Select/Special Topics in Physics
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Lecture - 7
Angular Momentum in Quantum Mechanics
Half-Odd-Integer and Integer Quantum Numbers: SU (2) & SO (3)

Greetings, and nice to resume our discussion on Angular Momentum in Quantum Mechanics and we are going to talk about the S U 2 and the S O 3 groups, today we will also talk about angular momentum matrices.

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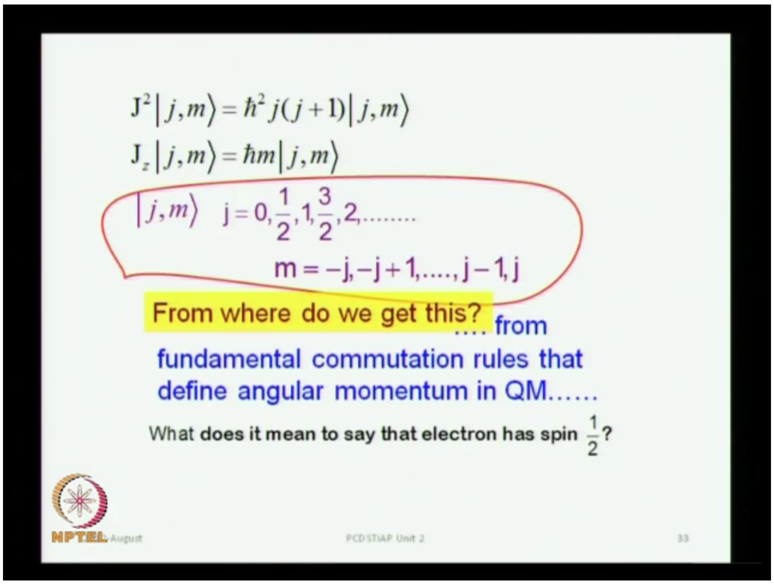

And the question that we discussed last time was this that you have J square and J z, which can be simultaneously measured and the Eigen values of J square are h cross square j into j plus 1. What kind of values can j take and we learned that j can take 0 half 1 and so on, half odd integers as well and for each j m can take 2 j plus 1 values going from minus j to plus j in shapes of 1, we learned that. We got it by simply working with the commutation relations, simply the fundamental commutation relations, which define angle of momentum, we did not put much else.

No additional postulates no additional you know mathematics or no additional physics, no additional quantum mechanics, it was mostly this and a little bit of common sense, which is and fortunately not very common. But, that is that is about, what all that we put in right and we got these results that j can be 0 half and so on and this has something to do with the fact that, we should understand, what we mean when we say that the electron is got spend half. So, we got half quantum numbers, but we did not use any relativistic quantum mechanics, we did get half integer angular momentum, only from non relativistic quantum mechanics and only by using angular momentum algebra.

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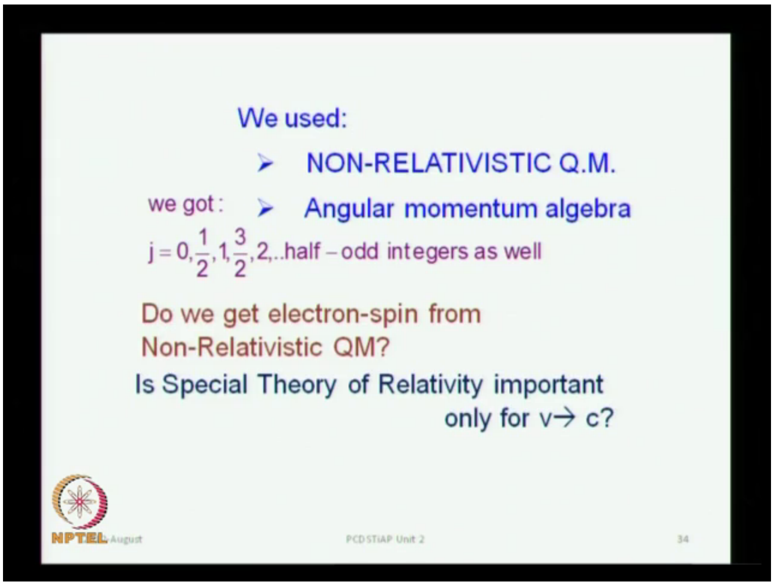

Now, this gave us the half order integer quantum numbers alright, but it does not mean that, we get electron spin from non relativistic quantum mechanics, it would be misleading that one can get electron spin from non relativistic quantum mechanics. I want to emphasize this point, because I have had students, who have sometimes been confused by this issue, because they do get half integer quantum numbers from non relativistic quantum mechanics.

And I would like them to understand that, this does not mean that you get the electron spin from non relativistic quantum mechanics, you can, you will get the electron spin as half angle, as half h cross only from relativistic quantum mechanics. You must use Dirac equation, it is the consequence of the special theory of relativity, it is based on the fact that Galilean transformations are no good, you have to use Lawrence transformation, it is because the speed of light is constant.

So, these are the things, which go into the Dirac equation and unless you do that, which is what we will do in unit 3, we will do it systematically and we will find that the electron spin comes neatly out of it. What it also means is that, if you are talking about an electron at rest or moving at the speed at which you walk or even I walk, which is like a 1000 speed, at which you walk may be right.

At any low speed, if you are talking about the electron spin, there is no escape from relativistic quantum mechanics, you cannot say that, you can, you are going to use relativity, only if you are dealing with very high speeds. Because an electron at rest exists along with its charge and mass and along with it is intrinsic angular momentum, which can come only from relativistic quantum mechanics.

So, the consequences are built into it and sure enough, you do need Lawrence transformation, when you are dealing with you know objects at high speeds, but then even if you are dealing with objects at low speeds. If you are talking about phenomena, such as electron spin angular momentum, then you do really need a relativistic formalism to be consistent. One can always plug in things, in an ad hoc manner, but that is not how it comes systematically.

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And the discovery of electron spin is really very interesting and we know that electron spin is very closely associated with the Pauli matrices and Pauli introduced a true valued quantum number. This was in the connection of assigning quantum numbers to different atoms in a periodic table, that you have the periodic table and how do you fill in the electrons you know cells, all of you have heard about the aufbau principle, so 1 s 1 then 1 s 2 then 2 s 1 2 s 2 2 p 1 2 p 2 and so on right.

So, you go on filling till that cell is filled and you had to do it in some kind of systematic manner and to explain the filling of electrons, in various cells in the periodic table. Pauli came up with a 2 valued quantum number, he said that n l and m n l and m, do not suffice to explain the filling of electrons in the periodic table. And you must have an additional quantum number, which can take 2 values and he did not say that these 2 values must be plus half and minus half.

He did not say that this refers to electron spin, he just said that, there has to be an additional quantum number, which must have 2 values. Those 2 values could be alpha and beta x and y or whatever, these are 2 quantum numbers and they must be moved to explain the periodic table, so this was Pauli in 1924.

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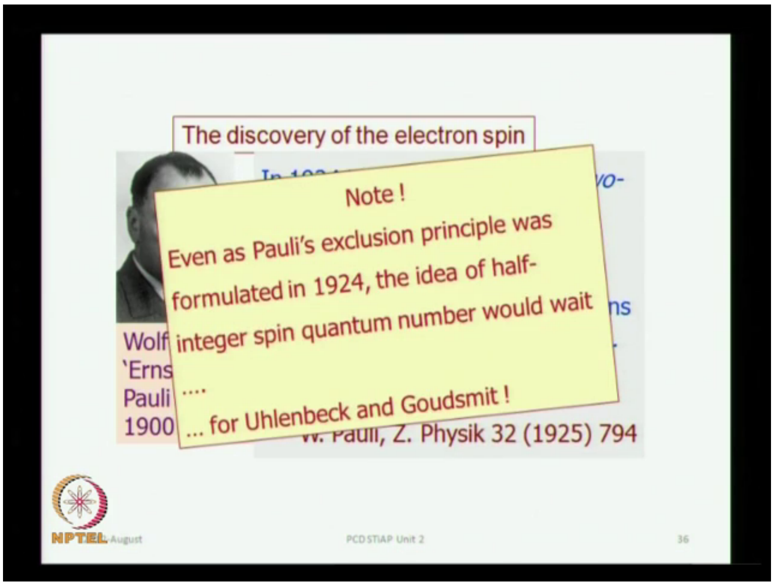

And he really had no idea at that, time about, the electron spin as a half angular momentum and this idea had to wait till, it was proposed by 2 experimentalists.

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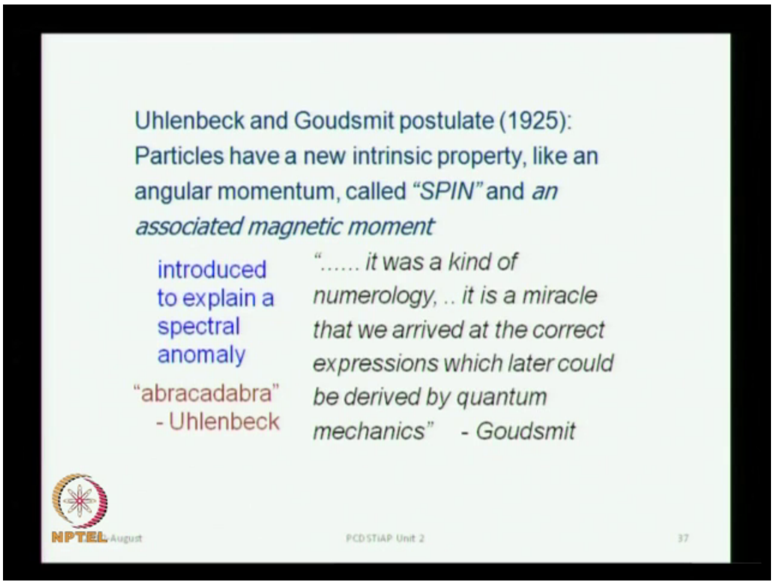

Uhlenbeck and Goudsmit and what they did was to propose the electron spin in 1925 and associated with this magnetic moment, the reason they did it is because they were trying to explain certain anomalies in certain spectra that, they had observed and to introduce this, they passed this idea. It came from nowhere, a brilliant intuition, you might call it and they proposed that there has to be an electron spin, it was so much out of the blue that uhlenbeck actually called it like abrakadabra [FL] that, it just comes from somewhere, but it works. There was no model no theory to explain that and Goudsmit said that it was some kind of numerology, it was a miracle that we arrived at the correct expression, which later could be derived from quantum mechanics. So, it just came out of nowhere, this was in 1925.

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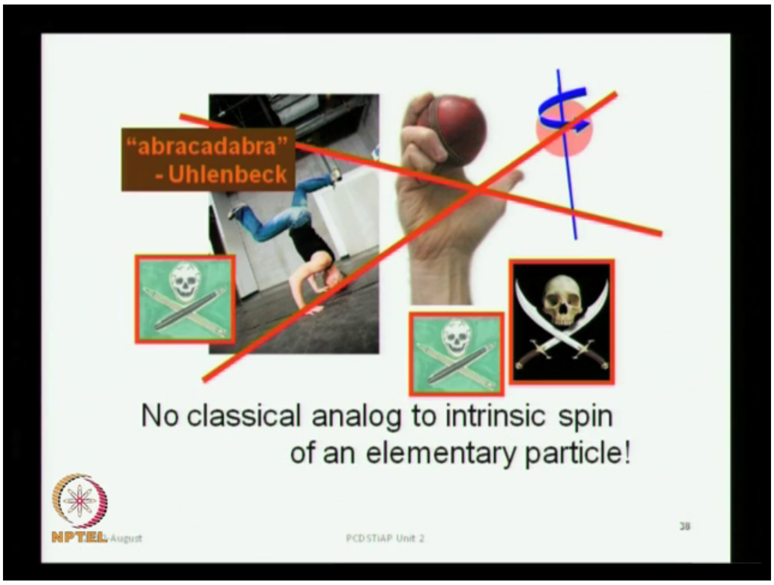

And very often, we assign these pictures to the idea of electron spin, which are all wrong as I mentioned earlier as well, that there is no classical analog to any of this any of these pictures.

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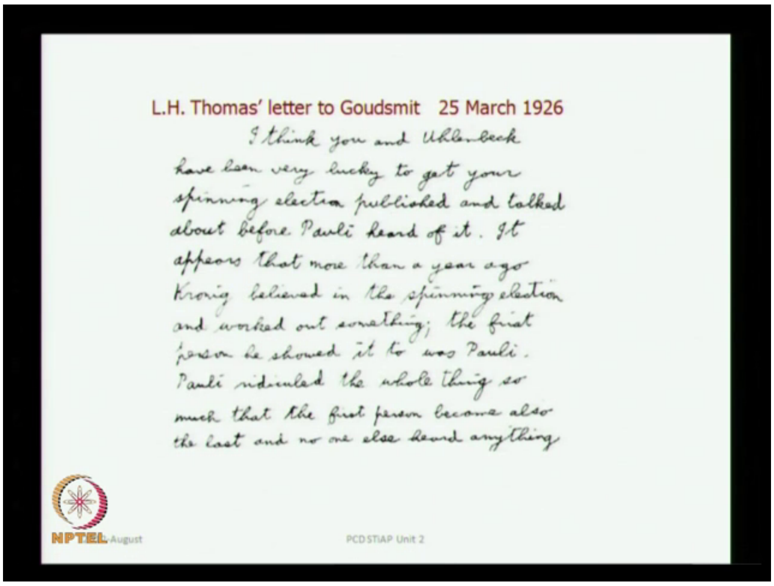

And I will also like to draw your attention to a letter by Thomas to Goudsmit and this letter is available in the internet, if you just Goolge, I am sure you can find it and I downloaded, I copied it from there. And what Thomas wrote to Goudsmit is that, I think you and uhlenbeck have been very lucky, to get your spinning electron published and talked about it, before Pauli heard of it.

It appears that more than a year Kronig believed in the spinning electron and he worked out something and the first person he showed it to was Pauli. Pauli ridiculed the whole thing, so much that, the first person became also the last 1 and no 1 else heard anything of it, so that was Pauli response to a suggesting, which came from Kronig, which was before Goudsmit and uhlenbeck proposed it.

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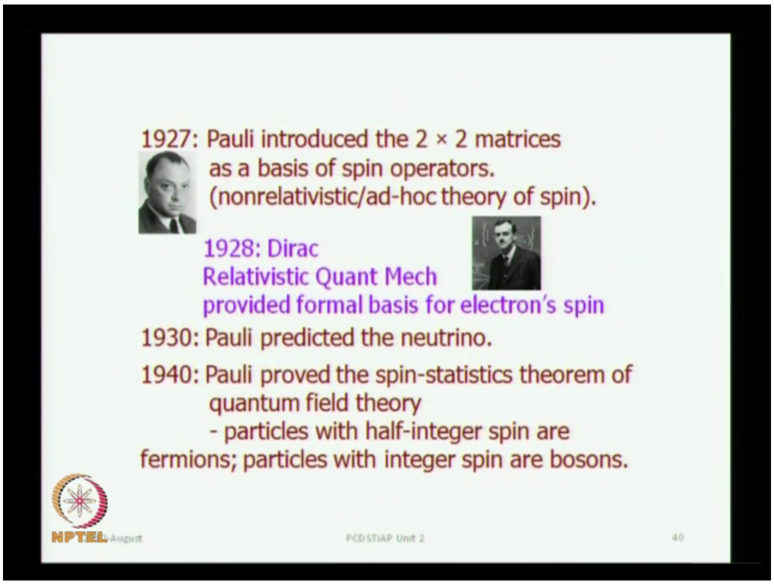

And then later in 1927 Pauli introduced the 2 by 2 matrices, which we call as Pauli spin matrices, which are connected with the electron spin as all of you know from your first course in quantum mechanics and we are going to meet them, today as well. And then in 1940 Pauli proved the spin statistics theorem, which is very easy to state and very difficult to understand that particles with half integer spin are fermions and particles with integer spin are bosons.

Feynmen says in his lecture somewhere, that this is one of the few theorems in physics, which can be very easily stated and extremely difficult, to prove and to understand, but that is a different story altogether. And then in 1928 Dirac came up with the relativistic formulation of the quantum mechanics and the electron spin comes, very neatly out of it as we will rediscover in unit 3.

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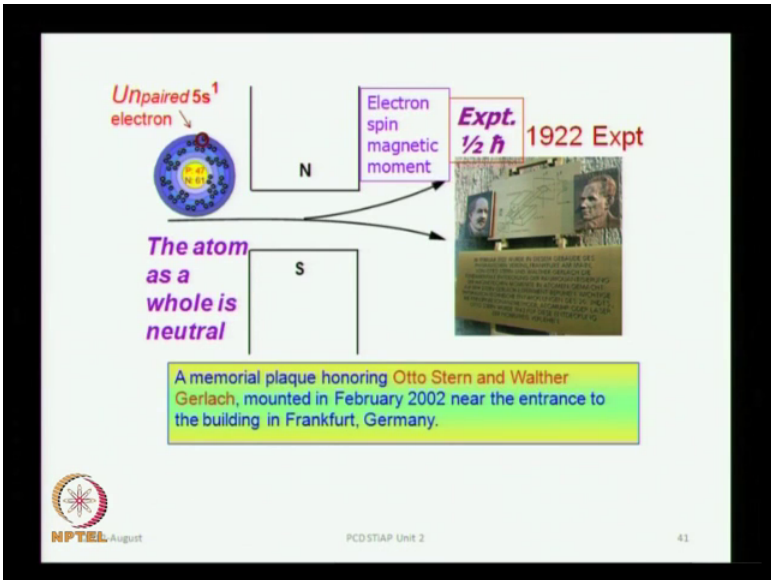

So, the electron spin is a reality, it has got 2 states and the experimental verification came before all of this it was known, in an earlier experiment done by stern and Dirac, that when you pass silver atoms in a magnetic field, they sort of spread out, they come out and this experiment is known to most of you.

And this comes from the spin angular momentum of the outer most unpaired electron in silver atom that belongs to group 1, so it has got the N S 1 configuration outer most configuration and that N S 1 is unpaired. So, that is a single electron and it is the magnetic moment of this unpaired electron, which provides these 2 states in the magnetic field, now if you did this experiment with electrons.

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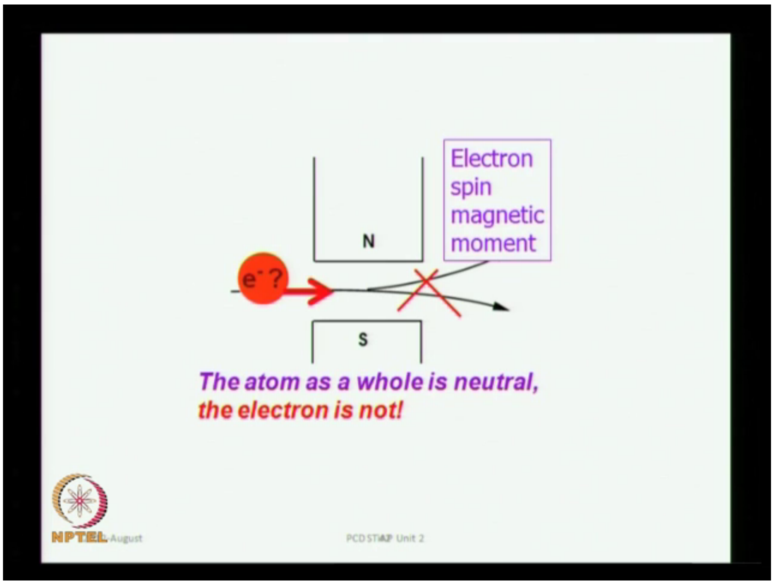

Instead of silver atoms, you send in a beam of electrons, you have a electron gone Gerlach and you fire in a stern Gerlach magnet, they do not separate out, as up spin and down spin. The reason it does not happen is that electrons are charged particles, silver silver atoms are neutral particles, so when you send in a silver atom, the outermost electron has got a magnetic moment, which interacts with a magnetic field. But, when you send in an N l an electron the electron being a charged particle, also experiences a Lawrence force, which is charge times v cross b right.

The charge force Lawrence force on a charge is q into v cross b, o, the v cross b term, which is an additional force, on the electrons, which is not there on the silver atoms, silver atoms are neutrals. So, q is 0, but for the electron, q is the charge of the electron, so there is this additional force and then there is a little complicated not, so complicated, if you sit down to work out the algebra, but I would not spend time detailing it, but I will just mention the result.

That when you work out the consequence of this Lawrence force the charge times v cross b and try to play the consequences in a formalism, which is consistent with uncertainty principle, then it turns out that this kind of separation is not possible. So, this argument you can find out in some of the books, I will be happy to share some references.

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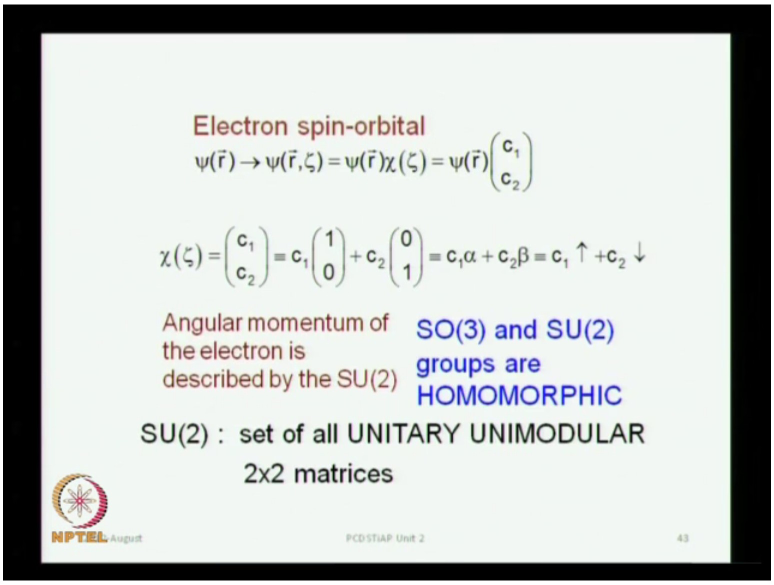

But, we will you know, share the historical, you know narrative on this and pick up the discussion on electron spin orbital, which has got a 2 state and you have got a special coordinate r and a spin coordinate zeta and you write this as a spin orbital. This is the orbital part and this is the spin part, sometimes called as a spinner and this has got 2 states represented by c 1 and c 2, which are the coefficients of the 2 pure states that, the electron spin can have.

The pure states are either alpha or beta or 1 0 and 0 1 or up and down or whatever you call it, these are 2 states and this is your spin v function or the spinner as it is called. And the angular momentum of the electron, which is associated with the spin is referred to the SU 2, rather than SO 3 and I will tell you why you have to have this difference.

So, S U 2 is another group, it is a unitary Uni modular group of 2 by 2 matrices, SO 3 you have already met in the context of rotations generated by components of the angular momentum vector right. And these 2 groups are Homo morphic, which is why they have some similar properties and they are important to us, but they are not isomorphic, there is a certain similarity.

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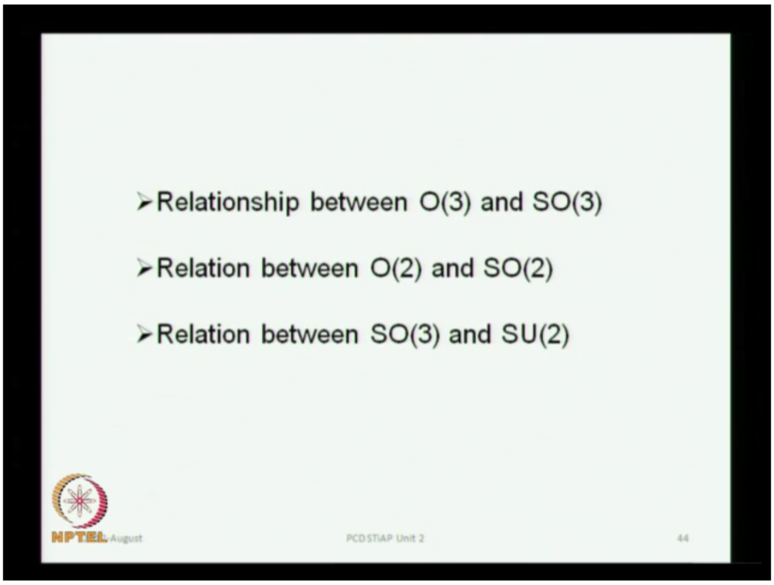

And I will demonstrate the relationship and we can do it in the context of understanding S O 3 and S O 2 or we can also have a look at O 3 and S O 3 and you know, some of these group properties become relevant, for our understanding.

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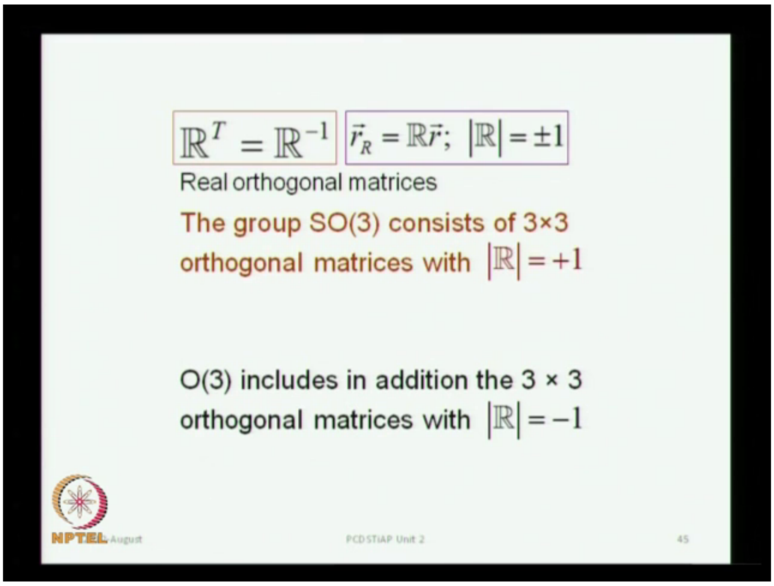

And let me remind you that, you deal with orthogonal matrices, in the case of rotations and these matrices will have determinants, which are either plus 1 or minus 1 and for the case of S O 3, you pick that subset of matrices. Those rotations, those matrices, whose determinants are plus 1, that is what makes it special orthogonal group, that is what refers to the S in S O 3.

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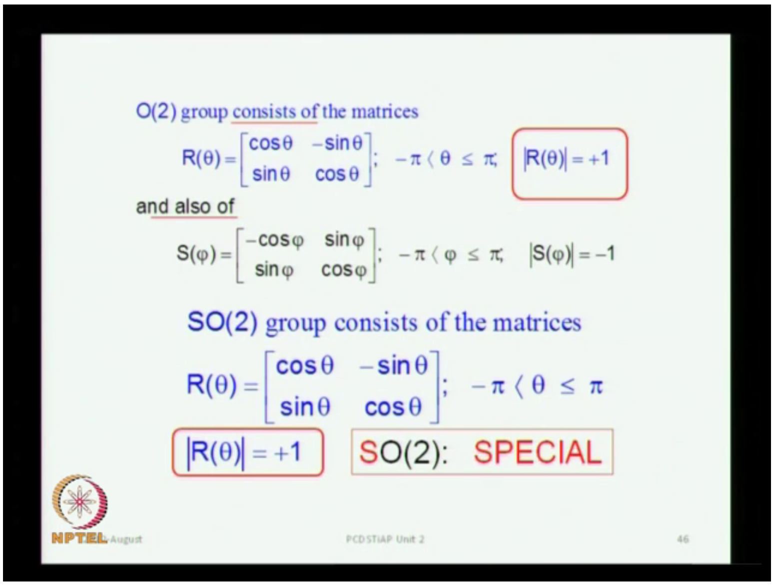

Now, you can have a similar story for the O 2, which also has got explicit representation, which you can see very easily and you can see that the determinant of Cos theta minus sin theta, sin theta Cos theta is plus 1, whereas, if you have the 1 1 element with the minus sign, then you have the determinant to be minus 1. So, if you take the subset of those matrices, which have got the determinant to be plus 1, then you get the S O2. So, these are very simple relationships, that we are working within the context of S O 2 or S O 3 and this is what makes our group special, that you are picking a subset.

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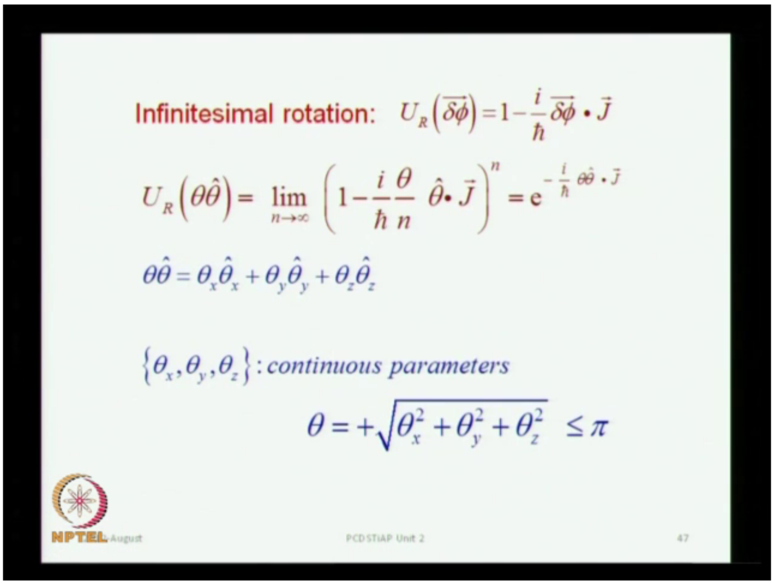

So, now when we deal with rotations our infinitesimal rotation is generated by the angular momentum operator, we have identified this operator and our unit 1, we know that it is 1 minus i over h cross delta phi dot j delta phi being an infinitesimal angle, which is a vector. Finite angles are not vectors infinitesimal angles are vectors, now what you are going to do when you work with finite rotations, now you can express a finite rotations theta, this is no longer an infinitesimal rotation.

But, you can take a finite angle theta break it into 9 piece into n pieces, so that each piece is tiny and then let this operate n times and let n tend to infinity, now what you are doing is to break it into such tiny pieces, that you will have infinitesimal rotations, which you have no difficulty with. And this series sums up to this exponential series, now I have been careful not to put a vector bar on theta, I write it rather as the magnitude of theta times the unit vector, because I do not want to recognize the finite angle theta, as a vector.

So, I write it as a product of a number times a unit vector, so it has got a certain mathematical sense that, we understand through this limit n tending to infinity of the operator for infinitesimal rotations, acting n times, n tending to infinity. So, that is the mathematical sense, that we are all comfortable with, now these angles can be changed continuously, which is why they belong to the lee group.

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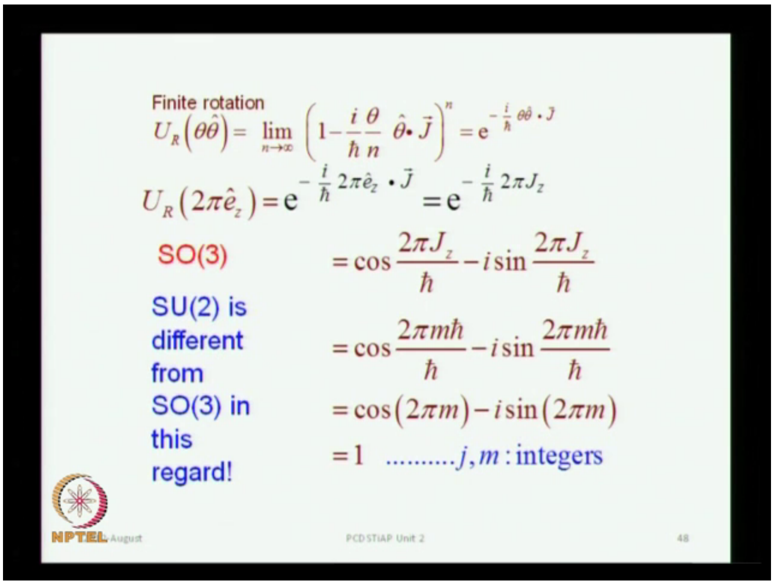

And you can work with this algebra, for finite rotations and now, if you consider this finite angle to be 2 pi, 2 pi is of course, a special angle, because we always think that when you take any object and turn it through 2 pi, you are going to recover the original state right that is your expectation. So, you take this angle theta to be 2 pi and look at this operator for finite rotation through angle 2 pi.

So, now, this is theta is 2 pi, let us say it is about the z axis, so this unit vector theta carried is e z unit vector and now, you have 2 pi times the j z operator, which is the z component of the angular momentum. Now, this is cosine 2 pi j z over h cross minus i sin 2 pi 2 pi j z over h cross and if you operate this on the angular momentum Eigen states, you know that j square j z have got, simultaneous Eigen states that you have the Eigen value equation satisfied by these operators.

So, you can operate by these operators on angular momentum states and you will get the corresponding Eigen values, Eigen value of j z is m h cross, which in the case of rotations, j are integers like the orbital angular momentum right, these are integers m goes from minus j to plus j, so they are also integers. So, this rotation operation through 2 pi is equal to the unit operator 1 as would expect, now this is the story coming from S O 3 S U 2 is different in this respect.

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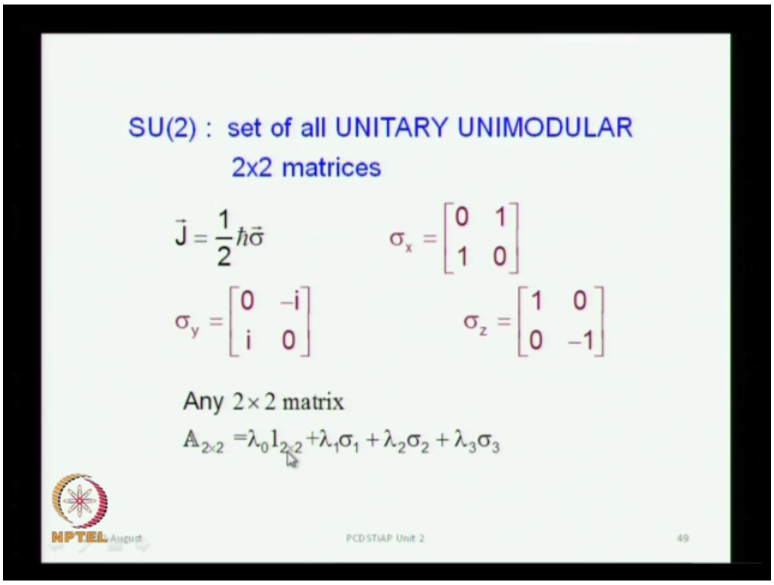

Let us see, how now S U 2 is a set of all unitary unimodular matrices, which are 2 by 2, the poly matrices are classic cases, in fact, they give you a basis for any 2 by 2 matrix, along with the 2 by 2 unit matrix. So, if you have any unitary, any 2 by 2 matrix, any matrix does not matter, what you can always write it as a linear super position of the 3 poly matrices and the 2 by 2 unit matrix, that is what is meant by a basis. So, you can always write any 2 by 2 matrix in terms of these matrices.

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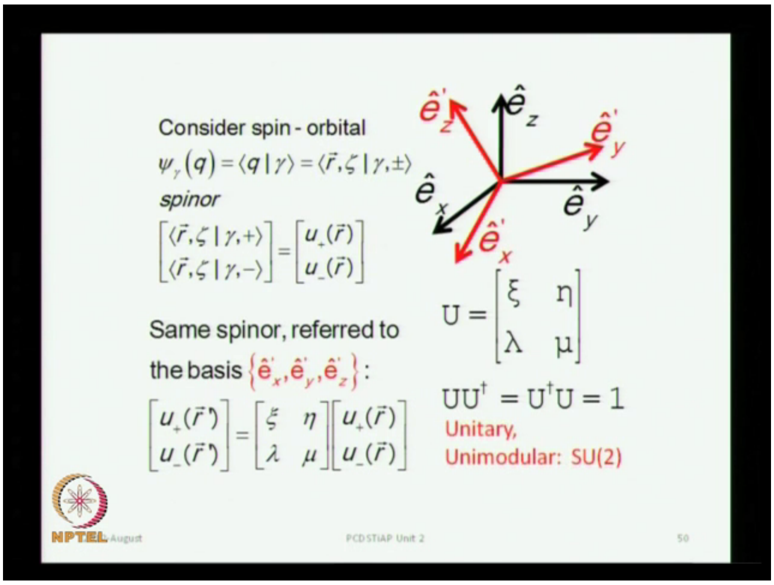

And if you refer to a certain spin orbital, now you know what a spin orbital is it has got a spin part and an orbital part, the 2 component function is called a spinner and these are the 2 components, here the position vector r is with reference to a certain coordinate system, which is e x e y e z, which is a Cartesian coordinate system. Now, what would happen, if you refer the same spinner, the very same spinner to a different coordinate system, which is rotated with respect to the previous 1.

So, we have the spinner 2 components and we refer the spinner now, to a different coordinate system, which is rotated with respect to the previous 1, as I have shown in this figure and the new coordinate unit vectors are e x prime, e y prime, e z prime. The new spinner is obtainable from the first spinner by matrix multiplying it by this 2 by 2 matrix xi eta lambda mu, all of these are complex numbers. So, you can, but these are unitary matrices, so they will satisfy this relation that, the joint is the same as the inwards that is what makes them unitary and they also have modulus 1, they belong to the S U 2.

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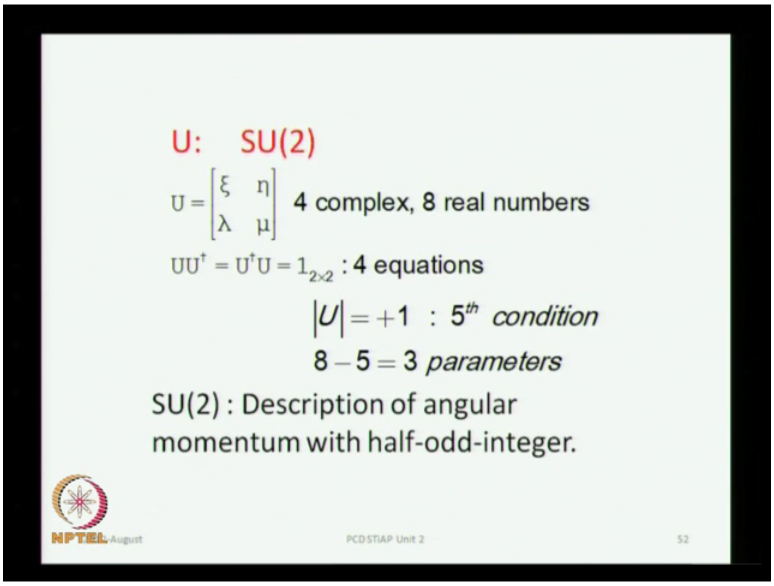

So, you do have 4 equations here, that U dash U must be equal to 1 and you also have another constraint, since the modulus of U is equal to 1, so out of the 8 real numbers that, you work with in 4 complex numbers. Each complex number has got 2 real numbers, the imaginary part is as real as the real 1 right, so you have a 5th condition and you really have only 3 parameters as 1 would expect. And the these are the matrices, that you invoke, when you are dealing with the angular momentum with half odd integer, like the electron spin, that is described by S U 2 and not S O 3.

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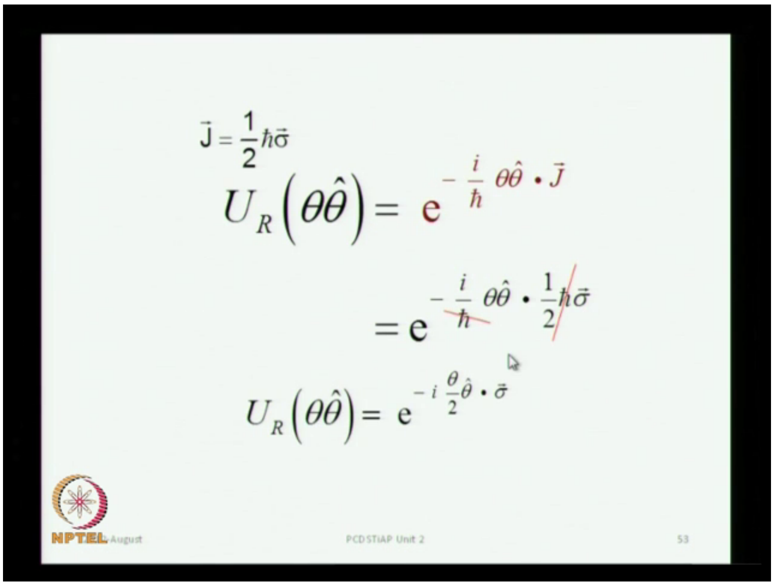

So, let us take this electron spin, which is half h cross sigma, sigma being the poly matrix vector and if you now look at the generator for rotations, using the same relationship, but now for j equal to half. So, now, this j is half h cross sigma, now let us see the fun, this is really interesting, because you have done the same thing for the rotation operator now, the h cross cancels, you now have theta over 2 times this.

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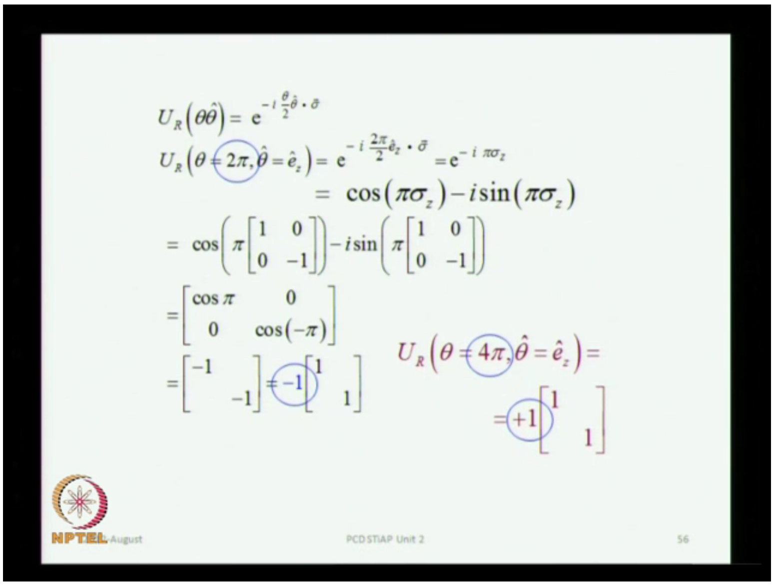

And if you now take the same finite angle 2 pi, which you had taken earlier, take theta equal to 2 pi, so this angle theta by 2 becomes 2 pi by 2 and then sigma dot e z will give you sigma z. What you get is cosine and sin of pi times sigma z, sigma z you know what they are, 1 0 0 minus 1 right, so you get the cosine and sin over here, so you get Cos pi and 0 and 0 and cosine minus pi. But, cosine of pi and cosine of minus pi are both equal to minus 1, so you get minus 1 times the unit matrix, you have rotated through 2 pi angle, you do not get the unit operator, you get minus 1.

And if you want to recover the original state, then you need a rotation through 4 pi, because if you do it twice, then minus 1 into minus 1 is plus 1, even in atomic physics right. So, you have to carry out this rotation twice and this is what distinguishes S U 2 from S O 3.

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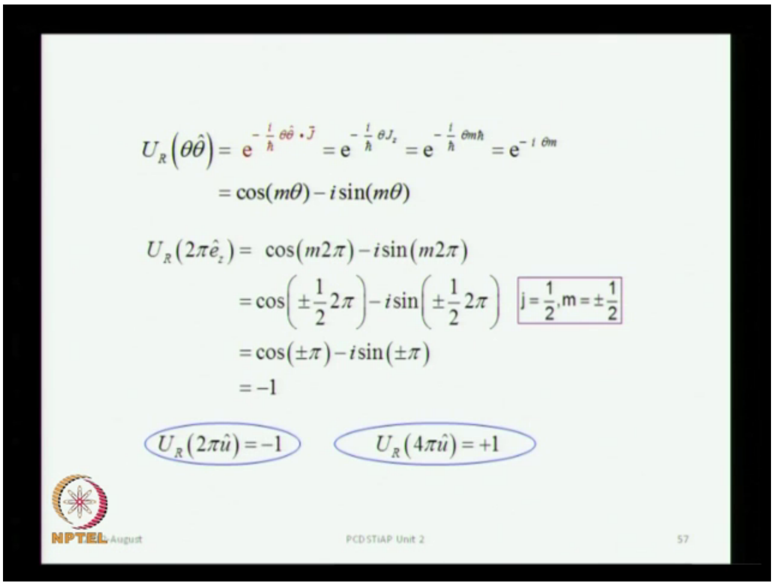

Now, you can see it in another fashion same result essentially, you go ahead and replace this operator by the j z operator and get it is Eigen value, which is m. But, m now for spin half is either plus half or minus half and you get essentially, the same result that this operator for 2 pi rotation will be minus 1, it is the same thing. And you will need to rotate this state through 4 pi to get plus 1.

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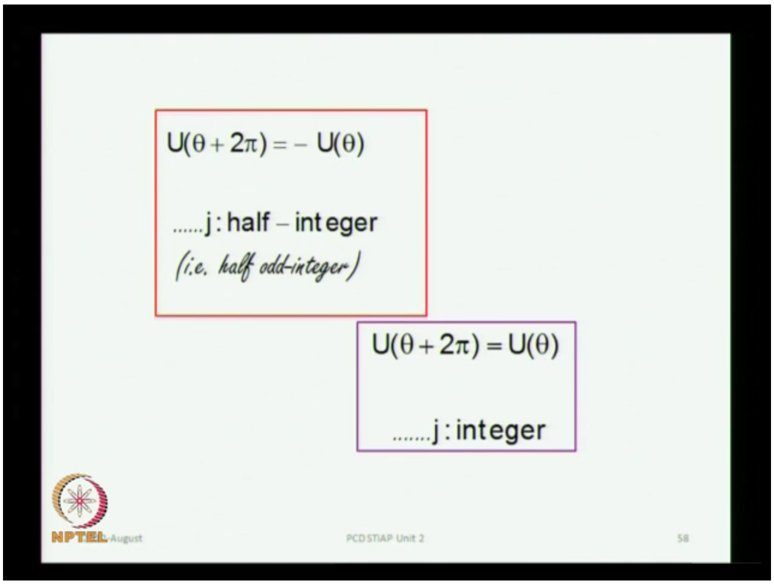

what it really means, is that when you are dealing with half integer quantum numbers, you need to work with S U 2 or else with S O 3.

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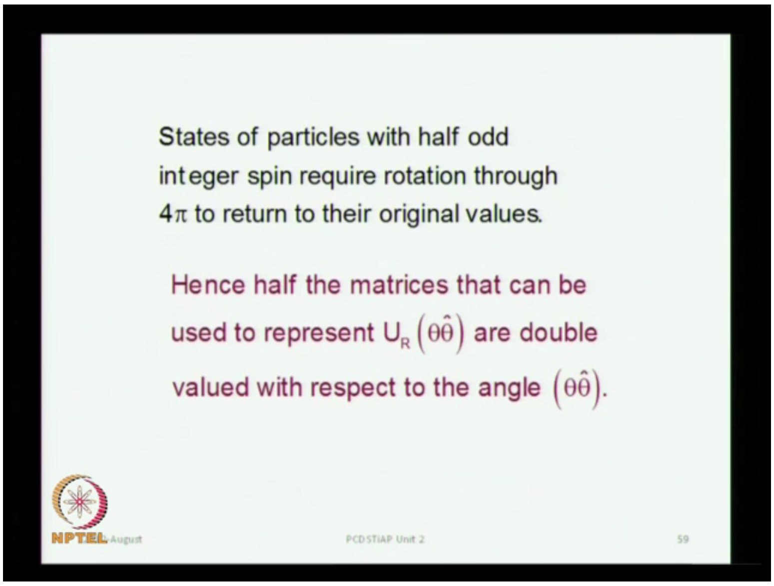

And states of particles with half odd integer spin will require a rotation, through 4 pi, which is a problem with shifts quantum mechanics or many other books in quantum mechanics, you know and it is very easy to show the results. It also means that, half of the matrices that can be used to represent this rotation operator, they are double valued with respect to this angle theta, because you need 2 of them.

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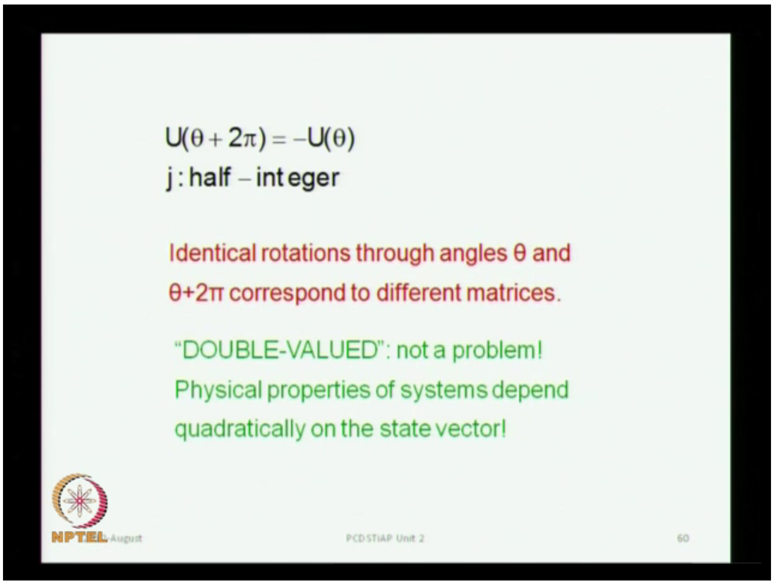

So, you can have identical rotations, through angle theta and also through theta plus 2 pi, which corresponds to different matrices, so that will happen and this is not a problem, because you really deal with quadratic quantities in terms of the said vectors. So, this normally does not lead to any problem, but so far as our understanding of the phenomenology is concerned, it is such an importance.

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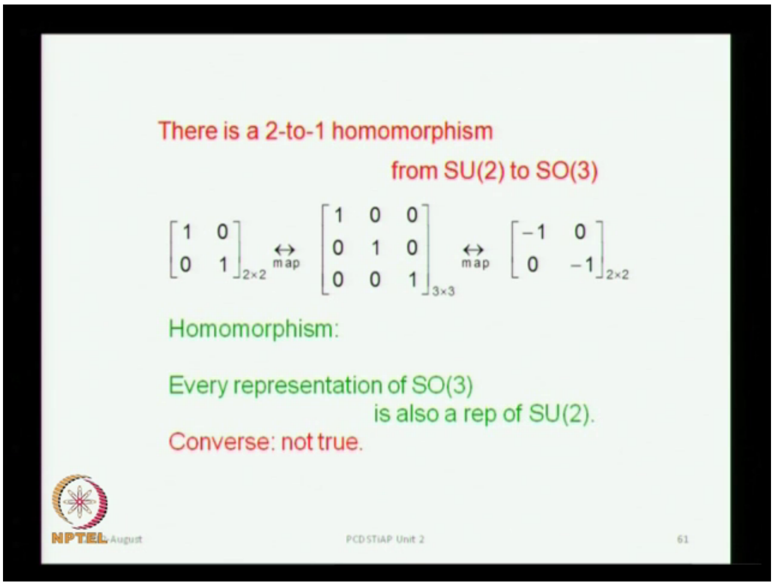

So, there is homomorphism not isomorphism between S U 2 and S O 3, the 3 by 3 unit matrix would correspond to both the 1 1 2 by 2 matrix of the S U 2, as well as the minus

1 unit matrix of the S U 2. So, there is this correspondence and it means, that every representation of S O 3 is also a representation of S U 2, but not vice versa, because you do not have a 1 to 1 mapping.

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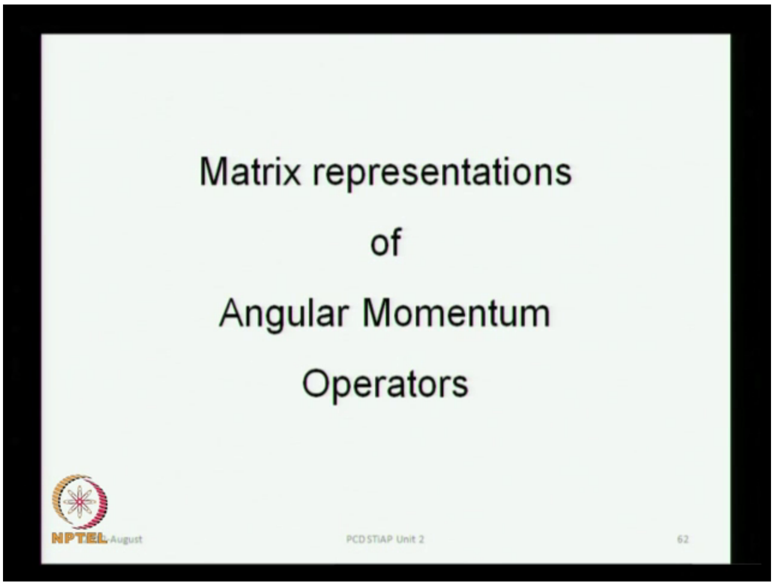

We will discuss the matrix representations of angular momentum operators now and you will see very soon, that this is very simple, but also extremely important.

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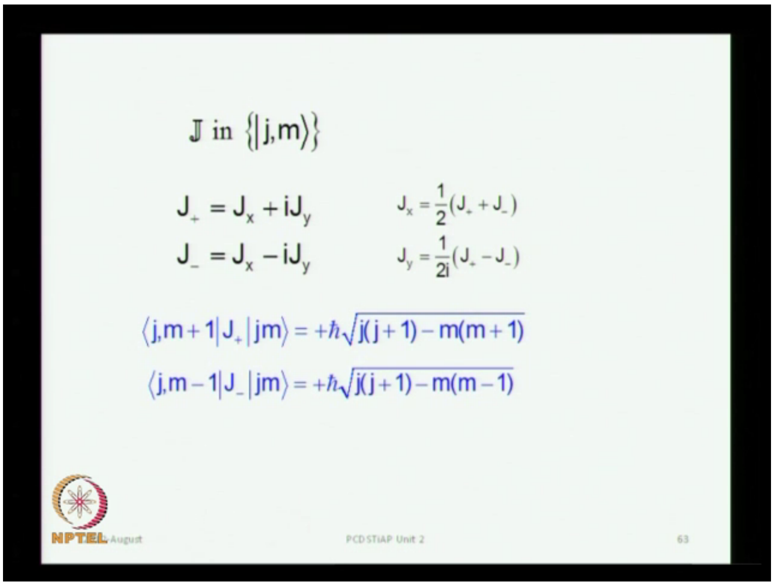

And it will be impossible to highlight, how important this really is, but you will certainly see it today, so these operators we have introduced earlier, in our unit 1, these are the step up and the step down or the raising and the lowering operators, in terms of which you can write the operators j x and j y. So, we also have these matrix elements, for j plus and j minus, which we have used in unit 1, so I will use these results straight away.

Likewise, you have the matrix element of j minus as well or if you get the matrix element of j plus and j minus, then since j x and j y are explicable in terms of j plus and j minus, you can get the matrix representation of j plus and j minus. So, this is always how you go about, getting matrix representations of j x and j y, j z is easy, because j z is diagonal along with j square. But, j x and j y are not diagonal and you should get them first in terms of the ladder operators and then use this straight forward relationship to get the corresponding representation for j x and j y.

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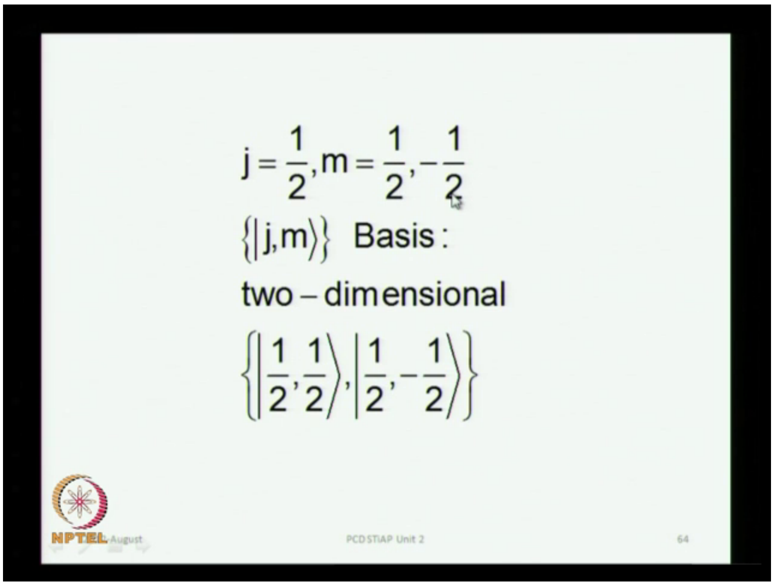

So, now, for spin half, you have got a 2 by 2 matrix representation, there is a 2 dimensional basis, the first quantum number is the j, which is half, the second quantum number is the m quantum number, which is plus half or minus half. So, you have got a 2 dimensional basis here.

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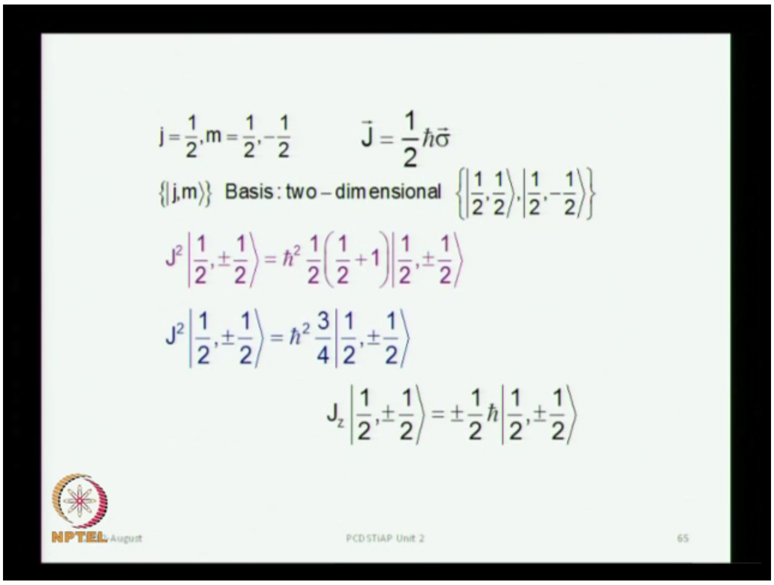

So, let us get the matrix representation of various operators j square is very easy, because it has got an diagonal representation, you know what the Eigen value is, which is h cross square into j plus 1, so half into half plus 1 will give you 3 by 4. And similarly, j z is also diagonal, so it has got an Eigen value equation, whose Eigen values are either plus half or minus half times h cross.

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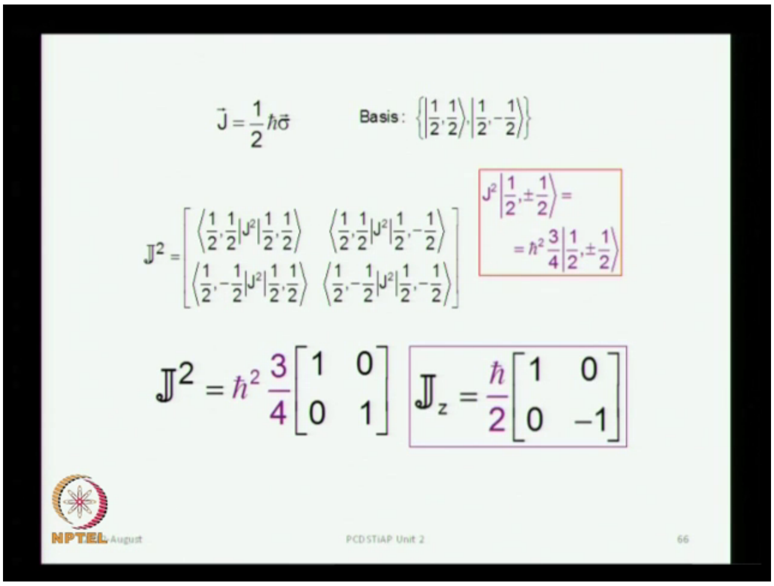

So, you write the matrix representation in this basis of J square, write the 1 1 element, the 1 2, 2 1 and 2 2, that is it, you get it all. And using the matrix elements and the Ortho orthogonal relationship of the base is set, you get the explicit matrix representation of J square and likewise, for j z.

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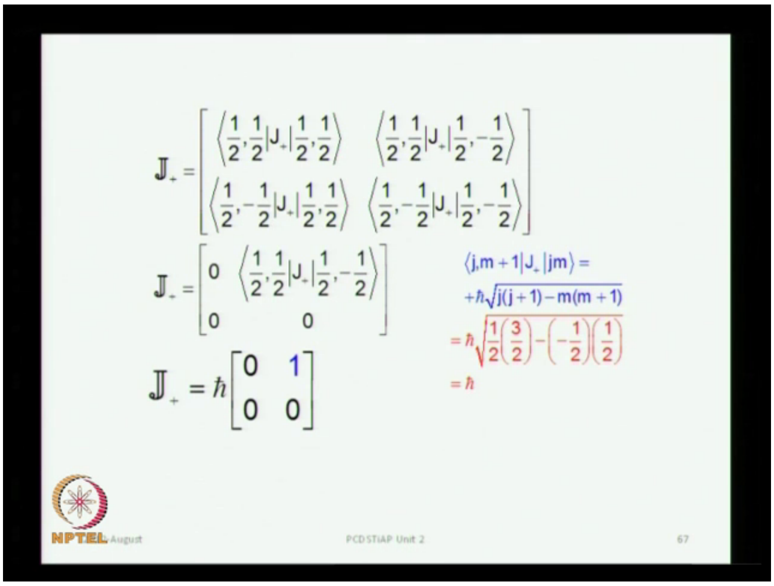

What about J plus before, we get J x and j minus, we will first get the matrix representation of J plus and j minus and we know, how to do that, because we have met these matrix elements. So, you get 0 and 1 1, 2 1 and 2 2 positions and only 1 2 is non 0, you plug in the value, which you already have and you get an explicit matrix representation of J plus, which is h cross time 0 1 0 0.

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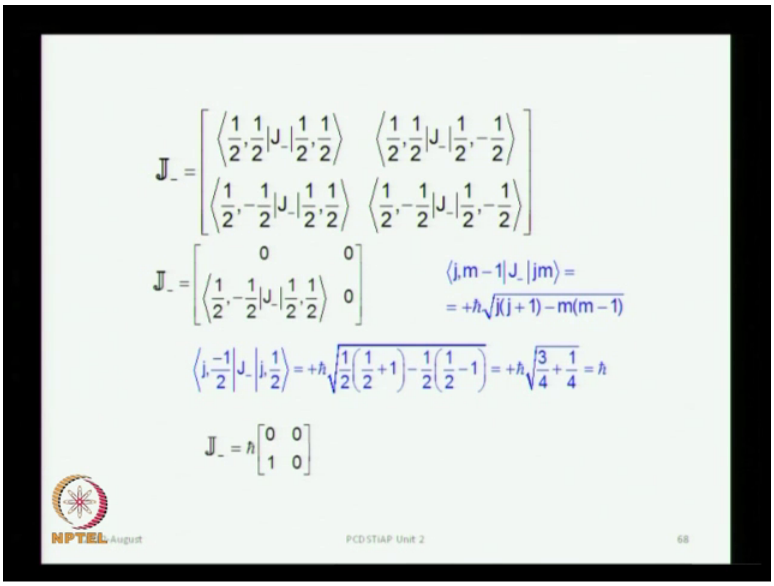

Similarly, you get the matrix representation of J minus by doing exactly the same kind of algebra and once again, you have the other relation for the step down operator and using that, you get the matrix representation for J minus, which is h cross 0 0 1 0. Now, you have the matrix representation of J plus and J minus both.

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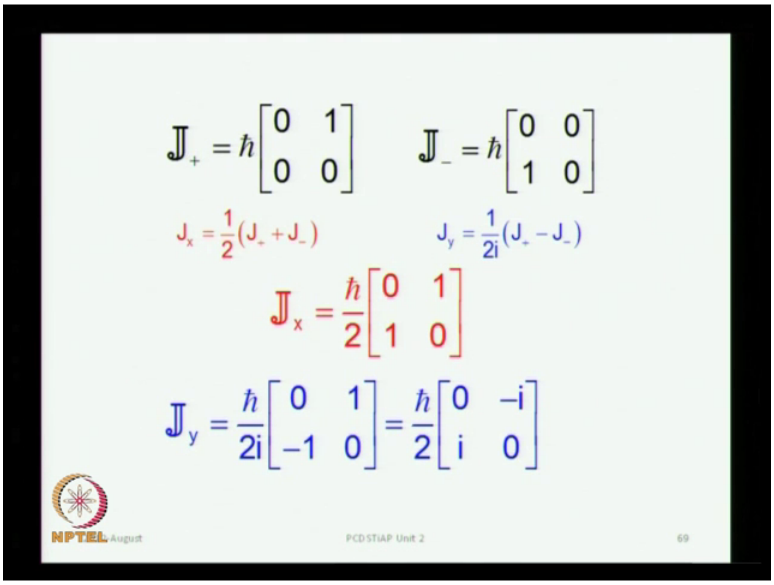

So, you can get the matrix representation of j x and j y. So, J x turns out to be h cross by 2 0 1 0 and J y is h cross by 2 0 minus i, i 0, which is where the poly matrices come in.

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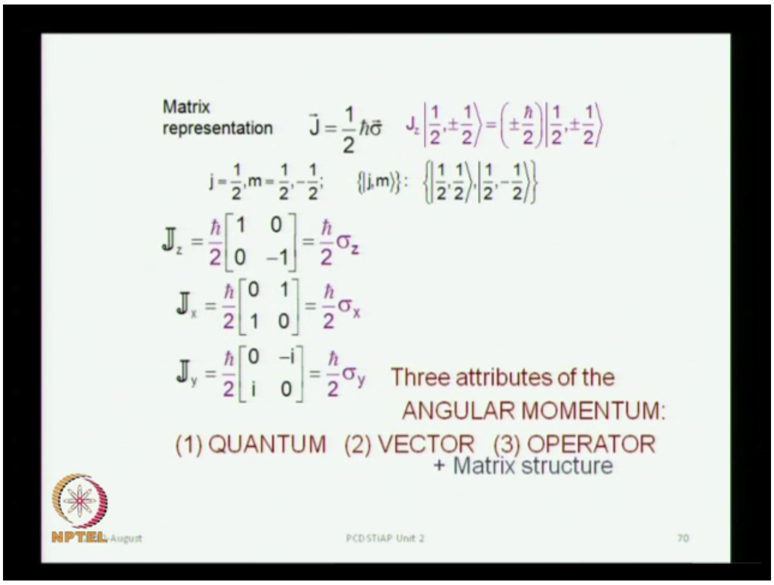

So, these are the poly matrices, the sigma z, sigma x and sigma y and you must keep track of all the attributes of the angular momentum the quantum nature, the vector nature the operator nature and now also the matrix structure. So, whenever you deal with these operators and that is what you are going to do lot in unit 3, you must keep track of all of these attributes, the mat matrix structure, the vector structure, the operator structure and the quantum structure all of this.

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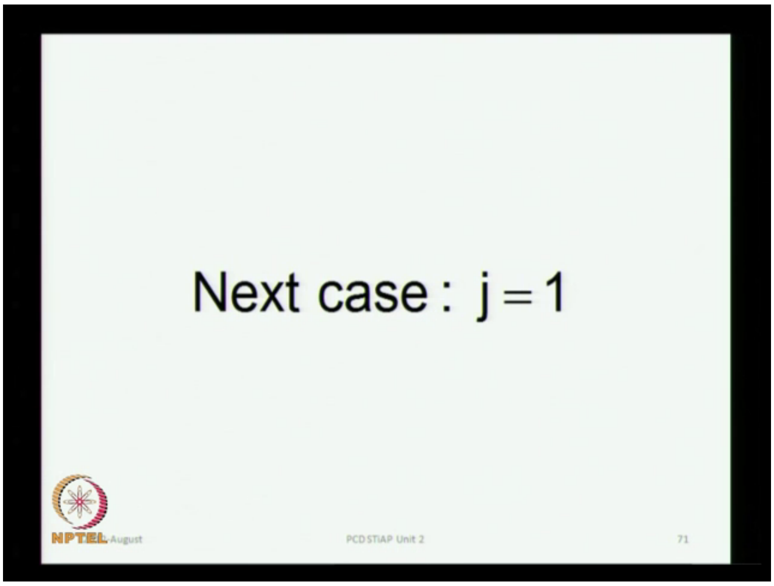

That you thing that you do can be inconsistent with any one of these features, what about j equal to 1.

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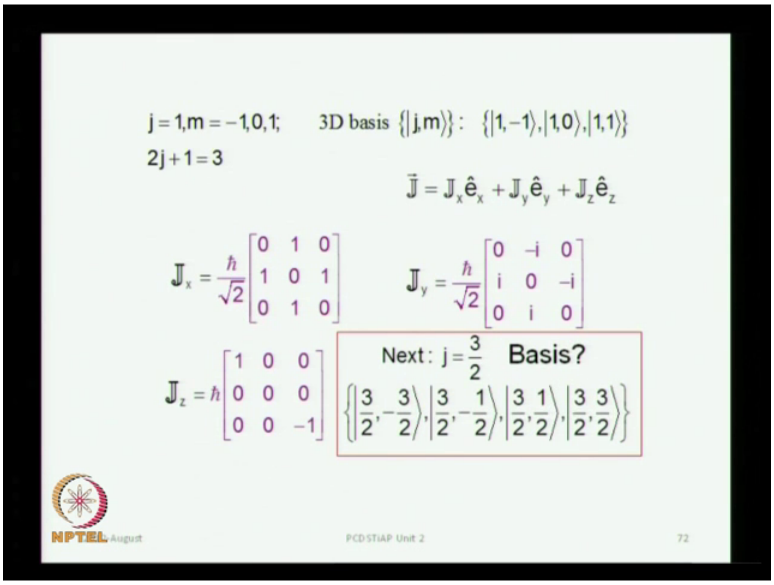

So, now, you have got a three dimensional basis, because for j equal to 1 m will take 3 values minus 1 0 and 1 3 is nothing new about it and you can get the matrix representation of all the operators j square j plus j minus then J x J y everything right. And I will like you to do this, for you self and obtain these explicit matrix representations, for j equal to half, j equal to 1 and for some other angular momentum just to get used to it for j equal to 3 half, for example. Now you will have a 4 dimensional basis, because m will go from minus 3 by 2 to plus 3 by 2 in steps of 1, so minus 3 by 2 minus half plus half and plus 3 have, so you will have a 4 dimensional basis and you will have 4 by 4 matrices.

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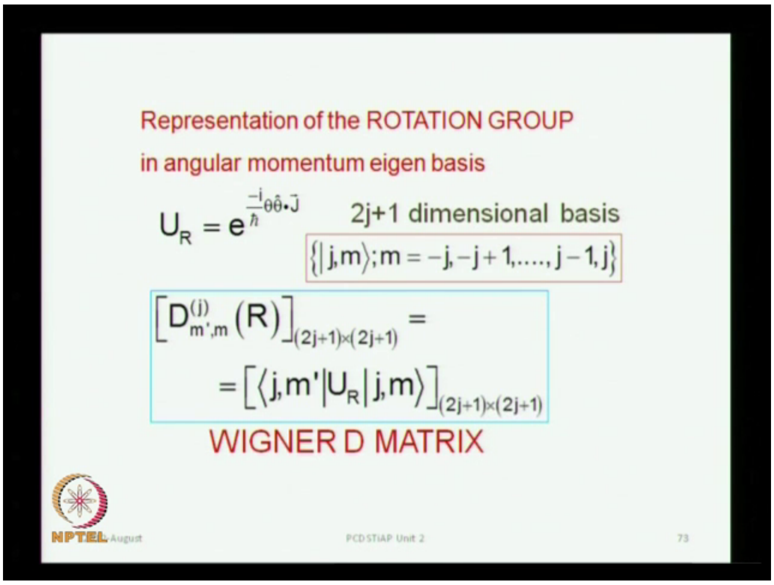

So, the size of the matrix also goes on increasing and these are the angular momentum and the angular momentum Eigen basis, these are the matrix representation of the rotation group. So, this is the rotation group and you can obtain, it is matrix representation in a basis, which is 2 j plus 1 dimensional, these matrices are known as Wigner D Matrices. And the Wigner D Matrices are matrix elements of the rotation operator, in angular momentum base as such, a very simple to obtain, for any value of j, we just saw how to get them, these are extremely important Wigner D Matrices, as they are called.

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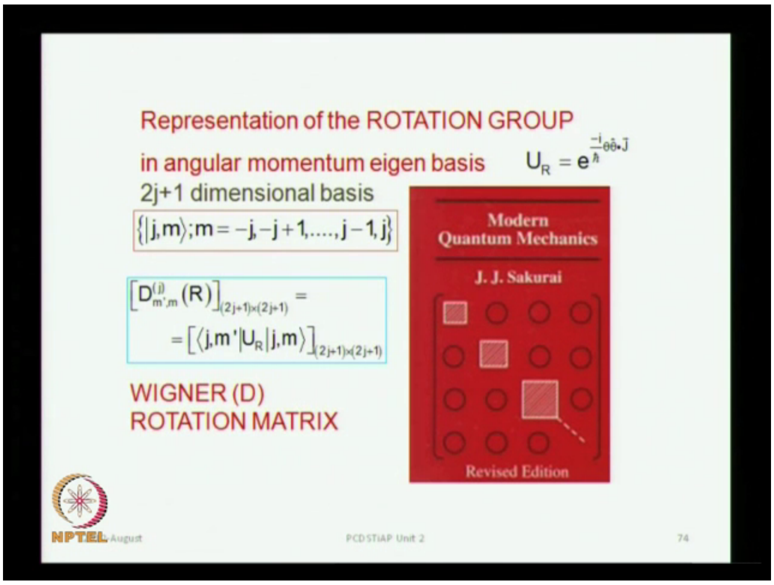

And they will all be 2 j plus 1 by 2 j plus 1 matrices, their size will grow with j and the reason, they are important, if it had not occurred to you yet, is that you would have seen them on the cover of Sakurais quantum mechanics book. And that book deals with, so many topics in quantum mechanics and what they pick out of all that to be placed on the cover are the Wigner D Matrices. So, they better be important and they certainly are you will find that, you will need tremendous expertise in dealing with these matrices.

You will need tremendous competence and if you have difficulty dealing with angular momentum algebra and commendation properties between angular momentum operators and lens operators and so on. Here is a classic book, which is used all over the world and it is telling you that, please learn this carefully thoroughly, have a good handle on this, that is precisely, what this cover is telling, you it is speaking to you now.

So, please develop tremendous competence and comfort with the algebra of angular momentum operators, angular momentum Eigen states, the lens operators, because S O 3 is a subset of S O 4, which is the collect symmetry of the hydrogen atom right. And I made an attempt to introduce you to these topics, expertise you will have to develop on your own.

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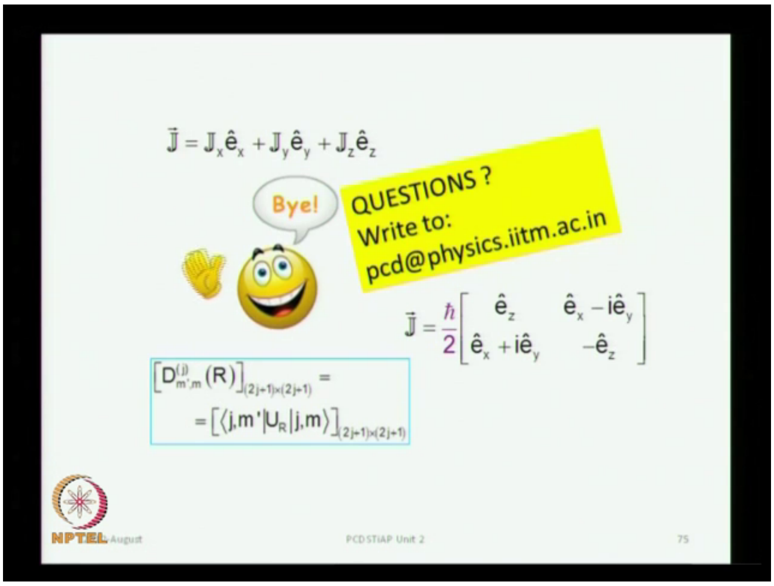

So, I guess, I will stop here today and we will continue the discussion from here tomorrow is there any questions, I will be happy to take.

Students: Can you please explain about, difference between electron spin and silver atoms.

Yes the silver atoms this is with reference to the stern and Gerlach experiment, now what happens is when you put a magnet in magnetic field, it aligns itself along the magnetic field, which is what you do in a magnetic compass that, you might wear on your wrist. Now, when you talk about classical magnets, you always think that, they align themselves exactly along the magnetic field that does not happen with real magnets, which are essentially quantum.

Because, the quantum magnets, have got a magnetic movement, which is proportional to the angular momentum is such an observable that you cannot get all the 3 components of this vector observable, you can get only one of the component along with it is size, which is given by j square, the size is given by j square. And one component is given by the Eigen value of J z, when you perform this measurement the system collapses into an Eigen state of J z and that is what you measure, but then it is not in simultaneously in an Eigen state of J y or J z or J x, it is in an Eigen state of J z right.

Now, you can have these 2 orientations for j equal to half, because m goes from plus half to minus half in steps of 1. So, the silver atom has got 1 unpaired electron, it is got it is like n s 1, it belongs to the first group and this unpaired electron has got a net magnetic movement. So, this magnetic movement can align itself, either along you know with, it will have 2 Eigen values, which is a plus half and a minus half and these are the 2 components, which gets separated out when you send a beam of silver atom through the stern Gerlach magnet now, so far so good.

Now, if you do the same with the electrons, the electrons do have this magnetic movement and the mu dot b coupling will give you the same kind of result, but that is not the only thing that is happening to the electrons. Because, in addition to this the electrons have got some difference with the silver atoms, the difference is in the charge, the silver atoms are neutral particles, there are how many 47, what is the atomic number, I had it on the slide actually, I believe it is 47.

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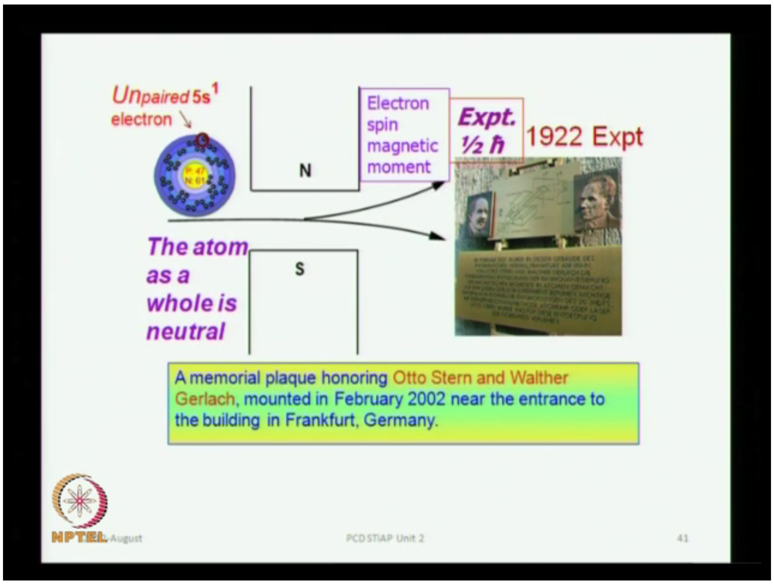

So, there are as many protons in the nucleus as the number of electrons, in the atom 47, it is written here in the middle of the atom, the number of protons is 47, the number of neutrons is 61. So, the 47th electron is outside, that is unpaired electron, which is giving you the magnetic movement, but the atom on the whole is electrically neutral, there are as many protons in the nucleus as the number of electrons outside it.

So, it is a neutral atom with a net magnetic movement, when you send in a beam of electrons, you have charge particles with the magnetic movement and the charge particle responds to the magnetic field, also through this q into v cross b term, which the silver atom will be insensitive to, because the silver atom the charge q is 0. So, there will be no q into v force on the silver atom, but you will have that force, on the electrons, now Kessler has got a book, I believe the tile is electron spin by Kessler.

And the very first few pages Kessler discusses this, that when you consider this phenomenology, of the Lawrence force on electrons, together with this stern Gerlach effect, which the electrons will also experience, because they do have this magnetic movement. But, in addition to that they experience this Lawrence force, so the combination is also constrained by the principle of uncertainty; that is fundamental to every process.

So, the combined effect of the Lawrence force and the uncertainty principle on the electrons is that they do not separate like this, so that is a little bit of algebra, which you can find in Kessler's book, any other questions.

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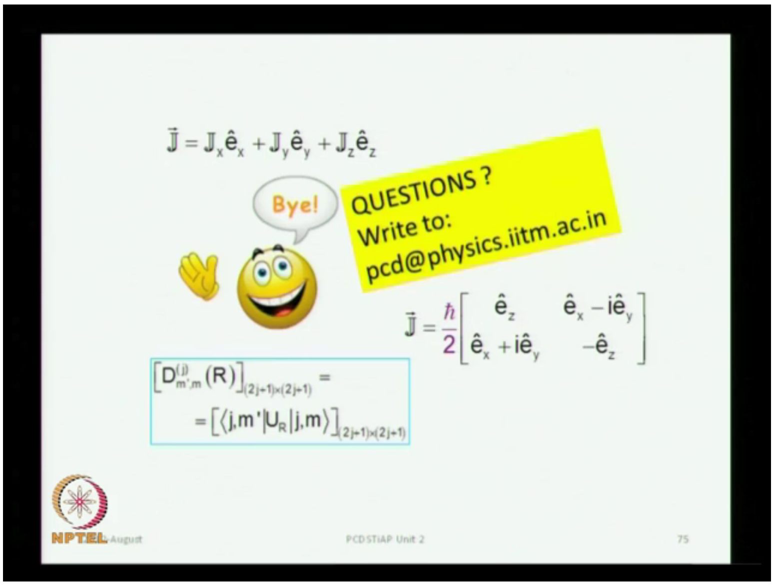

So, thank you very much.