Select/Special Topics in Atomic Physics
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Lecture - 38
Stark - Zeeman Spectroscopy

Greetings, we will look at the spectrum of the hydrogen atom or any alkali atom anything in the first group of the periodic table, they all have similar spectra in some respect of course, they are different in the details. But there are several common features and when we examine in the spectrum of the hydrogen atom, it does set up the procedures which we most employ in interpreting the spectra of the bigger atoms.

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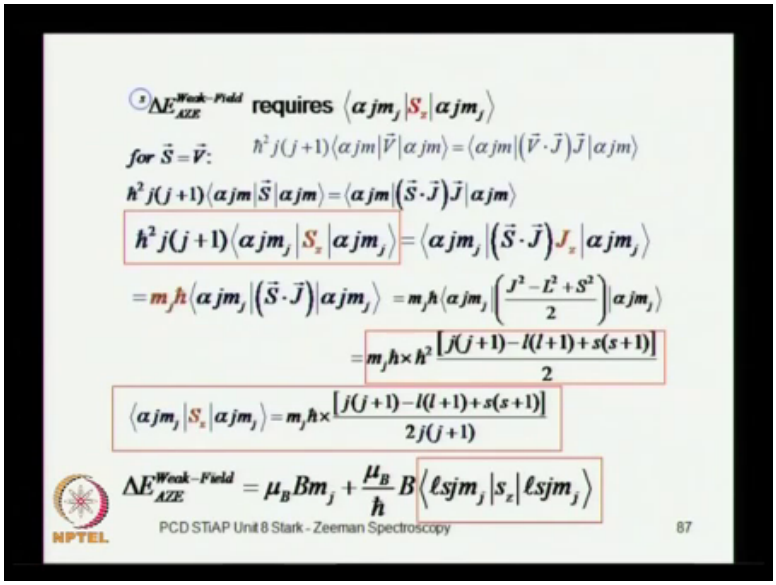

So, we consider the strong fill effect in the paches should back effect and then our active consideration now is on the study of the week feels Zeeman effect. And at these magnitudes of the applied magnetic field, the spin orbit interaction is the dominant interaction, because the magnetic field the applied field is weak. So, the internal effects take over and j and m are the good quantum numbers and not m l and m s, like this states or not eigen states of l z and s z, but there eigen states of j square and j z. So, that is what let us to determine the perturbative correction to the original unperturbed energy, at this correction requires us to determine the matrix element of s z.

Now, the difficulty here was that the quantum states that you are considering or eigen states

of j square and j z and not of s z, so that is the reason we had to find some mechanism to get this matrix element. And we found using two alternative procedures, one based on vector identities for vector operators, these are operators for which are irreducible tensor operators of rank 1. So, we use those identities and the other procedure which we followed was based on Wigner-Eckart theorem.

And using both the procedures, we are lead to relationship which gives us the matrix element of on arbitrary vector operator, no matter what it is, and you get it in term of V dot J and J. And if this we happens to be our S operator the spin operator, then we can use it and applied for this case, so we have S equal to V in our case, so now we get h cross square j into j plus 1. And here this matrix element, is the matrix element of the spin angel of momentum S, and this is equal to the matrix operator, matrix element of the operator at there is the scalar part which is the S dot j, and there is the vector which is the angular momentum itself.

So, this is the S dot J J operator, so now this part we can write this relation for each component, because this is vector relation and there are corresponding relations for all the three components S x and y and S z or of you can write it for the spherical components as well. And now the relationship for the z component, is that you take the z element of this operator on the right side S dot J is a scalar, so it remains as it is and you have the J z operator, and what comes out of this part is the J z operates on j m j and you will get m j times h cross.

So, that is a big advantage here, you get m j times h cross coming out of the operation by J z and this S dot J can again be written in terms of operators, whose eigen states are already involved in the bases set. Because, the operator S dot J is nothing but J square minus L square plus S square, because L plus S is what gives you J, so S dot J turns out to be J square minus L square plus S square, so all you do is to take the square of S plus L equal to J.

So, dot out S plus L with S plus L, and this is exactly what you will get, so you have the operator S dot J whose matrix element now you need, this is given by a some of three operators J square minus L square plus S square by 2. But, now this state is an eigen state of this operators, so what are the eigen values. So, you get h cross square into j into j plus 1 minus l into l plus 1 coming from here, and s into s plus 1 coming from here, so you get numbers of the right side now, and you are able to solve this expression.

And this matrix element which is the matrix element for S z is now obtained completely in

terms of these quantum numbers, and you can write it for this matrix element which is what you need. So, you take you cancel ((Refer Time: 05:55)) this h cross square with this h cross square, move this j into j plus 1 to the right it come in the denominator. And this is the result that we will looking for, this is preciously the term that you needed to get the correction for the weak field Zeeman manufacture, so this is the correction now, this comes along with the other term which was coming from the j term itself. And together with this gives us the correction, the perturbative correction when the magnetic field is weak and the spin orbit interaction and takes over. So, that is the dominating interaction and this is the energy correction that must be applied.

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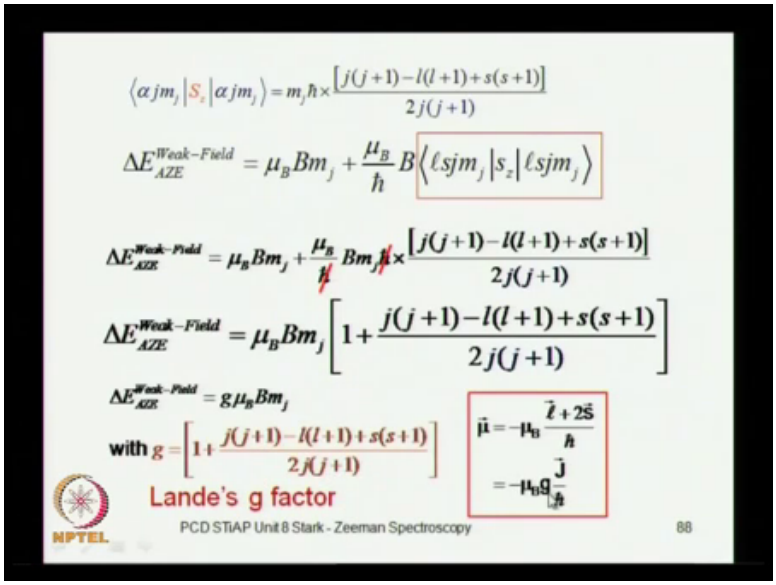

So, this is the expression that we were looking far and we have now been able to resolve it, so this is the energy correction the h cross cancels, and if you combine these two term there is the more magnet on times of magnetic field in both the terms. And m j in both the terms, so you extract this is the common factor and you have 1 plus j j plus 1 minus l into l plus 1 plus s into s plus 1 divided by 2 j to j plus 1, so there is this is the land is fact exactly. So, this is like g, this take exactly the same place as g did for the case of the orbital angle of momentum and also further spin angular momentum.

Because, there is the corresponding magnetic moment associate with the spin angular momentum, and with the orbital angel of momentum, and this is the effective g coming from the combination. And this is neither equal to 2 nor equal to 1, but it depends on the values of j

and l s is always half of course, s for electron is half, so it is half into half plus 1. And this factor is what is called is a Lande's g factor and this is what governs the energy splitting between the perturb levels as the result of the magnetic field, which is applied and treated perturbatively, when it is relatively weak and you can use the j m j quantum numbers.

So, which quantum numbers are the appropriate quantum numbers to be use is the dominant consideration here, and once you take the write bases you get the right results, but choose the choices of the bases is the critical factor here. So, this is a very similar expression for just the way we had the magnetic moment, which is proportional to the angle of momentum you got a same kind of consideration, but with the different value of g which is given by the Lande's g factor.

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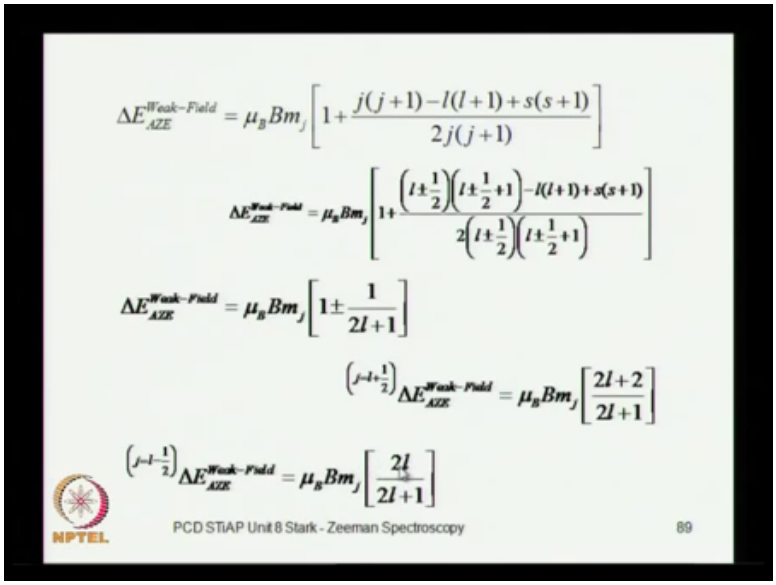

So, now the spin orbit interaction is s plus l, so s plus l gives you the total angle of momentum j which can be either l plus half or l minus half, so you can put in the values of j the possible values of j which is either l plus half or l minus half, so there are two possibilities here. And for every j you put these two alternative values and you find that the energy correction depends on whether j is l plus half or l minus half, and accordingly the correction is either 1 plus 1 over 2 l plus 1 or 1 minus 2 l plus 1, I have certainly used s equal to one half in the getting this expression. So, when j is l plus half, the correction goes as 2 l plus 2 over 2 l plus 1, when j is l minus half the correction goes as 2 l over 2 l plus 1.

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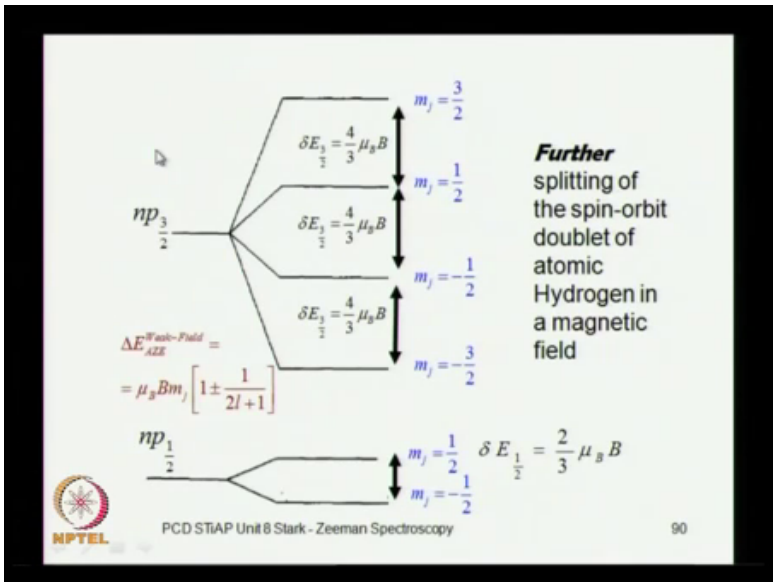

So, this is the resolution of the energy levels and the application of the magnetic field, if you look at the n p state, this is like the 2 p state in the hydrogen atom if you like, or the 3 p state in the sodium atom. Or if you take any of the group one elements, the outer electron is the n s 1 and when it gets excited to the n p level. So, these are the levels which are involved in the sodium atom spectrum, the famous d 1 to d 2 lines of sodium, they come from the transition to from 3 p 3 half and 3 p 1 half to the 3 s level.

But, now the 3 p 3 half level will get split into these four levels and the 3 p 1 half level get split into these two levels, so you will have the d 1, d 2 lines split to very many lines, and the same thing will happen to all the alkaline atoms spectrum. So, the rebellion spectrum or transoms or anything in the first group they will have, they will show this features.

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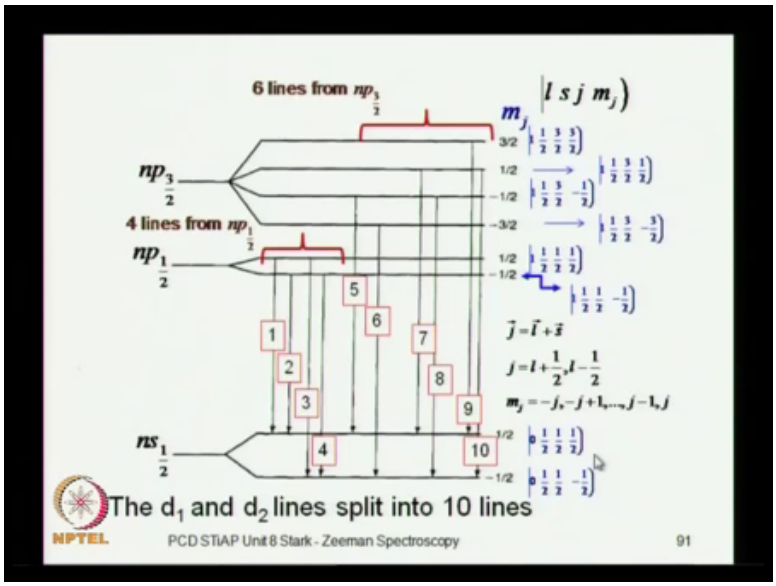

So, this is the resolution of this spectra, so n p 3 half splits into to these four levels, j is equal to 3 half, so m j can go from minus 3 half to plus 3 of in steps of , so minus 3 half minus half 1 half and 3 half, so this are the four levels into which the n p 3 half state splits. And the n p 1 half state splits into these two which is corresponding to m j equal to minus half and plus half and likewise, the n s half levels split into these two corresponding to m j equal to plus half and m j equal to minus half.

And the d 1 d 2 lines split into 10 lines, so what were originally only 2 lines, now we will show of s 10 lines, so there are 4 lines coming in from n p 1 half and we are sketching those line which correspond to the dipole selection roles. So, the line which are possible under the dipole selection rules, there are 4 lines which come from the n p 1 half, but n p 1 half is no longer or single level they are two of these. So, two come from the upper level corresponding to m j equal to half and two come the line number 2 and 4 come from the lower level, which corresponds to m j equal to minus half.

And likewise the transitions from n p 3 half splits into these 6 lines, and these are the 6 lines which come from the n p 3 half levels, so these are all the transitions which take place corresponding to the dipole selection rules. Now, the question is, first of all we have to write these quantum states for j m j quantum states, in terms of the m l m s bases, so these are the j m j quantum numbers. So, l is equal to 1 for n p, for all the p orbital's l is equal to 1, s always half, j for this state is 3 half and m j for this state is 3 half, so these are the l s j m j quantum

numbers for this upper most level.

For the next level the l s j m j quantum numbers are 1 half 3 half and 1 half, the last quantum number is the m j quantum number. And this way you can write the j m j quantum numbers for all of these 4 into 6 into 8 there are 8 levels for which you must identify the quantum numbers. And these are the l s j m j quantum numbers for the p states and then for the s states, l is equal to 0 for the s orbital, so l is 0, s is always half and j is always half, because l is 0. So, j can take only one value which is half and m j can take two values, which is either plus half for the upper one and minus half for the lower one, so these are the l s j m j quantum numbers for these eight states.

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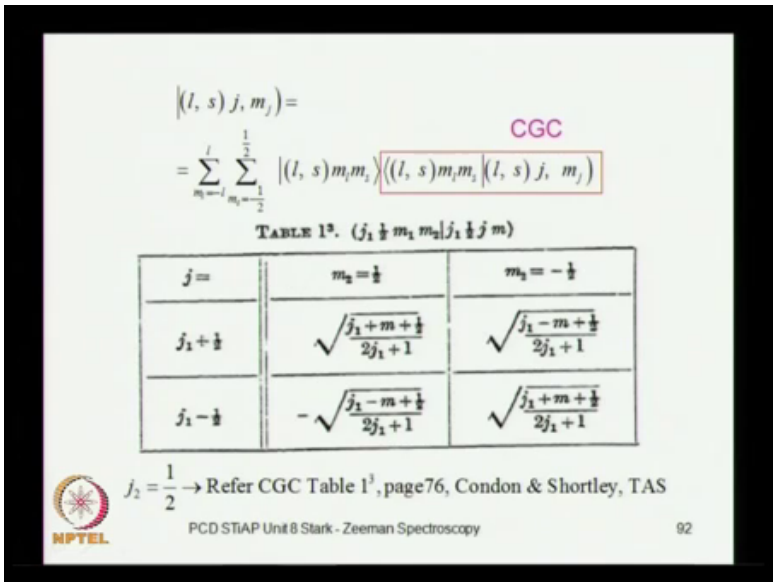

And you can write these coupled vectors j m j, in terms of the m l m s bases by looking at the Clebsch-Gordan coefficients, so here you are coupling l with s, and s we know is half. So, the appropriate table to be used is the table number 1 from corner is shortly which we have with us, it is also uploaded on the course webpage, so this is the table of correspond of the Clebsch-Gordan coefficient that we can use. And using these tables we can write the j m j quantum numbers, in terms of the uncoupled direct product of uncoupled vectors.

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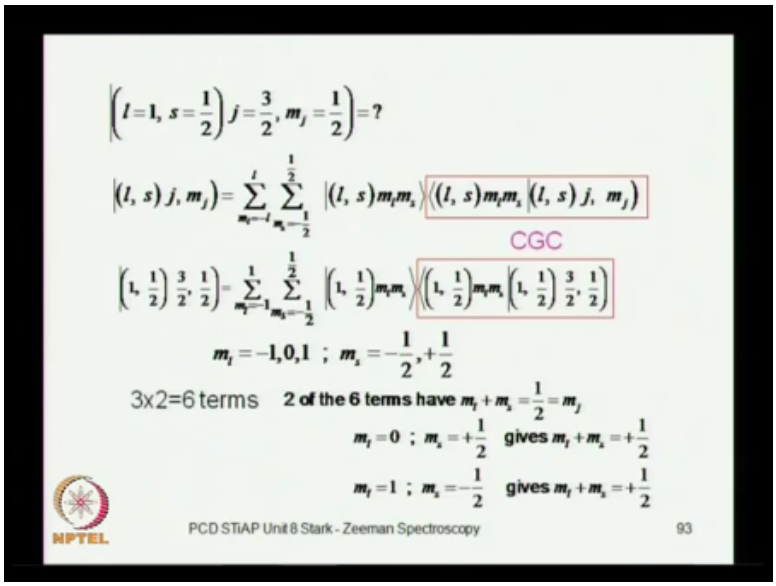

So, let us take let us illustrate this for one of these, so let us take the case when j is 3 half and m j is 1 half just to illustrate one of these, and what will these be, so your expand it the uncoupled bases along with the Clebsch-Gordan coefficients. And here is the sum over m l going from minus 1 to plus 1, and m s going from minus half to plus half, so how many terms will we have on the right hand side, 3 into 2 we will have 6 terms.

But, the Clebsch-Gordan coefficient will vanish and less m j is equal to m l plus m s, so out of the six terms you really do not have to find the Clebsch-Gordan coefficient for all the six terms. You can find the coefficient only on those cases for which m l plus m s will give you m j and that means, that there are only two terms which you need to consider and these terms are those corresponding to m l equal to 0 and ms equal to half or m l equal to 1 and ms equal to minus half. So, these are the only two terms that you need to consider, because both of them give you m l plus m s which is equal to half which is the value of m j here. So, now, you need to find the Clebsch-Gordan coefficients for these two terms only.

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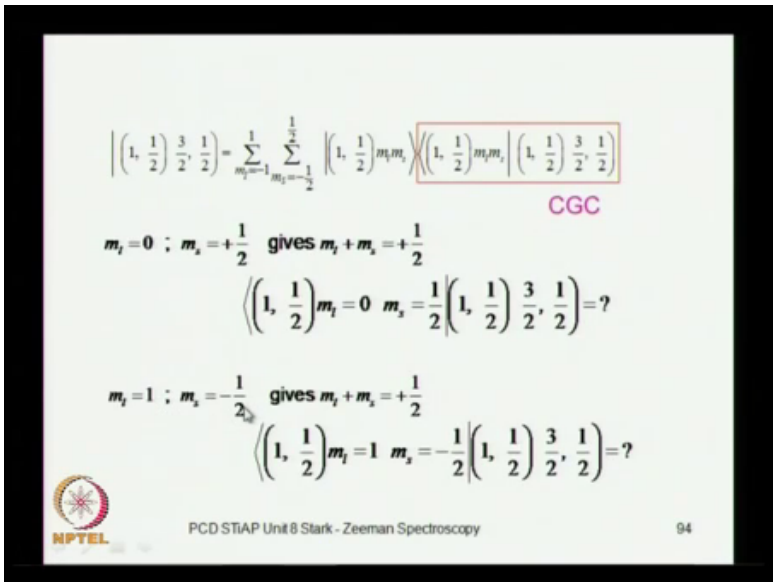

And let us take one of those, so let us take the case when m l is 0 and m s is equal to half, so for this what is the value of the Clebsch-Gordan coefficient. And you have to find the corresponding Clebsch-Gordan coefficient also for the other case, in which ml is equal to 1 and m s is equal to minus half. So, these are the two coefficients that you want to determine from the table.

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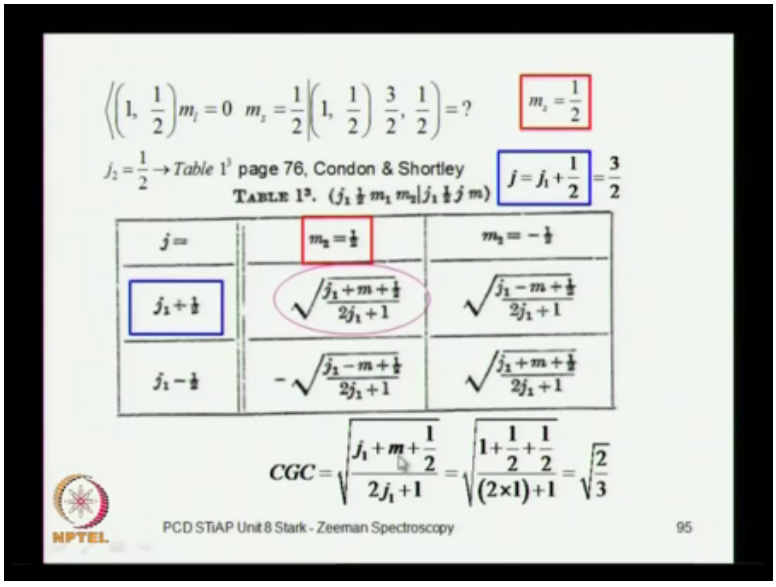

So, let us take one of these and notice that m s is equal to half, so you can look at the first column here, this is the first column, since m s is half. And then look at how j is related to j 1

and you find that you have j equal to 3 half this is the value of j, which is 3 half and it is coming from j 1 plus 1. So, when do you have j equal to j plus half you have it in the first row, so first row and first column is what you must look at, so this is the matrix element that you must look at.

And all you now need to do is to plug in the quantum numbers in this formula, for the Clebsch-Gordan coefficients, you can always determine the Clebsch-Gordan coefficient from first principle using the recursion relations that you have learnt. But, these Clebsch-Gordan coefficient tables are available in all books and quantum mechanics, there are available in the internet, they are available as Clebsch-Gordan coefficients, or is n j symbols and so on. So, you can take it from any source and then plug them in, so this after you plug in these quantum numbers, so j 1 m and half, so you put 1 half and plus half and you find that this coefficient terms out to be root 2 over 3.

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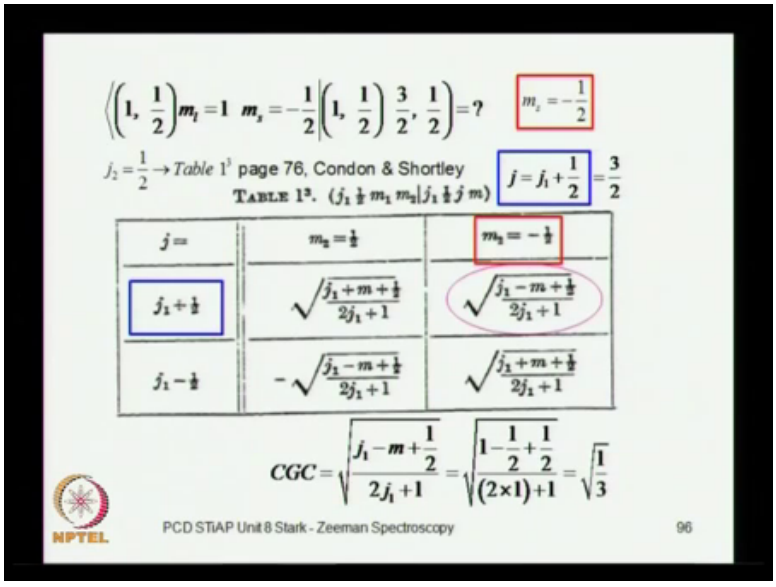

Now, you need the other one and in this case and m s is minus half, so you must look at the second column instead of the first column, but then you continue to have j equal to j 1 plus half. So, you look at the first row and this is the formula that you must used, so what you get from this, you get root over 1 over 3 and actually you could a guess that, because the some others squares must be equal to 1. But, then you would not have got an necessarily the correct face, so that is why you have to use the this table, otherwise you would not get the right face.

You would get the magnitude root 1 over three from the normalization, but not the phase it

could be either plus root 1 over 3 a minus root 1 over 3. So, normalization cannot be used as the method of find the coefficient, it should always be used as a check, because if you get this wrong, the normalization would tell you that is wrong.

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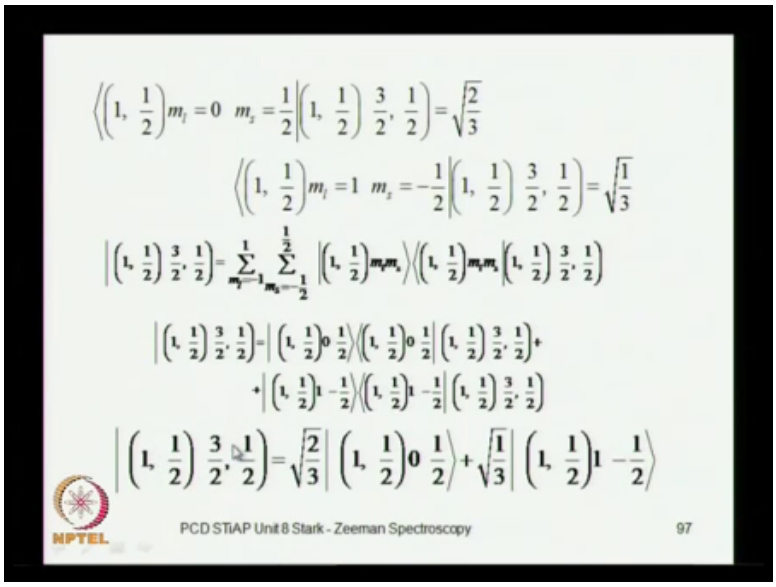

So, these are the two coefficient that we needed one is root 2 over 3 and other is root 1 over 3 and using this coefficients in this expansion, so out of these six terms only two terms will contribute, these are the two terms. And when you see, so many numbers sometimes you fill dizzy, and that is when it helps to look at these angle of brackets, because this side as got a rounded bracket this side has got an angle bracket.

So, you keep the track which was uncouple part and which is the couple part, because that is part of the reason, when I introduce the Clebsch-Gordan coefficient are the very beginning are used this notation, not that it is the mandatory most books do not use it, but it is very useful when you look at expression of this kind. Because, there are, so many number and you really feel busiest to what is what and which side is what. So, it helps to keep track of which is the this is the angle of bracket here, now this is the circular bracket and this is the angular bracket here. So, this is the uncoupled part, so this is the m l m s and that this is j and m j, so when you are dealing with practical application the notation of this kinds sometimes useful, which is why I introduce written unit 2.

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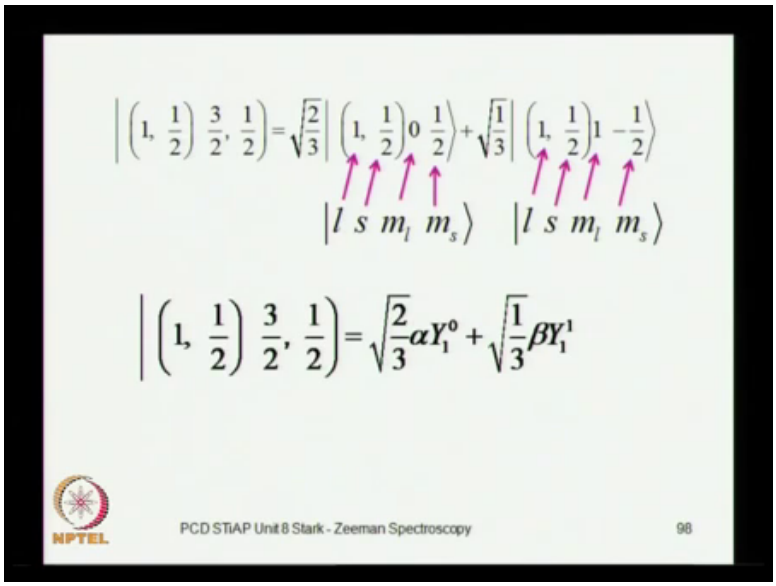

So, these are the two terms and this is your expression for this state, but then there are you can write it in not just the Dirac notation, you can also write it as the Schrodinger notation is well. Because, what you have for l equal to 1 and m l equal to 0 is the spherical harmonic y l m, which is the spherical harmonic for l equal to 1 and m l equal to 0. So, likewise this is also spherical harmonic for l equal to 1 and m l equal to 1 and this is 1 minus of, so this is really the spin down state and this is s equal to half and m s equal to plus half, so this is the spin up state.

So, the first term is a product of the spin up state of which I have written as alpha, the second term involves the spin downstate which is beta. And then you have got the spherical harmonics y 1 0, coming from this 1 and 0 and y 1 1, this is l equal to 1 and m l equal to 1. So, you can write this is a linear super position, this of course the coordinate representation of this vector. But, you can go from the Dirac notation to the de Broglie Schrodinger notation back and force, just by taking the coordinate presentation of the vector, so this is the linear super position of the j m j states.

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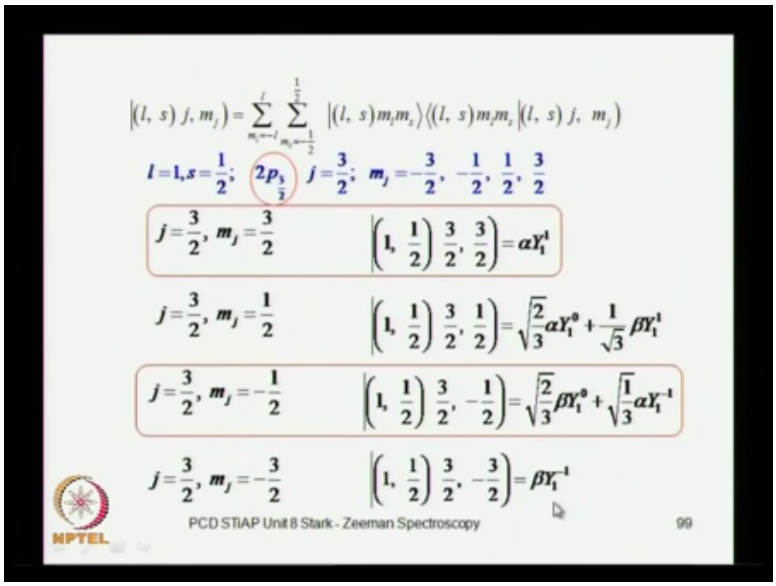

So, let us write this for all the eight states that we are concerned, we found that the 2 p 3 half split into 4 straights, 2 p 1 half into 2 and 2 s one half also into 2, so there are total of eight states for we should write this expression. Four of these come from our 2 p 3 half and by getting the coefficients you can write this directly, there is only one term over her, there is only one term which can contribute to m j equal to 3 half there is none other, so this is unique this is, therefore this is got a coefficient of 1.

And then you have m j equal to 3 half, 1 half minus, half and minus 3 half, so m j equal to half will give you this, m j equal to minus half will you give you a similar linear combination. But, notice that this is the superposition of alpha y 1 and this is beta y 1 plus alpha y 1 minus once, so this is the different super position. And how to get it, illustrate for it one of these and you can use the same procedure use the Clebsch-Gordan coefficient and get all the states written explicitly. So, m j equal to minus 3 of again is a unique contributor to this which is got a coefficient of unity which is beta y 1 minus 1.

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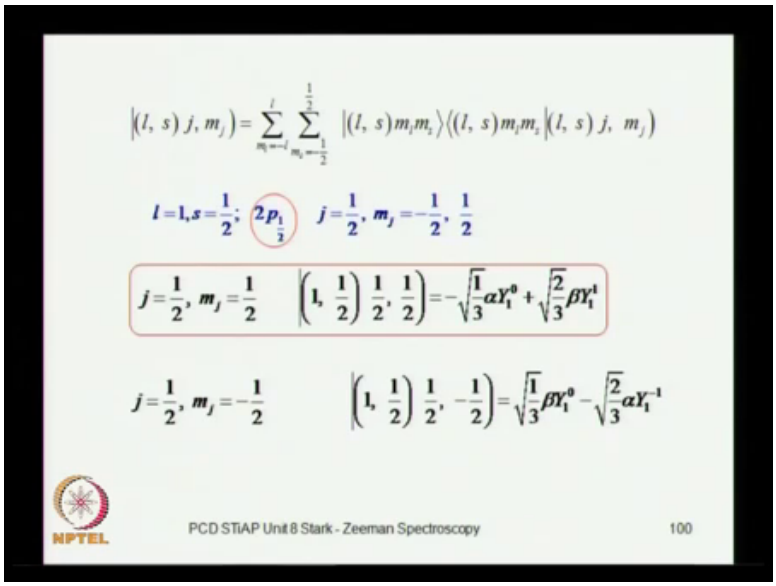

What about the 2 p 1 half states, this is got two states, one with m j equal to minus half and the other with m j equal to plus half, so this is what you get for m j equal to plus half and then you get another term for m j equal to minus half, which is the different combination. Mind you have got a minus sign here and minus sign here, so the phase of the coefficients is of important, and this is the phase that you would miss out, if you did not use the Clebsch-Gordan coefficient tables correctly. So, these are two states for 2 p 1 half.

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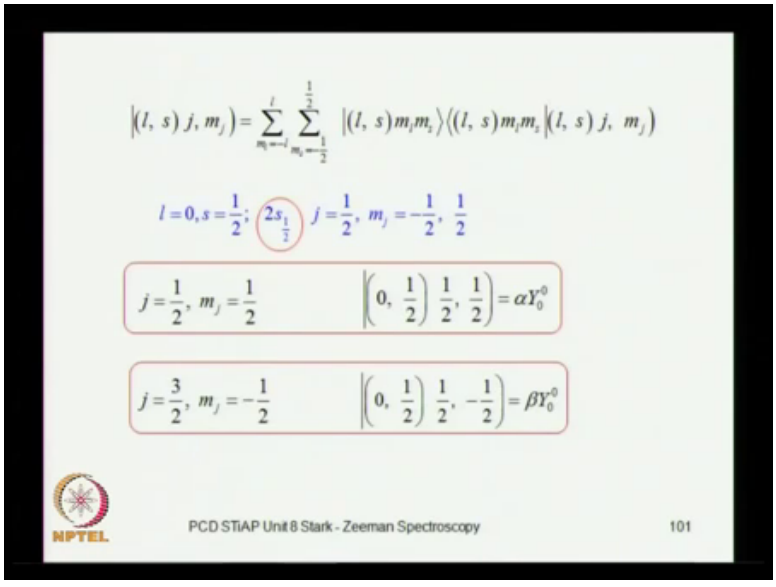

And likewise, there are two states for the 2 s 1 half, so that gives us all the eight states.

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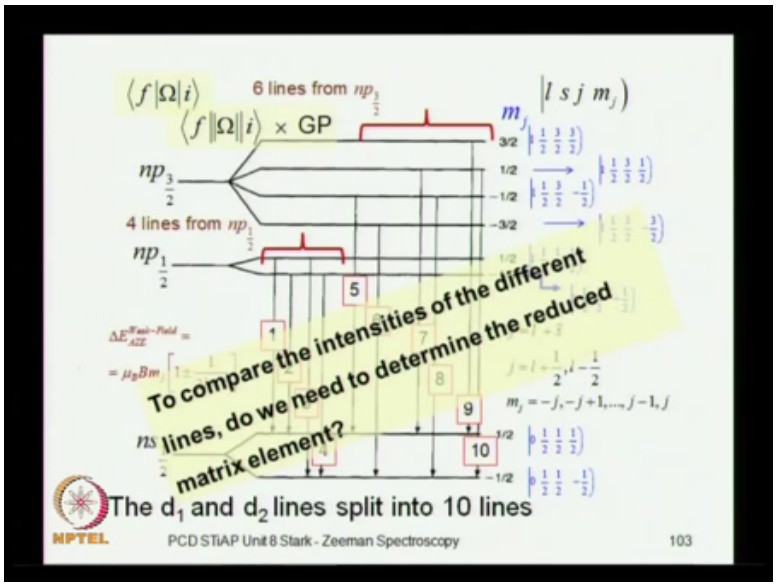

And now you can look at the transitions between various states, these are the tent transitions which take place between these eight states as we have seen, according to the dipole selection role. And as a spectroscopes as an experimentalist, when you carry out your observations, you would be interested in looking at the intensity of the this lines. If there is the transitions first of all there will be a certain intensity that you will measure, if there is no transition the corresponding intensity would vanish.

And you will need to calibrate your spectra meters, you will be measuring intensities on a certain relatives scale when you do the calibration and so on. And you will be interested in comparing the intensities of transitions from one initial states to a final state, and that is the effected by the transition matrix operator, the transition operator here is omega. So, this is just the general expression for an interaction omega which is responsible for transition from i to f, and what this matrix elements gives you is the probability amplitude, that this transition will take place. It is moldiest square will give you the probability and when you multiplied by appropriate contents and so on, you will get the line intensities.

So, the information about the intensity of the spectrum is contained in this matrix element, and we know that this matrix element by the Wigner-Eckart theorem can be factor into a physical part, which is the reduce matrix element and the geometric part, which includes the Clebsch-Gordan coefficients. So, to get the transition intensities, if you just look at the Wigner-Eckart theorem, it would seen that you will need not only the Clebsch-Gordan

coefficient, but also the reduce matrix elements.

Because, this matrix element has got these two factors and on the phase of it would appears, as if you will need both the reduce part as well, as the geometrical part. Now, this as a turns out is not necessary, if you are interested in comparing the line intensities which is what the common of interest of it. Because, anyway there is a certain calibration which is a involved, there is the certain normalization of the intensities, these intensities are normalize with respect to one of them and then you really measure the ratios of intensities.

So, we will now studies this question that do we really need the reduce matrix elements, and in some cases in spectroscopy in a good number of cases you really do not. Because, when you take the ratios they get canceled out, and that is what we are going to the discuss now.

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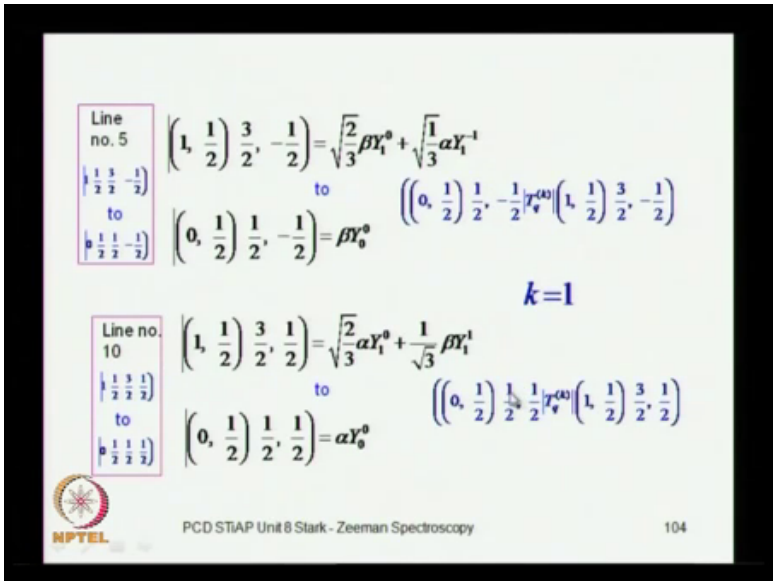

Now, so let us take two of these spectrum lines, so will take just to illustrate this argument will take line number 5, which is from this level to this level. And take line number 10 which is from this level, which is 1 1 1 half 3 half 3 half to where is it go, it goes all the way here which is the lowest level which is 0 half 1 half minus half. So, let us take these two cases, the first one is line number 5 which is the transition from 1 half 3 half minus half to 0 half half minus half state.

So, these are these are the expression in terms of the uncoupled bases, this is the matrix element that you want to study, this is the transition operator whatever it is, we know that

these transition in the dipole approximation are induced by an operator of rank 1, that is all you really need here. You do not even need it is explicate form all, you need is that it is the dipole it is the vector operator of rank 1, it does not matter if you looking at the length form of the matrix element or the momentum form the matrix elements it does not matter.

Because, that only thing that is the importance is the rank of the operator, so this rank of the operator is 1, so you are going to put a equal to 1 in your Clebsch-Gordan coefficient. And this is one of the transitions that we will study, and we will compare it is intensity with the intensity of line number 10, which is the transition from 3 half 1 half state to 1 half 1 half, so let us see how this works out. So, this transition matrix element is represented by this operator, which typically is the dipole operator, operator of rank 1 and this is the operator which is sandwich between the initial state and the final state, and you got a similar expression for line number 10.

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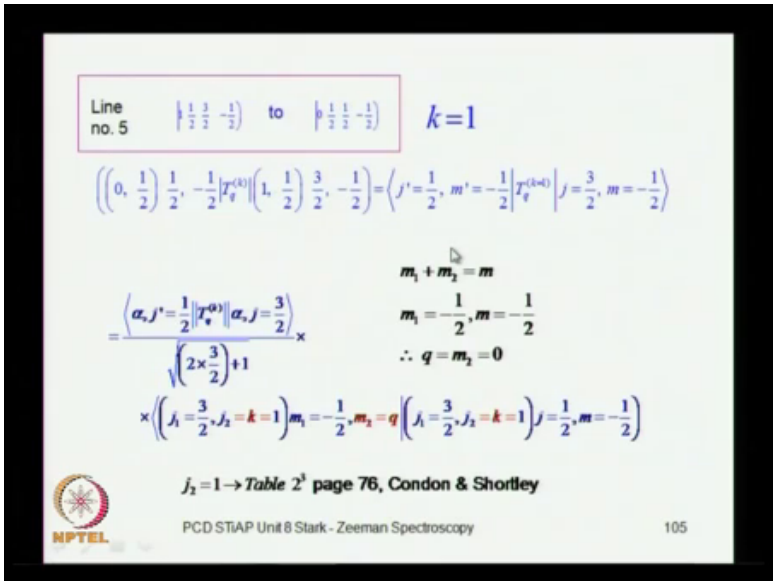

So, let us first take the line number 5 and this is the matrix element that you want to study, now this is the matrix element of in irreducible tens or operator of rank 1. You can resolve it using the Wigner-Eckart theorem as the product of the physical part, which is the reduce matrix element and we have define it with the root 2 j plus in the denominator, that you can define it differently also it does not matter. So, this is the reduce matrix elements part and geometrical part, which is the Clebsch-Gordan coefficient.

And this Clebsch-Gordan coefficient, what we know about the Clebsch-Gordan coefficient,

what about quantum numbers we know we should plug them in. So, we know j 1 which is 3 half we know j 2, which is the rank of the operator which is k we know that is equal to 1. We know that m 1 is equal to minus half in the state, this is m 1 this one and we know these quantum numbers what we do not know is m 2, so since we know that we are coupling angle momentum j 1 equal to 3 half with 1, which is equal to 1 j 2 comes from rank of the tenser operator which is responsible for transition, you will use table number 2 from corner Schrodinger.

So, this condition however, must be satisfied that m n plus m 2 most be equal 2 m, so this tells us that q must be equal to m 2, because both of these are minus half, so m 2 will have to be 0. So, q must be 0 and you can plug in q equal to 0 over here, so this comes from the selection rule which makes a Clebsch-Gordan coefficient non zero, so you use everything that you have learned and put it all together.

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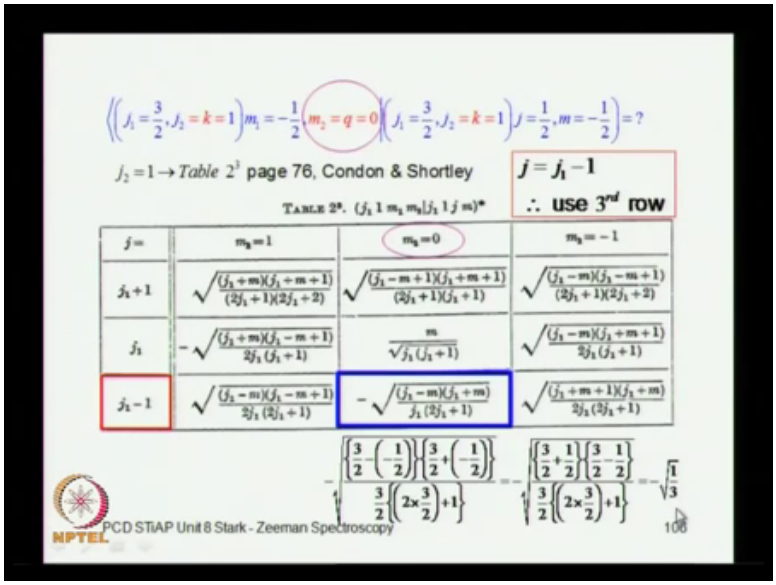

So, you have this table, table two for j 2 equal to 1 and we have got all the necessary quantum numbers, we have m 2 equal to 0, so m 2 equal to 0 means that you should use this middle column here. And what else do we have, now we need the relation between j and j 1 and what is that this is the third row that you must use, because j is equal to j 1minus 1, j 1 is 3 by 2 and j is 1 half. So, 3 half minus 1 is equal to half, so this j this half is equal to 3 half minus 1, so you must use a third row and this is the coefficient that you need. So, now, plug in the quantum numbers j 1 m and so on, that will give you value of the Clebsch-Gordan

coefficient, it comes with an appropriate sign And if you just plug in the numbers in turns out to be minus root 1 over 3.

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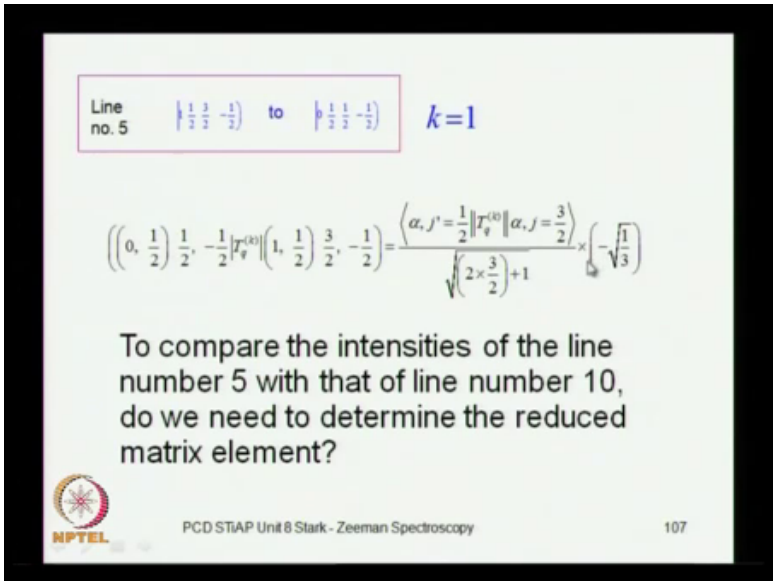

So, this is the coefficient you needed to write the geometrical part of line number 5, this is the physical part, this is the reduce matrix elements. Times the geometrical part which is root of 1 over 3 with the minus sign, and we will now ask what is the corresponding expression for line number 10.

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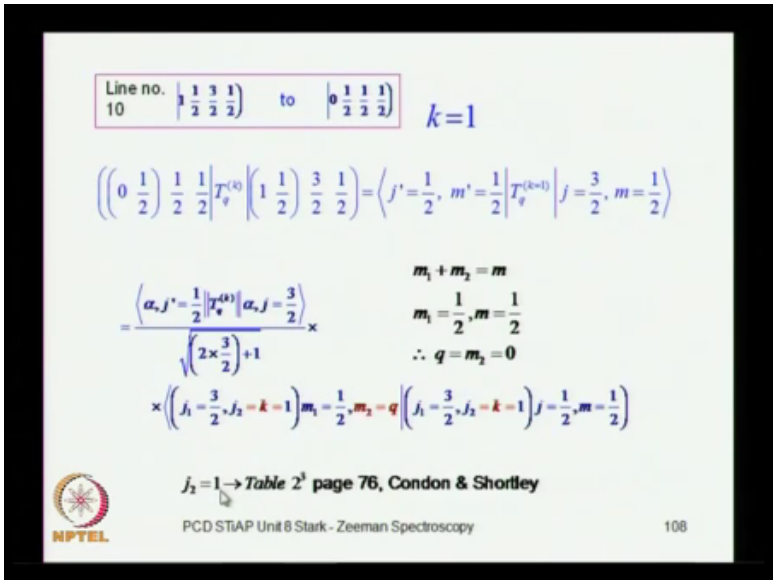

And for line number 10, this is the transition for line number 10, from 1 half 3 half of to 0

half half and half, again we know that k is equal to 1, so you break it into the physical part and the geometrical part put in the quantum numbers, put k equal to 1 over here for find that, in this case also the m 2 quantum number must be 0. For the same reason that in the previous case you had both m 1 and m equal to minus half, in this case both m 1 and m are equal to plus half. So, there is the similar, but it different reason, which is responsible for the fact that m must be 0 and using the table for j 2 equal to 1.

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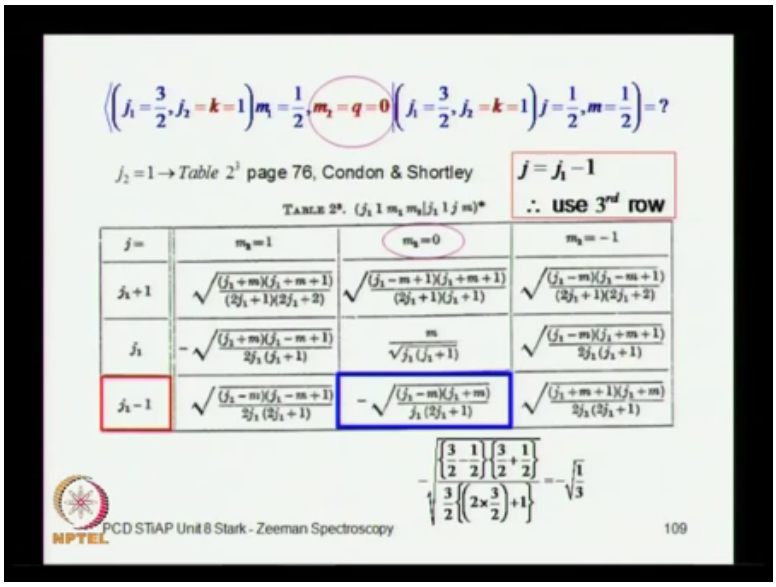

Since, m 2 is equal to 0 you use the middle column, which is the column for m 2 equal to 0 and then you will see this difference this is j equal to half, this is j 1 equal to 3 half the difference is 1, so it is j 1 minus 1, which will you give you the value of j. So, again you must look at the third row and you look at this expression and plug in the quantum numbers, again it turns out to be root 1 over 3 with a minus sign, but for a different reason. In the previous case you have two factors in the numerator and the under root, and the first factor was 3 half plus half and the second factor was the 3 half minus half; in this case these two position are swapped, but the numerical value turns out to be the same.

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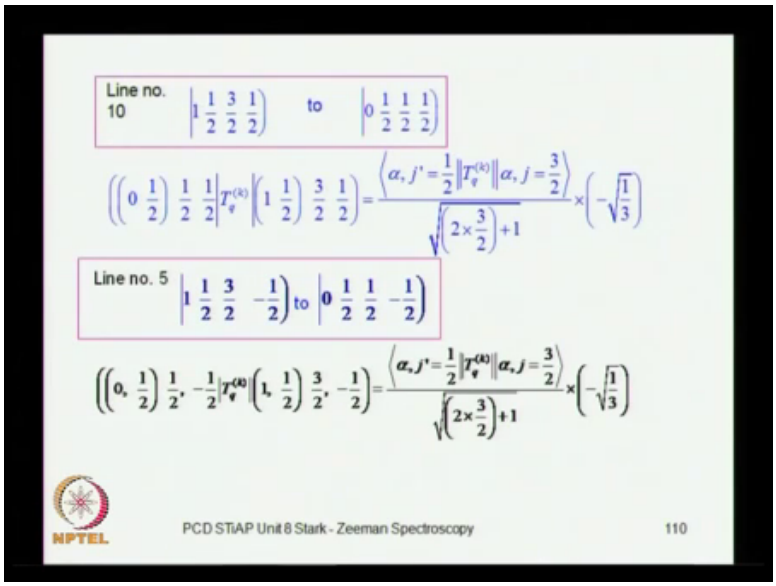

So, this is the matrix elements, which you have factored into the reduce made part and the geometrical part, which is minus of 1 over root 3, for line number 5 we had already found out what the factorization was. And here also you had minus of root 1 over 3, so what is our conclusion, when we take the modeless square and take the ratios the root 2 j plus 1 would cancel that does not surprise us. Because, they could have been observed in the definition of the reduce matrix element any way, but then the reduce matrix elements themselves cancel, is the same reduce matrix element which appears in more.

So, the ratio is then given only by the ratio of the Clebsch-Gordan coefficient, so without looking at the explicit form of the reduce matrix elements, that we have discussed the explicit form of the reduce matrix elements in some cases. For example, when we study photoionization, we actually determiner those in integrals, we plugged in the dipole operator, found what is the transitions probability from the initial state to the final state. We put in the radial functions for the hydrogen atom, so all of that had to be explicitly done, in this case we have not have to do it.

So, without even looking at the reduce matrix elements by taking advantage of the Wigner-Eckart theorem, we find that those terms which contribute to the reduce matrix elements, they cancel each other. And then all you need to consider at the ratios of the Clebsch-Gordan coefficient this particular case, they happen to be equal, so we can conclude easily that the spectroscopes going to sign, see these lines to be equaling tense. So, without actually

calculating the reduce matrix element which otherwise, is certainly required, when you look at this matrix element.

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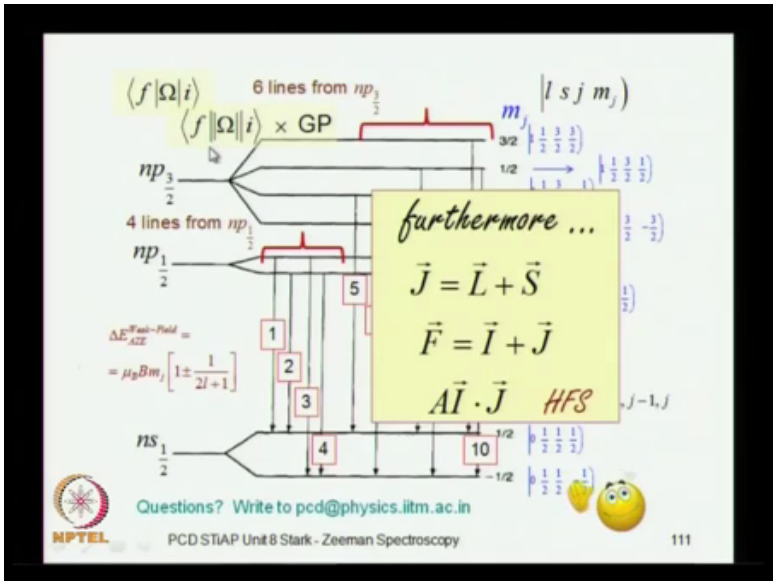

Because, this matrix element is given by a term like this, there is the transition with this operator is responsible for which effects the transition from an initial state to final state. And if you were to determine this explicitly, you will certainly have to evaluate all the space integrals, but by exploiting the Wigner-Eckart theorem, we could factorize this matrix element into two part. The first part is the reduce matrix element, second is the geometrical part and then we find that when you are looking at comparison between the intensities of various lines, which is the most common situation that experimental concern with, the spectroscopes are concern with.

Because, anyway they are going to do some standardization with respect to some normalization, so this normalization can also be observed in the normalization, when you do the calibration of the intensities. So, this is the power of Wigner-Eckart theorem which is an extremely powerful theorem in all branches are spectroscopy, whether it is atomic spectroscopy, molecular spectroscopy, nuclear spectroscopy, no matter what when you look at transitions of condense matter form one state to any other.

The Wigner-Eckart theorem is an extremely powerful one, and it gives you a excellent very powerful of handle on estimating the intensities of the transitions. But, then of course, there is more to follow that weak, started out with this non realities Schrodinger equation we had the l

and m quantum numbers, then we learnt that the speed of light is not infinite it is constant. And we must accommodate all consequences of that, and what comes out of that is the Dirac equation, what comes out of the Dirac equation is the electron spin.

So, the angular momentum is then no longer just the orbital angular momentum, but l plus s, which gives the total quantum number total angular momentum which is j, but even that is not the ultimate angular momentum of atomic system. Because, a nuclease is got a spin, the nuclease contains elements of articles the protons, neutrons, and they have their own internal spin properties they are fermions, protons and neutrons. And depending on the number of neutrons, on the number of protons the atom may have a net angular momentum, which is either integral or half integral and then you have either boss atoms or fermi atoms.

So, when you consider the nuclear spin, then the nuclear spin I will couple to the net angular momentum J and you will get another angular momentum, which is the total angular momentum inclusive of I and J. And this is a relatively weak interaction as one would expect, because the nucleus spin will involved not the bore magnetron, but the nuclear magnetron. You will remember, that the bore magnetron had the mass and the denominator, now you will have the mass of the nuclease in the denominator, and the mass of the nuclease is much larger than the mass of the electron.

So, that makes the nuclear magnetron much smaller and as a result of that, this is a relatively weak interaction nevertheless it is an important one. And this is what gives rise to the hyper fine structure coming from this interaction I dot J do just like to hand, the L dot S interaction, you now have the I dot J interaction which gives you the high prefine structure. And then this high prefine structure, which lead to further spreading of this Zeeman levels, and that splitting will be very small very tiny, but that is very nice.

Because, you can have very nice control when you look at those transitions, and some of those transition in the high prefine spectra of alkaline atoms are very common transitions. That you can control in laser cooling and other exponents and atomics physics like bioscience and condensation and so on, so today’s class we conclude over here.