Classical Field Theory
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Lecture – 19

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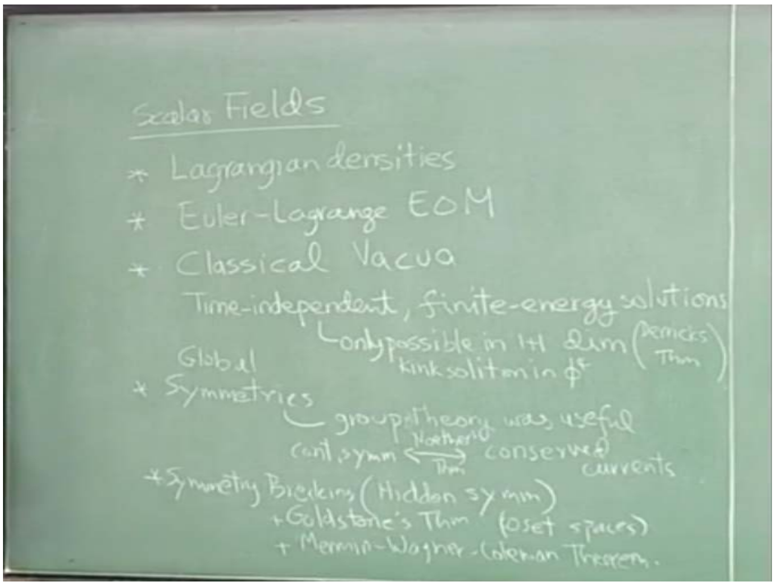

Also make it, it is also a check list for you about the check, what you know, what you do not know or maybe I should check learn something again. So, what we started out was mostly we did scalar fields. So, we started out with 1 scalar field, and then we graduated to systems with many, many scalar fields. And we saw how to write Lagrangian densities for these guys, and then we learn to given an Lagrangian density to work out the Euler Lagrange equation of motion. And then we also little bit of playing around with solutions.

So, we looked at classical vacua, these where we define these to be the lowest energy configurations. And we also looked at other solutions like you know we look for things like time independent finite energy solutions and we saw that this was only possible only possible in in one-dimension one spatial in 1 plus one dimensions. And it was not possible in the high in any higher dimension. And we saw a nice example of it we saw the kink solve it on of the kink solution in phi 4 theory. We saw that and so this was called Derricks theorem or whatever. Then we also study symmetries and we saw how groups group theory was essential in describing them or useful. And then we we did symmetries we also saw that continuous symmetries implied conserved currents and this was called Noether's theorem.

And then we discussed symmetry breaking or what we call hidden symmetry and this was nice think we saw there was Goldstone's theorem. And so here, here another mathematical concept which we are seen in an abstract setting came into use we saw that when you have continuous symmetries which are broken down, then your the that there is a use for the cosets spaces. So, Goldstone's theorem is best stated in the language of cosets spaces cosets or cosets spaces. And we also saw some brief discussion on what happens in, in quantum field theory we dint prove anything ,but we sort of saw that their where issues with I mean discrete and continuous symmetry breaking where different there was the dimension dependence statement. And that was the Mermin Wagner Coleman theorem.

And of course, the wave we derived Noether's theorem it was valid for not just scalar fields we dint make any assumption on about the about the field. And so clearly it holds for must more general situations, but the wavy we went beyond scalar fields was the wave we went beyond scalar fields.

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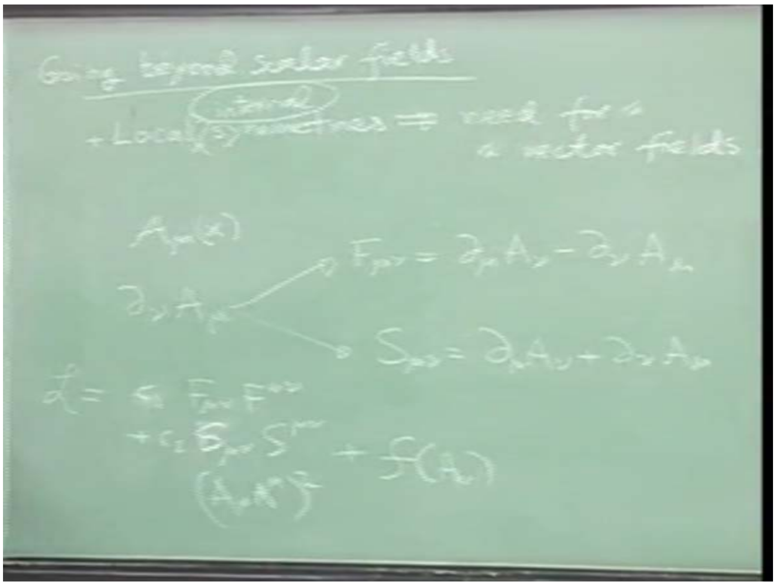

So, all these symmetries which we looked at we call them very defined them to be global symmetries, because they did not depend on space and time. So, the route we followed is the route is to look at local symmetries will call, we only looked at internal symmetries and this implied the need for a gauge field or a vector field or vector fields. One may think do I need to go through this method? Can I ad hoc say my basic field is a vector field, nothing actually there is nothing wrong with that, it turns out that so. Suppose I started out with a vector field for electro magnetism say and it is perfectly a good field except that if you try to write, so try to write a Lagrangian for this guy what you what you would end up getting is. So, you would say am around 2 derivatives. So, taking this already has 1 index, so I could write things like, so this would be an object which has a derivative of this thing.

So, we have to construct scalars out of these guys and I could since this is just a second right answer I can break it up into symmetric and anti symmetric parts. And the anti symmetric part I will call it F mu nu and the symmetric part let us just call it S mu nu. So, if I try out an Lagrangian the most general Lagrangian which would involve 2 derivatives of these things would have, would have some coefficient here let us call it c 1 F mu nu square plus c 2 S mu nu S after mu nu. And you cannot have a cross term, because that vanishes, one is symmetric; one is anti symmetric if you try to contract them you get 0.

So, you get 2 different kinetic energy pieces plus anything you now plus arbitrary function of A mu's as long as it is some Lorentz scalar for instance I could write something like A mu A mu whole square perfectly. So, this is what should have done if you just try to use the ideas that you had for a scalar field. You just saying you have a vector field and you now that you want a Lagrangian density to be a scalar field and you just go ahead, put in the restrictions and derivatives etcetera exactly like we did. And then you see already that at the level of kinetic energy there are 2 different possibilities.

And so in fact you could go ahead and say can I construct quantum field theories out of this, and people actually spend lot of effort trying to quantize these theories with this kind of terms. And these theory turn out to be sink quantum mechanically. So, in quantum mechanics you know what is sarcasm? You want to have a Hilbert's space in which everything is positive definite, you do not move on states with negative non. And they call ghost or whatever so they these, these theories had lot of problems with that those kinds of issues. And so if you just write adhoc, write a theory like this as for as we know today that these, these do not make sense quantum mechanically. So, but as if you want to write a classical field theory you can go ahead and do this, but what you get out of local symmetries is something very different. It says that there is an extra thing which is gauge invariance or local will call gauge invariance or we call it local invariance.

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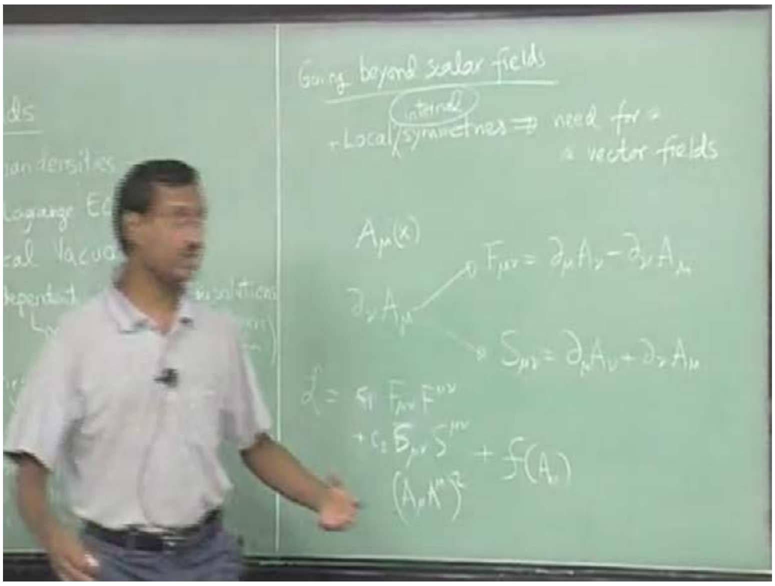

So, the extra ingredient that you need when you come to vector fields is that we call it the gauge principle. And this we know from electromagnetism; electromagnetism, the fundamental objects where electric and magnetic fields and it was a A mu. So, we saw that if you get 2 different so in electromagnetism know that if you give A mu of x and another A mu prime of x which is A mu of x plus some D mu of some scalar of. Both of them give the same electric and magnetic fields and we know that they are, we would say that these two are gauge equivalent they are the same. So, you have to impose the so in other words so this so you require so and we also got the same transformation by converting global symmetries to local symmetries. When I did the S O 2 it looked exactly like electromagnetism, but we will see that in more general cases it means something it gives something much more general called young means.

But right now we are not getting into that so the idea is that we have to put in the gauge principle, what does the gauge principle do? First thing it does it tells you c 2 has to be 0, it implies that c 2 should be 0 and it also says that there is a function like this is again; obviously, not gauge invariant it is not invariant and local symmetry transformations or gauge invariants I will keep using these two interchangeably and it also says that f should be 0. So, if you restrict yourself to two derivatives, this is it; you do not have any more freedom. This is the only term you could have written with the derivative restriction if you high derivatives, of course you could write more terms. But like I discussed in the beginning of this thing even for scalars we never looked at higher derivative thing this is the restriction we are putting in it. And there are situation where you go beyond two derivatives, but we will not in this course.

So, you can see that we need one extra ingredient to generalize this whole structure to include vector fields. And vector fields come naturally in this sort of when you try to do a local this it comes to a gauge invariant part so is not just this you also this is just the kinetic energy for the gauge field. But there are interactions in other words there are cubic terms, so anything any, any term which is not quadratic in the fields we will think of them as a interactions from the view point of quantum field theory, you always expand about thinks about a quadratic and the rest should treat as a interaction, so it is a very perturbative quantum field theoretic view point.

So, we will say that if there is a cubic term or a quadratic term, we will think of them as interaction. So, if you have a scalar field with the term like m square phi square that is not an interaction that is a mass term. But is there a phi fourth term that is an interaction, because it involves 4 fields. You also know that the equation of motion becomes non linear then. So, so similar, so now if you go back and look at what we are done for the S O 2 example you naturally see things.

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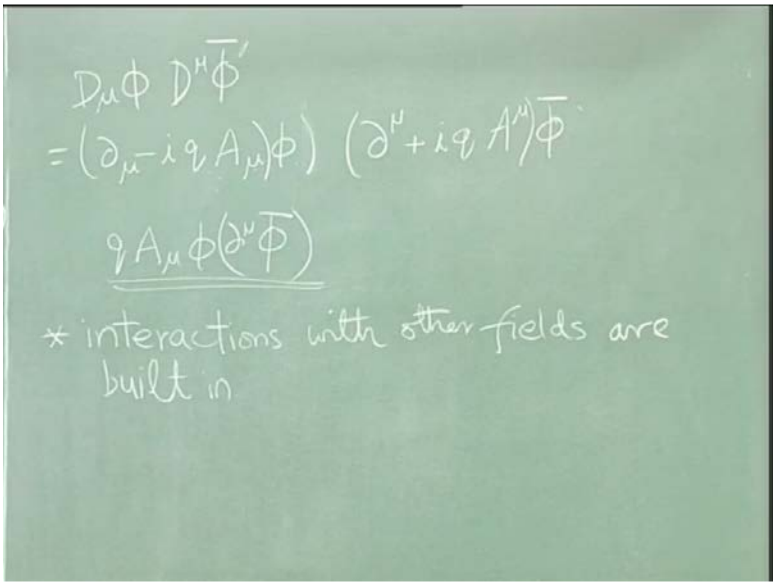

So, when you look at terms like this D mu phi D mu phi star, phi star or phi bar what did I call it? Phi star or phi bar; phi bar I call it phi bar. If you expand this out you will see there are terms. So, if you lets write this out, it is D mu, we did expand it last time, but what I what to point out is say take a term like this q A mu D mu phi bar. So, you have this is the term which is like this. So, you can see that this particular term is, is cubic in the fields it has two scalar fields and one more field so interactions so the gauge fields will does two things, are we now. So, so you can see that even so you one part is K E is this you can see this gauge principle tells you this, but it also does not sort of let you free lance and write random thing it gives so interactions also built in.

So, in some sense if you are a minimal list in the sense, if you do not want to do anything more than beyond this sort of a this thing this is where the you, you take this kind a approach very seriously unless you are some other way of constructing situations where you get some other interactions which are gauge invariant, but they are very more much more complicated. And so that that there are situations where this happens, but it this straight ahead approach actually works well and it, it works so well that it leads you to a frame work where you can describe all the basic interactions in basic field rather the interactions forces in nature we can describe all of them except bar 1 guess which one gravity; gravity there is an issue, but forgetting gravity for the moment.

In fact, you are so successful that there is something called as standard, a model standard model which is based on extending this to more complicated settings. And by the way in this whole course there will be no fermionic fields that is, because I do not want to add a flexure of indices as if that is not enough. So, the idea is to simplified and this an undergraduate course. So, I do not want I mean this is an essential simplification, but in some sense all the concepts can be illustrated without that. So for instance we you would have seen in your assignment that you can discuss spin of a photon or helicity of a photon. You can discuss without I mean permanent I am not the anything which carries spin. So, the key point is interactions with our built in and so.

And the thing about the standard model is that the more or less bulk of it was done more than 20 years ago and I mean write now people are only building more and more things to checks test, you know what are called precision tests of the standard model to look beyond the standard model to know what is coming beyond that. But so far as, as we can tell all experiments in high energy physics this approach is been more than enough that I mean it is shocking or amazing or whatever I do not know what, what to use. But this is the status and that is why we will we will in this course we will take this view point. And now we will push it further through examples where the gauge group is not just an a abelian S O 2 or u 1, but could be an e group. So, what will do to, to understand? So, the, the next part of this course will be will; will include settings where we have vector fields as well as scalar fields in your theory.

And we will input will convert global symmetries to, to gauge or a to local symmetries and see what happens? And will go through this whole thing, you know for instance we saw ready for a billion case that if you are in a setting where symmetry breaking happens, what happens in the presence of a gauge field is rather different from what you saw when it was a global symmetry so leads you the it is mechanism. So, we will see interesting things and we will also see how that that we could go around ricks theorem, one easy way to understand it the ricks theorem assumes at the only fields in the thing was scalars.

And there is no gauge principle out there or anything and, and the thing is again there will be a nice feature which will come whenever you are looking at finite energy solutions, time independent I do not call them I mean allowed to call them soliton's, because soliton's are suppose to have some non dissipative nature. But I will on and off key calling them soliton's and I would recommend you read Mermin Coleman's edited lectures. He gives a very nice description of what should be a soliton? But I will be much more relax about my use of the word soliton. When I say soliton you should have in mind time independent finite energy at least these two, these two conditions we will use them interchangeably.

So, will see that that, that in every dimension a soliton has a different does not a like it is like just as an, a new twist in every dimension and it is sort of you get more space to do something new. And so we go from 0 1 2 3, and even 4 where we will sort of take time and make it Euclidian's. So, the space will be like r 4 you get something call instant terms. So, that will be sort of the final part of this course. So, before we do that we need little more structure and the structure was already evident when we try to understand, try to write a good guess for the how to parameterize the fluctuations in when you have spontaneous symmetry breaking.

And the, the structure that underlies that goes by the name of lie algebras, and we have seen it in various forms I have given you hints throughout this lectures. Now, we will put all of them together, and we will see that there is a structure called a lie algebra is just a collection of whatever ideas you already know. So, I will just put them together and then you sort of write something more abstract from that and which I can go on from that.

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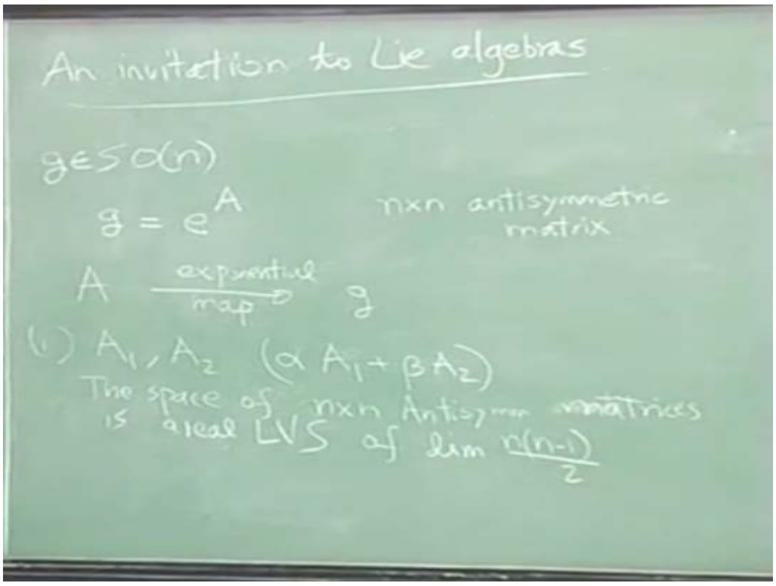

Sort of maybe I should call it an invitation to the algebras because I am I am not really going to. So this starting point is following thing which we already know some things. So, we saw that any S O n group element can be written, let us say you have a group element of S O n, we saw that that could be written as g power A where A was an n by n anti symmetric matrix. And so that the point here is that what it you tell you is that instead of looking at the group element I could work equally well work with anti symmetric matrices. In that in that what this tells you if give me an S O n matrix I can I mean if you give me a group element by doing the inverse so there is the.

So, let us put it this way so there is this there is a exponential map which gives you the group element. And there are some features of, of A which are not that in g, so if I gave you two elements and asked you how what is their composition rule? It is pretty complicated not that it gets easier out here, but at least we can write formally something and in some cases we can actually write the answer first. So, this so the exponential map is a very important ingredient in, in this whole connection. So, what we will see is that first point is that let us lo at this if you give me 2 anti symmetric matrices at least for the next 2 half boards where I use the symbol A, it just means anti symmetric matrix. So, one property is you can see that if I take alpha A 1 plus beta A 2 that is also an anti symmetric matrix.

So, this is roughly the structure of, of what vector space so these. So, let us write this out so the space of anti symmetric matrices n by n anti symmetric matrices with real entries for now is real LVS, LVS is short form for linear vector space of what dimension. So, what they will see is that lie algebra is just kind of a linear vector space with some extra structure. And the extra structure is there is basically what come from the axioms of a group, the first point is that if you give me 2 elements g 1 and g 2.

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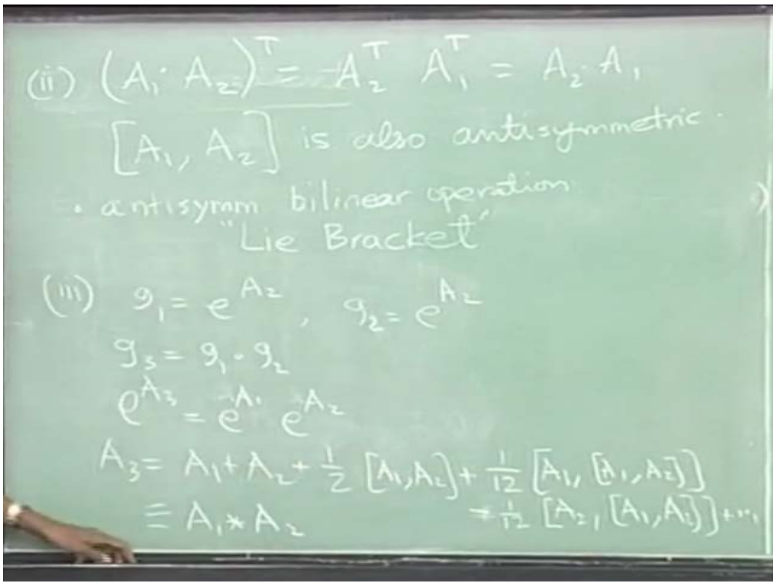

So, before we even look at that the key point is, suppose you look at the product A 1 times A 2, if A 1 and A 2 are anti symmetric is the product anti symmetric no its easy to see it is not, because if you take the transpose it is A 2 transpose, A 1 transpose and these are minus of each other. So, it is equal to A 2 dot A, it is not A anti symmetric there is no structure, but if you take the commutator it is indeed. So, let us lo at the commutator and what are the properties? It is linear in each of its arguments and its anti symmetric it is an anti symmetric bilinear operation. So, we have a, a linear vector space as one more extra structure which is something which eats up to things it linear arrangement of its argument it is an and its anti symmetry if you exchange this thing this we will this has a formal name it is a lie bracket.

So, it is so the abstract statement is there is a linear vector space which is with a lie bracket structure there is little bit more structure which I have to explain to you. So, let us call this the lie bracket, but in this course and in most physics applications lie brackets can be you can bliss think of the lie bracket has being replaced by the appropriate commutator, because that is what will happen. Now the second, third part actually comes through the, the fact that we know that there is a group. So, if you give g 1 and g 2 are 2 elements and suppose g 3 is the equal to g 1 dot g 2 then g 3 is also an element of the group.

So, it should be writable as e power some A 3, but the product of 2 exponential matrices as we saw earlier have you seen it before in this course. So, there is something call the baker gambol hoister formula and for once let me write out to the next order piece. Again the way I think of this thing is to put two lambda s here it is like a formal book keeping device and say that write out bunch of terms. And and I will say first order in lambda, second order in lambda that will give you a nice way of tracking things. And keeping track of when you multiplying things to check things, you know that if you keep these to correct some order you can only verify this to that order.

So, what you should do here is to work out you can show. So, I will not prove it I will just write the write something, put a lambda here, put a lambda here go ahead and expand it and then start collecting terms and re exponentiating it and that will give you a 3 you know it has to combine back in to that things. So, so the exercise which is a rather tedious one is to show that A 3 of course, this is the easy part. And the next part should I have seen in quantum mechanics usual, but often is half A 1 times A 2, but I am going to write out the next order piece and for once I need a piece of paper I do not want to mistakes, it is an infinite series it does not terminate. So, but this is something very, it is a concrete statement it is says if you give me 2 elements A 1 and A 2 I will give you a unique element A 3 it is an infinite thing. And let us just define this to be some operation which will write us A 1 star A 2, the reason to write it like this is because just as we define the lie bracket so it is made up of objects in lie brackets of those things.

So, in the abstract setting there is something which we will call as star operation which gives you which, which is giving you the composition rule. But and I would recommend that you work out this piece it is a good practice to play around with it. And if you there are I mean there is not a very clean way of writing this whole thing there are some generating function methods etcetera. But at first I can see that that I have in not personally found a use for things beyond this level, up to this yes I have used it and maybe I leave an assignment to do. So, so you can see now what so the so we have a linear vector space equip with the lie bracket and this kind of operation which tells you the composition rule. But there is still is something more in the story.

So, if you go back to the group definition it that should be an identity. So, what is the identity element here; The 0 vector in the vector space, if you take the 0, so that is the identity element, so that is there inverse is easy, inverse is not a problem. But is there any other property that we have to.

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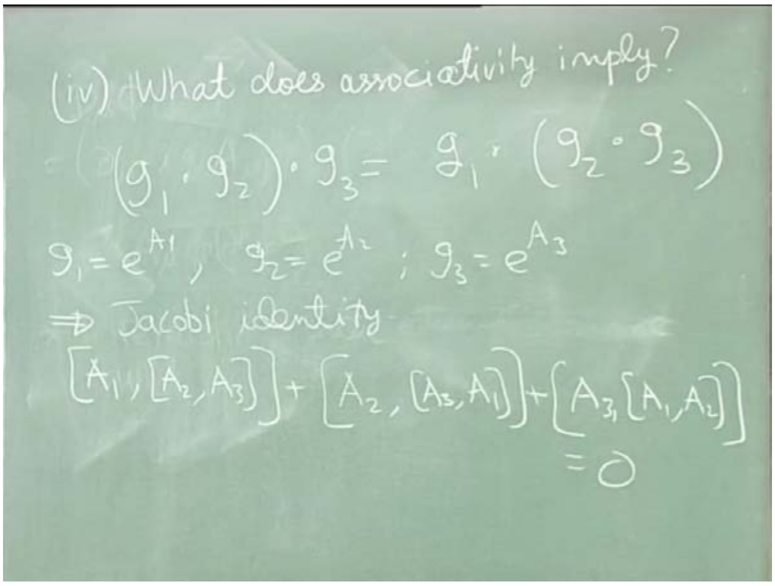

Associativity; so that is the last property so we, so for so what about what is associativity imply. So, it implies some extra structure on this, this lie algebra whatever we are defining we I mean after we put all these structures in only then we have defined there what we mean by a lie algebra. So, now, now comes the, so the thing is associatively what does it you tell you? You have g 1 dot g 2, you do it first like this, this should be equal to this is at the level of group and you have given this star operation. So, that tells you how to compose them so this, so we already have achieved that.

So, we have to put all these things together and ask what is this imply? So, this implies the following. So, let us write again it is exactly like this let us say that g 1 is e power A 1 g 2 is equal to e power A 2 and g 3 is e power A 3; g 3 here is not like this composition rule it is just some 3 different elements. And if you go ahead and work this out and you can check that what this implies is something as follows. So, for matrices you know this is true, if you give me any 3 matrices they satisfy the Jacobi identity.

So, this is just the so it is now we have a definition of what we mean by a lie algebra it has all these structure. So, it has it has a linear it is a linear space, but need not be real by the way it will be complex also then. So, if it is a real vector a real vector space we say it is a real lie algebra we, if we say it is a complex vector space we say it is complex lie algebra. And and dimension of the linear vector spaces defined to be the dimension of the lie algebra. So, it is a algebra is a linear vector space over some field acute with the lie bracket with this kind of composition rule it satisfies the Jacobi identity. So, this is the definition of a lie algebra. So, what I have done here is to take our definition of S O n and see all the properties of the factor it was a group and you are sort find that you get this structure. So, one more important point is that all these group for instance we in one of your assignments.

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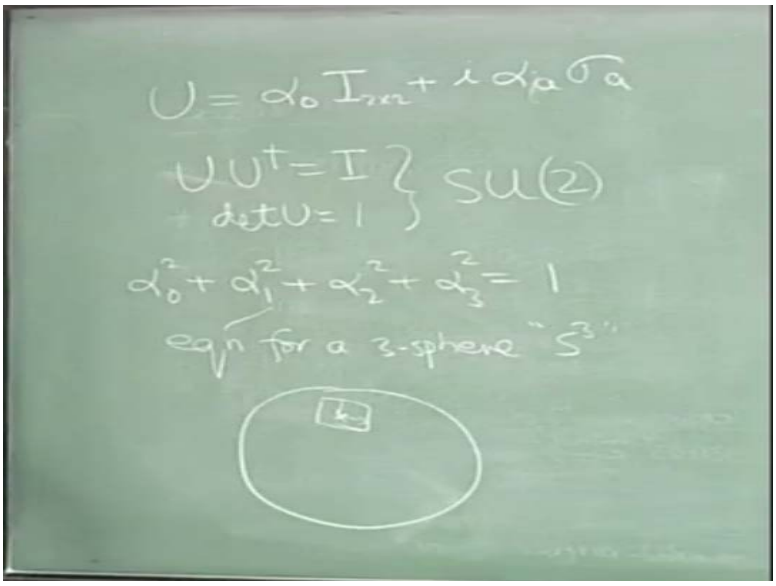

You I think something like this. So, I, I gave you a arbitrary 2 by 2 matrix with 4 numbers alpha 0 alpha 1 alpha 2 alpha 3 say sigma is for the poly matrices. And I asked you the conditions under which U U dagger was equal to 1 and determinant of U equal to 1. So, this define the group which we already seen it is a group S u 2 and if you worked out in your assignment you will see that first thing it implies is that alpha 0 and alpha a have to real. And further alpha 0 square plus alpha 1 square plus alpha 2 square plus alpha 3 square should be equal to 1. So, what it says is that the set of unitary S u 2 matrices S u 2 is group elements is in 1 to 1 correspondence with the points on the 3 sphere, because this is exactly 3 number 4 number subject to 1 relation. So, this is an equation for a 3 sphere if this not that this would be a normal sphere.

So, what you see is that all these groups these are these are called lie groups, because the space the parameter space is actually nice manifold. So, in this case it is a 3 sphere and locally and what this the lie algebra tells you is sort of gives you locally what the directions are, if you know what it is like the tangent space to this space. So, you pick a point here it tells you all the different directions you make it look like. So, it is a S 3 is a 3 dimension manifold so you locally it looks like 3 space at every point you put some it is it is a curved space, but let us not get into that. But the point is there are 3 directions and the, what are those 3 directions those are exactly, if you look if you come back to S u 2 it is actually similar to S O, S O 3.

So, in, in fact, this discussion will tell you that you have to look at there is a certain difference where you come to the group, but at the level of. So, level of lie algebra they are the same S O 3 also has the same structure it is just here we would look at 3 by 3 anti symmetric matrices and it is 3 dimensional and this is also 3 dimensional. So, this will give a lie algebra. So, as groups S u 2 and S O 3 are not the same, but as a lie algebra you would see that there I there is an, in other word locally it looks like. So, you can ask what is the precise. Can you tell me the difference between S O 3 and S u 2, the answer is very simple it is S u 2 with an identification at least this is group manifold it has an identification identify antipodal points. So, it gives you some space do you know what it is called r p 3 or something like that.

So, it is S 3 with antipodal point identifying I, you cannot picture it if you want you can write it as an equation, you identify the opposite points. And so so in other words S O 3 group element, every group element there will have 2 things; one if you pick a point a locally they are the same, but it just at this point and it is antipodal point which you which is on the other side will not be will be the same as a group element. You will see all these things in your assignment and some notes, but. So, what I want you to remember is that the lie algebras are much simpler, because there is a vector space structure in that so you can add subtract, you can do lots of things nice stuff what you will could do with vectors, so it makes life easy and. So, we have in this course through, through your assignments and through my discussions you have seen actually 3 families of groups, lie groups.

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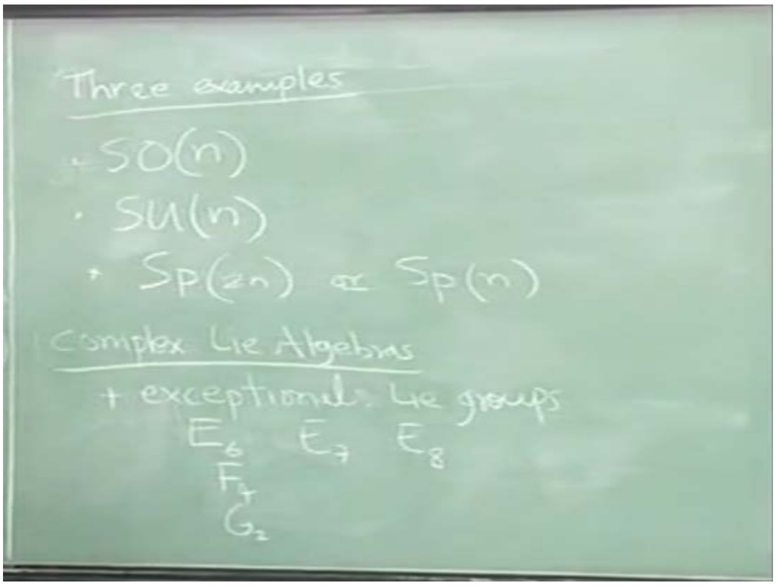

So, three examples which was have seen by examples I do not mean one example is actually each one is a family. So, you have S O n that you have seen, you have seen also S U n these are n by n unitary matrices, but you also seen this in lactic matrices. And now, now books you should be careful some books have will write S p n and some others will write as S p 2 n. And the only way to solve the ambiguity is to ask what S p 1 stands for if you if there is S p 1 that is same as S u 2 which you can prove or yeah or you look at. So, you just look at the first case and you distinguish what their notations so this is I mean there is no fixed notation on this. But the key point is you have seen all these things. So, they are all the examples of lie groups and the interesting thing is these this is not all, there are few more which are not in this list. And the classification is better when you go to the complex so complex lie algebras real's are more complicated.

So, you find that you get these 3, but plus there are exceptionals there are called exceptional, because they are not part of any family, they occur only for some special cases. And (( )) classification has some may numbers there is there is a b c d, so these you here you separate out S O 2 at the even case and the odd cases. So, they form a b c and d, I will not tell you which one is which you go back and check so covers these things. But you have E F and G, so E then F 4 and G 2. There are some finiteness conditions, so that is we will find that you will end up with only these kind of these are the only examples of finiteness, simpleness, there are some conditions you need to put in.

But basically this is the, it and the amazing thing is you know in other words you know, you know just about 99.99999999 percent of the group lie groups you already know you have seen it in some form or the other. And these you will never see in this course, so we will go back to only this. So, now coming back to the algebras in, in general now we know that it is a linear vector space. And we can use some of the tricks we learnt in the when we were working with linear vector spaces, what did we do we said we will choose a basis.

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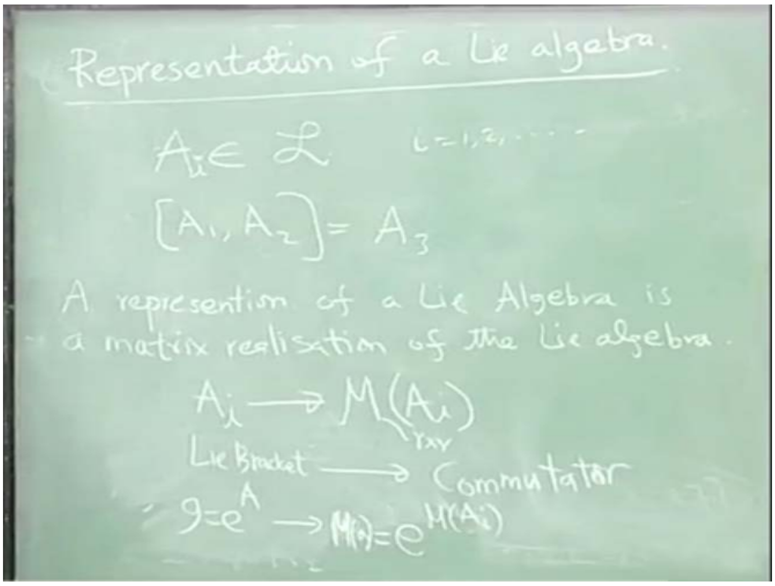

So, first thing is before we do that I will discuss what I mean by representation of a lie algebra. So, a lie algebra is I told you there is this whole structure which is there and so the idea is that, so let us say that a belongs to so let me use script L, a belongs to script L means that it is an element of a lie algebra l and it has all the standard properties. So, so the thing is that we could have, suppose A 1 A 2, so A i let us say for all suppose we have the lie bracket of this thing.

Now, if you find that so a representation for groups we saw that if you give the there is an abstract group multiplication law we realized it by acting on it by some matrix. Similar thing is to find so a representation of lie algebra is in a simplest way it is a matrix realization of the lie algebra. So, in other words if you give me the abstract element I will give you some matrix and let us say this is some r by r matrix, such that so and lie bracket gets mapped to what? What should lie bracket get mapped to? The commutator. So, it like chicken and a (( )) started out with a actually we started out with a n dimensional representation of the lie algebra of S O n. And then we work out or abstract definition of the lie algebra. Now, we are going back to this the reason is that that is not the only realization of S O n every time we wrote a tensor of S O n, we are actually getting a different realization it is a different representation of the same think. So, lie bracket becomes the commutator, and of course is has to respect the lie algebra structure I mean that is implicit in this statement.

So, this is what you mean by a representation and you would say this is the representation of dimension r, where r is the size of the matrix and it should be a faithful representation. For instance there in one representation which will work trivially suppose I say that every A i gets mapped to you now a matrix of 0s, it will satisfy all your conditions very trivially, but that is. So you would like to get rid of those things and there will also be a notion of a reducible in the sense that you cannot break it down to something smaller. And you will see that again, now the thing is the group element.

So, the corresponding group element g I will be realized as just e power m of A i in this representation so so we will write something like this here. So, so g where I have written as e power A, and this will go to m of g that means the matrix realization of g will be equal to e power m A. And in physics we never, we never ever do the abstract stuff, we really always work with some representation and so we will we will stay will that I mean So, now, now we can come back to linear vector space setting. So, I just want you to remember what a representation is...

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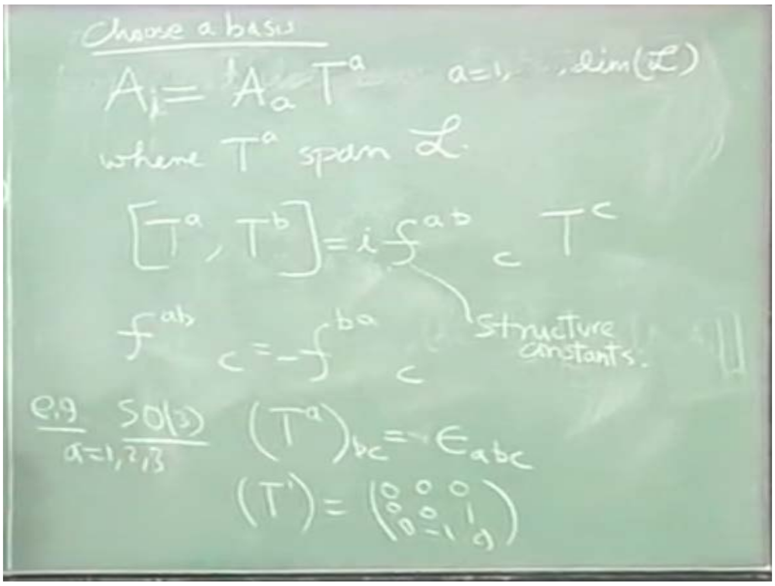

Now, we will go back to a lie algebra. So, if you give me a bunch of A i's since it is a linear vector space I can choose a basis. And so we can write any A, so choose a basis you can write this as A a T a, where T a span the linear vector space and a goes from 1 to to dimension of L. If you take S u 2 you will have 3 such generators or S O 3 you will have 3 generates. So, this is what you get, now we can ask what, what so the thing is now, because of the linearity of the lie bracket this structure all the data which we have in the lie bracket is captured by just by looking at what it does to the basis vectors that is a big simplification. Always we saw that any structure we put on the linear vector space life became easy, because we could only work with basis vectors and we could.

So, what you do is I could ask, what is the lie bracket of T a and T b? This has to be an element of the lie algebra. So, it should be which again can be rewritten in terms of these guys. So, I can write it as some summation over c f a b c T c again repeated index is summed over, again I respect upper, lower etcetera it is this. And now there is a essential distinction between physics literature and math's literature, in the math's literature you do not put anything out here, in the physics literature we put an i. So, and this f a b c's they have all the data of the lie algebra, because it tells you of course, it is basis dependant. But but the key point is that this has knows everything about the lie bracket structure and we know that it has to be...

So, what do we know, we know that f a b c has to be equal to f minus of f b a c, because that is the anti symmetric property of the lie bracket. I will leave it as an exercise or it will show up in one of your assignments to show what the factor this satisfy so the T is themselves they have to satisfy a Jacobi identity. So, you take T a T b T c and write out the Jacobi identity and you see that it will imply a condition for the f's.

So, these have a name they call this structure constants. So, we will give I will give you an example, now and it will be in a in terms of a representation, and so then I will write T a's as matrices and so T a now so in a So, the thing is what I will do is I will I will use the same symbol T, I would not say I would not do this m of A i, but I will write a matrix for you. So, so let us take the case of S O 3 and I will write out for you a as a matrix, and I will give you a 3 dimensional representation so a so S O 3 a will go from 1 2 3. So, there are 3 of them I have to give you 3 such anti symmetric matrices I will give you a very simple basis which is. So, b and c here are the labels of the matrix so T 1 will be the matrix, so let us look at what T 1 is T 1 will have 0s in the 1. It is an anti symmetric matrix, so it has only these elements so you can see yourself and you can. So, exercise for you is, is to check what the structure constants are.

And again I must warn you the way I have given you you will get without the i. If you want to get an I, if you want to put an i back in here I think you put a minus i here. So, this is like the physics literature versus math literature whenever you do a representation you would look at, we will choose the matrices to be homician in physics while in math's literature that is skew symmetric or anti symmetric. So, that is what is this i you can see I mean that this is convention dependant of course, whatever it is. But this is the fun thing to just go ahead take this particular example, where you know these things.

And you do not have to sit and do matrix multiplication and it is better to work with index notation, it just comes down to work you out epsilon, epsilon equal to delta, delta when rewrite it is it is basically I mean it is a little bit of a pain to do that, but you have to do its good for your soul so. So, kindly check this and get these structure constants. And there will be an assignment coming which, which will be covering which we let you play with a little bit more I mean glory details especially S u 3 I mean you would see there is eight-dimensional. And there is something called the Gellman matrices and you have to play with them.