Classical Field Theory
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Lecture – 28

We got a bird's eye view into the, what is called the standard model into the strong sector of the standard model.

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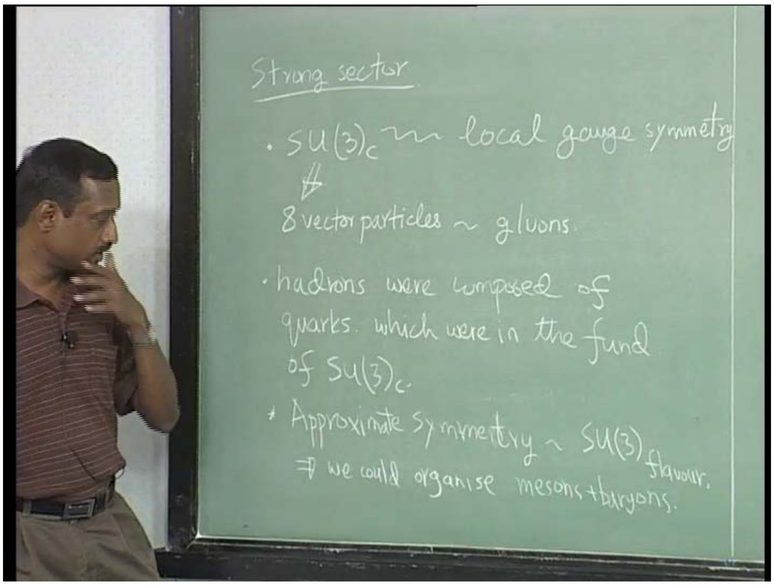

Strong sector

- $SU(3)_c \rightsquigarrow$ local gauge symmetry
  $\Downarrow$
  8 vector particles ~ gluons.
- hadrons were composed of quarks. which were in the fund of $SU(3)_c$.
- Approximate symmetry ~ $SU(3)_{\text{flavour}}$.
  $\Rightarrow$ we could organise mesons + baryons.

So, in the strong, we saw that the relevant gauge group was s u 3 color and this was a local gauge symmetry and so and this is sort of implied that, there were 8 particles spin one particles or 8 vector particles called the gluons. And we saw that the hadrons, basically particles, which feel the strong force were composed of quarks and we saw that the quarks, which were in the fundamental of s u 3 c.

So, these were the sort of things and but, we also saw something very nice, it was using a approximate symmetry, which was if you had 3 quarks, there is an approximate symmetry, which was an s u 3 flavor. And we could we found that this was able to organize mesons and baryons but, it turns out that, there are many more quarks more than just these three.

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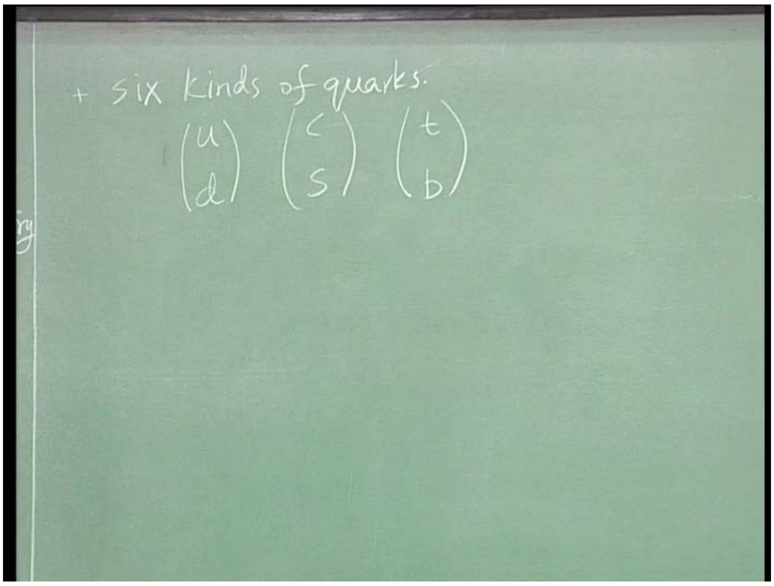

+ six kinds of quarks.

$\begin{pmatrix} u \\ d \end{pmatrix} \begin{pmatrix} c \\ s \end{pmatrix} \begin{pmatrix} t \\ b \end{pmatrix}$

And so, we will see, they are actually at least we will explain that, there are 6 kinds of quarks, which are organized into doublets of some s u 2, which we will see in a short while. But I am being little bit what do you call, imprecise but, that is this is just to give you charm and strange, and top and bottom. So, there were 6 quarks and but, this is the most massive guy and the so this in some sense, these, these are very massive.

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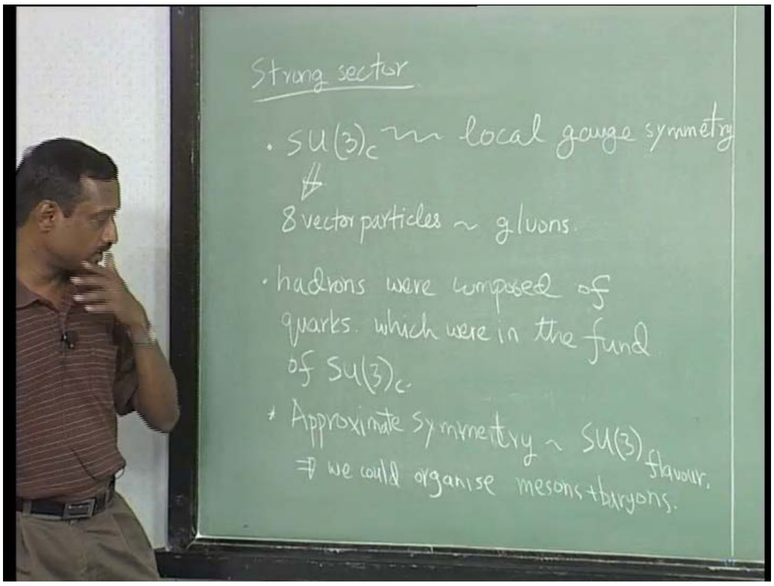

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And so, if you want to talk of quarks, which have low energies, which to construct the mesons at some so, mesons were quark anti-quark pairs. So, if you have a if you want to construct something, which has a b b bar, just to create a b b bar, you require twice the mass of that thing. So, that is a very high energy object, to create that so, it is much easier so, interms of interms of masses these are lighter than these are lighter than this, roughly speaking.

And in fact, s u 3 flavor is actually explicitly broken by the factor, these do not have the same masses for instance, in a obvious manner. So but, but now, you could say that, you once you able to track mesons, can we actually be clever and try to use the factor. We know some masses for these guys and work backwards and work out the, so you pick an octet for instance and you, you will find that because the masses are different. You would expected the, the mesons in an octet need not all have the same masses, can we predict them in terms of some three parameters.

And in fact, this is a very, very successful way of preceding this thing but so the input would be just a few numbers, like the masses of these particles plus some knowledge of the strong coupling strength. With these things, you can actually have good estimates of your masses and it is by I mean, you I would recommend that, you look through it. It is a wonderful idea of looking at something, for which we do not have a complete understanding of things.

But nevertheless, you know from symmetry grounds that, there are these constraint etcetera, you can put them in and you find that, you can do a good job. So, there is no field theory necessary, in some sense to do that so, this is, so this is one part of the standard model.

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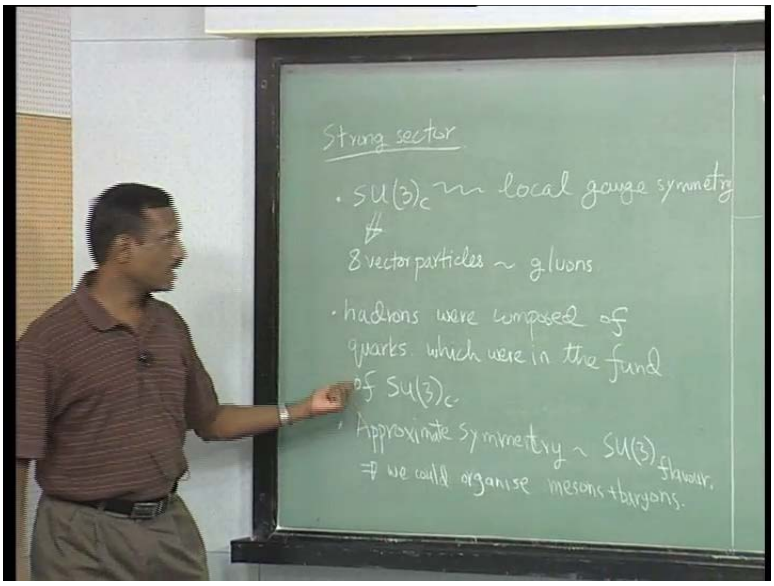

So, we looked at hadrons and this is the only time, I am discussing fermions in this course.

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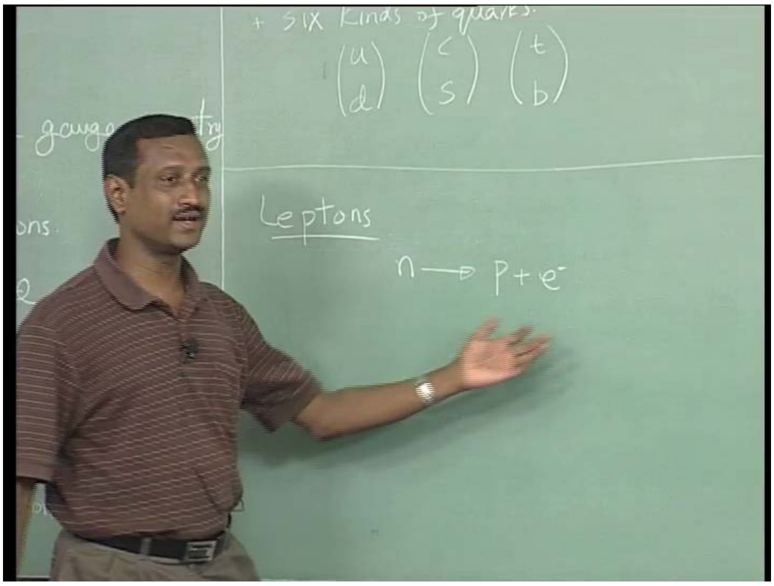

So, what happens is that, there are leptons and these are particles, which do not feel the strong force. But, they feel other forces so, for instance, we saw that, a neutron at some lifetime of around 15, 15 minutes or something like that. A free neutron it decays into, you would have thought it should decay, you know just based on charge conservation it can decay into a proton, may be an electron.

But what people found is that there was some missing energy and then, there was a puzzle, should we think that, you know should we think that, conservation of energy is violated or something like that.

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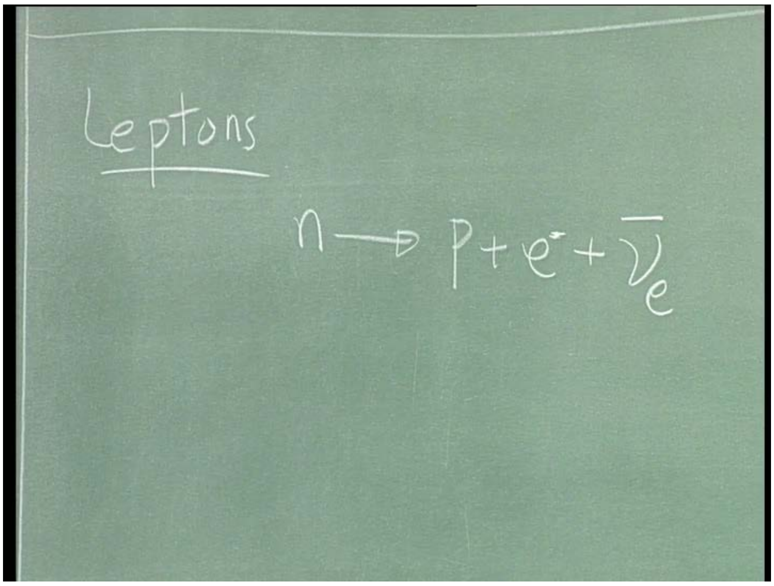

But, it turns out that does not true but so, somebody was brave enough to predict new particle called the, I guess it is a bar, anti electron neutrino. So but, all these so, the thing is that so, the thing is that, you find there are more particles etcetera and this is actually the forces involved in this are correspond to something else and not these are not strong, because these do not feel any strong interactions, these two particles, this should there should be something else, which is doing this.

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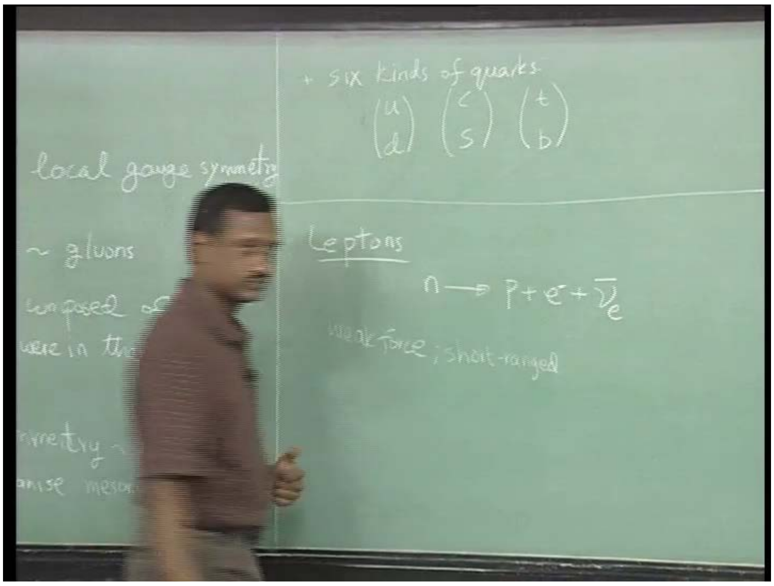

And that is, that is sort of, let me something called the weak force, it is a weak force relative to say, you know electromagnetism or something like that, that is why, we do not say, also short ranged.

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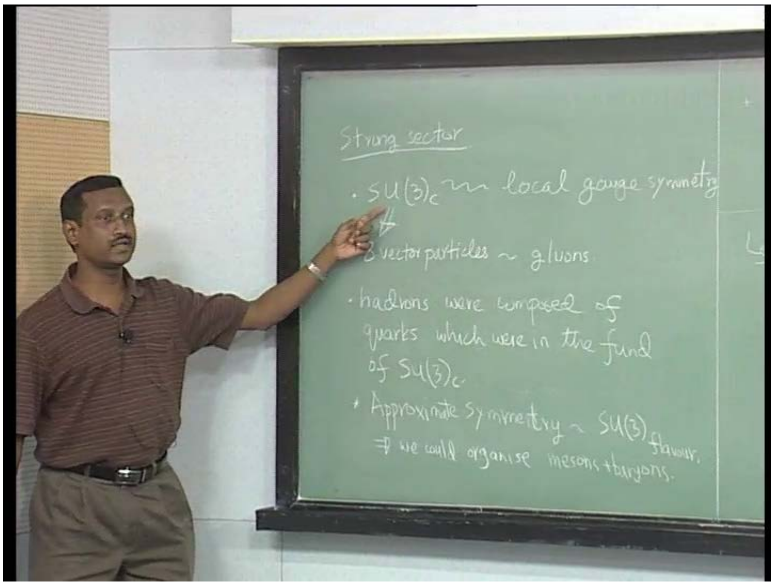

By the way, the strong sector if you look at this, this does not explain, why it is short ranged, just writing this thing, it should be a I mean, it should be mass less, etcetera. So, you might one way of doing it is to say, may be there is some higgs mechanism, which would become make it massive, that is not the way it is going to happen. But, that is exactly the way it is gonna happen out here, for the weak force, it will be short ranged because, the the vector bosons pick up some mass, we will work out detail of this mechanism in in the in this lecture.

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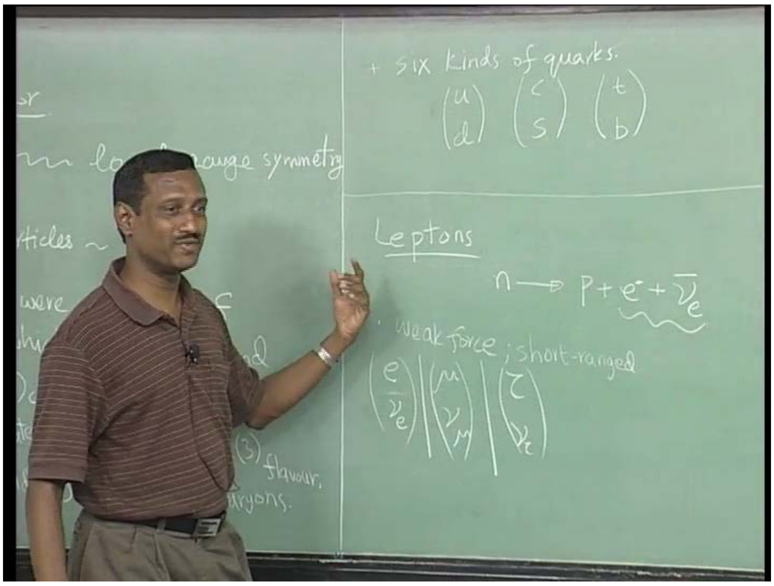

So, it is short ranged and so, the idea is to understand this force. But, the key point here is that, they are these particles, a whole bunch of particles like this. So, you find that you have electrons, electron neutrinos and the antiparticles you also have, what you what people found is there was mu, nu mu and tau nu tau. So, these are all leptons so, these are exactly like the electron but, it is more massive and this is also like this, it is more massive.

So, in some ways, it is sort of let me put them all together like this, that is the sense in which, they are paired up again and the point is that, now you see, that there is a beautiful symmetry with the with the hydronic sector I mean, the quark sector and the the lepton sector. There are exactly 3 such things, 3 out here so, these sometimes they are called families so, this is called the u d e nu is the first family, c s mu nu mu can be one family, t b tau and mu tau and scientists are very clever.

And they tried to look for some larger symmetries like s u 5 or something, which we would mix all of them together and that, and that has some amount of success. But, it has also there, I do not think there is a given model in in some sense, as we will seen in this towards the end of this lecture that, experiment lags behind theory in some sense. And today for instance, the l h c is verifying stuff, which is known from now ages.

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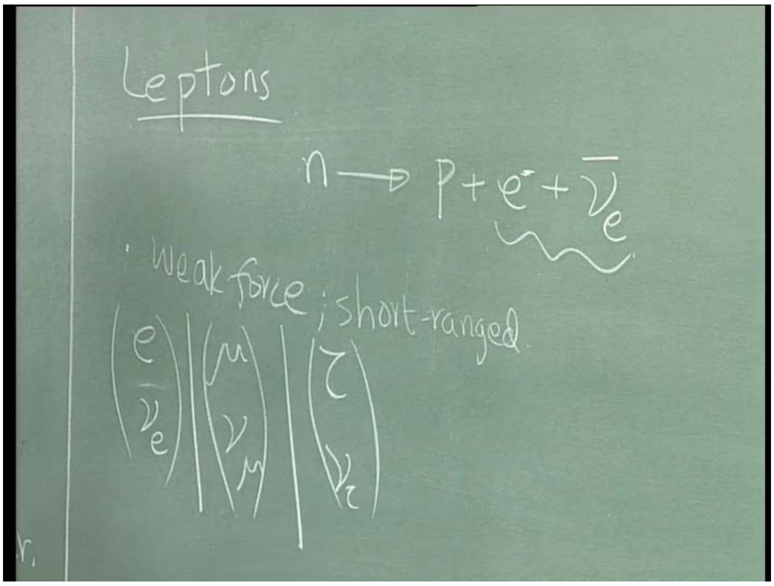

So, so what you find is, these are the kind of leptons that you have and so, these are again, I have written something as doublet. But, there is a technicality involved I mean, in the sense that, one usually breaks up electrons into left movers and right movers and one only one of them pairs up with the neutrinos in the standard model. But, again things have changed from from interesting sources for instance, the sun is known be a good source of neutrinos.

And we understand, how the reactions of the of the in the core of the sun rather well and so, we so, there is a prediction, theoretical prediction based on, you know things like normal reactions, in the lab, because, you do not think that the reactions inside the sun are going to be any different from what, we doing in the lab. Because, it is the same material, same substance so, cross sections will be the same, degrades will be the same, etcetera.

So, you can predict how much, how many neutrinos you should see in the lab but, it turns out that that, the number of neutrinos we see, is very small. By the way, neutrinos are really extremely weak I mean, they can just (( )), it is very easy for them to just go through the whole hours without interacting with anything. So, you can see, they are very

very hard to detect and so, they were really detected the first time, around as a missing mass and you you sort of say that, there is this particle.

So, these are very very light particles so, the original guess was, they were mass less and (()) in the sense, they were only something called left movers. But, there were no right movers or vice versa, it depends on, I do not know what the convention is, so, that is not relevant. So, coming back to, what I wanted to say is that so, neutrinos are very hard to detect and I just lost the thread of what I wanted to say. So, what we will see today is, to understand a little bit more about weak force and how, these become short ranged.

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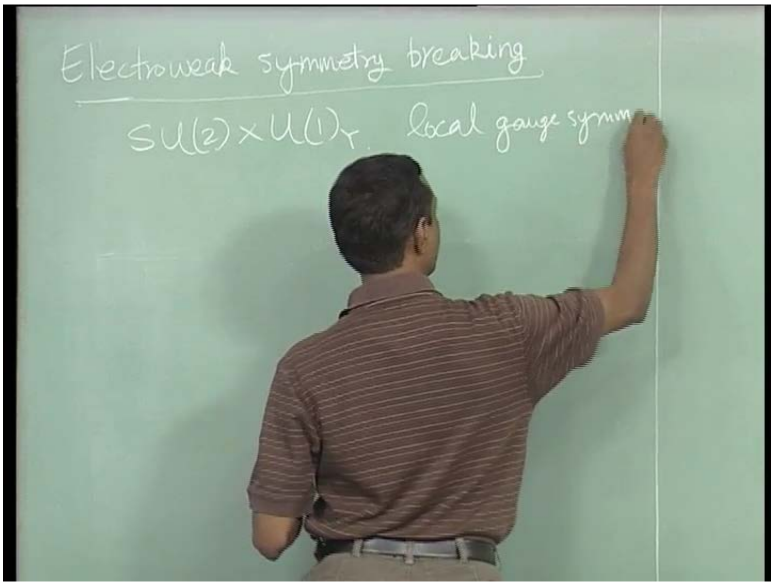

So, this is called the electro weak symmetry breaking and the group the relevant gauge group is going to be s u 2 weak curl u 1 Y, sub-script Y. So, this u 1 is not supposed to be the electromagnetic u 1 so the so, these are all, these are local gauge symmetry.

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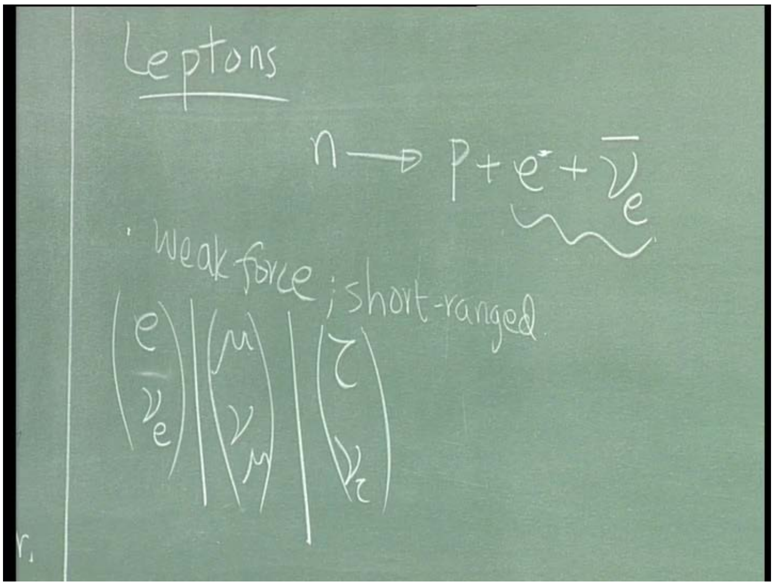

And now, I remember what I wanted to say, what I wanted to say is that, this doublets have to do with only one set of movers. If you are exactly mass less but, the thing is that the, today we know from experiments of the solar neutrino part, we saw that the number of neutrinos we observed was less. This is also another thing called the atmospheric neutrinos or the so, both of these things they were some puzzles, which we knew, how to resolve there is a I mean, there is illusion involved giving masses to the neutrinos but, rather small.

So, they have plenty of bounce on what, the total masses of of these particles can be and thereof I mean, it is rather small, it is of the order of e v I think for instance, the neutrino mass, etcetera. So, for instance, the solar neutrino puzzle is resolved by saying that, some of them get converted to some other flavor, I do not work in this area so, I do not know the litigated details of that, it has been many years since I looked at it.

So, but, the key point is that, we as theories we understand how to handle all these guys, even if they have little masses, we know how to do them. But, the reality is that, you should know what nature chooses and so, this is a very theoretical course so, we we will just look at mechanisms and we need to prove that, this mechanism is indeed.

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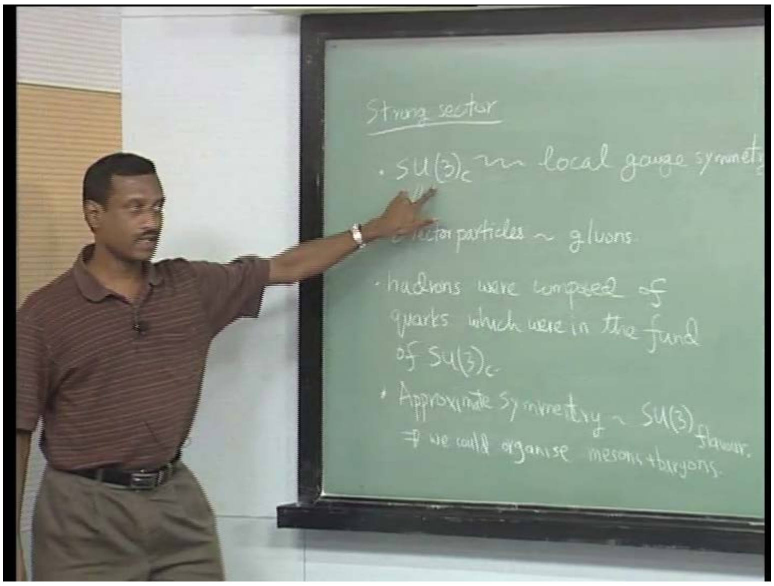

Strong sector

- $SU(3)_c \rightsquigarrow$ local gauge symmet
- ector particles ~ gluons
- hadrons were composed of quarks which were in the fund of $SU(3)_c$
- Approximate symmetry ~ $SU(3)_{\text{flavour}}$. ⇒ we could organise mesons + baryons.

Now, we see that, the so called standard model has 3 local gauge symmetries one is s u 3 c, the other two are these s u 2 and u 1. So, in principle, what one has in mind is, if we go at very, very high energy energy scales all these symmetries which are hidden or broken will will become obvious.

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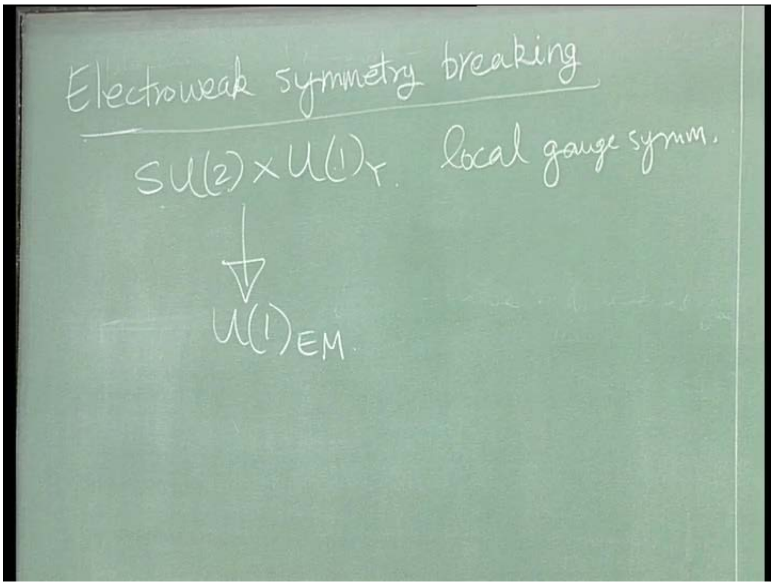

Electroweak symmetry breaking

$SU(2) \times U(1)_Y$. local gauge symm.

$\downarrow$

$U(1)_{EM}$.

So, what happens here is that, the symmetry breaking mechanism is that, this combination is broken down to u 1 E M. Now, let us just do some counting s u 3 has 3 gauge bosons, u 1 will have 4 and the so, what we will see is that, this is so the higgs mechanism will give masses to 3 of the 4 and will leave only one thing, which is mass less. And that combination is what, we should call the electromagnetism because, that we know that, we are I mean, even at lower energies, we do see a photon, which to to a great accuracy has zero mass.

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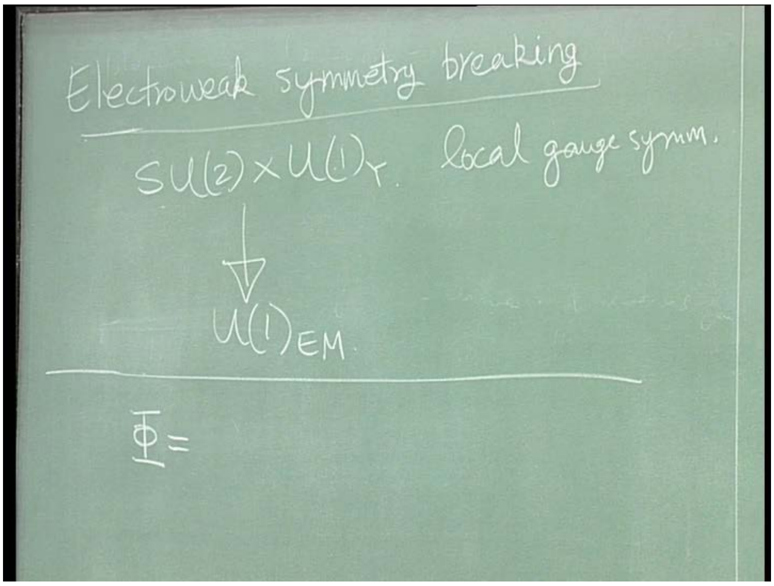

Electroweak symmetry breaking

$SU(2) \times U(1)_Y$. local gauge symm.

$\downarrow$

$U(1)_{EM}$.

$\Phi =$

So, let us understand, how this works and so it works in a in the in some ways, this is the simplest way of doing it, there are many other ways of doing it. So, so what we do is, we consider a complex s u 2 doublet so, here all these guys, you know all these doublets, which I have written are doublets of this s u 2. And roughly, it will look like so, the generators should look like t 1 t 2 t 3, which we use. So, to the approximation, this there was an s u 2 sub group sitting inside, this s u 2 flavor that, roughly has is related to this we will see little bit more.

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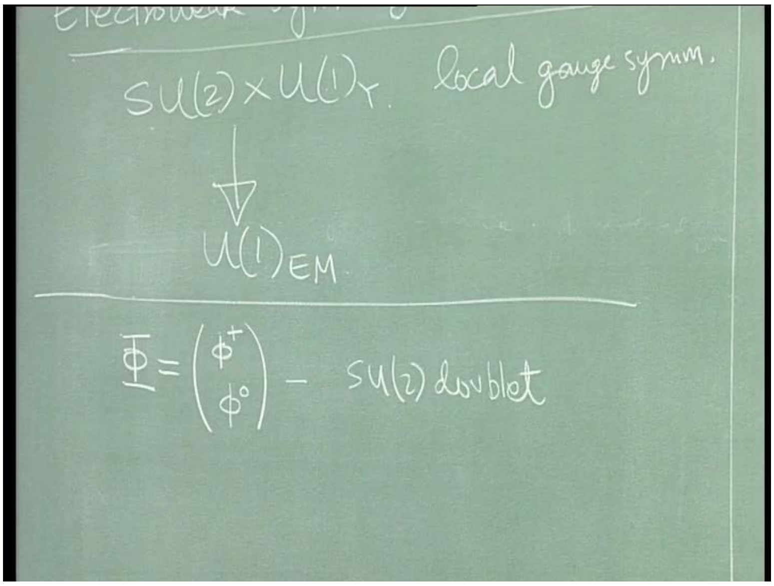

$SU(2) \times U(1)_Y$ local gauge symm.

$$\Big\downarrow$$

$U(1)_{EM}$

$$\Phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} - SU(2) \text{ doublet}$$

And then, instead of calling it phi 1 and phi 2, I call it as phi plus and phi 0, it is a doublet. So, it is a s u 2 doublet and the u 1 is just the diagonal, u 1 it just acts in on this thing so, I will just write the transformation.

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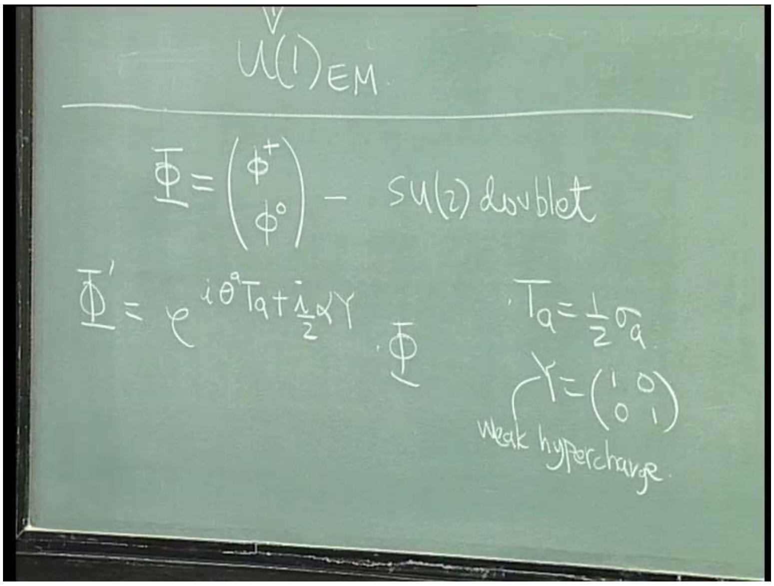

$U(1)_{EM}$

$$\Phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} - SU(2) \text{ doublet}$$

$$\Phi' = e^{i\theta^a T_a + \frac{i}{2}\alpha Y} \cdot \Phi$$

$T_a = \frac{1}{2}\sigma_a$

$Y = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

Weak hypercharge

It is best written that way phi prime will be the transforming as e power i theta a t a plus I by 2, may be some angle alpha times y at t alpha, where t a is just the poly matrices, half of the poly matrices, it is just the so, this theta is are the s u 2 transformations and this is just alpha is the u 1 transformation, y is just the identity matrix. So, that is like saying that, the y charge of phi phi and phi plus and phi 0 is 1, is plus 1.

And this half is so, this is just to fit certain notations, y is sometimes called weak hyper charge, I just call it the hyper charge. I do not have freedom in in these definitions because, these are standard so, I have given you the transformation so, you can see this is just the statement that it transforms like a doublet, under this s u 2 and this just says that, the diagonal. yes. (( )). No, no no they will become the u one charges, u one under electro magnetism. It is not this u 1 Y.

Yeah, it is not this u 1 Y that is what I am saying, the u 1 chargers are plus 1 plus 1 if you want or if you want to put this half into that, you can say half half. Now, we just need to so, this is just you go ahead and make this into a local gauge symmetry and you write out this thing.

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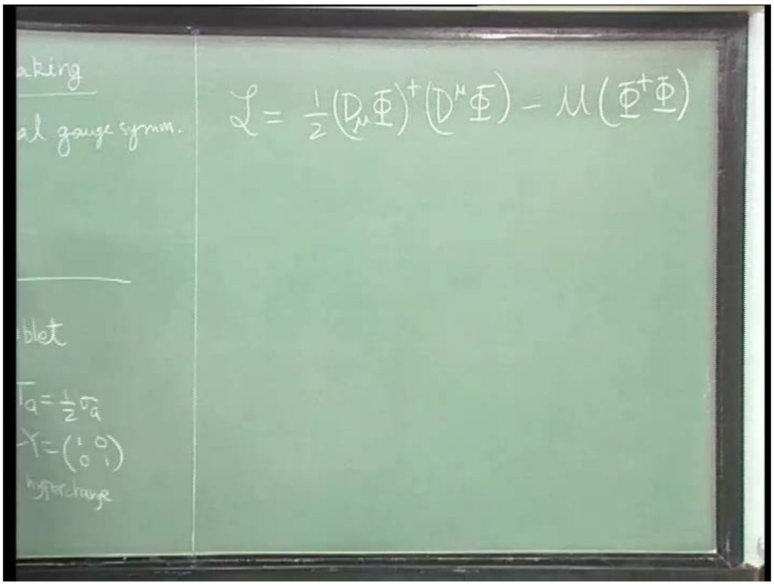

So, the Lagrangian would be and and then, so this is what, we will put and so, let me just do one more thing, we can also put kinetic energies for these two things.

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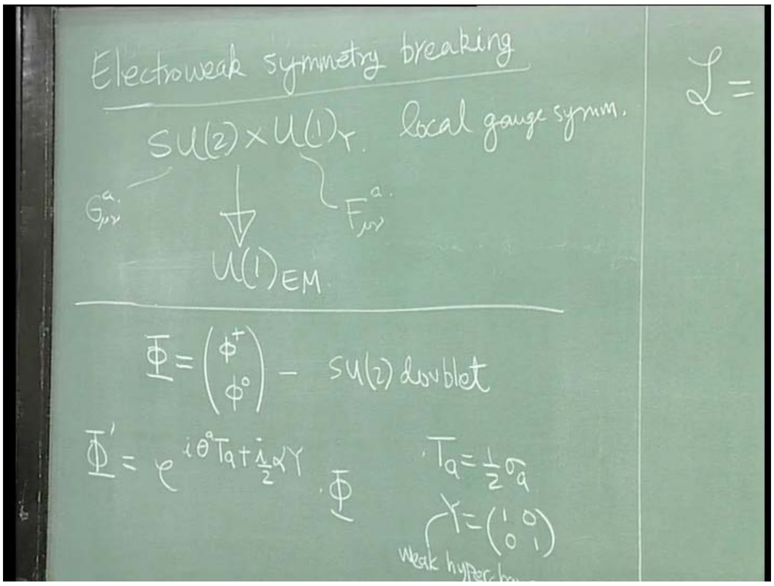

Electroweak symmetry breaking

$SU(2) \times U(1)_Y$ local gauge symm.

$G^a_{\mu\nu}$ $F^a_{\mu\nu}$

$\Downarrow$

$U(1)_{EM}$

$\Phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix}$ - SU(2) doublet

$\Phi' = e^{i\theta^a T_a + \frac{i}{2}\alpha Y} \cdot \Phi$ $\quad T_a = \frac{1}{2}\sigma_a$

$Y = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

Weak hyper

$\mathcal{L} =$

So, what I will do is, I will call the field strength for this, I will call it g mu nu and for this, I will call it f mu nu a.

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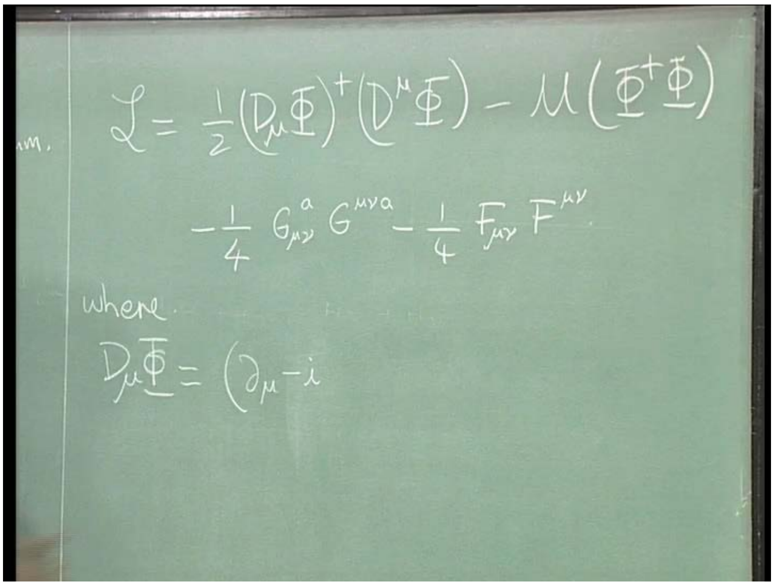

$$\mathcal{L} = \frac{1}{2}(D_\mu \Phi)^\dagger (D^\mu \Phi) - U(\Phi^\dagger \Phi)$$

$$-\frac{1}{4} G^a_{\mu\nu} G^{\mu\nu a} - \frac{1}{4} F_{\mu\nu} F^{\mu\nu}$$

where

$D_\mu \Phi = (\partial_\mu - i$

So, minus one fourth and where, I need to define a whole bunch of things, d mu of phi is the covariant derivative, with respect to all these things so, it will be D mu of phi let me pull the phi out minus i. So, note that I have not put the coupling constant here, I rescaled things so, I will get it, I will get them out here.

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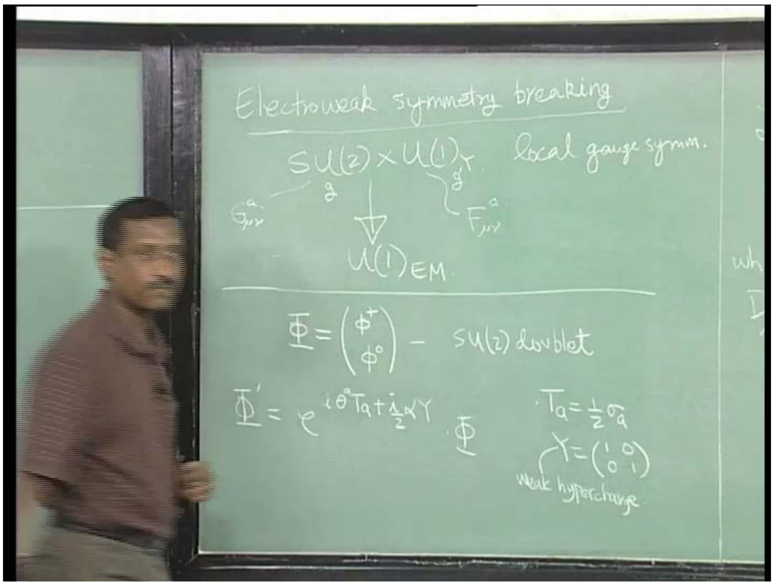

Electroweak symmetry breaking

$$SU(2) \times U(1)_Y \quad \text{local gauge symm.}$$

$$G^a_{\mu\nu} \quad g \quad g' \quad F^a_{\mu\nu}$$

$$\downarrow$$

$$U(1)_{EM}$$

$$\Phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} - SU(2) \text{ doublet}$$

$$\Phi' = e^{i\theta^a T_a + \frac{i}{2}\alpha Y} \cdot \Phi \qquad T_a = \frac{1}{2}\sigma_a \qquad Y = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

weak hypercharge

So, I will introduce two coupling constants little g for this and little g prime for this.

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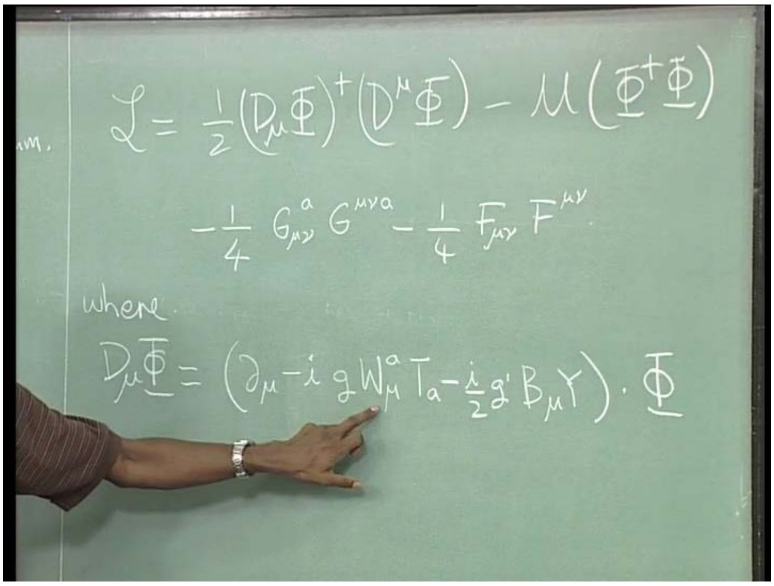

$$\mathcal{L} = \frac{1}{2}(D_\mu \Phi)^\dagger (D^\mu \Phi) - \mu(\Phi^\dagger \Phi)$$

$$- \frac{1}{4} G^a_{\mu\nu} G^{\mu\nu a} - \frac{1}{4} F_{\mu\nu} F^{\mu\nu}$$

where

$$D_\mu \Phi = \left(\partial_\mu - i g W^a_\mu T_a - \frac{i}{2} g' B_\mu Y\right) \cdot \Phi$$

g times so, I need to use, if you remember for the u 1 case, I mentioned that you can see, that the charges, matrix of charges would look like a T. So, this is just that so, Y and T are assume in a row out here so, this is just that.

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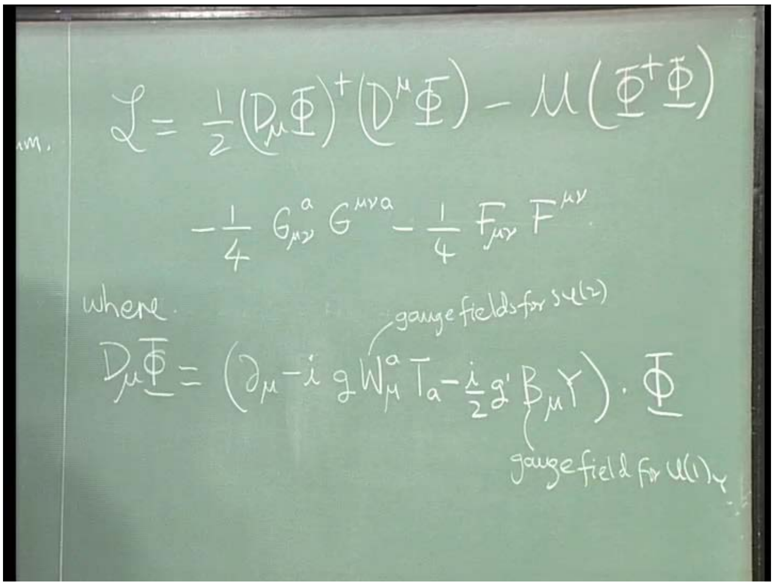

am.

$$\mathcal{L} = \frac{1}{2}(D_\mu\Phi)^\dagger(D^\mu\Phi) - \mathcal{U}(\Phi^\dagger\Phi)$$

$$-\frac{1}{4} G^a_{\mu\nu} G^{\mu\nu a} - \frac{1}{4} F_{\mu\nu} F^{\mu\nu}.$$

where.

gauge fields for su(2)

$$D_\mu\Phi = \left(\partial_\mu - i g W^a_\mu T_a - \frac{i}{2} g' B_\mu Y\right)\cdot\Phi$$

gauge field for $U(1)_Y$

So, W mu is the, these three are the gauge bosons or gauge fields, for s u 2 and this will be the gauge field for u 1 Y. So, they are just local so, this is so far nothing, just carry over whatever, we did for non-abelian gauge theories out here. And this for the abelian gauge theory, just putting them together, next thing is to just define these things.

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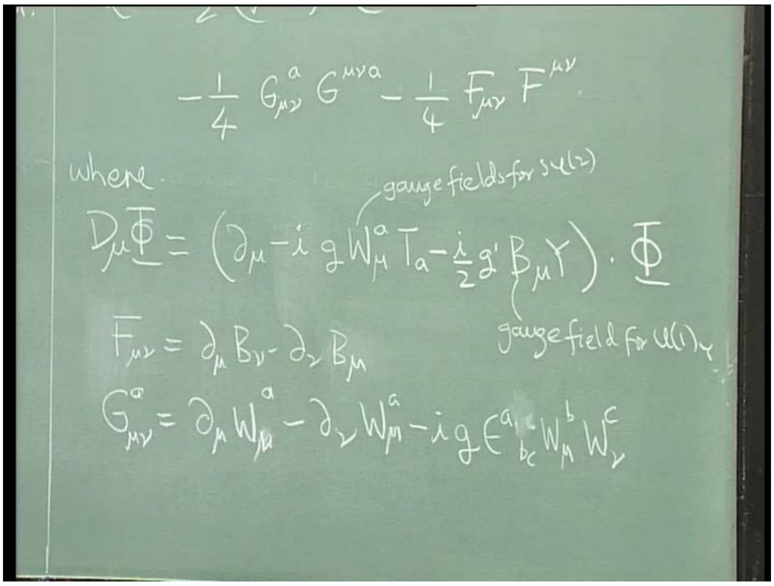

$$-\frac{1}{4} G^a_{\mu\nu} G^{\mu\nu a} - \frac{1}{4} F_{\mu\nu} F^{\mu\nu}.$$

where.

gauge fields for su(2)

$$D_\mu\Phi = \left(\partial_\mu - i g W^a_\mu T_a - \frac{i}{2} g' B_\mu Y\right)\cdot\Phi$$

gauge field for $U(1)_Y$

$$F_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu$$

$$G^a_{\mu\nu} = \partial_\mu W^a_\nu - \partial_\nu W^a_\mu - i g \epsilon^a{}_{bc} W^b_\mu W^c_\nu$$

These you are worked out in an assignment so, let me do the easy one first is there any question, is there anybody raise a question. So, this is the piece but, this is a non-abelian gauge field so, we will get some extra pieces out here so, f a b c which would be in this case, epsilon a b c W nu mu nu. It does not matter what is this, I am not 100 percent sure about the sign here by this but, this is what you should get.

So, just remember that, the g comes here because, think about it, when you what did you do, wherever you say W you put g W. So, you would have written a g w here, g w here but, this is W square so, we will get a g square. One you can pull out and that, will cancel the g square out here so but, there will be this thing. This is a very important way of writing it because, what we are doing here is, writing the kinetic energy in standard form.

And this g, here is really the coupling so, what you would do in perturbation theory is to expand things out and work with g g as a small object and you do computations. So, this is actually much more trailer to computations, etcetera so, if want to fix masses, etcetera this is the way to do it. Because, otherwise you would be missing factors of g in the mass even though, we are in 3 plus 1 dimensions and g is dimension less, is still a number. and so, you need to keep track of that.

So, that covers all these things, I just need to, what is it we need about you, we just need you is something such that, it has an minimum at a non-trivial value of phi which is, phi naught equal to 0. We could choose my favorite potential but, the only potential that I seen to know to make that work but, actually that is not very relevant. And so, I am not going to say that, I will just say that the thing is minimized by some value where, I will write out what phi should be. You just ask me questions; if there are any terms in there you do not understand or go straight ahead.

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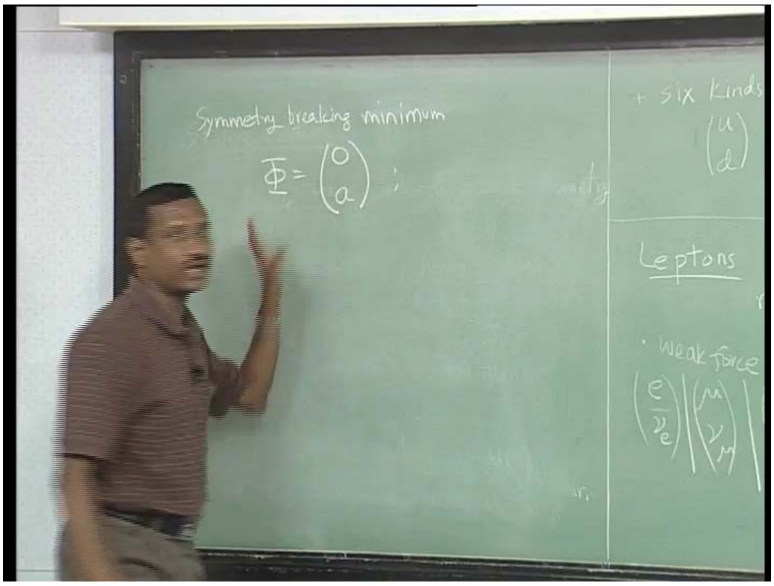

So, what we do is so, let say that, the the symmetry breaking minimum is given by same phi equal to 0 and some number a and which was it to be real. So, this is what we do and once, we had so, this is what we would have done, if you writing the vacuum solution for that particular user.

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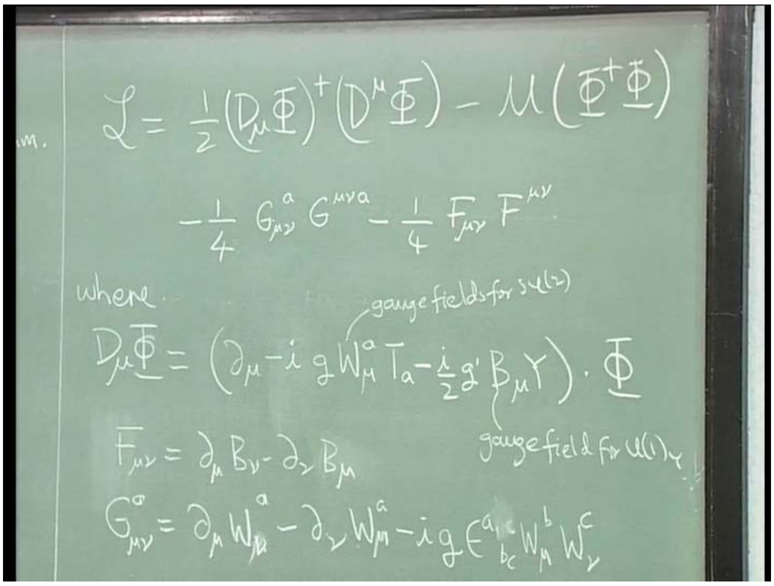

$$\mathcal{L} = \frac{1}{2}(D_\mu \Phi)^\dagger (D^\mu \Phi) - U(\Phi^\dagger \Phi)$$

$$-\frac{1}{4} G^a_{\mu\nu} G^{\mu\nu a} - \frac{1}{4} F_{\mu\nu} F^{\mu\nu}$$

where

$$D_\mu \Phi = (\partial_\mu - i g \underbrace{W^a_\mu}_{\text{gauge fields for } SU(2)} T_a - \frac{i}{2} g' \underbrace{B_\mu}_{\text{gauge field for } U(1)_Y} Y) \cdot \Phi$$

$$F_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu$$

$$G^a_{\mu\nu} = \partial_\mu W^a_\nu - \partial_\nu W^a_\mu - i g \epsilon^a{}_{bc} W^b_\mu W^c_\nu$$

So, a is determine of course, by the minimum the region where, you can see that, that implies phi dragger a is a square.

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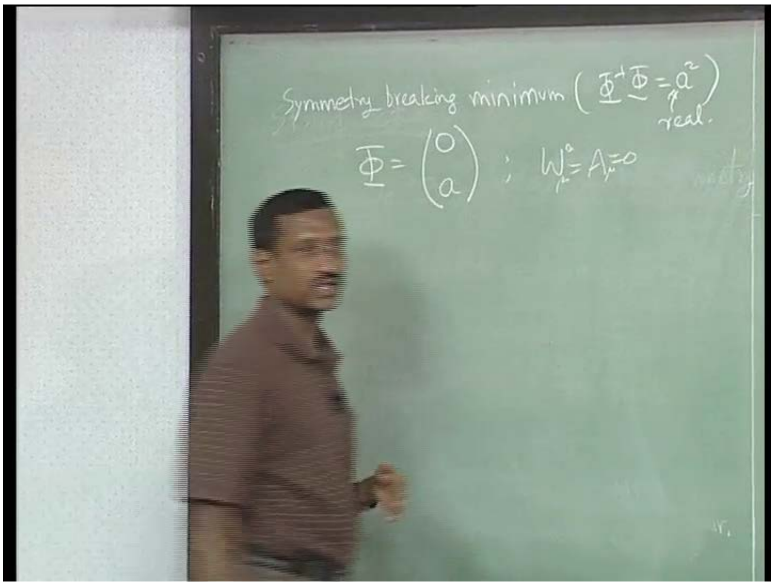

So, the minimum occurs at say, phi dragger phi equal to a square, this is real so, the thing is that, once we turn on gauge fields, the minimum this vacuum solution would be a solution with W and A also being zero. This is something we have seen again now, what we have to do is, to to get the masses we have to write out fluctuations and we could be clever right.

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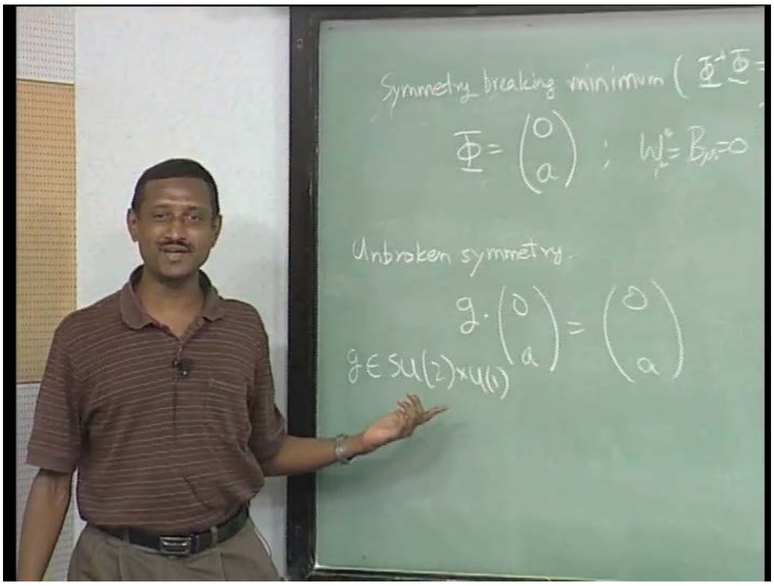

So, write out to get masses, first thing is that so, first let us understand the unbroken symmetry. So, I have to show that, you get a u 1, what is the unbroken symmetry so, to the unbroken symmetry would be given by the one, which where, act on g or a and give you back 0 a, where g is some combination of these guys yes, thank you, the reason I am not using a mu is because, we will I i think for once, I will use a mu for the electromagnetism. So, so we need to do this but, this looks very complicated to solve but, we can solve it in a very simple manner.

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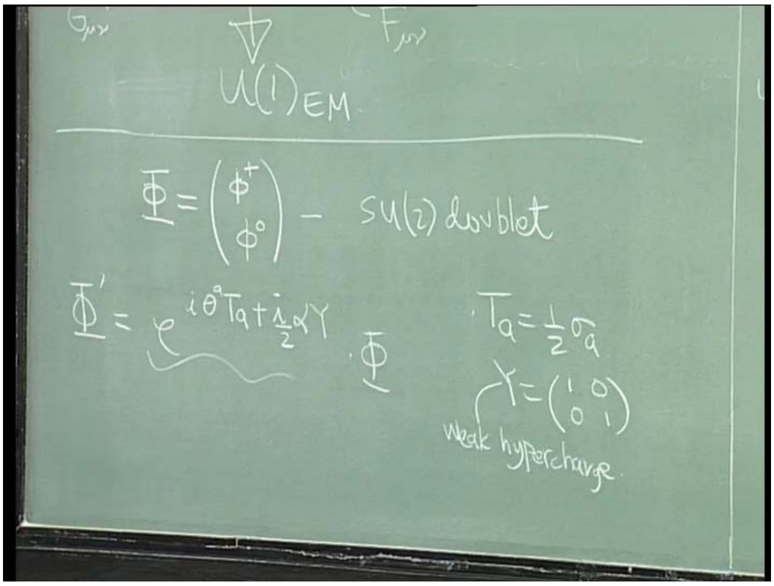

We just need to see that the g is this kind of element, this is a typical element, you act on that and so, all you need to do is, to ask what is the linear combinations of of T and Y, which acts on this, which goes to so, you just it is easy to see that.

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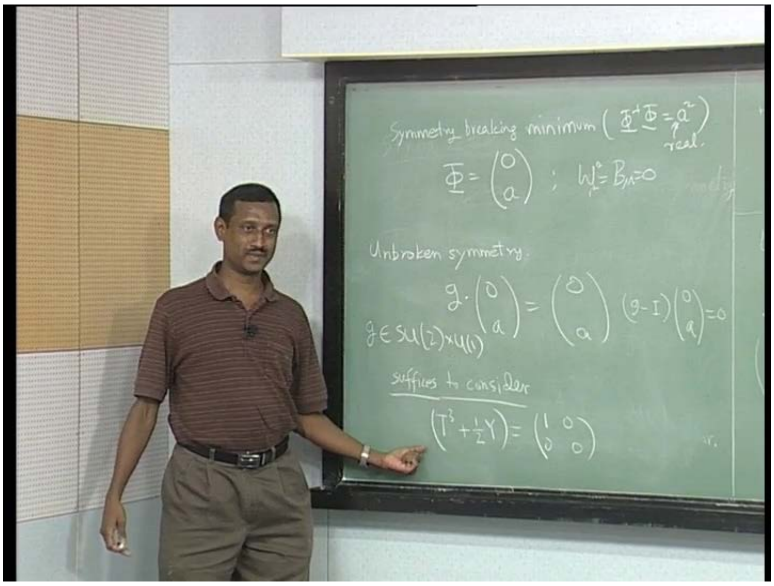

So, suffices to, to consider this is the advantage of to, find the elements of the lie algebra which on, which annulate this. Because, g is one plus something so, this is the one so, you can say, this is g minus 1 equal to 0. So, you want to and you expand it out so, it is better to think of this as, g minus identity of 0 a equal to 0. So, we just need to look at this thing and it is very easy to see that T plus, T minus will not work or T 1 T 2 will not do it.

But neither will T 3 neither will Y, but a linear combination and that combination is very easy to write. It is this combination, interms of matrices T 3 is half minus half so, this will become 1 and 0 and trivially, it annulates this. So, this is the unbroken linear combination interms of the lie algebra generators and no other combination all the others of this thing so this is the unbroken.

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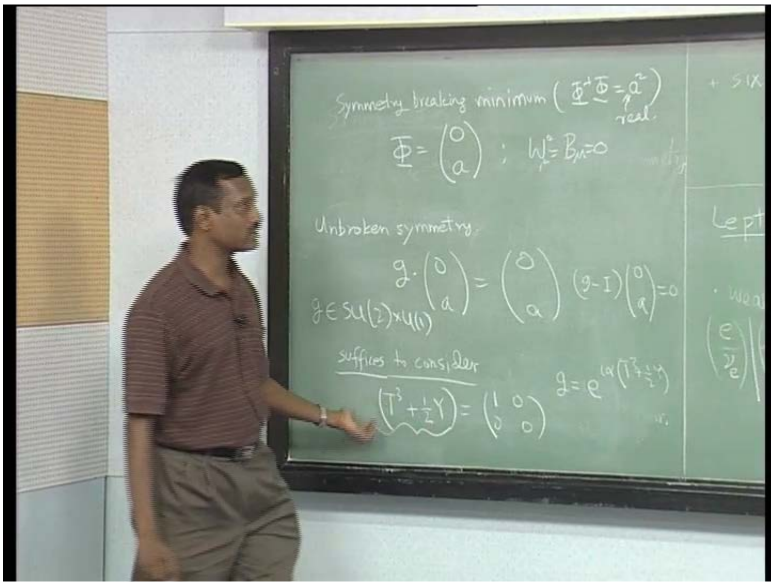

So, in other words, g e power whatever, you put these things, you choose theta 3 to be equal to so, what do I mean by that g, which is equal to e power some I alpha into T 3 plus half Y, will do the job, will be an invariance. So, this proves that so, we gets that there is a u 1, which is unbroken so, you can see that the advantage of having a lie algebra is the symmetry breaking.

Even checking for symmetry breaking is reduced to some actually finding out the it is just a problem in linear algebra more likely, it is enough to do for small element. And then, you are done and by exponentiation, I get the more general valuation so, from general things, which we have seen so, we expect.

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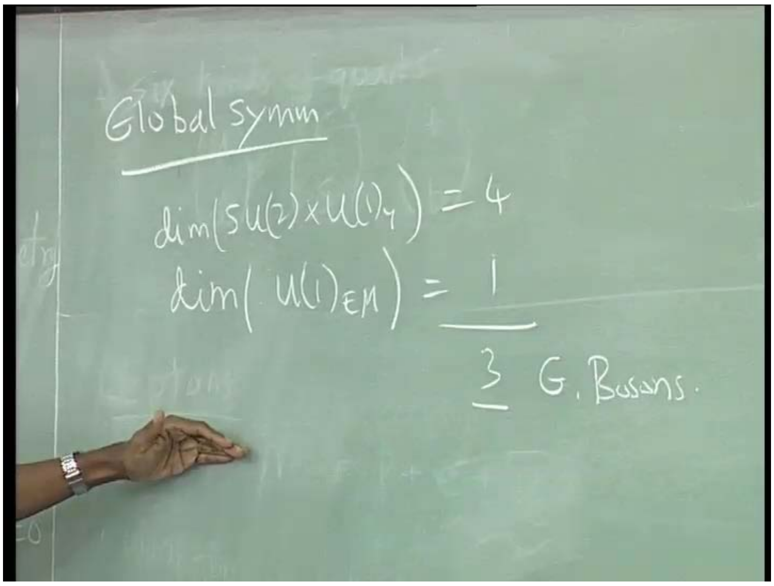

Global symm

$\dim(SU(2)\times U(1)_Y) = 4$

$\dim(U(1)_{EM}) = 1$

3 G. Bosons.

So, for global case, global global symmetry what is it, we have how many goldstone boson should we expect. So, we have s u 2 mod u times u 1 broken to u 1, will have 3 because, dimension. So, so this is 3 plus 1 is 4 and dimension of g h, which is u 1 E and M is equal to 1. So, you subtract these two, you get 3 goldstone bosons and the higgs mechanism will tell you that, these goldstone bosons will three goldstone bosons will give masses two three of these linear combinations. And one of them will remain massless, that massless guy will identify it with electromagnetic gauge field so, now, we need to do, we need to put meet on to this.

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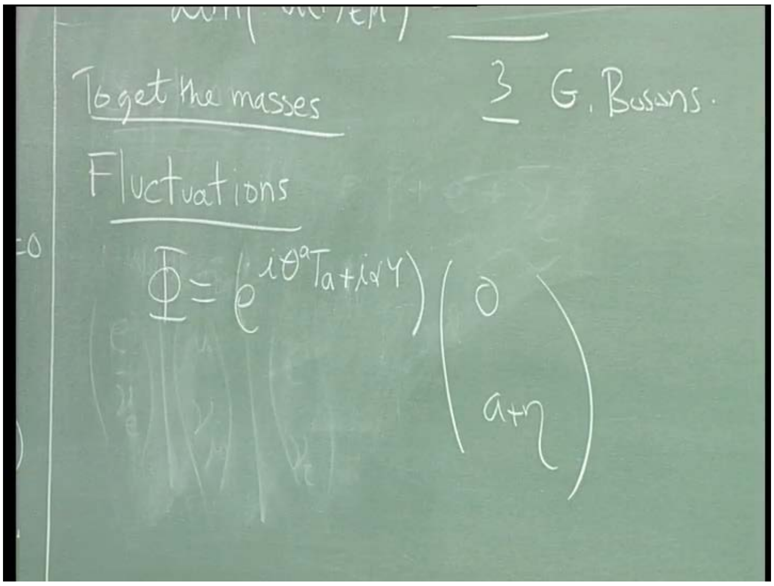

To get the masses

3 G. Bosons.

Fluctuations

$$\Phi = \left(e^{i\theta^a T_a + i\alpha Y}\right)\begin{pmatrix} 0 \\ a+\eta \end{pmatrix}$$

We have to, we know what to expect but, we have put numbers and see what we get so, just me let let me remind you so, first step would be to do fluctuations, to get the masses. We will consider fluctuations and so, we can see that, in the local so, you would write phi prime, phi as some e power, write the most general group element e power i theta a t a plus i alpha.

Instead of that, I can actually say that, I should get the other combination of something like that acting on 0 a plus some eta. First point is that, there are not 4 parameters here because, one called linear combination that is, this combination annulates everything. So, I should remove infact, I do not need to so, I can just get rid of in that sense, I can just get rid of this Y, I can trade and rewrite in this sense.

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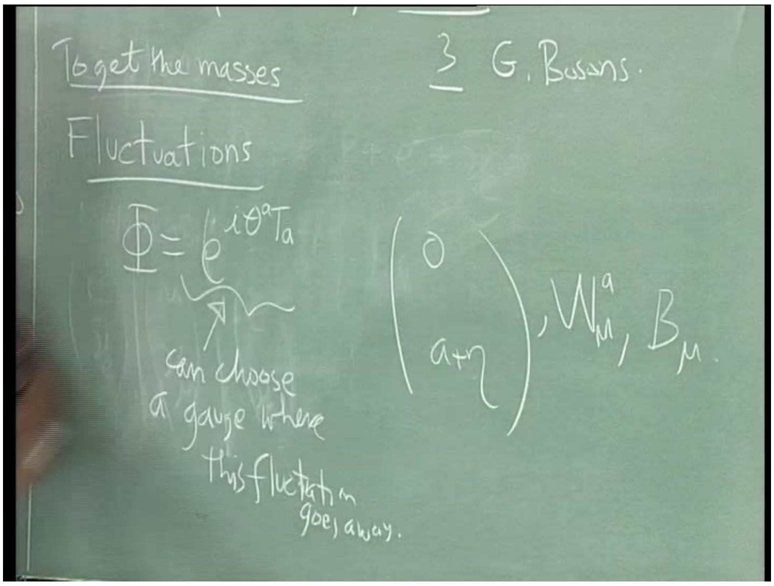

So, but, the point is that so these fluctuations so, you can see are exactly 4 fluctuations and in the local thing, we can shift this and get rid of this. So, can choose a gauge at this fluctuations goes away so, we will just write so we will write the phi as 0 a and eta is some real guy. So, it is eta is a real fluctuation and of course, there is the gauge fields so, there is W mu a and B mu.

So, they were originally 0 so, I do not need to 0 plus something, I just call whatever I get as a fluctuations, fluctuation plus this gauge transformation part, which to remove the get rid of that, this is what we get. So, the first thing is to realize is that now, so, I can I can drop this piece in the gauge. So, I just need, you can see that to leading order, this phi here is just 0 a plus eta, what I will do is, I will do it in two steps, I will forget about eta. We will do the eta masses later so, we will just put 0 a so, we will compute.

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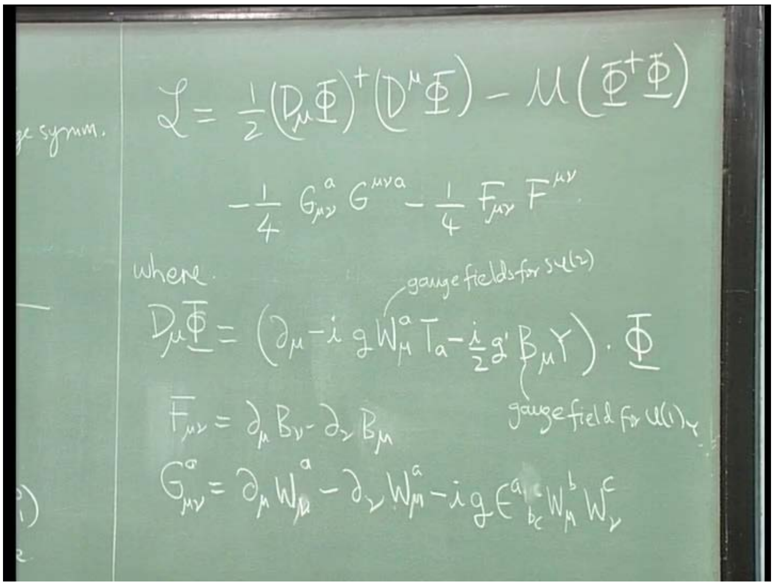

$$\mathcal{L} = \frac{1}{2}(D_\mu \Phi)^\dagger (D^\mu \Phi) - \mathcal{U}(\Phi^\dagger \Phi)$$

$$-\frac{1}{4} G^a_{\mu\nu} G^{\mu\nu a} - \frac{1}{4} F_{\mu\nu} F^{\mu\nu}.$$

where.

$$D_\mu \Phi = \left(\partial_\mu - i g \underset{\text{gauge fields for } SU(2)}{W^a_\mu} T_a - \frac{i}{2} g' \underset{\text{gauge field for } U(1)_Y}{B_\mu} Y\right) \cdot \Phi$$

$$F_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu$$

$$G^a_{\mu\nu} = \partial_\mu W^a_\nu - \partial_\nu W^a_\mu - i g \epsilon^a{}_{bc} W^b_\mu W^c_\nu$$

So, then this term, if you look out here d mu on some constant, a is a constant so, this vanishes. So, this will just give you these kind of terms, let just go ahead and play with it a bit, I can erase this.

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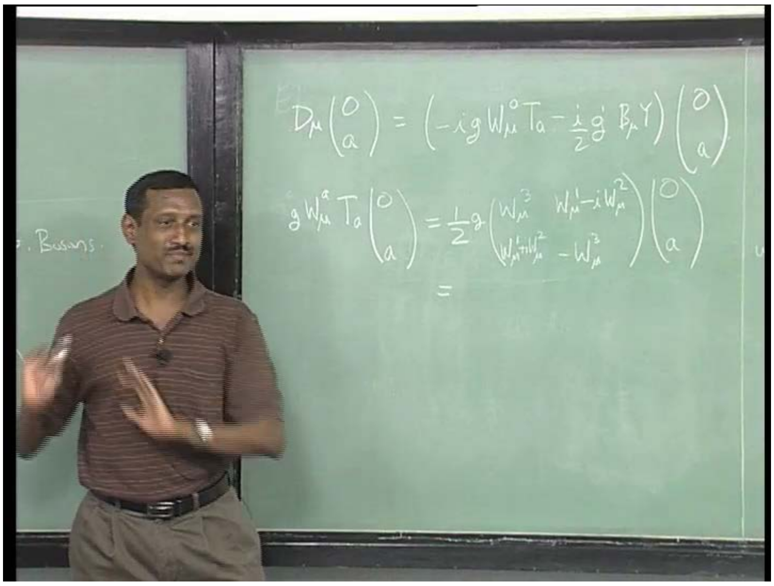

$$D_\mu \begin{pmatrix} 0 \\ a \end{pmatrix} = \left(-i g W^a_\mu T_a - \frac{i}{2} g' B_\mu Y\right) \begin{pmatrix} 0 \\ a \end{pmatrix}$$

$$g W^a_\mu T_a \begin{pmatrix} 0 \\ a \end{pmatrix} = \frac{1}{2} g \begin{pmatrix} W^3_\mu & W^1_\mu - i W^2_\mu \\ W^1_\mu + i W^2_\mu & -W^3_\mu \end{pmatrix} \begin{pmatrix} 0 \\ a \end{pmatrix}$$

$$=$$

So, what I am going to do now is, I want to work out what is d mu of 0 a, in principle, I can do it with a plus eta but, I do not want to do it for now. So, what is this we need to work out but, this is nothing but, the derivative acting on this is 0. If I put the eta, they will give me a D mu of eta so, that will be minus i g and t x where, half sigma s.

So, we can just I can work out what g W mu a T a acting on 0 a will be, this just half of poly sigma matrices. So, I can just pull that out, half g into again this was this expression was there in your assignment, previous assignment so, you get something like that. And let me just call this combination W W mu plus, if I wanted to do things, I should put root two's etcetera so when I look at the mass term I will remember that.

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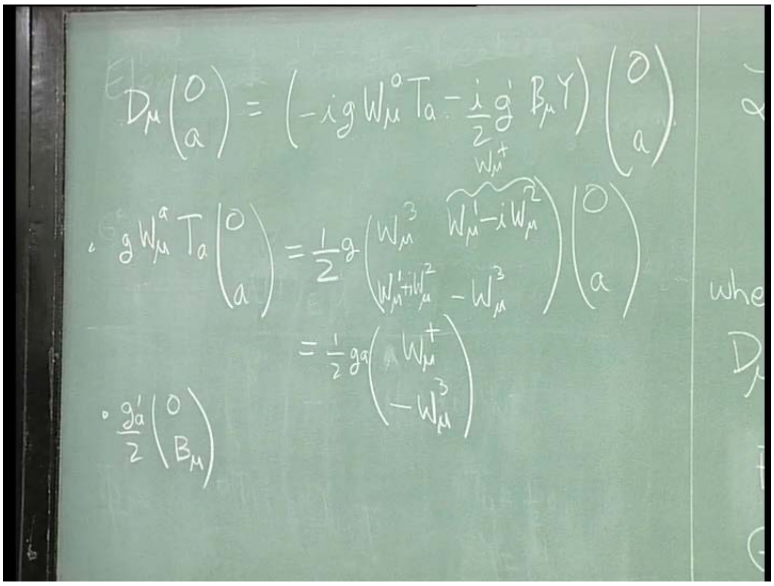

At this point I will just leave it as this so, this is just what I will call, W mu plus so, this is equal to half g into a so, I can even pull out this a, I get W mu plus here. And this term will give me minus that is, this piece, first piece I am pulling out, a minus I and next one will be, this is not easier. It is just g prime over 2 into Y, which is just into 0, I am just jumping steps here that is, this piece because, Y is a diagonal matrix. So, any diagonal matrix multiplying 0 a at this, I pull out the a, this is what I get so, you can see that, if I add, I just need to add these two things and multiply by minus i, I get what I need.

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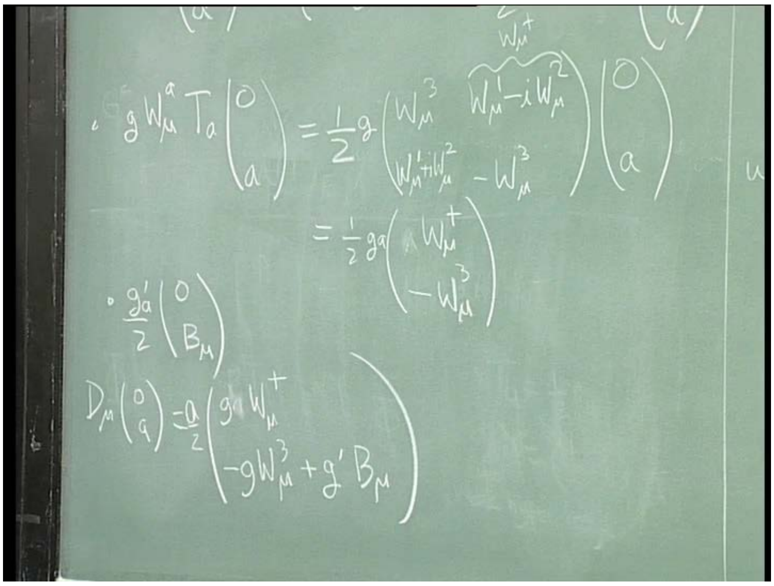

So, I get D mu of 0 a is equal to, you can keep the halves outside, g a, even the a I can pull out, g so, this is what I get. You can see that, only one linear combination of W mu 3 and D mu is coming out here, there is an i minus i, which will disappear. So, what I have to do is, mod square of this minus I will go off, important thing is W mu, the plus will become star of that is, W mu minus so, we just work out, what we get for this.

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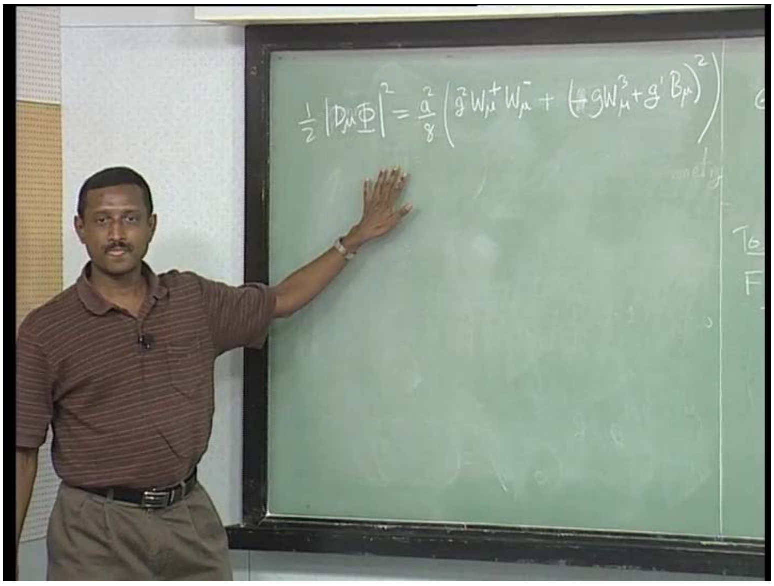

So, now we see so, what we get is a square by 8 into that is, this piece minus why did I do this 8, there was half from this, is there any half I have forgotten anywhere? No, I have not, good. Now, I just do not want to mess up that is all, minus what is there a half or I mean, I cannot see what I have there, half half half, I am I am perfectly fine so, this is what I got.

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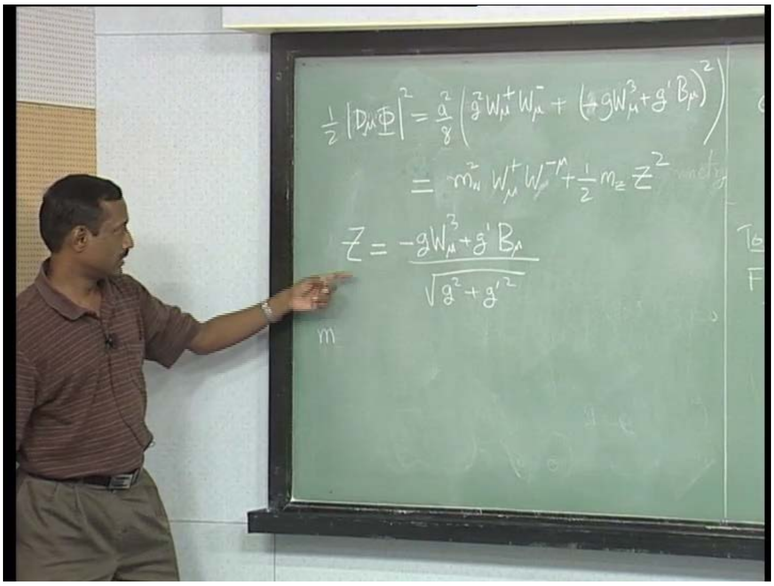

So, this I will equate to the following, why am I using capital m, I have no idea. Let us use small m, m W square, it does not matter, you correct for that, that is true, legally speaking that is required. So, I am correcting it out here and then, here what we will see so, this I would like to be half, I will define something called m z z square.

I have not defined, the whole bunch of things I have to define, only thing I have defined is W mu plus and W mu minus. I do not put a half here because, of the reason that, where is that because, this I should have a root 2 in the definition, to get it correct. And so, what is z, is its actually nice it is just more or less that combination but, I have to sort of make it. So, just comparing things we see that, m so, z is so, what we have is 3 bosons call them, W mu plus W mu minus and z, these pickup masses.

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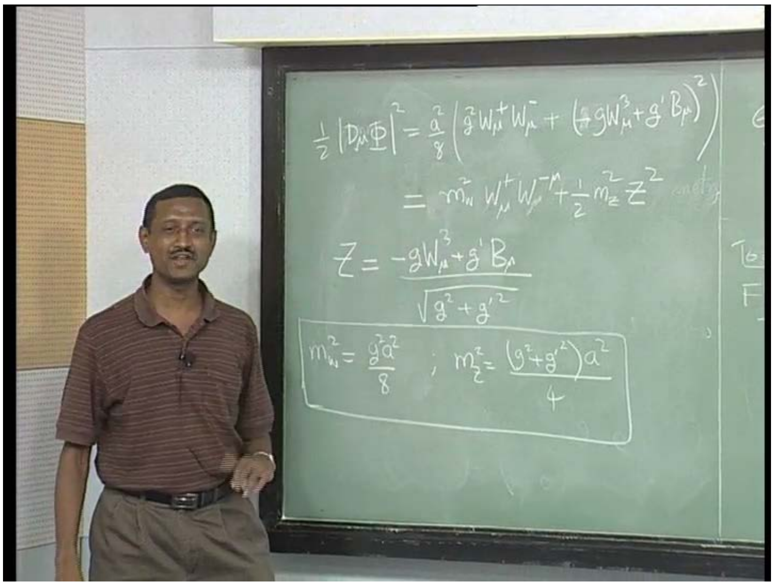

$$\frac{1}{2}|D_\mu \Phi|^2 = \frac{a^2}{8}\left(g^2 W_\mu^+ W_\mu^- + (-gW_\mu^3 + g' B_\mu)^2\right)$$

$$= m_W^2 W_\mu^+ W^{-\mu} + \frac{1}{2} m_Z^2 Z^2$$

$$Z = \frac{-gW_\mu^3 + g' B_\mu}{\sqrt{g^2 + g'^2}}$$

$$\boxed{m_W^2 = \frac{g^2 a^2}{8} \; ; \; m_Z^2 = \frac{(g^2+g'^2)a^2}{4}}$$

The mass for the W’s square is just, just by comparing is upon 8 and then, m z square so, m z square will have this thing now. I am little bit unsure about this, this could be 4 and 2 depends on, the there are some conventions. So, which I which are often, I do not remember but, what you can see here is that, this is a concrete thing that you see.

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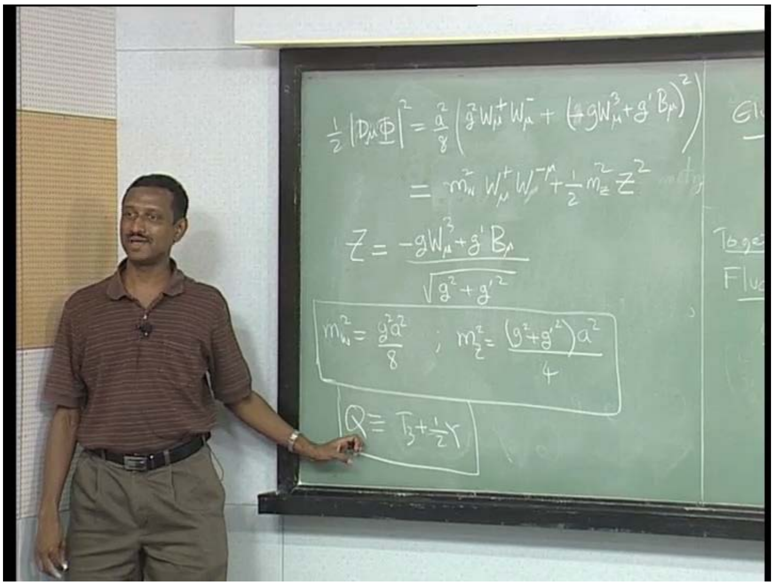

$$\frac{1}{2}|D_\mu \Phi|^2 = \frac{a^2}{8}\left(g^2 W_\mu^+ W_\mu^- + (-gW_\mu^3 + g' B_\mu)^2\right)$$

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$$\boxed{Q \equiv T_3 + \frac{1}{2} Y}$$

And I forgot to mention that there was a combination, which the unbroken combination, these are the charges so, we will define Q, we will define it to be T 3 plus half Y, do you see anything familiar in this, what was that gellman E C M relation. Now, going back to the original definition of phi, you can check that the first one so, this may, this is exactly, which was 1 and 0. We says that, the guy below has 0 charge and the upper guy has charge 1 so, this so this Q is nothing but, the electromagnetic charge. So, we just need to work out, what is the combination, which is which becomes A mu and that is easy to see.

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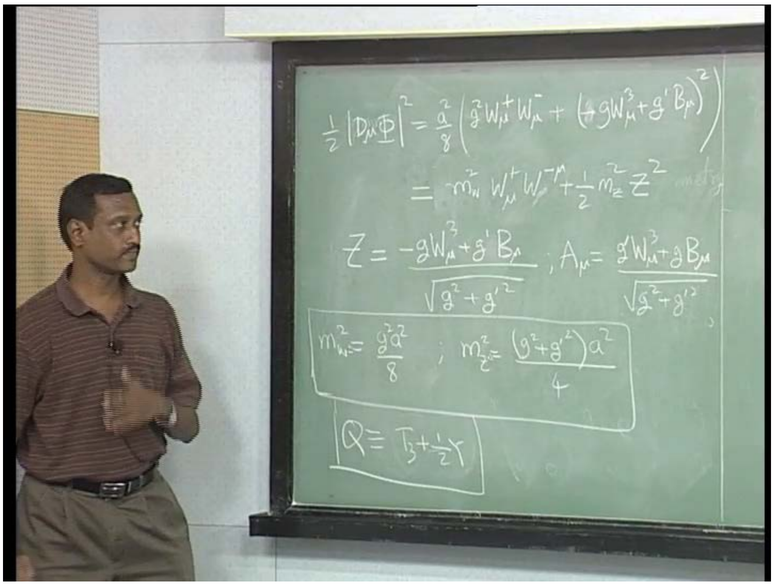

A mu is just the orthogonal combination to this and I think, you just should exchange g and g prime and change a sign here. So, g prime, the angle is nothing but, now, this also you can it is easy to see that, the W mu plus and minus will have no Y charge. So, T 3 eigen values, which is what, they are plus and minus so, the plus fellow will have electromagnetic charge plus 1 and this will have charge plus minus 1 and this will have charge 0 because, it commutes it comes from the T 3 and y sector.

So, this, this, this whole thing says that, you should find two, two charge charged and u 1 bosons with masses given by this and you also get some, you get another vector boson, which is charge less, which has this. So, usually it is, this is written z 0 and this could be W plus and minus.

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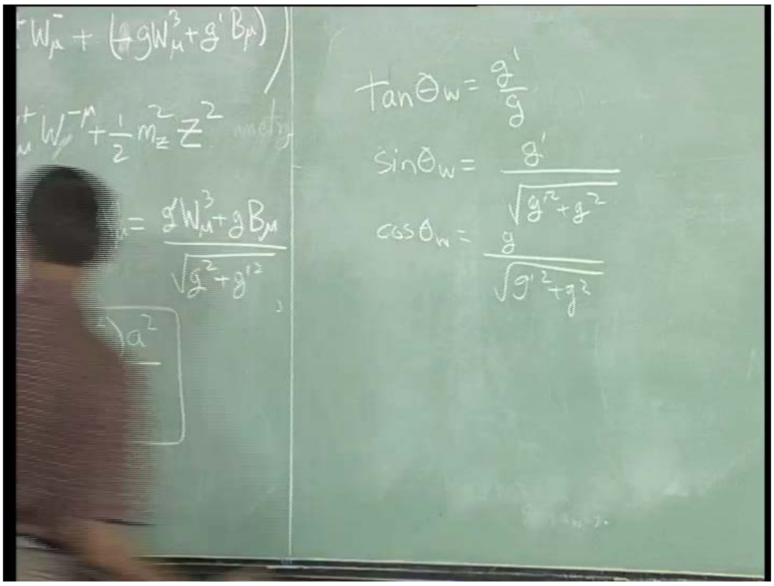

So, the the angle tan theta W is defined to be g prime over g, let me just check this, I do not want to write some wrong formula. So, you can see that we can write, this would imply that, sin theta w is so, the sin theta upon cos theta so also same.

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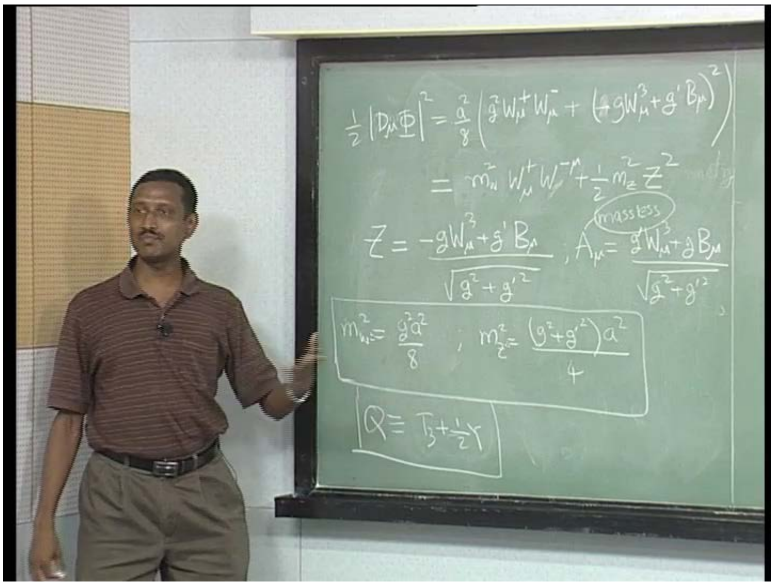

So, you can see that, this is this for instance is minus cosine theta W, W mu 3 and this thing. So, it is just a rotation by an angle theta, W in this phase of D mu and, and this is the photon, which does not appear in in the masses so, this is massless. Now, you see that, this relation is what I had remind when I said, you know it looks so, there is nothing wrong with thinking of this T 3 as related to the other the the T 3, we saw in the flavor thing, etcetera and it does not mess up anything because, u and D sit in a doublet obviously.

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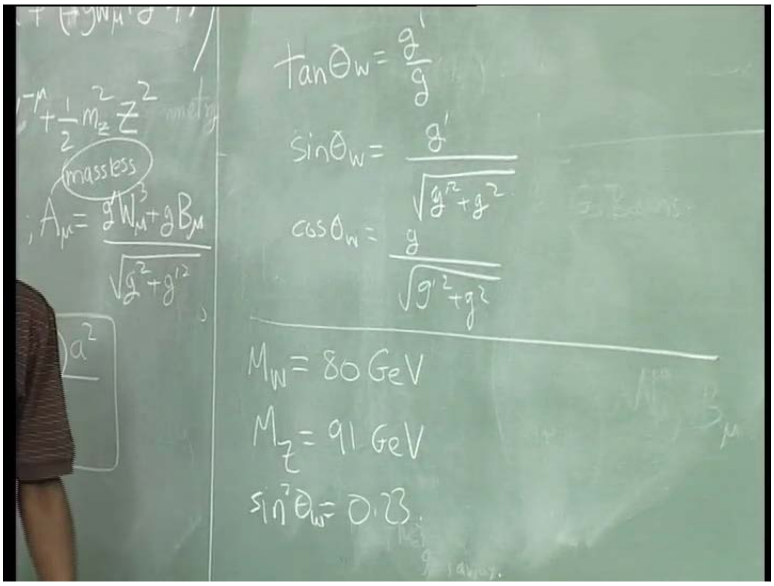

So, we can look at numbers and the numbers are nice M W is a 80 G e V, M Z is around 91 point something, let me leave it at that 91 G e V and sin square theta W is around 0.23. So, these are experimentally measured things so, you could turn things around and see that, these are also, you can see that what are, what are all the observables? Observables are the masses of M W and M Z these are things, which we observed and then, you can, you can see that for instance, what can we see M W by M Z square, the ratio of them will be g square by this thing. So, that would be sin square theta W so, now you can see that, the experimentally observed things, really fit into this thing, and it really fits this pattern.

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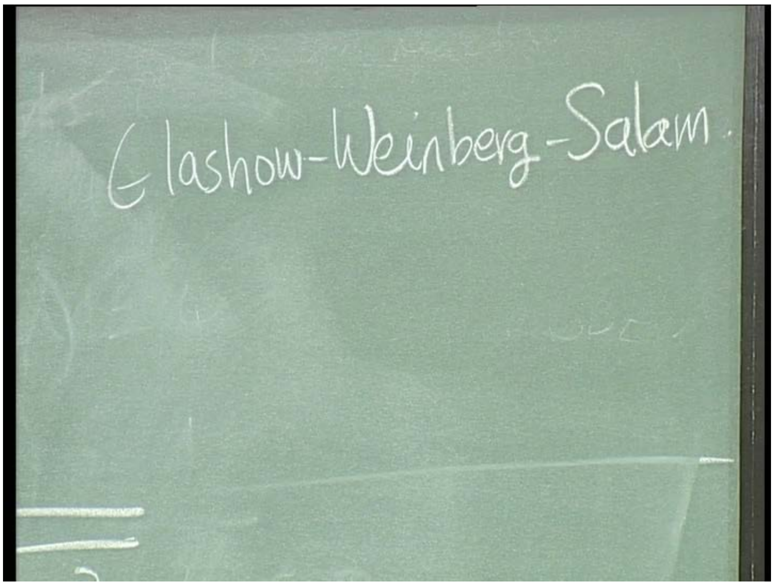

So, this goes by the name of Elashow Weinberg Salam model and I must tell you these models were proposed the the s u 2 cross u 1. Certain parts of it was proposed well before the higg mechanism was known and it took, I suspect it took, it will early 60’s I think, took enormous courage on their part. And people did not even know that, the quantum theory was good so, very brave people. So but, now there is still one more guy, whose mass we have to figure out and that is, the eta.

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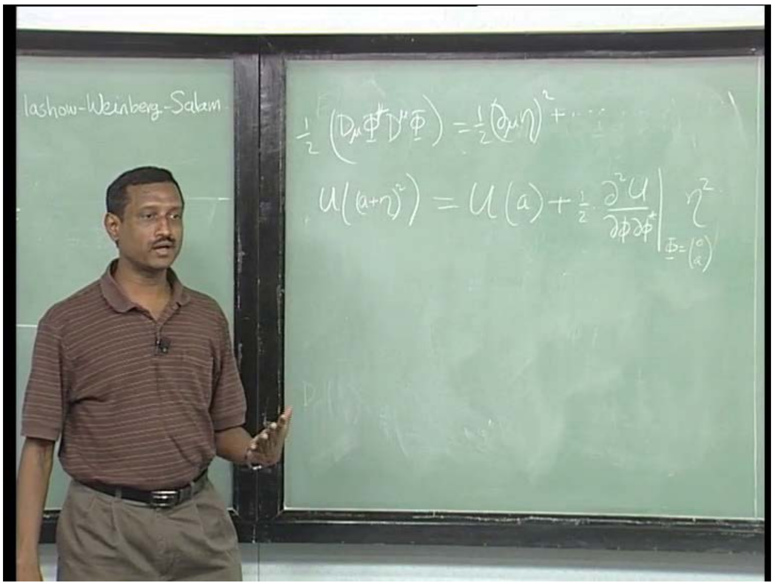

Now, I will look at so so, this will give you one bunch of term, this will give you half of that, will give you D mu of eta whole square plus, maybe a interactions, etcetera plus plus more terms higher order pieces. But, this will give the kinetic energy piece and then, there will be pieces with eta, with the gauge bosons, these are interactions in the field theory sense.

But now, what does this give you so, here we will see that, you will get u, what is, what would phi dagger phi b, it would be a plus eta whole square. Now, we have to expand this to the second order in eta so, this will be equal to some u at a plus, the first order piece will go out and then, you will end up. So, we need to take a second order derivative of this of evaluated at whatever you know phi equal to with a half may be. Now, so, this is a part so far, I mean all these things did not involve, it only involve the value of the vacuum expectation value, this thing. And we did not need anything, any details of the potential but, now, we need the curvature of the potential at that thing and that, tells you the mass of the eta square.

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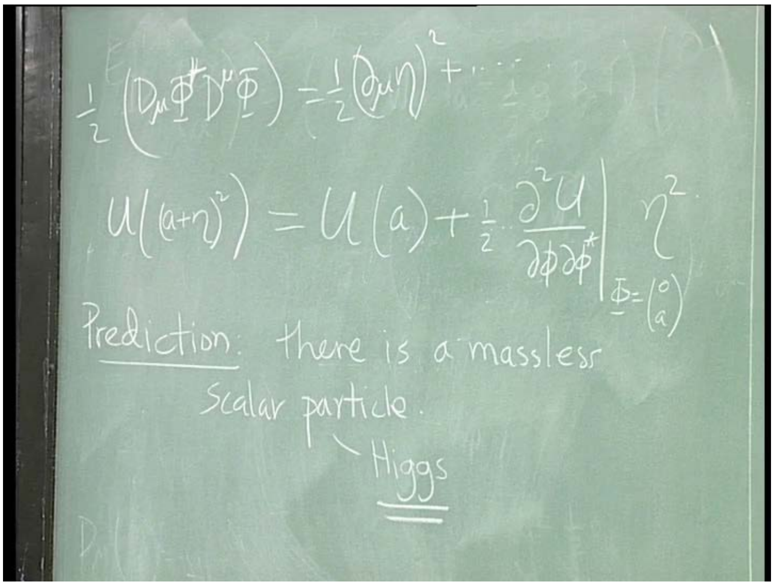

$$\frac{1}{2}\left(D_\mu \Phi^\dagger D^\mu \Phi\right) = \frac{1}{2}(\partial_\mu \eta)^2 + \cdots$$

$$U\left((a+\eta)^2\right) = U(a) + \frac{1}{2}\left.\frac{\partial^2 U}{\partial\phi\,\partial\phi^*}\right|_{\Phi=\binom{0}{a}} \eta^2$$

Prediction: there is a massless scalar particle.

Higgs

So, the prediction is of this, if this model is correct the prediction is, there is a massless scalar particle corresponding to eta, this particle is called the higgs, and a lot of money is being spent at LHC to actually find this particle. But you can see that, there is no way the, this data that we have the fact that we have observed all these particles. We know these things, I am giving you precise numbers, that will give a, you can work things around and you can get for me what is…

But that is not going to help you, fix the mass of this particle because, this depends on something else. So, you can go ahead and put that lambda phi 4 kind of theory that thing, which I write there lambda mod phi square minus a square whole square, expand it and you will find that, it depends on lambda and lambda is not fixed by anything. So, there is no prediction, you can make for this mass now, people have other ways of putting constraints, observance, these things and so, that comes to my experiments.

But theoretically, there is no way, we can predict it this, this is a serious problem, I mean you do not know, is it low, is it high, I mean these are the kind of things. And in fact, recently in the news, I saw a sort of possible masses where, in fact I think familiar people ruled out some segment and you have bank in the middle of somewhere. So, there were range of masses I think, 100 to 200 GB where, you know somewhere in between it is. So, that the open question is, is it a light, is it below that range, is it above that range, these means different things.

And so, we so far, we have never observed a fundamental particle, which is scalar we have observed lot of scalar particles (( )) are examples of that. But, they are not fundamental, they, we know that they are made up of quarks but, we have not seen a single scalar particle, fundamental particle. And so, in some sense, this is a holy gift and may be, may be that is why, it is called the god particle so, I answered my own question.