Select/ Special Topics in Atomic Physics

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Lecture No. # 06

Angular Momentum in Quantum Mechanics

Greetings, we will begin unit 2 today, this is an Angular Momentum in Quantum Mechanics. We have already introduced angular momentum as such in unit 1, because the whole machinery of the rotation symmetry of the hydrogen atom was intimately geared to the angular momentum algebra. So, we will develop it further, but in more formal way now and some other results that we had sort of anticipated or assumed, we will establish them more vigorously now.

In particular, there are some questions, which sometimes are quite interesting for example, one might asked, is relativistic mechanics needed only when you are dealing with particles, which are moving at extremely high speed like relativistic speeds 0.9 C, 0.8 C. These are relativistic speeds and if you are not dealing with very high speed, do you need relativistic mechanics at all, this is the question which sometimes people ask.

And very often people believe, that you do not really need relativistic mechanics, very often you say that, you do not need quantum mechanics if you are dealing with microscopic objects. If you are dealing with big objects this water bottle, the laptop, the furniture in this room, motion of trains and aero plane and so on, you do not really need quantum mechanics. You need only when you are dealing with small tiny particle, atomic particles, elementary particles and so on.

When often believes or when often asks, do you really need relativistic quantum mechanics if you are dealing with the electrons spin, because you can deal with half angular momentum quantum numbers in non relativistic mechanics. So, do you really need relativistic quantum mechanics, so these are some questions which people ask. And one really needs very precise answer and I will believe that, you already have some of these answers and if you do not have these answers, I expect you to find these answers in this class.

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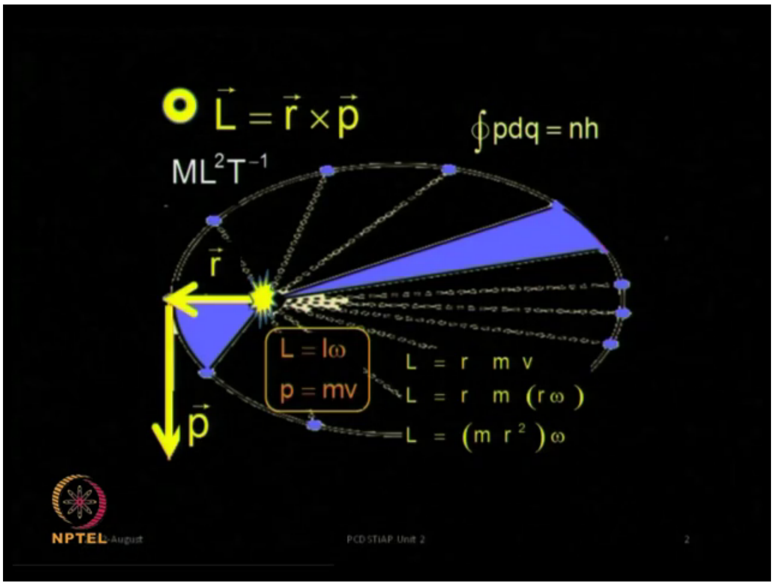

Now, angular momentum in classical mechanics is very well known thing and you have the Kepler problem, the two body problem. And angular momentum is defined as r cross p and it has got a magnitude, which is r times the momentum, so it is sometimes called as moment of momentum for this reason. You can see that, it can be written as a product of the moment of inertia and the angular velocity and the angular speed in a form, which is very similar to linear momentum, which is a product of mass times the linear speed.

And then, in the Bohr model of the quantum mechanics, when you go over to quantization, it does appear once again as this line integral of p d q which is quantized, which was done in Bohr's model of the old quantum theory, but that is valid only for periodic orbits, not for open orbits.

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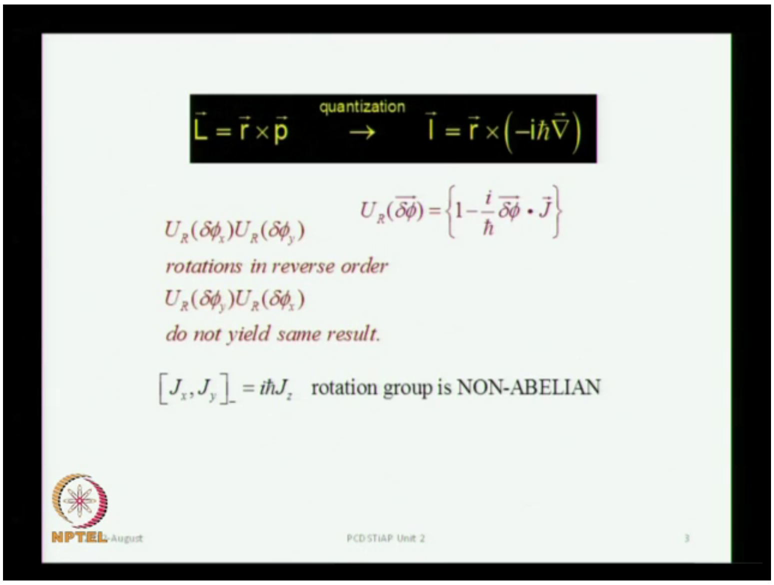

Now, if you quantized this, you replace the dynamical variables by corresponding operators and then, instead of p, you get the quantum operator, which is minus i h cross gradient operator, this is one way of quantizing the system as such. What it leads to is that, when you connect it to the generator for rotations, which angular momentum is and you know this for classical mechanics as well. What happens is that, if you perform rotation about x axis and y axis in one order and then, do it in the reverse order, you do not get the same result, I refer to this exercise in number 1 unit as well.

As a result of this, the operators in quantum mechanics J x and J y do not commute, the rotation group is therefore, is non abelian. And this commutation relation is what we proceed, to take as the very definition of angular momentum in quantum mechanics.

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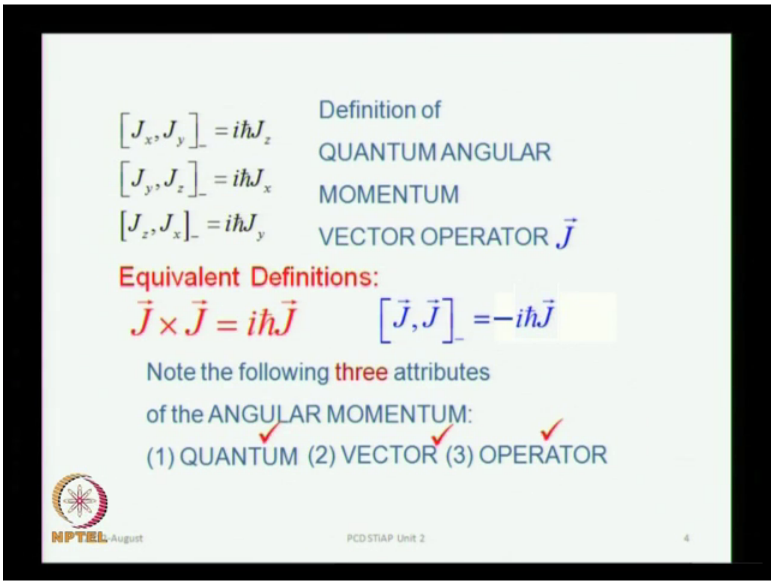

So, the definition of angular momentum in quantum mechanics is given by these commutation relations. I have shown in this class that, she found that J cross J turnout to be 0 and that J comma J also turn out to be 0 while doing the algebra. So, I think it is obviously some careless mistake that she would have done and I hope that, all of you have worked this out to convince yourself that, the quantum J operator does not commute with itself.

The commutator of J with J is not 0, it is i h cross J, likewise the cross product of these two operators is not 0, so these are equivalent definitions of the angular momentum. And it has got these three attributes that, it is the quantum creature, it is a vector creature and it is an operator. So, all of these attributes must be respected when you deal with the angular momentum operator.

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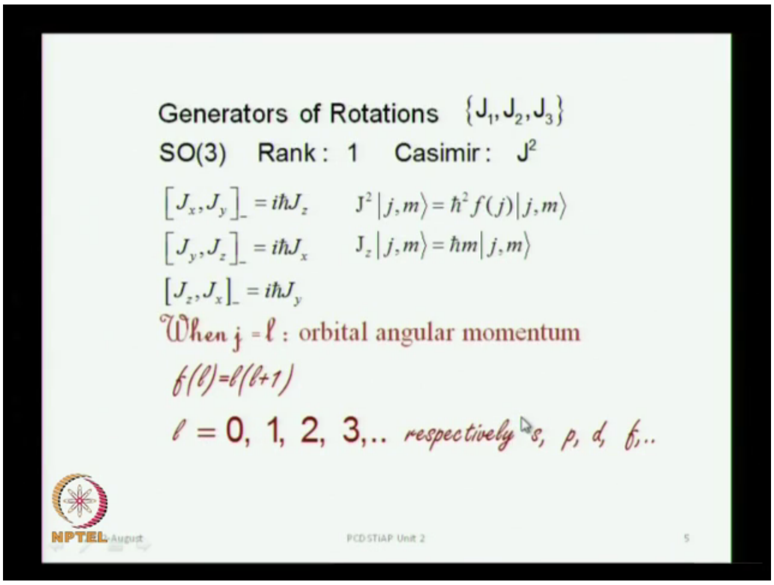

Now, we have already equated ourselves with a fact that, these are generators of rotations and then, of the SO 3 group, whose rank is 1, whose Casimir operator is J square. And because, J square commutes with every generator, J square and J z can be simultaneously diagonalized, what it means is that, these correspond to simultaneously measurable quantities, their compatible measurements.

You can therefore, have simultaneous Eigen values of these two operators, you can perform a measurement of J square. Then, perform a measurement of J z then, come back and perform a measurement J square and do it in any which order and the Eigen values will remain intact. Because, what the measurement does to a quantum system, is that the system collapses into the Eigen state of the measurement. And once it collapses into that Eigen state, it stays in that as long as it is not disturbed by another measurement, which is not compatible with this.

If it is disturbed by the measurement, which is not compatible with this then, it will not remain in that pure state, it will have to be expressed as a linear superposition of the entire basis center. So, this is what it means and as a result of this, the Eigen vectors can be labeled by two quantum numbers, both of these are the good quantum numbers j and m. And what we have done is, we are going to find out for ourselves, what this j should be and what this m should be.

So, I am aware that, you have some earlier knowledge about this from your previous courses, you know that j can be either 0 or half or 1 and three half and so on. You also know that, m goes from minus j into plus j in steps of 1, what we are going to do is, to establish this on first principles. So, we will not assume that, we have already know what j will be like, we are not going to assume that, we already know that, m goes from minus j to plus j.

But, we will do a simple analysis and discover that, this is what it will have to be, it cannot be anything else. So, it is not something like a result that we have take for guaranteed or make assumption or anything, we will see from very simple analysis and the most beautiful thing about this is that, in physics you like to get results based on this smallest number of assumptions. You should not add any additional assumptions and what I am going to demonstrate here is that, the algebra of angular momentum that you see in the screen, that just a definition of angular momentum.

It is defined by, how it is components commute with each other, this is sufficient to lead to the consequence, that j will have to be in number, which is either 0, half, 1, etcetera and m will have to be number, which is from minus j to plus j. So, this will come automatically just from the very definition of the angular momentum, which is here, nothing else has to be used, nothing has to be invoked, nothing has to be assumed. You have already done the Schrodinger equation for the hydrogen atom and from the solution to the spherical harmonics you know that, when you are dealing with orbital angular momentum, L can take these values.

Now, these are integers, there are no half integer here when you are dealing with the orbital angular momentum, there is no half integer. But, in general, these half integers are admitted in the angular momentum algebraic scheme.

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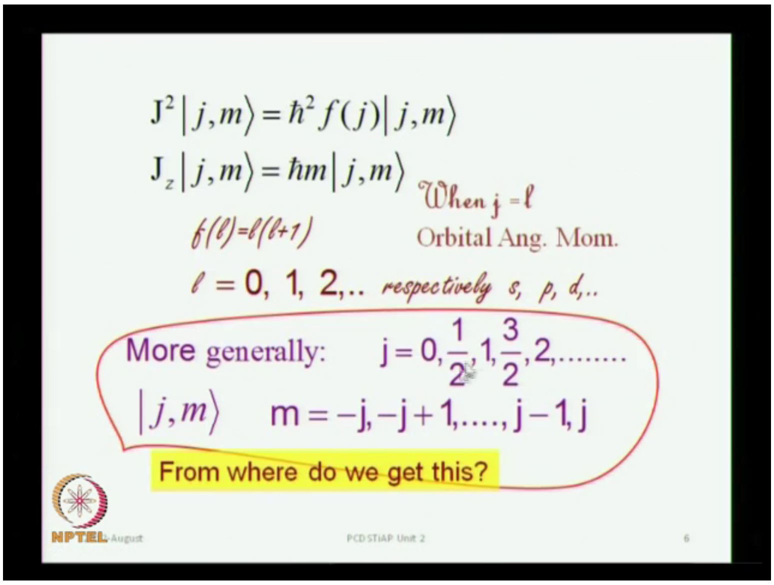

And we have to find out, from where these really come, there are some related questions, as to how we get this.

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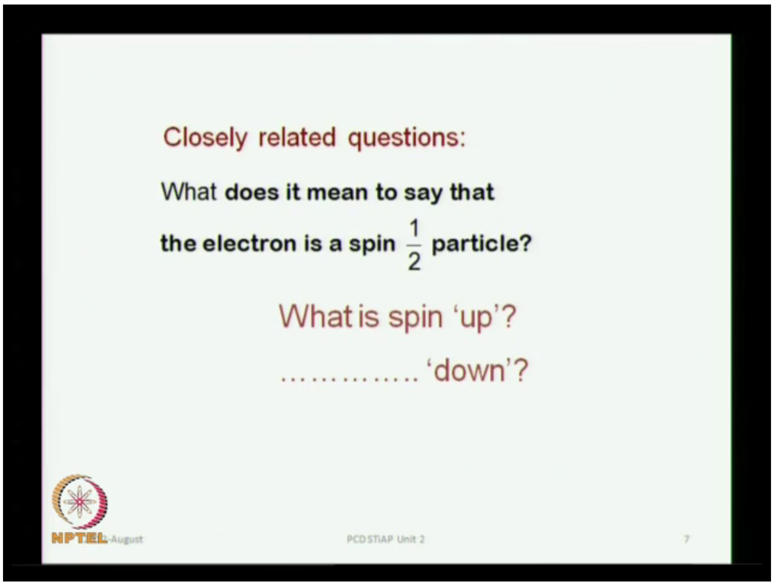

When we say that, the electron is spin half particle, what exactly do we mean by this, what does it mean to say that, the spin is half and spin is down. These are some related questions and we will look for very regress answer to this questions.

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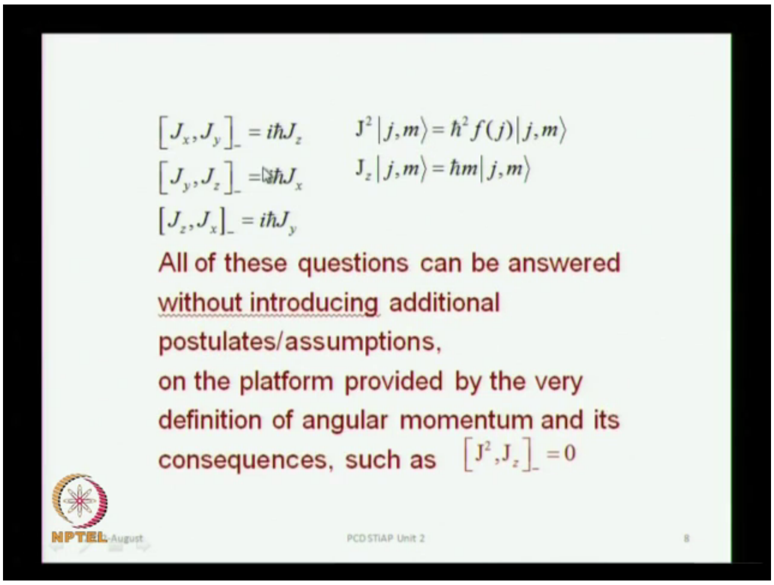

And all of these questions will be answered based on this very simple defining relationship, nothing else is needed. You do not have to make new assumptions or additional postulates.

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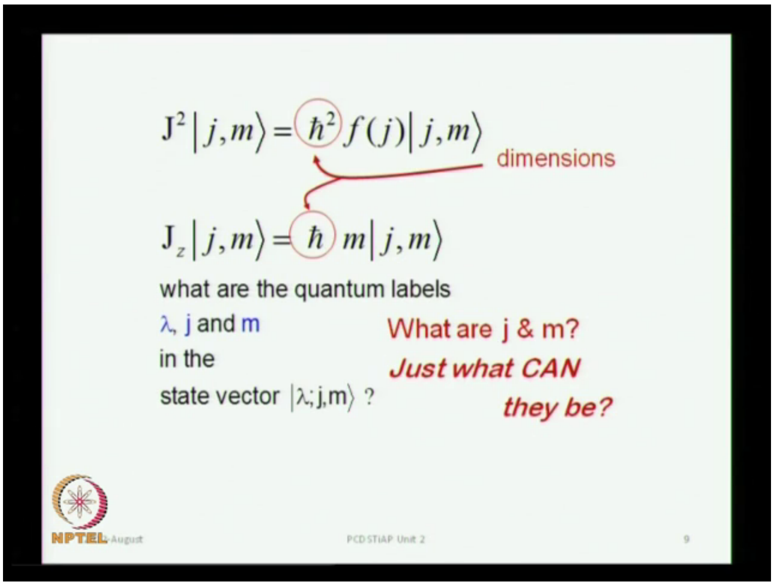

So, these are our basic Eigen value equations, we know that J square and J z will have simultaneous Eigen states, they can simultaneously diagonalized. So, you can write Eigen value equations for both of them and used the Eigen values to label of states. This is what we have learnt already in unit 1, that your label state vectors by Eigen values of those operators, of which these are Eigen vector. So, the Eigen vectors are label by j and m, where the labels come from the Eigen values.

From the Eigen value, I extract the dimension separately, so this is J square, so there will be a dimension of angular momentum square in the Eigen value, which I extract in units of h over 2 pi. And then, I am left with a residual Eigen value, which will be just a number, which will be dimensionless number, whatever it is, which I call as some functions of j and used that label j to describe the vector. Likewise, for J z, I have extracted the dimension h cross over here, which is angular momentum dimension, which is h over 2 pi rather than h, that just a matter of convenience.

And then, there is an additional label which is m, but at this stage, I really do not know what m is, now do I know what j is, that is something which we have to figured out. There can always be an additional quantum number like lambda, it will come from Eigen value of some other operator, with which J square and J z both commute and there can be one or more such operators. So, lambda represent the family of all these additional operators, which also commute with J square and J z.

For the hydrogen atom, you already know it is a Hamiltonian, but it could be something else. So, we ask what can be j and m, they can be only one they turn out to be, cannot be anything else.

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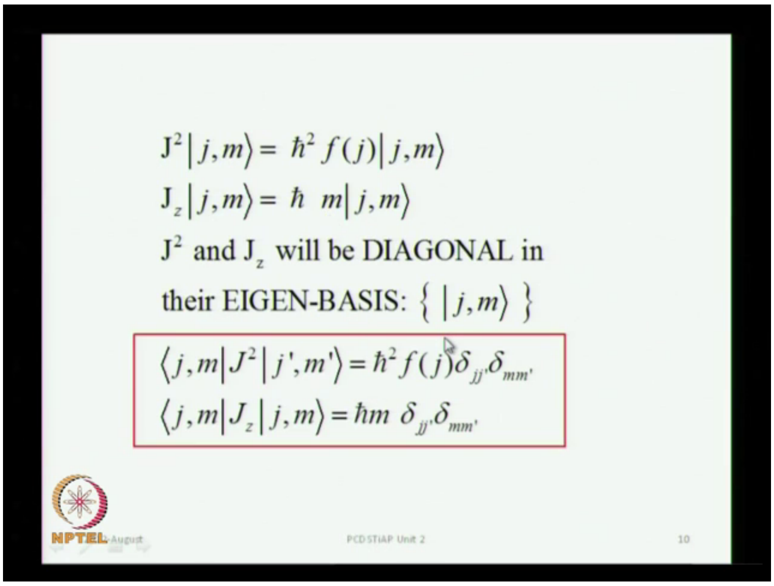

So, we will find out what j and m turnout to be, now we know that, in this basis J square and J z both must be diagonal, because they can be simultaneously measure. So, if you determine a matrix element of J square in the J m basis, it must be diagonal in the j index as well as in the m index, that is what is meant by a diagonal representation. So, both of these matrix element, matrix element of J square as well as matrix element of J z must be diagonal in j n m index, which is I have represented here. The rest of it is coming straight from this h cross f j and h cross m, fair enough.

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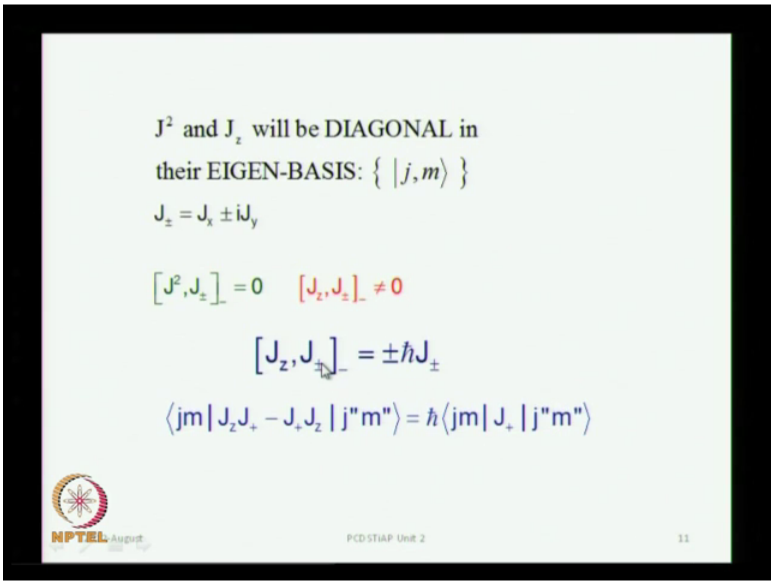

So, we know that, now if we construct these operators J plus and J minus, which is the sum and difference of J x and i J y then, it is clear that, J square will commute with J plus and J minus, but J z will not commute with J plus and J minus. And if you just use a original algebraic equation, this is the point which I am emphasizing repeatedly, that all your using is the defining relationship for the angular momentum. The J x comma J y is equal to i h cross J z or J cross J is equal to i h cross J, just these definitions do not use anything else.

Do not use what you know from other sources, just use this and using this you find that, J z does not commute with J plus and J minus. And using these basic commutators between J x and J y, you can easily show that, this commutator is not 0, rather it is plus or minus h cross J plus or minus. So, this is very a simple algebra that you can work out, all you do is, plugging the original commutation relations.

What it means that, if you take the matrix element of the left hand side which is here, in two states j double prime and m double prime and j m. It is equal to the matrix element of the corresponding matrix element of right hand side and I am taking the case of J plus, just to demonstrate this argument.

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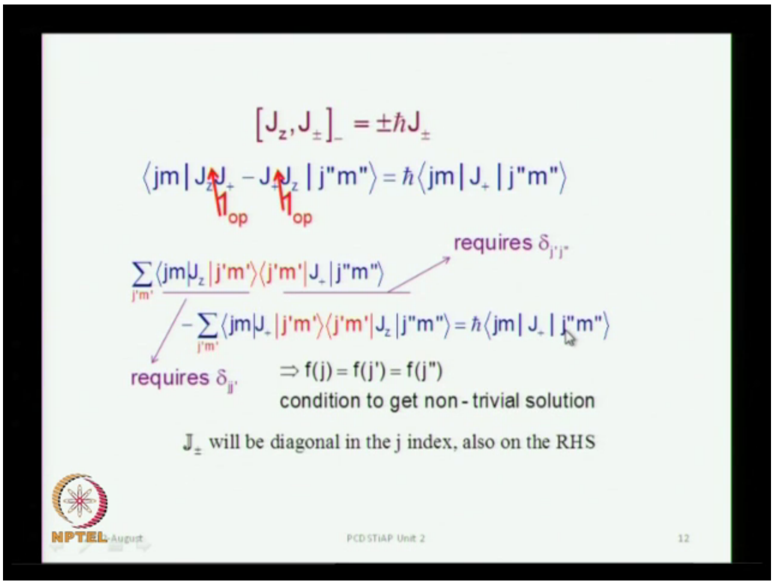

So, here you have this relation, this matrix element is equal to matrix element of h cross J plus, h cross is a constant which I have extracted out, I can always insert a unit operator here, I can insert a unit operator here as well. You know how to resolve the unit operator, so resolve the unit operator in this basis, j prime m prime, j prime m prime, this is the ((Refer Time: 16:34)) summed over j prime and m prime. This is the resolution of the unity, which I have exploded and what you immediately find is that, when you look at this matrix element, this is a factor here.

The first term is product of two scalars, the first one will require a delta j j prime, because J z is diagonal in j index. Likewise, the second term requires orthogonality of j prime j double prime, because J plus is also diagonal in the j index. So, we can exploit this and what it essentially means that, the function J and j prime and j double prime, these are must all be equal to get non trivial solutions, they must be equal to each other, that automatically follows. And this would be the case on the right hand side as well, so this J and this j double prime must also be equal.

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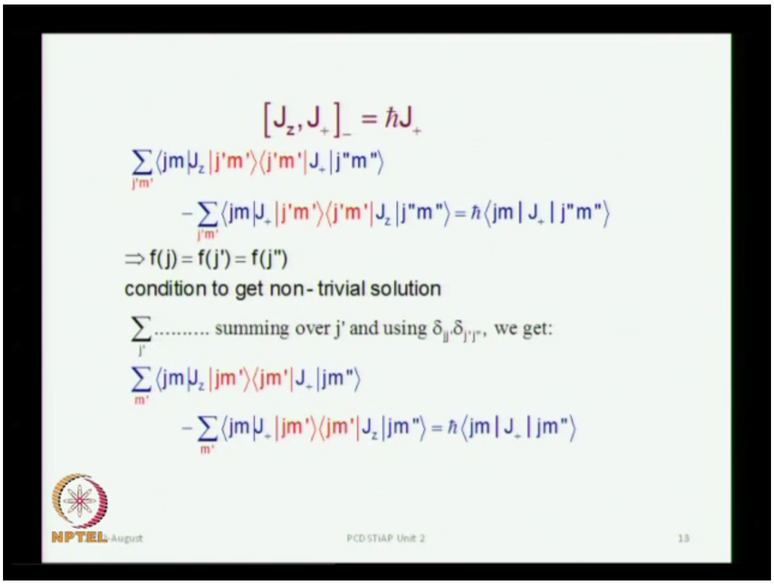

So, here we have the main result and now we proceed by carrying out these sum over j prime and we now have this orthogonality between J and j prime and j prime and j double prime. So, you are left with the sum over m prime, all the sums over j indices have been carried out. And what remains, we will have all the js, J, j prime, j double prime, all of must be equal to each other, for which I used a single symbol, which is J itself, so let us take this result to the top of the next slide here.

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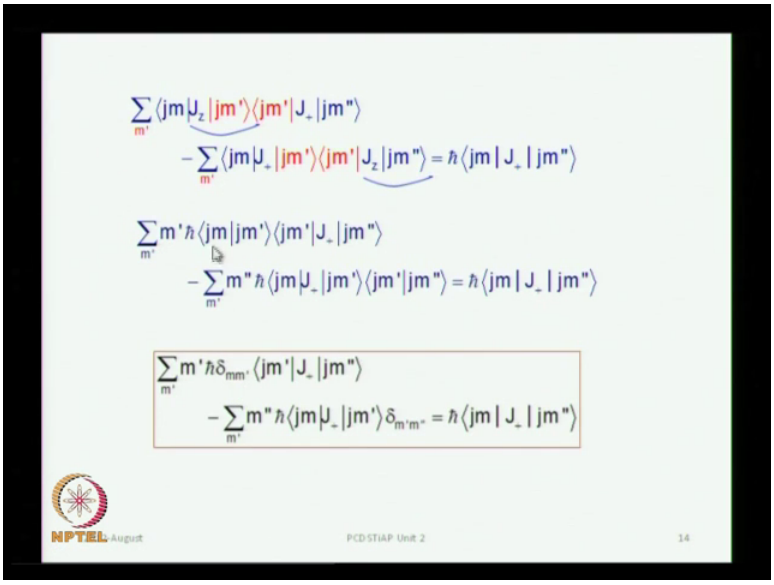

And now, you have J z operating on j m prime and J z operating on m double prime over here. So, the result from the first one is m prime h cross from the Eigen value equation, so this m prime h cross comes here, from here you get m double prime h cross, which comes over here, there is a minus sign which drops here. And now, you can see that, there is an orthogonality between m and m prime and then, you can sum over this m prime to exploit that orthogonality and reduce this whole sum.

So, it is very straight forward, you get this orthogonality between m and m prime, because these are orthogonal states or the normal states.

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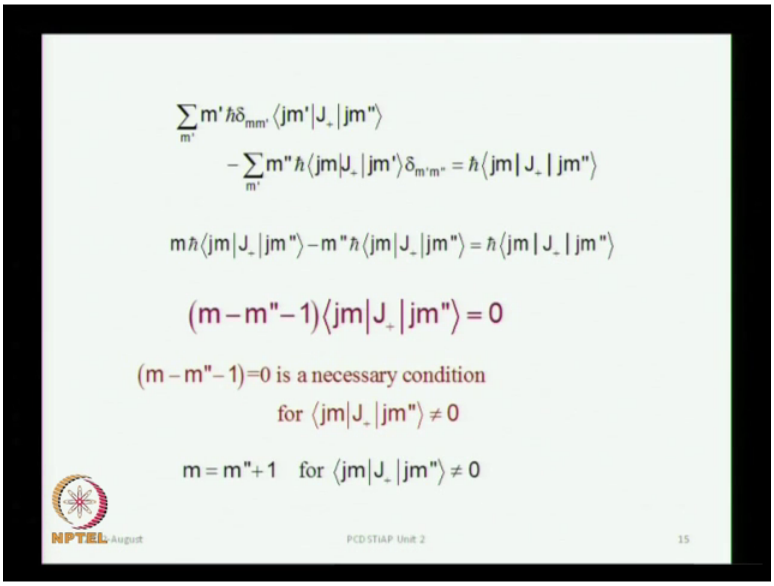

And now, you exploit this orthogonality, carry out the summation over m prime and you are left with only one term now and what is that term, it is m h cross times this element, in which m prime has been set equal to m. And in the second term, you have summed over m prime and the only term that survives is the one, in which m prime is equal to m double prime. So, you have got m double prime over here, so this is the result that you get.

Now notice, that this factor matrix element is common to all three terms, it is matrix element of J plus and corresponding states in all three terms. So, you can factored it out as a common factor and you get a very simple relationship, which is a product of two terms which vanishes which means that, at least one of has been 0. And the condition that the matrix element of J plus is not 0 is that, the other side factor must be 0. So, essentially it means that, m minus m double prime minus 1 must be 0 for the matrix element of J plus to be non trivial, otherwise it is a trivial 0.

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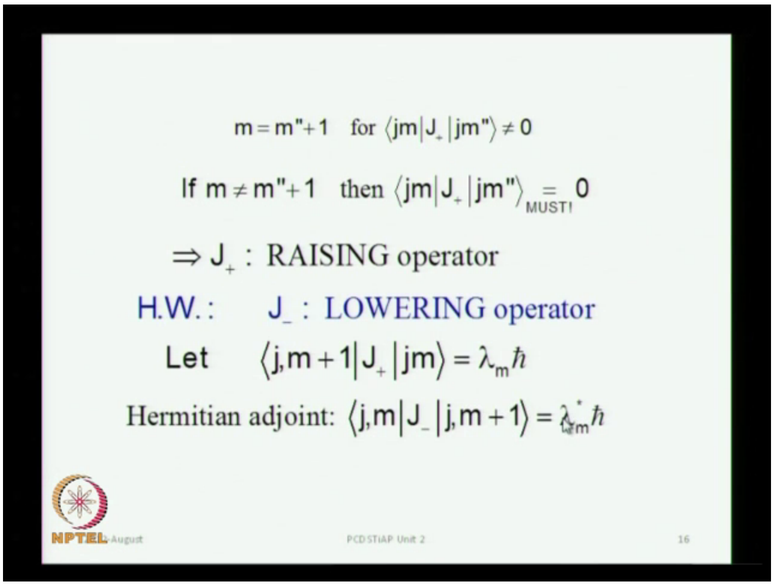

In other words, m must be equal to m double prime plus 1 and this is a necessary condition that emerges from a very simple analysis, which offers this interpretation to the operator J plus as a raising operator. Because, this the index on the left must be in excess of the index on the right by unity, that is necessary condition involves. I will leave it as very simple exercise for you to find out that, J minus is lowering operator, do the same analysis, it will come out automatically.

So now, you know that, you have a non trivial value for this matrix element, only when this index is in excess of the index on the right side by 1. So, you plug this m as m plus 1 when this is m and this will be some scalar, it will have dimensions of angular momentum, which you extract in units of h over 2 pi. And then, you are left with an unknown quantity, which is some number lambda m and we have now to find, what this lambda m must be like.

So, if you now take the Hermitian adjoint of this relation, what is a Hermitian adjoint of this relation, it is what you have over here, term by term. So, you get lambda m star h cross, this is no new algebra which is introduce over here, all you have done is to determined the Hermitian adjoint of the previous relation.

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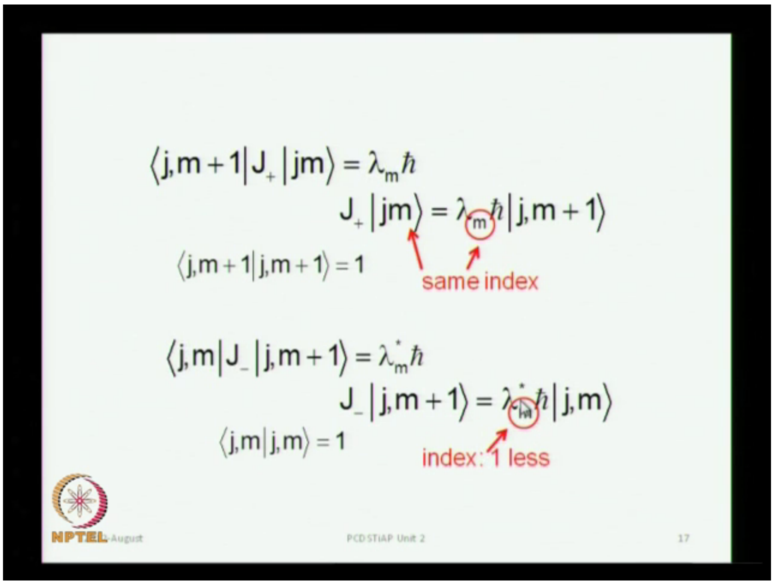

So, what it essentially means is that, J plus would raise the m index to m plus 1 and scale the new vector by lambda m h cross. It will not give you an Eigen value equation, it is not an Eigen value equation, it gives you a new vector, whose m index is in excess of previous one by unity and which is scaled by a factor lambda m h cross, but we have to find what lambda m is, this belongs to the orthogonal basis.

Likewise J minus, you have an exactly identical relation, in which J minus lower this index by unity from m plus 1 it will go to m and it will be equal to lambda m star h cross, which is come from the adjoint relation. So, J minus reduces the m index by 1 and scales it by lambda m star h cross, once again the angular momentum has been factored out. Notice that, when J plus operates on this, this index and the subscript on lambda is a same, whereas in the case of J minus, it is different. Here, this index is m plus 1, but the subscript on lambda is one less, keep track of that.

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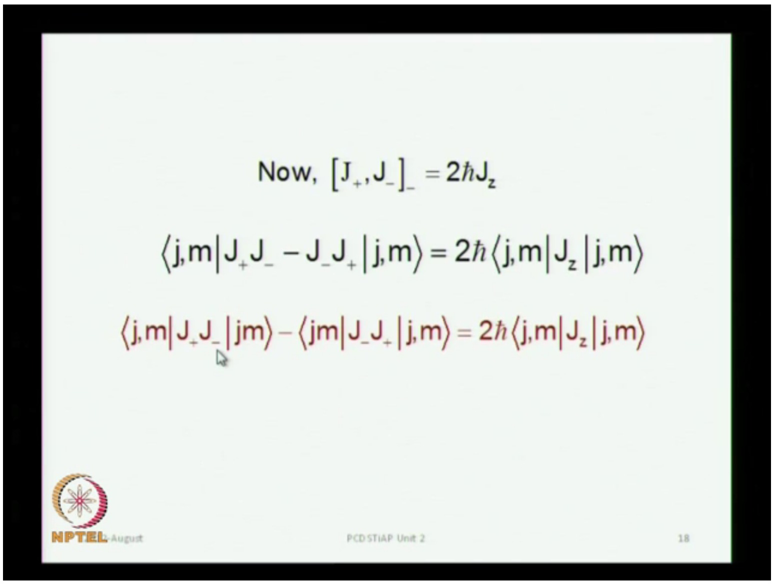

Now, this is a basic commutation relation, which again comes from the basic fundamental definition of angular momentum operators, it is very easy to establish. You take the matrix element of this, a diagonal matrix of element of state J m, which is equal to the corresponding diagonal element of J z. You separate these two terms, so you have two matrix elements on the left hand side and now, you operate on J m by J plus J minus term by term. First by J minus, which will lower the index and then, by J plus, which will increase that index, you do it stepwise.

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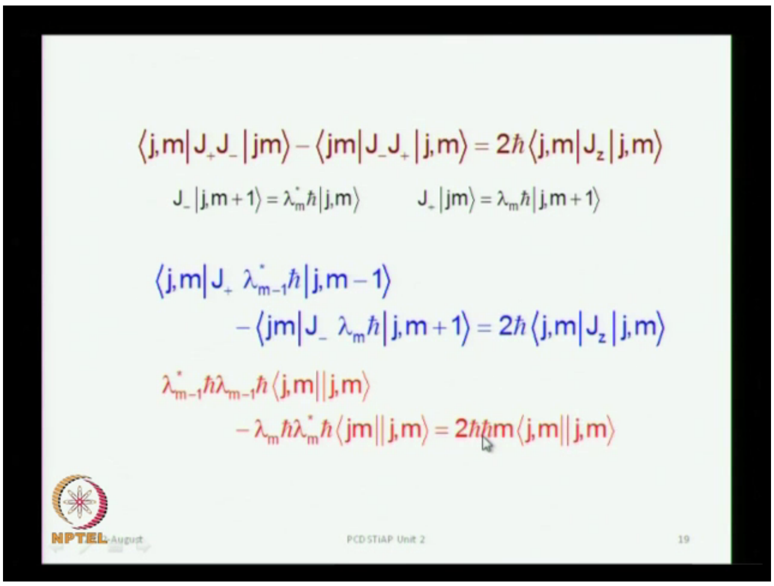

So, you know how to do that, because J minus lower this index by 1, so J minus would lower this index to m minus 1 and the subscript on will be lambda minus 1, it is a same as this index over here. So, this index will be lambda m minus 1 star, whereas in the second term over here, J plus operating on J m will give you lambda m h cross. So, this subscript will be mm, so this is lambda m, this is lambda m minus 1, keep track of that. Now, you work with J plus and J plus when it operates on this, it will increase this index from m minus 1 to m but then, it will give you the same index as was before, so it will again give you lambda m minus 1.

So, in the first term, you get lambda m minus 1 star lambda m minus 1, you will get this modulus square of lambda m minus 1 in the first term. Likewise in the second term, you get the modulus square of lambda m, of course there are h cross square terms appearing everywhere that is, h cross square in the first term, h cross square in the second term and h cross square in the third term as well, because when J z operates on J m, you get m h cross. So, that h cross with this h cross will give you this h cross square over here and you can cancel the h cross square in all the three terms.

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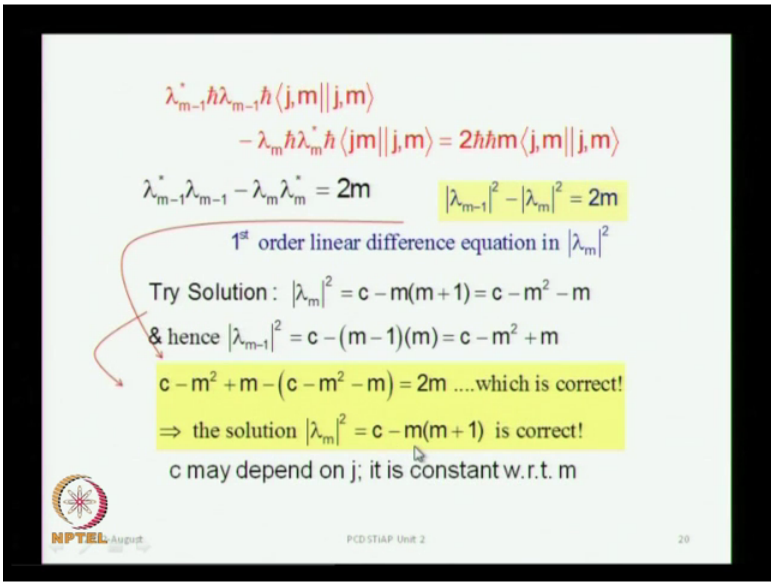

So, let us do that, you cancel the h cross square in all the three terms, here you get just the normalization overlap and what you have is a simple first order linear differential equation in lambda. You began with just a basic angular momentum commutation rules and you arrive at a very simple first order linear difference equation in lambda. So now, it is not so difficult to solve it, there are well known techniques of solving first order linear difference equations.

And you can try the solution that, lambda m square is equal to c minus m into m plus 1 and if you try this solution, the same prescription for lambda m minus 1 gives you c minus... Here, instead of m you get a minus 1 and instead of this m plus 1, you get m and you get c minus m square plus m over here and you get c minus m square minus m over here. And you can convince yourself that, this is the correct solution, because you can plug in this right hand side over here and this right hand side over here and asked yourself the difference turns out to be twice sum, it does, so our solution is correct.

So, this is our solution to lambda m square, c must be a constant, it cannot depend on m, but it may depend on j, nothing stops it from depending on j at this point and that is the question that we will leave as open at this stage.

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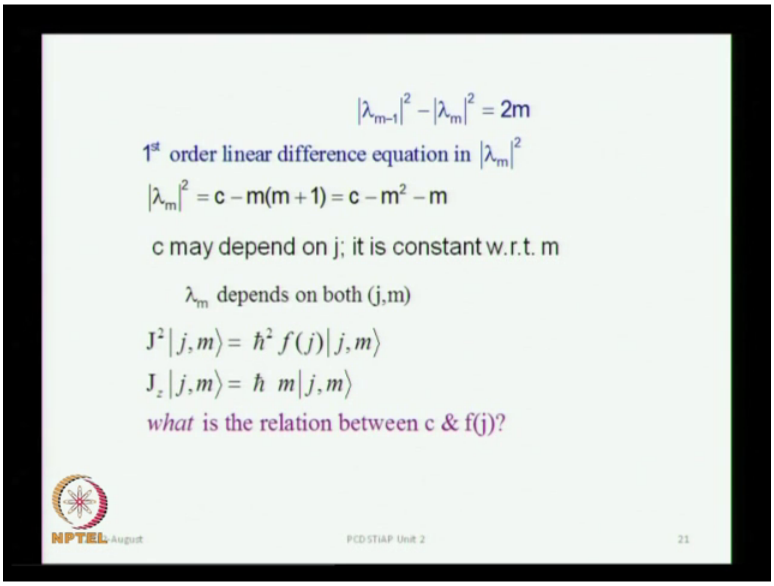

So, this is our solution to the first order linear difference equation and lambda would depend not just on m, but also on j. Because, lambda depends on c and c may depend on j, so you have to remember that. Our question now is, what is the relation between c and f j, f j was Eigen value of J square, that is what we want to detect.

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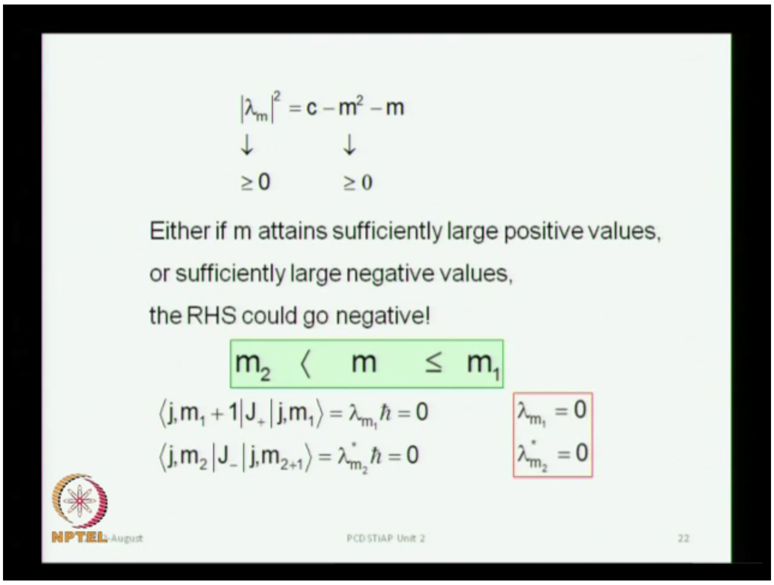

Now, let us have a look this relationship now, now this is an exact equality, the left hand side however cannot be negative, because it is square of the modulus, there is no way the left hand side can go negative. So, the right hand side also cannot go negative, which means that, in the right hand side you have three terms, are you subtracting m square, which is also cannot be negative. And if this relation is to remain valid, the necessary condition is that, m cannot take arbitrarily large positive values, nor it can take arbitrarily large negative values.

It must be confined to a certain range, it will have an upper limit, it will have an upper bound and it will have an lower bounds. So, these bounds project themselves automatically, they are not artificial, we just have to find what these bounds are. These bounds are necessary conditions and it means that, m can have a maximum value and it must always be greater than a certain lower bound. So, there are these bounds to m, but we do not know what these bounds are, that is what we will find.

Now, what we do is that, if this is the upper bound m 1, m 1 is this upper bound then, you cannot raise this any further, that is a very meaning of the upper bound. So, J plus operating on j m 1 must vanish, likewise J minus operating on this cannot bring it down to a lower value m 2, because there is a certain lower bound. So, these relations come automatically which means that, lambda m 1 must be 0, lambda m 2 star must also be 0.

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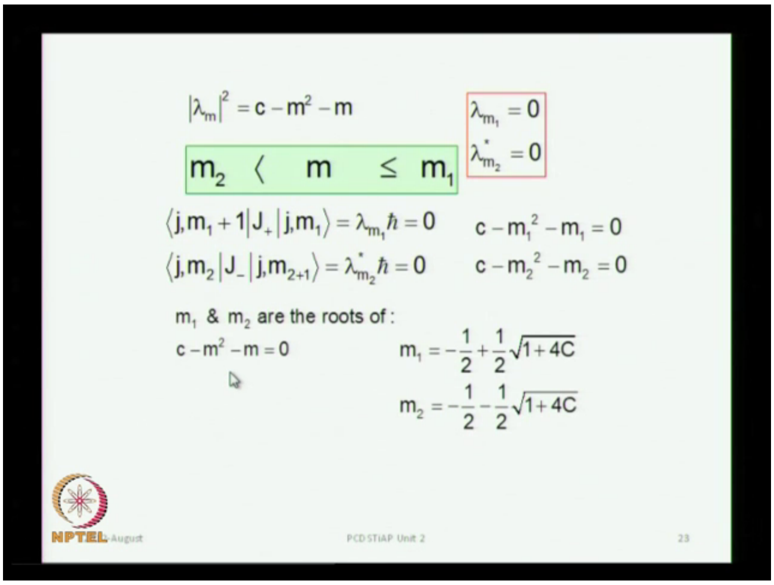

And we will now exploit this, to find what these values must be, now we do have these relations, because this relation is satisfied by both m 1 and m 2. What this means is that, m 1 and m 2 must be roots of this quadratic equation and this you know from your elementary school, this is the quadratic equation, find the roots and these are the roots of this quadratic equation.

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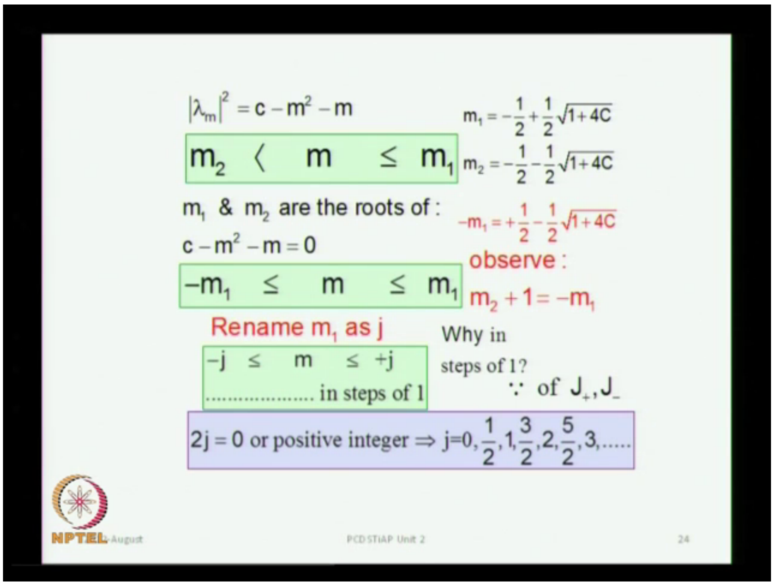

So, m 1 is equal to minus half plus half times root over 1 plus 4 c and m 2 is minus half and minus half times root over 1 plus 4 c, so these come as natural roots of the quadratic equation. Notice that, m 2 plus 1 is equal to minus m 1, from these the relationship between these two roots is very simple, m 2 plus 1 if you add 1 2 m 2 from this minus half, you get plus half which means that, m must be confined to this region. Now, this we have established, the only thing we have used is the commutation relation of J x and J y.

And m 1 is some number such that, plus m 1 and minus m 1 are attainable bounds at upper limit, plus m 1 and the lower limit, it is minus m 1, these are attainable bounds at the top and at the bottom. And why you want to called it m 1, you can called it anything, it is just an label, it is just a name, it is just a symbol and this is what, we relabel as j. This is the new name for m 1, so m now belongs to the range minus j to plus j, where j is a new name of m 1.

We still do not know what j is, we are not claiming that, it must be 0, half, 1, three half, it will turn out to be so. So, j is the new name and m belongs to this range in steps of 1, why in steps of 1, because you can go from 1 to the next through the raising and lowering operators with changes the m index by 1. So, it has to be in steps of unity, because of the raising and lowering operators, so we know that much. What it also means is that, 2 j must be either 0 or a positive integer, because this can go in steps of 1.

So, the number of steps will have to be in integer, so 2 j can be either 0 or a positive integer and this is what, gives us the result that, j can be either 0 or half or 1 or three half and so on. So, half integer quantum numbers come automatically from the angular momentum algebra, we are not used anything else. And they have a natural place and then, we still have to find out what this f j is, so let us complete that and come back to a discussion on this point.

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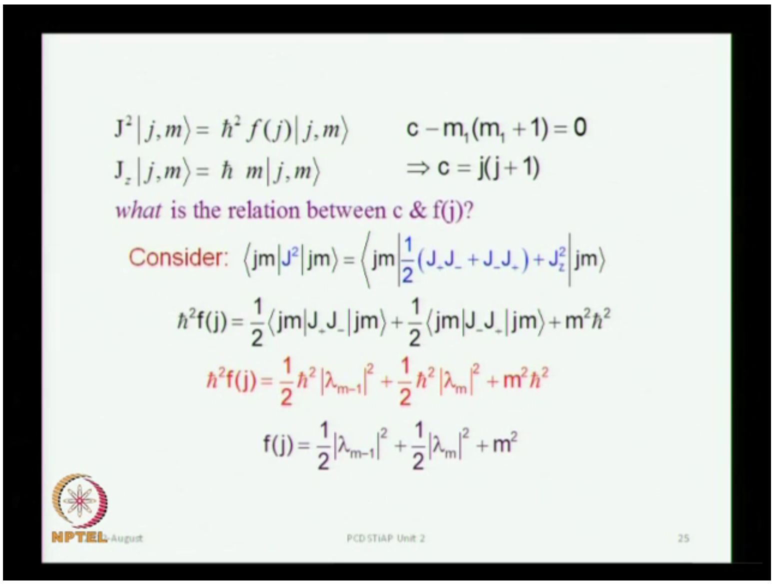

We have to find the relation between c and f j, so that should be emerge a few steps, we already know that, c minus m 1 into m 1 plus 1 is equal to 0. We means that, c must be m 1 into m 1 plus 1, but m 1 is now renamed as j, so this is a relation between c and j. Now, if we consider the matrix element, the diagonal matrix element of J square and J square you can express in terms of J x square plus J y square plus J z square. So, this is just a matter of simple rearrangement of these terms, you can see that already.

And you have the Eigen value J square, which is f j, from which of course the angular moment of J square has been extracted, h cross square over here. And now, you use a lowering and the raising operators, using the earlier forms you find that, you will get a term in modulus lambda m minus 1 square, from this term and from this term, you will get modulus of lambda m square, very easy to see that. Now, all you have to do, is to cancel h cross square, which appears in all the terms, which connects the Eigen value f to the lambdas.

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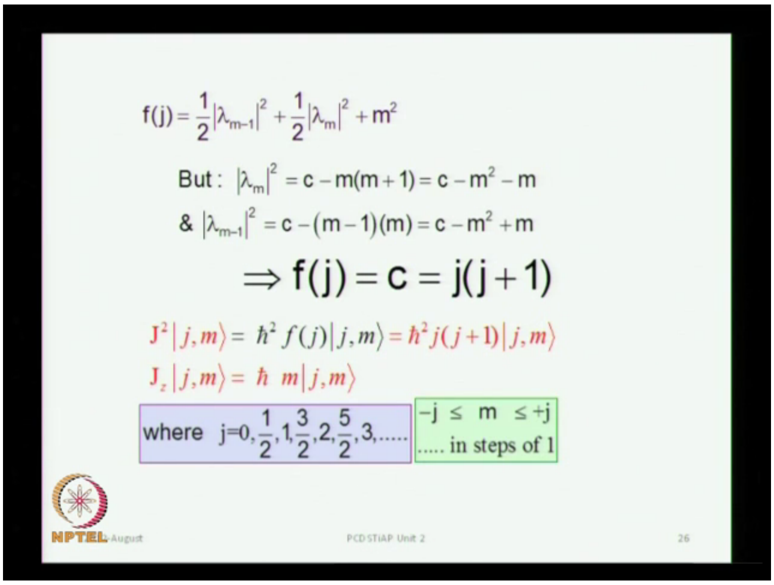

And the lambdas we know, because the lambdas are in terms of c's, so lambda m minus 1 square is c minus m square plus m and lambda m square c minus m square minus m. So, you put this right hand side over here and you find that, f j must be equal to c, which we already know, is equal to j into j plus 1. So now, you have the result that, when J square operates on J m, the Eigen value h cross square j into j plus 1, you also know that j must be either 0, half, 1, three half and so on.

You also know that, m can go from minus j to plus j in steps of 1, so this is not something like a new postulate or new addition that you have to make to your set of assumptions. All this comes automatically from just the basic definition of the angular momentum, that angular momentum is a quantum vector operator, whose components do not commute. But, the components commute according to the prescription that J x comma J y is equal to i h cross J z and other two relations of this kind, nothing else has been used.

And that is something which I think, one should try to learn in angular momentum algebra that, what are the basic postulates, what are the basic considerations and then, everything else can be built from it. So, this is the fundamental thing, from which you can build everything else.

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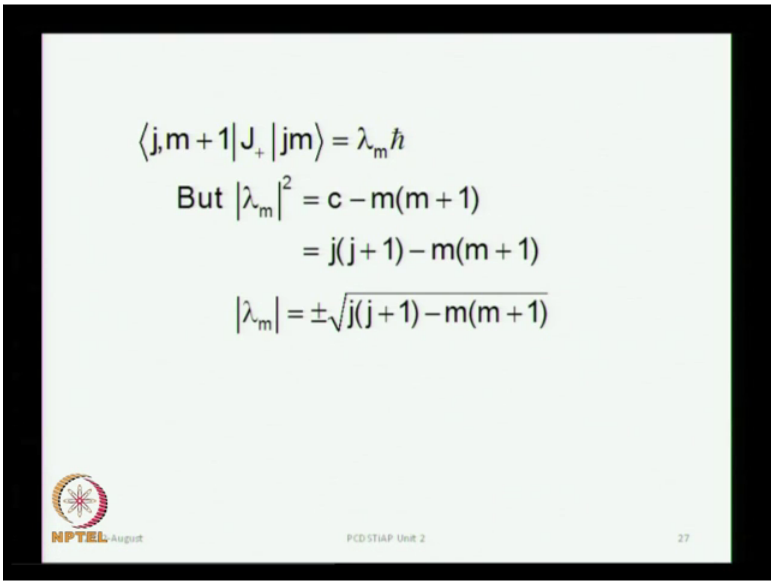

So, these results come automatically from angular momentum definition and now, the only thing that needs to fixed is a fact that, you have a relation for the square of the modulus, rather than the lambda m itself. So, modulus of lambda m can be plus or minus square root of this, so there is a little bit of ambiguity there. There is something that cannot be resolve from this analysis and it is a matter of convention that, one picks a phase convention, this is called as a phase convention in angular momentum algebra.

This is a very important one, because different contributors to angular momentum algebra sometimes make use of different conventions. And if you do some analysis using one scheme and the remaining analysis using another scheme and if the originator of these two schemes have used different conventions, you are going to mess up your angular momentum algebra. And I am alerting you for very good reason, because there are lots of practitioners in atomic physics, nuclear physics, would do make use of angular momentum algebra.

Are you are working with very involve problems and then, you need to use some results, which have been produced by one group, some other results produced by another group and if they have used different conventions and if they do not reconcile them to each other, you can end up into a very messy situation and this is not a very uncommon. So, please be alert to that, that always check the phase convention and we now have these relations with the phase convention that we have adopted.

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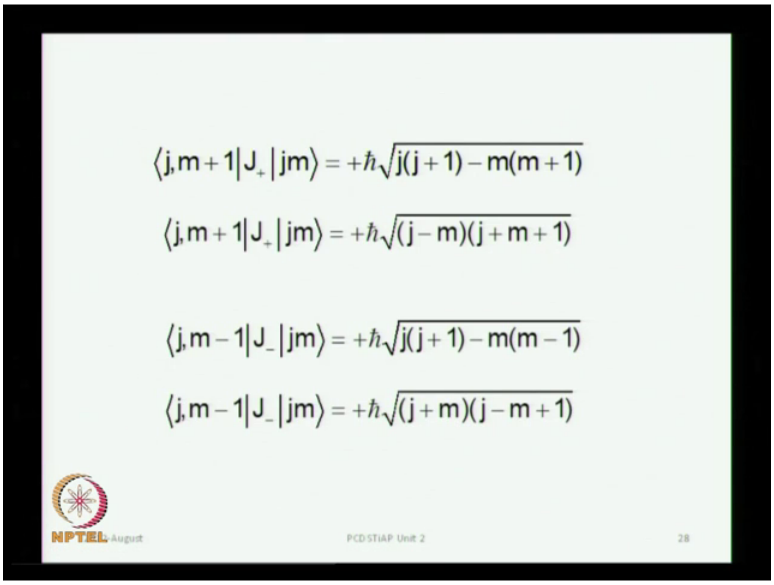

And you can write this j into j minus 1 minus m into m plus 1 also as, what comes another square root as product of these two factors. You have a similar relationship for J minus and these are our basic relationships that we will use in angular momentum algebra.

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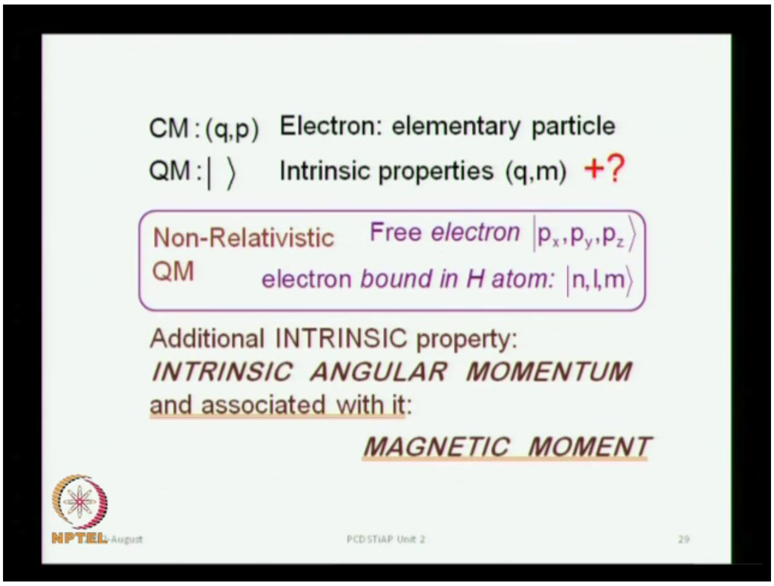

So, the main thing over here is that, in classical mechanics, you describe the state of the system by position and momentum q and p in quantum mechanics by a state of vector. Electron is an elementary particle having some intrinsic properties like charge and mass, this q is a position, this q is a charge. So, it has some intrinsic properties like charge and mass, but it may have some additional properties, not just these two. And it has this additional property, in non relativistic quantum mechanics, you can describe this state by these quantum numbers.

For a free electron, it is a different set of quantum numbers, which are the three components of momentum. For a bound electron in a hydrogen atom, these are the three quantum numbers like n, l and m, which come from Eigen values of the Hamiltonian L square and L z. But, what you have in addition to charge and mass is an additional quantity, which is an intrinsic angular momentum. And associated with this intrinsic angular momentum is also a magnetic momentum, so there is this additional property which the electron has.

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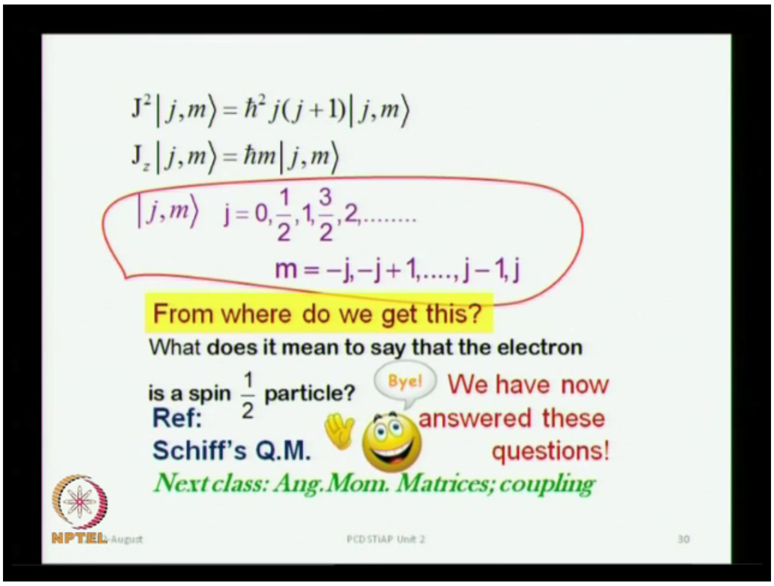

And you have this half integer quantum number for the electron spin and we have got it basically using angular momentum algebra, nothing else. So, sometimes people used this to say that, the electron spin has got a natural place in non relativistic quantum mechanics, we have not used relativistic quantum mechanics in our discussion today. We have not referred to the finiteness of the speed of light, we have not used a Lorentz transformation, we have not used a Dirac equation, which is the signature of the relativistic quantum mechanics right, you are not done any of that.

We have got half integral quantum number, which can be used for a electrons spin, now this does not mean that, non relativistic mechanics provides for half integer quantum number as the spin property of the electron. It accommodates this property, it does not provide for it, there is a difference between this and I hope you recognize this difference. The provision of the electron spin does not come from non relativistic quantum mechanics, it comes only from the Dirac equation.

You have to go to the relativistic quantum mechanics and we shall do so unitary and when we do unitary, when we do relativistic quantum mechanics, you will find that the electrons spin automatically comes out neatly out of it. Here, we have done angular momentum algebra, which accommodates half integer quantum numbers, it does not mean that, it assigns half integer quantum number to the electron, it does not do that.

The assignment of half integer quantum number to electrons spin is done by the Dirac equation, which we shall discuss in unit 3 and it is important to recognize this difference. Questions, I like to refer you to shifts quantum mechanics, where all this is nicely done.

Yes.

Student: ((Refer Time: 45:05))

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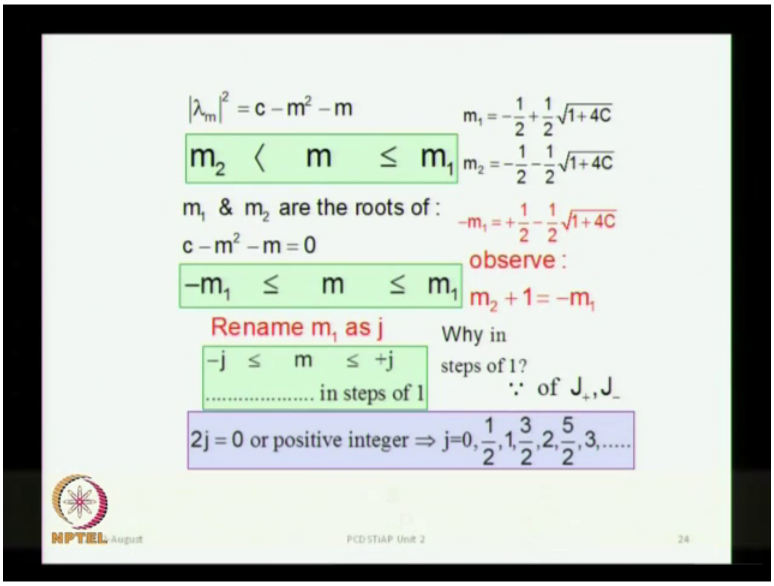

Yes, may it did not have any specific meaning, the meaning comes now, it is just a name.

Student: ((Refer Time: 45:34))

You can carry this name into the Eigen value, it is just a name and because, m can go from minus j to plus j and this process of going from the lower bound to the upper bound, has to be in integer steps. Because, that is mandated by the lowering and the raising operators, there is no other way to do it, but through integer steps.

So, the difference to j will have to be either 0 or some positive integer, which is what guarantees that, j can be either 0 or half. It is a very simple analysis and I really love it. You will find it this is straight from ships quantum mechanics but then, you do not have to learn it as an additional postulates. it comes straight from the basic definition of angular momentum algebra of what angular momentum is, any other question.

Student: ((Refer Time: 46:52))

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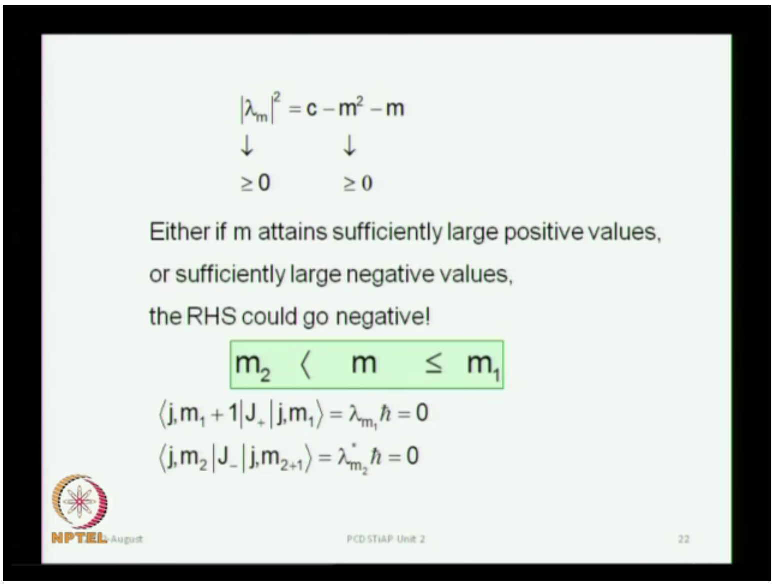

Yes, see from c minus m, you are subtracting m square, so if m becomes arbitrarily large and m square is definitely a positive quantity. So, whatever is c minus m, from c minus m if you subtract m square then, if m is sufficiently large then, this whole thing will become negative, because when you subtract a large quantity from a small number, the difference is negative, you know it better if you think of money. If you have a certain amount and you take out a big chunk of it, you will be in debt. So, to avoid that and you have to avoid it, because a right hand side cannot go negative, because a left hand side cannot go negative. The left hand side is a square of modulus, how can it be negative.

Student: ((Refer Time: 48:06))

Yes, any other question.

Student: ((Refer Time: 48:15))

Yes, it is a difference equation rather than the differential equation.

Student: ((Refer Time: 48:21))

Yes.

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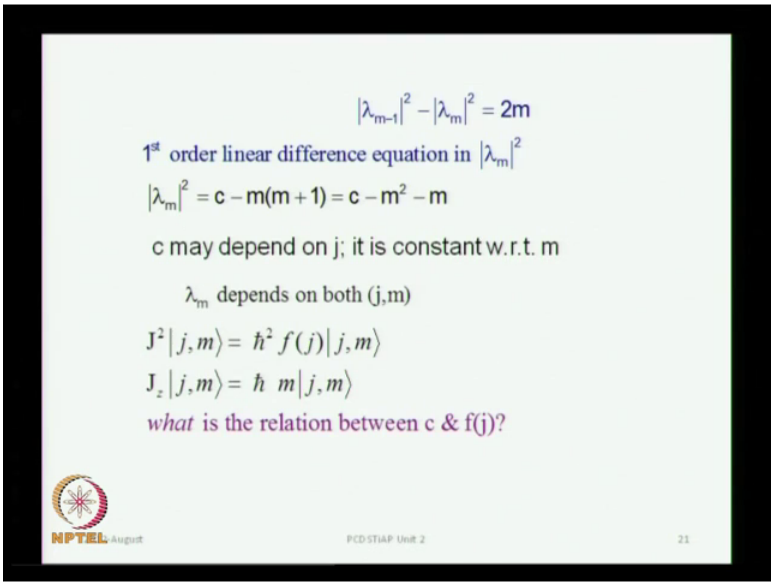

We do know, because you know these are usual ways of solving difference equations.

Student: ((Refer Time: 48:47))

Not to my knowledge, I think this solution is pretty much unique.

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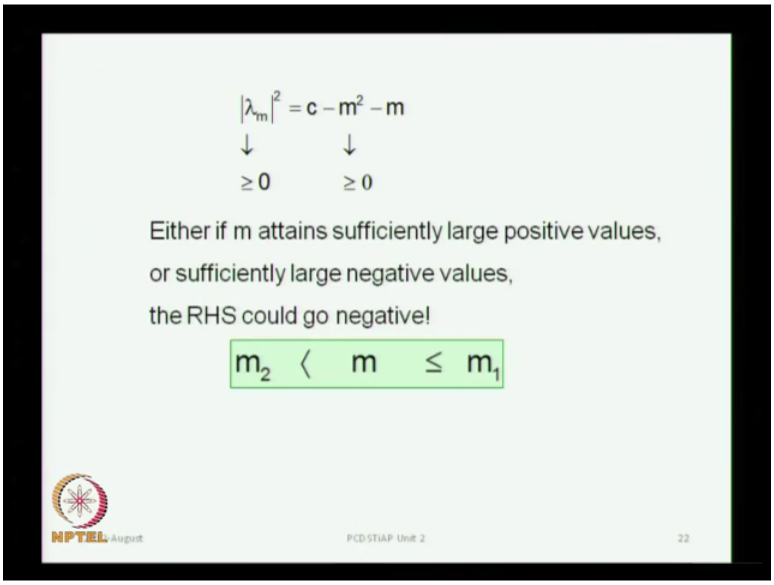

Any other question, Mandy, Anna question, so thank you all very much.