Classical Field Theory
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Lecture - 29

In the idea of the algebras, you have played around with it enough in your assignments. I guess you have done all that.

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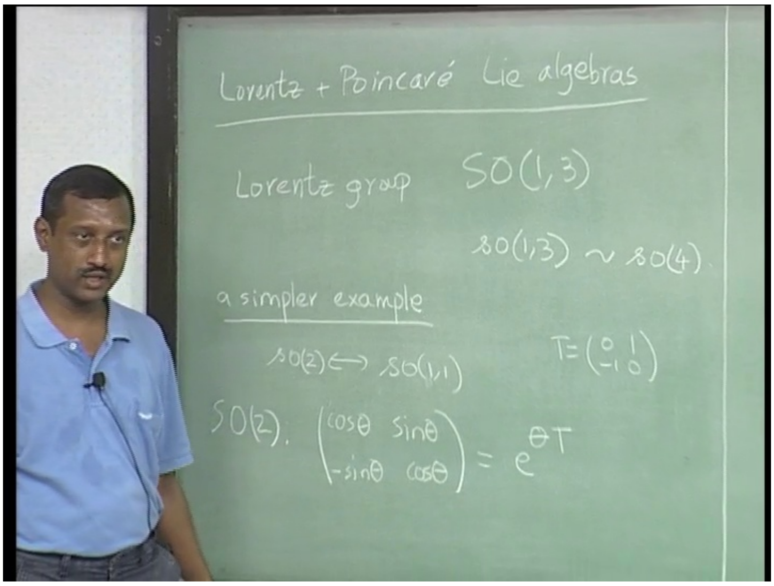

So, what I am going to do today is to actually discuss the Lorentz and the Poincare Lie algebras, some aspects which are actually relevant to field, field theory both classical and quantum. And so the first thing is to, one thing we already know is that the Lorentz group is the, is special orthogonal group. So, when by this I mean the continuous parts. So, we are looking at the special orthogonal group, this is the Lie group, and obviously there will be a Lie algebra which is, S O 1 comma 3, but the key point is as Lie algebras these are the same as, S O 4, and let me show you how this works for that it is enough for me to take S O.

So, a simpler example is to see how you will go from say SO 2 to SO 1 1. So, by this equivalence you remember when we discussed Lie algebras, it is a vector space we could think of it as over complex or reals. So, roughly what you will do is we go from, we start off with something which is in reals and we go to the complex domain, and then we sort of choose a different section kind of thing. So, let us understand how that happens. So, let us look at SO 2.

So, we know that any SO 2 capital SO 2 matrix can be written as cosine theta sine theta, where theta is some angle, and we know that this is equal to e power theta times T, where T would be a 2 by 2 anti symmetric matrix. So, this we know. So, now the question is, so the point I am going to make is that we can recover the booze matrix by taking theta and making theta to be pure imaginary.

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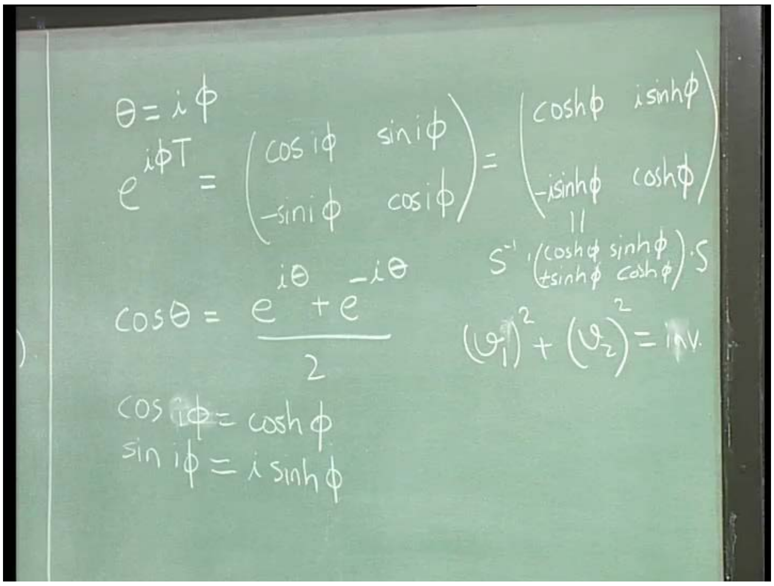

So, let us look at. So, the step we, what we do is we take theta, and we write it equal to i times some phi, so obviously I am, so you can see that what I am doing here, theta here was real I am complexifying it and then choosing something else which is this. So, now let us look at something like this. What is e power i phi of T, we can go back and look at it in this form, and write it as cos i phi sine w i phi minus sine i phi and cosine i phi.

Again, by going through the complex domain, we know that we can go from cosine the Trigonometric Functions to the Hyperbolic Functions, and in the complex domain they are the same. So, for instance we can look at cosine phi. So, cosine theta is defined to be e power i theta plus e power minus i theta over 2. So, if we just go ahead and put theta equal to i phi, we end up getting cosine. So, what we see is that cosine theta equal, cosine of i phi is equal to cos hyperbolic of phi, and similarly we will see you can show that sine of i phi is i sine hyperbolic of phi.

So, now we can see that we can rewrite this thing in terms of Hyperbolic Functions, you can see that the boozed, remember the booze matrix was looks similar to the that is it was something cos hyperbolic of phi, sine hyperbolic of phi and this thing. So, this almost looks like that except for this annoying factor of i's which hang around. So, let me ask you how do I get rid of this? Is there a way of getting rid of this i's.

Actually, not unitary, but a similarity transformation or a change of basis. So, it is useful to go back to the original definition. And so you would have written something like this, the norm would be for a Euclidian thing would have been v 1 square let me put it below v 1 square plus v 2 square equal to 1, equal to is, is invariant not equal to 1 is say invariant. But, for this thing we need to put a minus sign out here, either here or here depending on this thing.

So, that would have that can be achieved by taking one or either suppose I want a minus here I can take v 2 to i v 2, and that will be realized on this as a similarity transformation. So, I will just write here that this is equal to cos phi e hyperbolic phi minus sine hyperbolic phi, no minus the minus goes off, plus cos hyperbolic phi with some similarity transformation.

So, now you can see that similarity transformations are just a change of basis that is not a big deal, but what I am able to show for you is that these two Lie algebras are really the same, but as complex Lie algebras. So, when you go these things, obviously these two are not quite the same, if you now insist that I cannot take I keep phi real, then this is not the same as this that would be like, a like making a looking at it as a real Lie algebra.

So, but what we will do is I will always be thinking of going to the complex domain, and coming back and forth I will not specify that, and all the nice classification of Lie algebras is there only for the complex Lie algebras for the real once it is much more complicated not that is not that. So, this is the way it is going to go and you can see that the same thing will hold for this, even the change of basis will be obvious because you will have to put some i's in various places, but that, that is all there would be. And so you can see that this kind of is a nice thing as complex Lie algebras they are the same.

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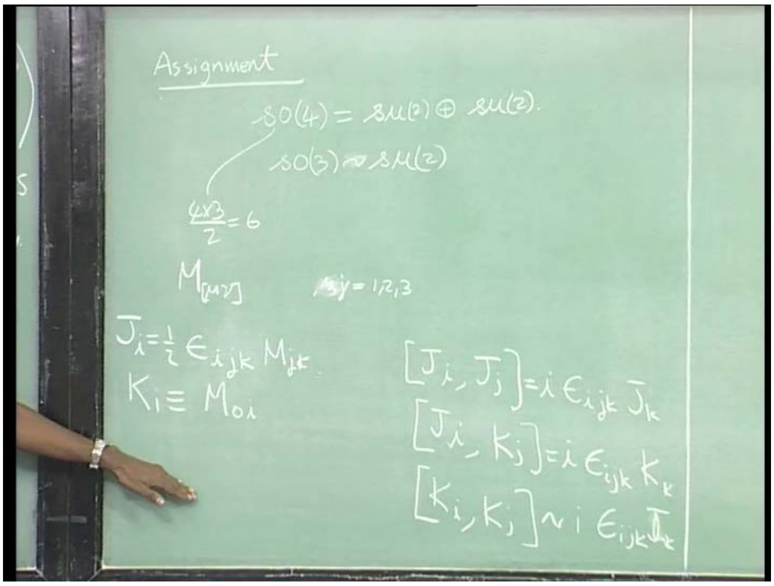

So, now the question is in your in your assignment you have shown, in one of your assignments, you have shown so 4 is two copies of (( )) 2, yes that seems to be a question there, no their groups are not the same one is this group is u 1. So, theta here takes values from 0 to 2 pi it lives on a circle while phi lives on a line it, there is no periodicity condition they are not the same group.

The groups were not the same that is why I am very careful. I mean, I have, you can see I have I have, done this, I am not saying I mean obviously, something which is compact like a circle is cannot become non compact I mean. So, they are different that is what makes booze different from this thing. So, at a group level they are not the same. And they are not the same as even real Lie algebras in some sense you need to, you need to the, you cannot put an i, if you are in the real domain.

So, this is just, this notation is carried over from the direct some these are also vector spaces. So, it like direct sum of vector spaces, and we also know that so 3 and su 2 are the same again, these two are equivalent as Lie algebras. So, let us use this symbol. So, intuitively we think may be one, you know given so 4, I can look at an so 3 sub group of that thing, you think that that would be one of these two s u 2's, and then there is something complicated coming out, and that you might have seen that, the that is not the case what really happens. So, the thing…

So, the generators here are, there are 6 of the generators because it is 4 into 3 by 2 which is equal to 6, and it is useful to write the generators as M mu nu, and its anti symmetric under mu and nu. And so now I am what I have in mind is really the Lorentz algebra, but let us write out. So, these are the generators, and there are 6 of them it is anti symmetric. So, this equal the notation is like this.

So, now the question here is what are these two s u 2’s, and you would have seen that first thing is to restrict mu and nu to take values only in the 1 2 and 3. So, that, that would be, so the notation will use for that is i j. So, we let us do this M i j, it is again anti symmetric, but I can define something which I will call j i, which is half.

So, this will be your usual angular momentum in three, three in non relativistic quantum mechanics whatever this what you would have written, but we know that we get these boozed guys when one of them is 0, and let us just call them K i. And so now the question is, so the thing is you can work out what how these things transform. So, one thing we already know is that J i J i. So, in this fashion writing things out J i J j is equal to i times epsilon i j k J k, this is just a normal thing which we know.

Now comes the interesting part what about j with k, and even this you can sort of interpret in the following sense up to, I mean not even a sign. So, we can think of J, this J as the generator of rotations, and this just tells you that this j is a vector, this index is a vector of so 3. So, that more or less tells you how K i should transform.

Now, comes the last bit which is K K. Now, so first thing it shows is that these three generators do not commute with j. So, these two imply that you have two bunches of generators which commute with each other, they do not even talk to each other in some sense. So, obviously this cannot be the one, and if you work this out I am not going to use equal to because I am unsure about the sign, but what is more important is that this becomes i times epsilon i j k l k J k. So, so what you can see is that there are 6 generators which is just a rewrite of the so 4 Lie algebra, but written in this form.

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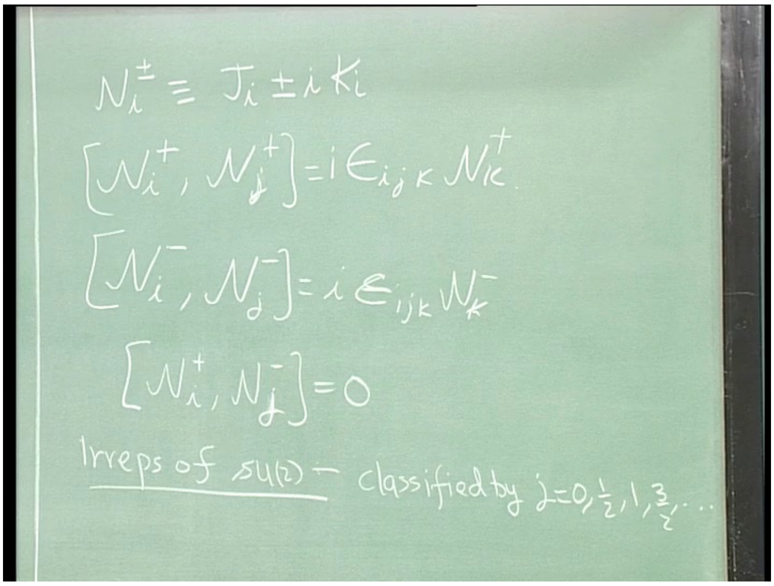

And we can and we get these things, and the nice thing is that you can define N i plus or minus to be J i plus or minus i K i, if you take these linear combinations what you find is that they decouple. So, roughly speaking what we will see is that N i plus N i j plus. Similarly, for minus there is another s u 2 Lie algebra, and the factor that they decouple implies what is that imply for N i plus and N i minus, the fact that I can write it this way they commute.

Suppose, just for a moment, K i K j gave you k k, it is still you would not say that, it gave two s u 2's, because j and k do not commute of course, I do not thing if I put this equal to k, it will satisfy Jacobean identity. I am just saying suppose, it acts as structure constants then it want. So, you need the… So, what we have is two separate s u 2's, so now if you are interested in representations of the Lorentz group what you end up getting is two s u 2's, and we know all about s u 2's.

So, there will be. So, irreps of s u 2 are nothing but classified by spin, by some number j which can take 0 for singlet half, for singlet 1, 3 by 2, so on. So, forth this one, thank you anything else missing good. So, now because there are two s u 2's, for each s u 2, I will have a j. So now, but here we already know for so 4 to tenser method we know how to construct irreps of so 4, but what I am trying to do now is asking trying to rewrite those guys in terms of these two s u 2 labels, and we have to work back and forth it gives you two distinct ways of writing the same thing.

And, this is very interesting, and kind of tells you that may be s u 2 is the only group we really need to know very well which is kind of true, actually s u 2 and the Heisenberg while algebra. So, these are the two things which I think one needs to know very well. So, what we will do is irreps.

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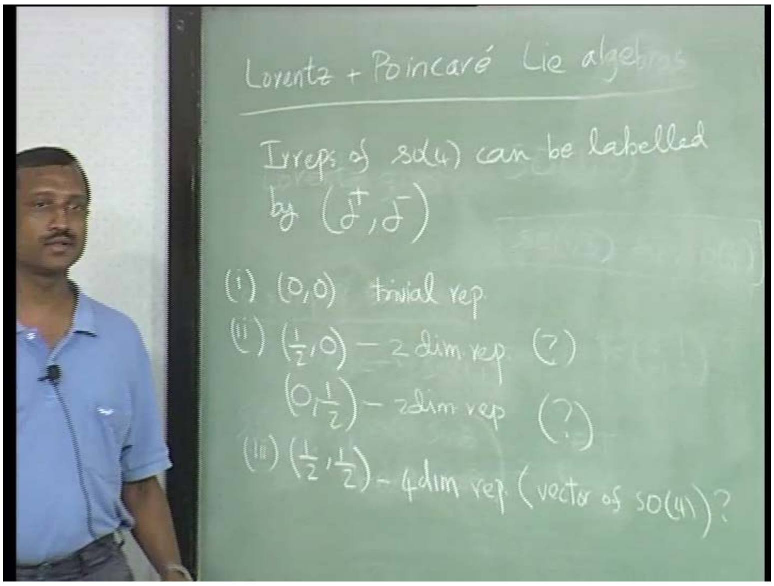

So, we have new this thing irreps of so 4, can be labelled by two numbers by j plus and j minus. So, it can. So, these are like commuting labels. So, you can give them this, so now for instance. So, let us we start working through these examples the first one is. So, I just write numbers 0 0, what would this be Scalar. So, it is just a Scalar one dimensional representation. So, this is a trivial representation.

So, now comes the, so let us do something else we can take a half and a 0, what would be the dimension of this representation, spin half would imply how many 2 0 will imply 1. So, 2 into 1 labels it will have. So, this is a 2 dimensional representation. Have we seen a two dimensional representation of so 4? What was the smallest defining representation which we used? How did we define so 4 first as what kind of matrices?

How did we define so 4 may be in the second or third lecture how did we define so 4? SO n say how did we define that the group S O n and? So, it was a n dimensional matrix. So, it is a n dimensional representation, so n equal to 4. So, we wrote a 4 dimensional representation, but this way of thinking about it is giving telling you that there is a 2 dimensional representation, little strange right we haven’t seen that.

So, this is a puzzle we do not know what this is, but we could also had 0 and a half. This is also two dimensional, but it is a different two, this is also two dimensional. So, let us let us try something else. So, what we are looking let us try to find the 4 dimensional representation. So, how would you. So, the thing is the interesting thing here is that suppose we considered this guy, this would be a half and a half, half is two dimensional this is also two dimensional.

So, it will have two kinds of labels this is 4 dimensional. Hopefully, this is the vector of SO 4, and there is a puzzle out here. Does anybody know what these two dimensional representations are? So, these are, this is called the spinor representation and this is what fermions carry.

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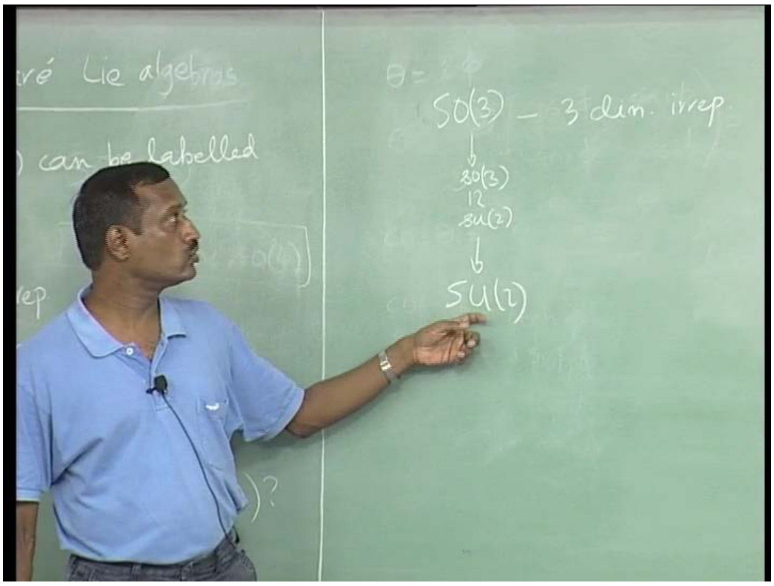

So, let me remind you it is there even in non relativistic quantum mechanics, we know that SO 3, what is the smallest representation non trivial representation its 3 dimensional, but we know that if you think of. So, let me write the group, but we know that this comes from the Lie group so 3, but the Lie group Lie algebra so 3.

But, the Lie algebra of so 3 is the same as s u 2 and s u 2 has spin half representation, has 2 dimensional representations. So, in some sense what you doing is going to a bigger group. So, you, so the SO 3 is not equal to the group s u 2 again, from here we can go to S u 2. So, there is a sense in which S u 2 is a, is a, is a covering group or whatever of S O 3. So, every element of SO 3 here will actually will have double this.

So, there will be a double valuation if you look at it from the S u 2 view point, and this is related to the fact that fermions when you go around by 2 pi actually pick up a sign. So, you so, you already have it even in non relativistic setting where when the minute you start talking about spin half representation, you actually are talking about going to the group S u 2 instead of S O 3.

So, that is exactly what is happening out here and one more thing you can see here is that if you go back and look at this thing what is J i up to factors, it is just an i plus plus N i minus, I do not know if there was a root 2 needed inside this, but so that is why I am using this. So, what you can see is that a normal generator of rotations is actually this thing, it is addition of angular momenta and we come back with this guy, and ask what is the spin of this thing in terms of normal rotation its half plus tenser 0, but this is just a half, this is also a half.

So, these are objects which we would have called fermions even in the relativistic, non relativistic setting you wouldn't distinguish between half 0 and 0 half, but that is the act in the relativistic setting is that clear. So, these, so this is, there this is exactly like this thing, but it is at a different level for so 4. So, so 4 also has these kind of representations. So, these are actually representation of what is called the spin four group, but let us not get into those details we are looking at only at the level of Lie algebras. So, that I can avoid this complications, at the level of Lie algebra I have all these things.

So, these are fermions, fermionic or spinor. So, these are left and right movers. So, we will call them these are called Weyl Spinors. So, one of them will be what is called plus (( )) and other is called minus (( )), but you would have seen in your when you did a relativistic quantum mechanics in Dirac equation, you had something called a Dirac Spinor, how many components did it have? How many did it have? 4. So, a Dirac Spinor is actually a reducible under so 4, it is the sum of these two guys. So, now we start getting the nice things.

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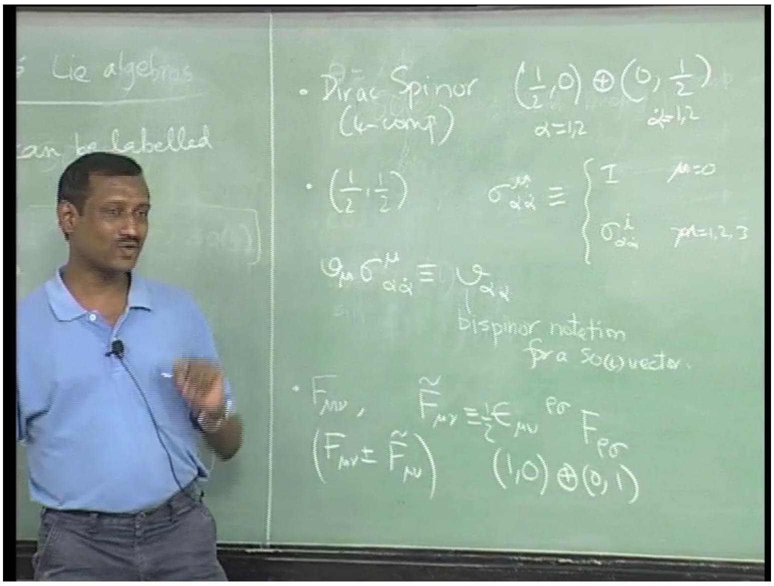

So, first thing we seen is that Dirac Spinor, which we know has four components is in this, thing I am not proving these things where at least I am trying to give you a flavor of where it comes from, and what conventionally what people do is to use an index alpha which runs from one, and two from this guy and differentiate it from the other guy by putting a dot. So, just these are just two dimensional indices.

So, now let us look at this something which has a half of this index and this thing will have an index which is alpha, alpha dot. So, let us look at these guys. So, half, half guy now that I am introduce notation will have alpha, alpha dot and I am claiming this as vector. So, you need something which will interpolate between alpha, alpha dot indices and the vector indices, and that is done by in the following sense.

We just take sigma m alpha, alpha dot which I defined to be for at m equal to 0, I call it identity I think and this is the identity matrix for m equal to 0, and the Pauli matrices sigma I, sigma mu right this is. So, this is a very interesting. So, the most natural, should I put with an upper mu, it does not matter leave this way. So, I can think of constructing a matrix valued object. So, what this tells you is that if you give me a vector v mu or in this case, a covariant vector. So, that I can contact with that, this is what I mean by that. So, it lets you go from the normal SO 4 vector index which is four indices to these alpha, alpha dot index, but it is also interesting if you think of this as a matrix.

So, then what happens is that this becomes a matrix valued it becomes a 2 by 2 matrix, and you know that matrices do not commute. So, this is related to something called quaternionic, so but from our view point it is just something which translates. So, this is called the bispinor notation, because it has two spinor indices bispinor notation for a vector. I will do one more example and I will stop for this part and to ask where does suppose I have an anti symmetric thing like F mu nu where does that sit.

So, now because you are in 4 dimensions we can break up you can define F mu nu you can take, you can define its dual F tilde mu nu by epsilon mu nu rho sigma F rho sigma. If you remember this was the one by the electric, and magnetic fields get exchanged for here to here that is all, but now because this is an operation if you do it two times tilde tilde will come back to itself, I can actually form combinations F mu nu plus or minus F tilde mu nu.

So, these will give you objects which are self dual, if you takes the plus guy under this operation, it will come back to itself the minus guy will go to minus itself, there might be an I here it depends on the conventions, so I will not get into that detail, but why I am trying to tell you is that this breaks up actually into two parts which is the self dual and the anti self dual part, this is something you can do only in 4 dimensions because you can see I am using something very special to 4 dimensions.

I am using the 4 dimensional times, because of that what happens is that these are actually reducible each of this the plus guy and the minus guy, now F mu nu had how many components six components. So, each of these will have three components and. So, what that translates into. So, spin one representation of this thing is three dimensional. So, this is one three dimensional and the other one is this thing. So, one of them will give the self dual part, the other will give the anti self dual part which one might be some even divide by root 2 or something like that.

You there will be obviously, even here probably I should put a half here and things like that, but I am just showing you schematically, this is not details can be worked out I mean you push me to a corner I will work it, but I think you should work it out apply two times and see what you get what where I should put a minus here, no it does not matter. It is really then I mean it is a question of what you will call self dual what you will call at the self dual, they will get exchanged that is not so important.

So, what is important is this point I want you to remember that a second rank anti symmetric tenser actually comes in this form, but again this is also we can ask how would we have seen this here is an example, we already saw a second rank anti symmetric tenser that under that broke up into two vectors j and k and what is. So, 1 plus 0 is a 1.

So, this is like a three dimensional vector 1 plus 1 times tensor, this is trivial in this. So, this is also 3, 2 vectors again, we see this sort of a thing and what we have done here also that plus minus i k is similar to exactly this, because if you recall there is an epsilon coming here. So, it is really the same story that we are going to n plus and n minus in terms of a representation theory this is exactly that. So, again I mention there was some root 2 here.

So, might be there, but again that is, that is, that is all a details for you to fix not for me I just tell you what it looks like I can fix it, but it is not done. So, now let us. So, this is this is all I wanted to say about the Poincare group, what we not about the Poincare, about the Lorentz group we have see that all irreps can be written in this form. So, let us now look at the Poincare.

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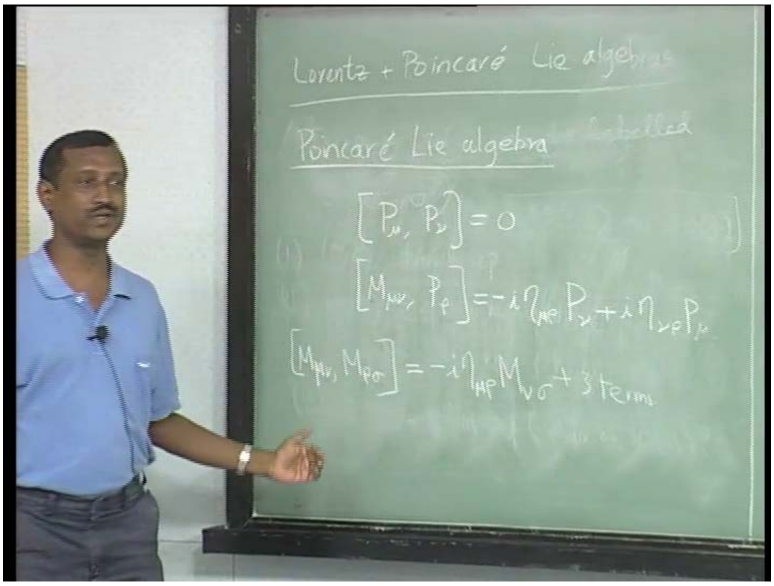

So, it is a 10 dimensional thing to be up to add to M mu nu, we need to add the translations.

So, I just, I thought let me at least write something correct. So, this is a vector. So, this is a generator of Lorentz transformations and we know that P transforms like a vector. So, it is similar to that it left two terms, but that is about it. P mu and using the same philosophy I can actually write now M mu nu M rho sigma, I can write this thing I just forget about the sigma for a moment, and then if you look at rho that is a vector index. So, we can see that.

So, I just close that and if you write that thing you will get I will write only one term, and then the rest you can fill. So, for instance look at rho here and sigma comes for the right. So, it just becomes i eta mu rho M mu sigma plus 3 terms. Now, the all the terms can be understood by saying you anti symmetrize with respect to mu nu, also with respect to rho sigma or you just say that this rho gave me two terms that is one vector index, then close this and then say there is a sigma gave me two terms and you write the other terms. So, this is also.

So, there is meaning in all these things. So, you do not never need to really memorize a lot, just the structure actually fits. So, the idea here is now what we are going to ask ourselves is how do we can, we write what how do we classify reducible representations of this group of this Lie algebra actually. So, but we got lucky in the case of so 4, it got mapped into something we know which is these two s u 2 is only thing which we know very, very well and we have taken for granted in this course. So, so the question is how would we go about doing this. So, before that, so let us go back to s u 2 and ask.

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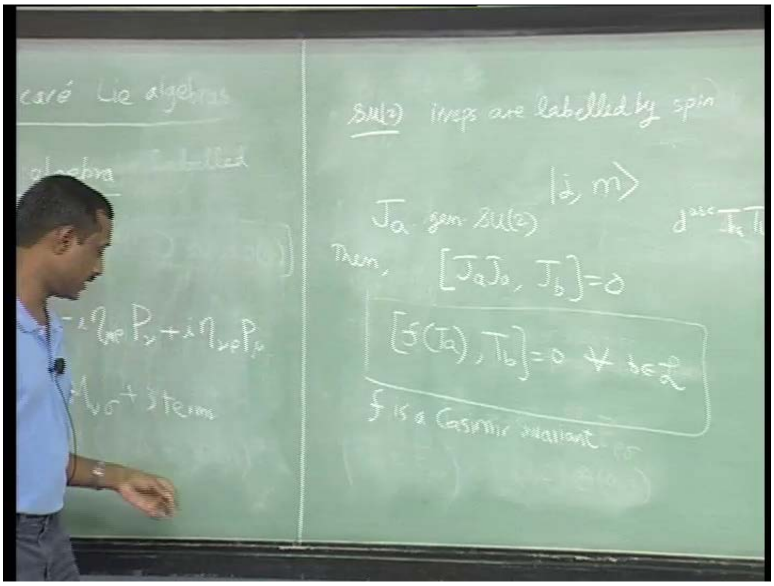

So, we I made the statement for s u 2 that irreps are labeled by spin. So, now the thing is that. So, in other words if you take a spin j multiplied m, it is actually it has a whole bunch of guys m running from minus j to plus j in units of in steps of one. So, now the thing is can we write some operator which will pick out will give us j for irrespective of it will be whether which element we pick and there is something, and that is a Casimir quadratic Casimir.

So, you let us say that T a’s are the generators or J a generator s u 2 then, it is easy to show that J a’s J a commutes with this thing. So, in other words because we know that these J’s can raise lower can may go from one multiplier to another. So, if you want something which is going to be independent of the m label it better be something which commutes with all the generators of the group they are called Casimirs such an example. So, anything. So, this is some functions of T a’s, which commutes with all them for all b belonging to a Lie algebra then we say that f is a Casimir.

So, for s u 2 we were done with j square there is only one such label what happens for s u 3, for s u 3 also you will find there is exactly this way except the a will run over 8 of them, you sum over all of them with the natural metric and then you get again, you get one such label, but s u 3 actually has another one which is a cubic thing. So, you go ahead and define something called d abc J a. So, let us write let us say T a T b T c, where d abc is something which you have to define or you can equally work out, what d a b should be the horrible way to do it is to actually work this out with the requirement that this will commute with the all of them. It turns out that for all the Lie algebras if the rank is r, it will have a r such casimirs independent casimirs.

So, for instance. So, this will give you a cubic casimir for this thing and that implies that irreps of s u 3 are labeled by 2 two numbers like one for s u 2 is that clear. So, what we have to do here is to ask can we construct guys with commute with everything and will use we use this as a way of understanding that. So, the first point is to. So, whatever we what else. So, commuting with M mu nu implies that it has to be a Lorentz scalar. So, is there a natural Lorentz scalar we can construct, that is another way of saying which will also commute with P. This is only other generator now the question to you is can you give me a Lorentz scalar with these guys obvious Lorentz scalar P square P mu P mu.

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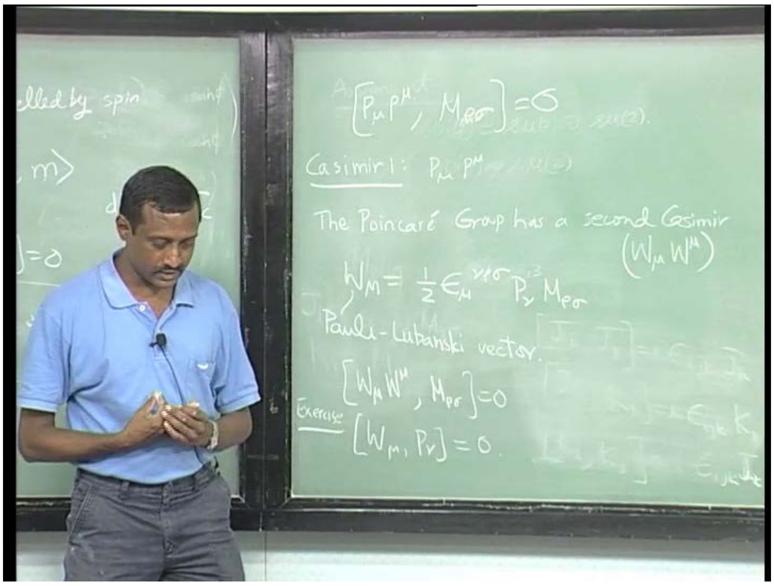

So, we can M mu nu I told you is the generator of rotations. So, without calculations this is a Lorentz scalar we know it, or you sit down you can do the calculation you will get it starting from this guy you can show that, that commutes. So, this is. So, clearly the Casimir 1 is P mu P nu. Now, the it tells out that this has one more Casimir. So, I have to construct it for you, it has a second Casimir.

So, for that I will define something called W mu, which is the following half. So, what is this is a vector co vector to be precise, but I can raise lower and do stuff. So, this has a name called the Pauli-Lubanski vector. Now, without calculation I can write for you it is what you call commutator with M mu nu, why is that it is just a vector. So, it is exactly has to a same as this except wherever we see P here we should put W. So, just put the Pauli-Lubanski vector, now can we construct a new thing new Casimir W square for the same reason as this thing, but we need to check but we need to check.

So, first thing is we know this is trivially true, but we need to check one thing that it commutes with the other generator which is P nu, this is only I am not proving it you can check this is exercise. So, now the now it is very clear that the second Casimir will be, and that is it there are no more. So, now what this tells you is that any irrep of the Poincare group will have two labels.

So, before we do that, so what we will do is that we will assume lets kind of look at objects, which can be in representations of the Poincare group. So, if we take that to be a fundamental symmetry in nature you would think that fundamental particles in nature should be in some irrep of this thing.

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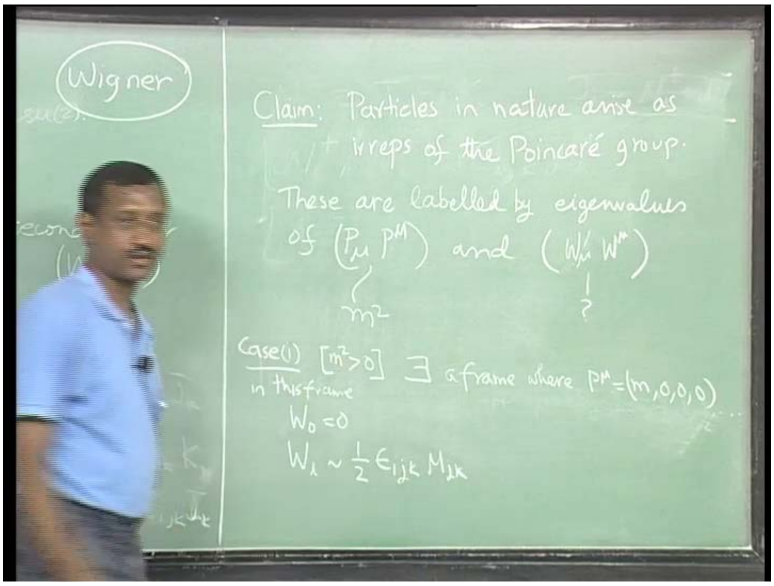

So, we will just say make a claim or whatever, particles fundamental particles in nature arise as irreps, by the way all these most of this discussion is due to Wigner arise as irreps of the Poincare group. So, let us and these are labelled by, these are labelled Eigen values of two things two Casimirs P mu P mu and. So, what would we call what is the natural thing to call P mu P mu, the Eigen value as the mass square right. So, this we will call mass square and the question is what is this? So, first step is to consider the case there will be three cases to consider, strictly greater than 0, this is the case of a massive particle.

So, in this case we can always there exists a frame, where P mu rest what is this frame called the rest frame. So, let us ask what is W mu in such a frame. So, in other words what that tells you is that P 0 is the only. So, now, if you see this, so first thing you will notice is that W 0 is 0 in this frame, but W i is up to some sign, its half epsilon i j k M j k up to a sign. I need a little bit of a theorem or whatever that the representation theory of realization of this Lie algebra on the space of functions or whatever is usually of this form.

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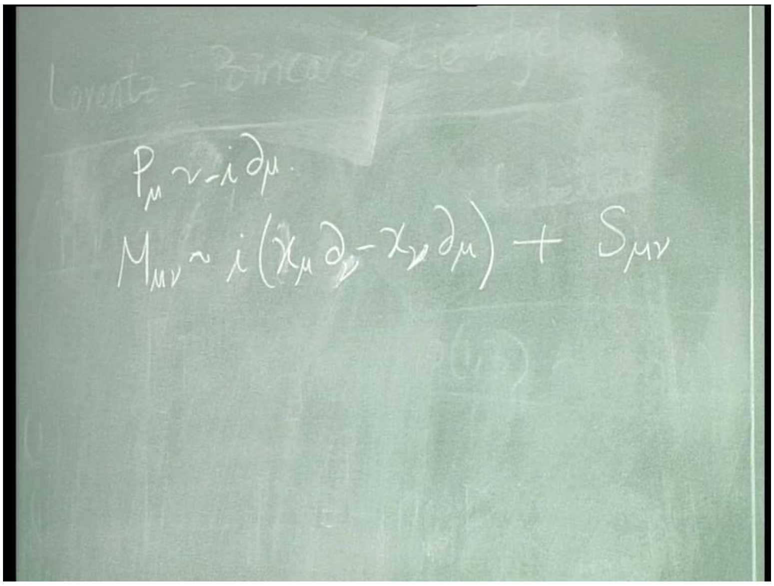

So, P mu is realized as minus i d mu, this is something we know m mu nu is realized as i, but it could have some internal part which we have already seen in the case of, in our field theory computations through no other thing that could be something else, which we will write as S mu nu. So, this is how you, how you would have a realization of this. So, if you assume you we have something like this and you take this value of M mu nu and actually wait we can even do little bit more let us look at M j k.

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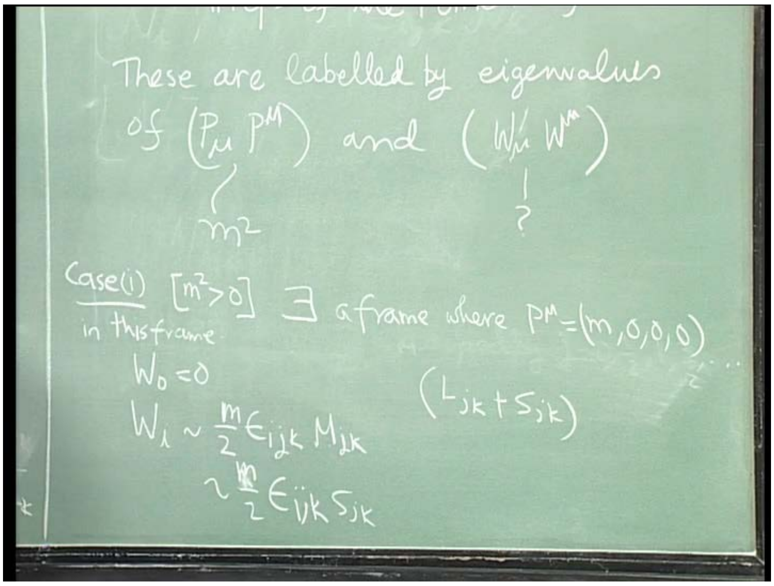

So, what would M j k be if you look at it. So, M j k will have two parts, which we will write one is the L j k plus L j k S j k, but what is L j k in a rest frame, it is 0. So, this actually can be rewritten as half epsilon i j k S k. S j k, what m factor there is a m, thank you because P 0 is m that is important. So, what this tells you is that in the rest frame, this is what happens and you can see that. So, this is what you will usually call spin.

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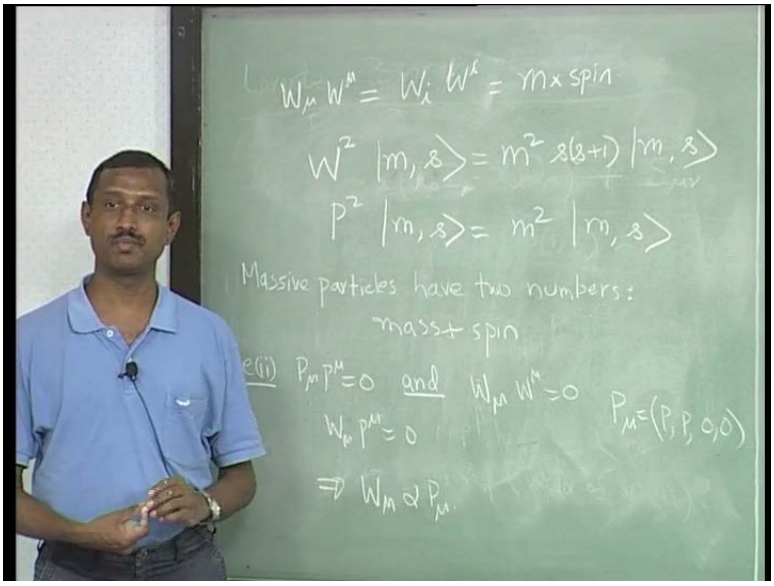

So, W i square is just S square. So, W i W mu W mu in this frame just becomes W i W i which is nothing but m times the spin of the part. In other words, if you give me any state, so any particle will be, will have two numbers one we will m, the other we will call as this thing, and this is just W square equal to m square s into s plus 1, of course P square.

So, implies at Massive particles have two attributes, two natural numbers if you wish two numbers associated with them one is mass spin. Is this clear, the second case is, when P square is 0, this is what we will call mass less particles, W mu also 0 if both of them are 0. So, this is what we have called mass less particle, but we will also put this condition in and it is trivial to check from the definition that W mu P mu is 0, just from the anti symmetry of the p's.

So, what you have is two 0 non vectors in 4 dimensional space and then because of that you can see that what is it, and there are orthogonal to each other. This can only happen if W mu and P mu are proportional to each other, is simplest way to that is to go ahead and say take P mu to be in some frame, where it is P P 0 0, and work out what you should get for W mu it is not like W mu, it is some arbitrary vector it should go back to the definition. So, you will see that. So, these conditions imply that W mu is proportional to P mu.

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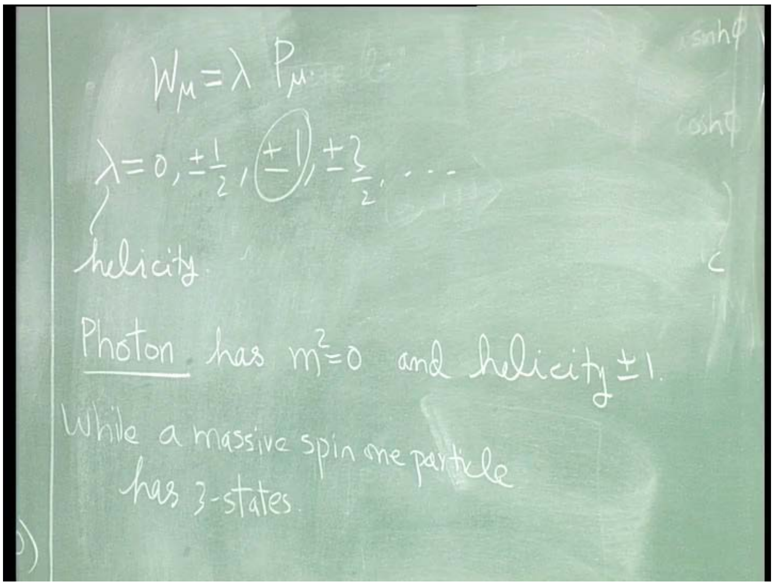

In fact, it will be not just that you will find that, W mu can be equal to lambda times P mu, where lambda of course, can be 0 it will be plus or minus half plus or minus 1 whatever. So, let us just focus on say this guy. So, this, so you, so given particle can either take only one of these set of values, it cannot be simultaneously half and one, but the sign can be possible because you can switch things around up to a sign. So, this is called the helicity.

So, a photon has m square equal to 0 and helicity plus minus 1. So, but if you take a spin one particle while a massive spin one particle has three states because it is in the spin one representation while this has two. So, this is something I have been telling you all along that master states are not represented by spin, but they actually carry something for helicity, it does not matter whether it is you know the analog of see the (( )) is spin half because the massive spin half particle has two, is 2 dimensional representation while even the mass less is two helicities.

I would not use the word spin actually I would use the word helicity which is plus or minus half, but they are the same I mean there is. So, there is no change in degrees of freedom, but you can see that if you go to higher things, higher spins you see that, that is not true. So, if you talk of graviton on which is spin two, it is not spin two it is mass less it suppose to be a mass less particle with helicity two is supposed to have two states, from this just representation theory view point and all fundamental particles we know seem to fit into this, so going back to our discussion of the Higgs mechanism.

So, if a photon has to become massive, it has to get one more degree of freedom that is exactly what is given by a scalar. Scalar is spin 0 which is one dimensional representation should gives you exactly the one which makes up the difference. So, whenever helicity is not half, the key point is that you can never take the m going to 0 limit is not the same, they are different irreducible representations.

So, the it is not a simple limit it is not just m being, but some parameter m being set to 0 because m is not a parameter, it represents the it gives you it is it characterizes a irreducible representation of the Poincare group. So, there are other possibilities for instance, P mu P mu equal to 0 and W mu W mu not equal to 0, but then you will find that the spin can take the here these were discrete values, here it can it will take a continue and we haven't seen particles in of this kind.

In our considerations and there are more things you could in terms of representation theory you could say that I could have P square negative. From again pure representation theory view point, and that is again not something which we have observed in nature. So, particles seem to fit into only these two cases I am done.