tour de Force due to Seiberg and Witten, I think around 1995. So, this is another non trivial example, where you know duality symmetries tell you, you expect this thing. So, the thing is that, consistency with these things and the factor, this has to be a certain kind of some analyticity properties, more or less fix the form of this thing and it turns out that this is determined by some other funny Torres or whatever, which comes into play and this as well 2 z, there is some Torres, which determines this completely. It is sort of, this is sort of an extreme case of a field theory, a quantum field theory being solved by, just consistency in some sense and taking the idea of soliton elementary particle democracy very seriously. So, you look at the solitons in this theory etcetera. So, I am done for today.