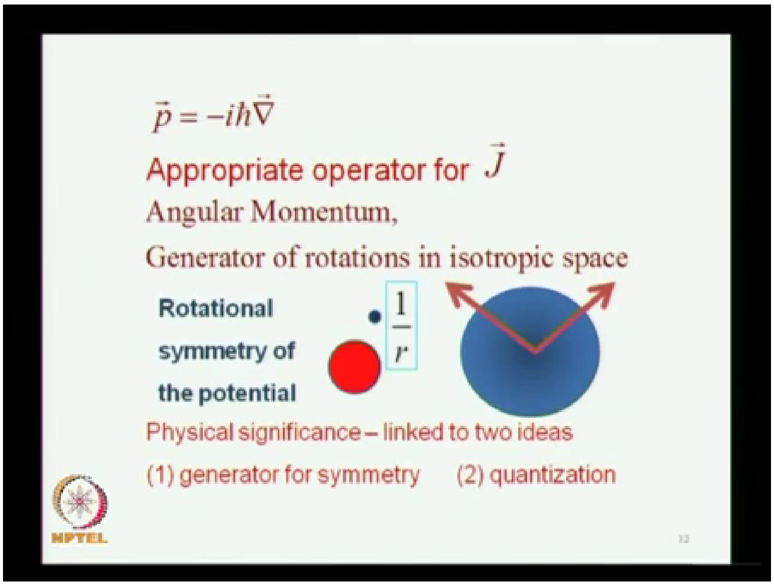
Select/Special Topics in Atomic Physics
Prof. P .C Deshmukh
Department of Physics
Indian Institute of Technology, Madras

Lecture - 2
Hydrogen Atom: Rotational and Dynamical Symmetry of the 1/r Potential

Greetings, we will continue with the quantum mechanics and symmetry of the hydrogen atom, the question that we began discussing is that how come the energy level of 3 p and 3 s, these are these energies are different, but for the hydrogen atom erase not the case. And this is the main question that we have began to address in this and we are going to discuss the operate of rotations, because the hydrogen atoms is got central field symmetry which is one over all coulomb field.

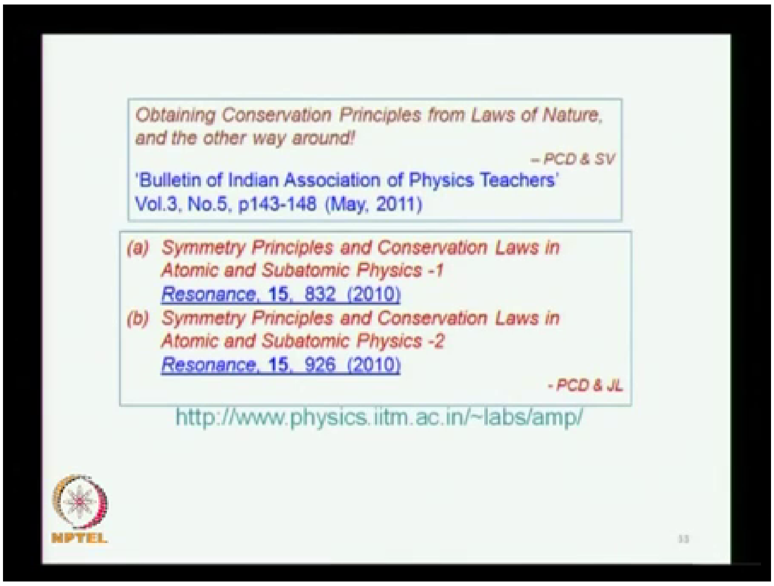
(Refer Slide Time: 00:56)



And we recall that the generator for translation in homogeneous specious momentum which is given by the gradient operator here and we need to find appropriate operator for the angular momentum. So, the liner momentum is given by the gradient operator which is minus cross gradient, and we have to find what is the appropriate operator for the angular of momentum?

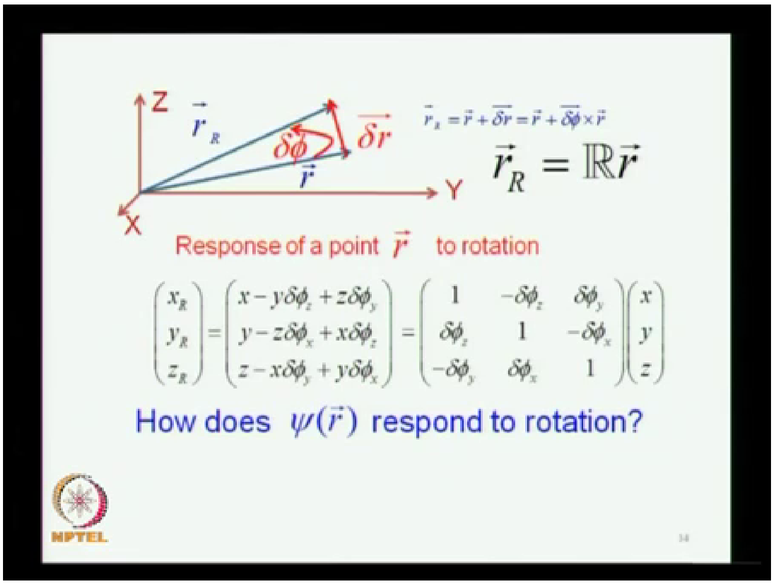
Now, the angular of momentum is the generator of rotations in isotropic space everybody knows what is mean by isotropic right. So, same tropic is different orientation different directions. So, no matter which direction you look at conditions are exactly identical in a isotropic space and the physical idea behind the angular momentum operator in quantum mechanics is governed by 2 centralized, 1 is quantization of course, and the second is symmetry that it connects.

(Refer Slide Time: 02:02)



And symmetry and quantization laws the play an extremely important role in our understanding of physics and I will like to draw your attention to these you know same your techniques popular articles which I will associated with the first 1 with student by name Shamala. And the second by my colic ((Refer Time: 22:24)) which probably all of you know and these articles are available on my web page. So, we are going to welcome to that page at your convenience.

(Refer Slide Time: 02:35)

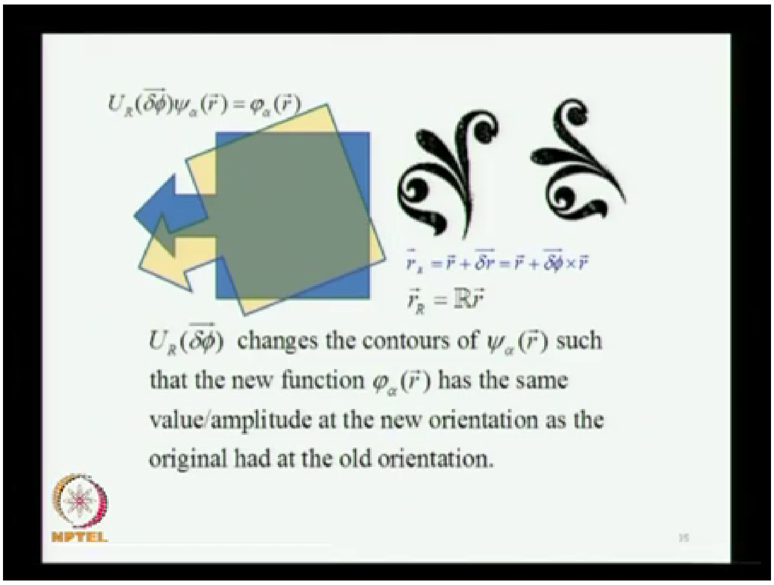


And we focus on rotations. So, you consider the rotations which takes the point from an original point to another point and the new position vector is r subscript r, which is r plus delta r and you can easily see, that this infinite symmetry displacement vector is given by the cross product delta 5 and I delta 5 is infinity symbol angle and that is the vector finite rotation and not n vectors, but infinitesimal rotations are.

So, this is infinitesimal rotation and we have written the response to rotation of a point in three dimensions space and you can write the new vector with Cartesian coordinates, which will be given by the coordinates of the vector r plus delta 5 cross r. So, it is a very straight forward thing to work out, you can also write this as a matrix relation as an operator matrix operator, operating on the original position vector. So, x y z is a original position matrix column vector and if you operate on this matrix.

So, you pre multiply by the matrix that you see, then the matrix multiplication rule will give you the new vector, so you can express this also as a matrix operation. Now the question before ask is how does a quantum mechanical way functions response to rotation or any function. Our interest will be quantum mechanical way function of a system, but in general any function no matter what is it, we not have anything to do quantum mechanics how does a function response the rotation, you seen how the coordinates of response rotation. So, the question is how does a function of coordinate. r is now the argument side and essentially what it means is that if you have a function with a certain you know control.

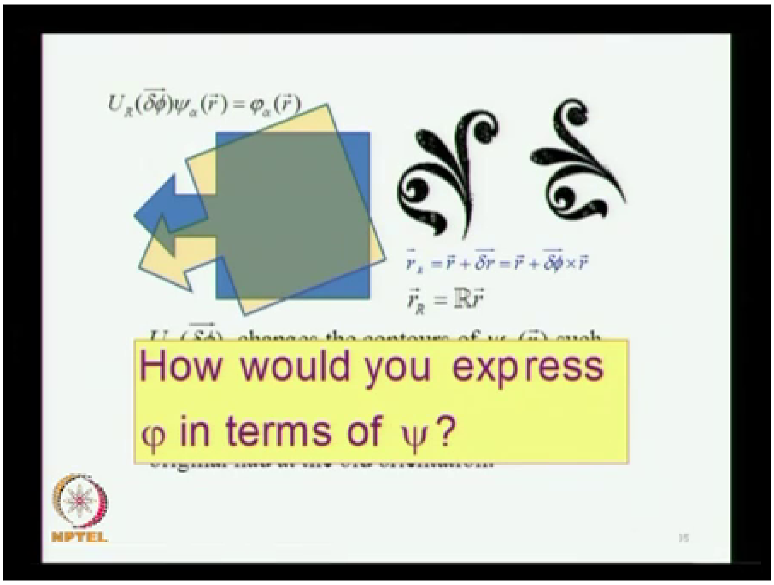
(Refer Slide Time: 05:01)



So let us consider some shapes and they can be any kind of arbitrators shape is you have 2 shapes on this in this figure. 1 you can describe as a square box with an array taking out of it these is some description which is possible. Simple description the other 1 is some arbitrary shape and it can be absolutely any arbitrary shape you can pick up a pebble from the science that are beechen you know that shape you talking about. Now the question is that under rotations how does this function which this types the controls, of this how would you response. So, what you have in mind is; obviously, some process of this kind.

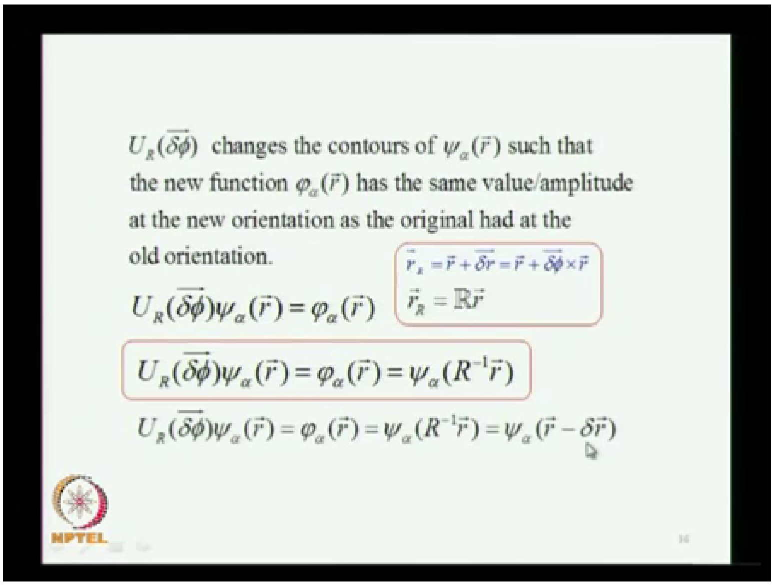
So you have a figure and you rotated. So, the meaning of rotation is very clear now essentially what the rotation operator does, this is the rotation operator U. It operates the original function 5 psi gives a new function pi and the reason it is rotation is because changes the controls of the function on which operators in such a way, that the new function, it has got exactly the same value and amplitude at the new orientation, as the old 1 original 1 had at the original orientation. That is mint that is what you mean by rotation there is no starching, there is no compression, there is no deformation right. It is a pure rotation, that is it this is the sense you have in mind.

(Refer Slide Time: 06:35)



So, now we ask the question how do you express, the new function psi in terms of the original function side. If you can find a recipe to get this, you would know prissily what the rotation operator is. So, you have to find out what kind of operator will generate this transformation.

(Refer Slide Time: 06:59)

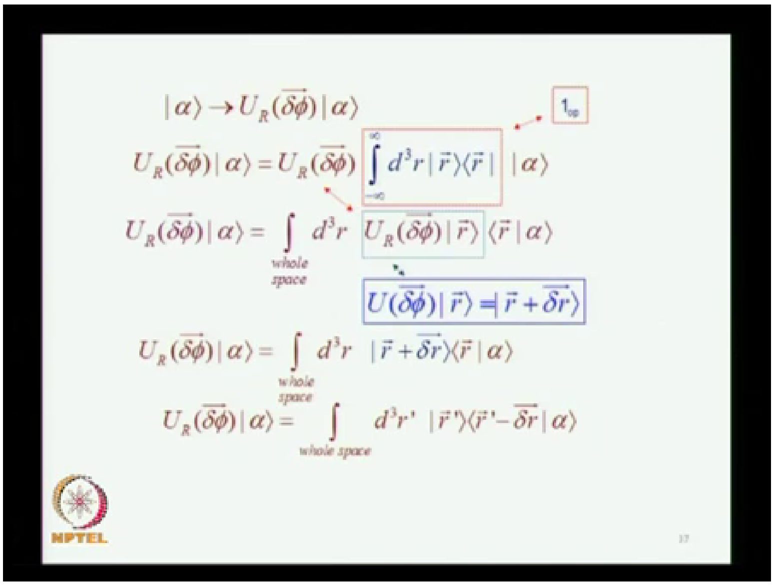


So, let us write that .So, you have the Rotation operator which operates on psi will gives you a new function and the reason there is no scratching, compression, deformation, which is involves, it means that the new function will have such amplitudes. At the new orientation as the original had at the old orientations. The only thing that has change is a orientation, which is the rotation which means that you can get pi completely equivalently you describe as a function of psi, but with an argument which is transform backward.

So, pi must be expressible as psi whose argument is r inverse of r and now we almost have solve, that is the connection. So, rest of it is very simple algebra. So, once you get this idea home, you know that this function whose argument is shifted r inwards this r inwards of r you know a psi of r minus delta r.

So, it is the value of the function at a neighboring point and you know from differential calculates, if you know the value of function certain point you can always get the value of the function at neighboring point, if you know the rate at which change from 1 point to the other, Right the derivatives something that you need.

(Refer Slide Time: 08:34)

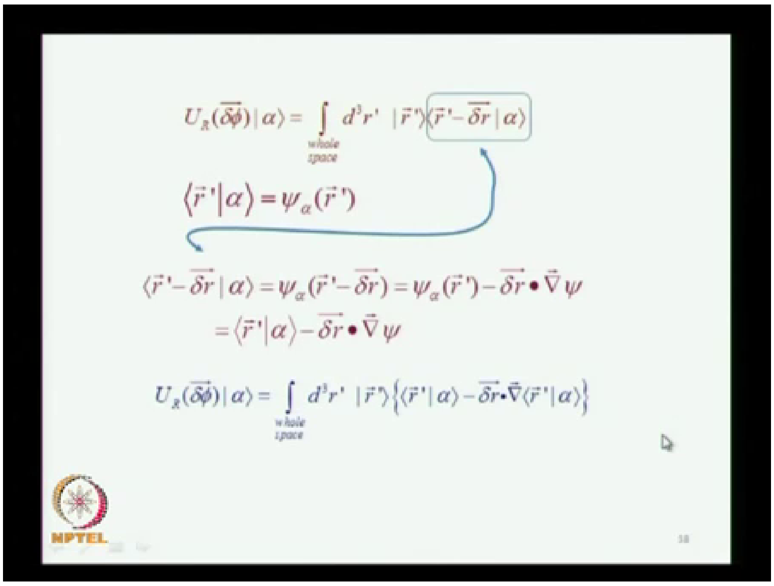


So, we will use that machinery and we can do it for arbitrary state vectors. So, alpha is some arbiter state vectors, in the heal burs no matter what it is and we are asking same question, how does it response to rotations. So, what we do is to insert the unit operator. We have the resolution of unity as you can see clearly right and now we operate on the Eigen ket r which is Eigen ket of the position vector.

By this rotation operator and we know that when this rotation operator operates on the Eigen ket position operators. It protect it shift to a new position which is r plus delta r. So, result is already know.

So, we plugged it in and Now, what we have integration of whole space of a vector scaled by this scalar, which is the position represent of the state vector r alpha, right and this is integral over whole space and the integration label always dummy label. So, instead of r plus delta r if you take r prime to be equal to r plus delta r, then the integration over r can be shifted to integration over r prime, but when you do that every argument are must be replaced by corresponding r prime appropriately, so this is the delta. So, what it means is that this functions, are alpha this 1 right. This will be written as the position representation of alpha, but the argument will be r prime minus delta r, is it clear, everybody.

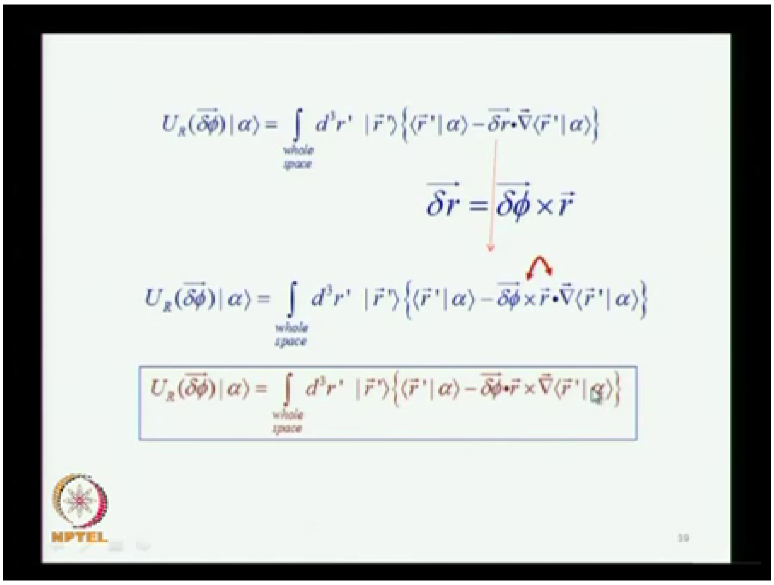
(Refer Slide Time: 10:37)



So, I bring it to the top of the next slide here. Now you got alpha at r prime minus delta r and you know the connection between the Dirac representation and starting ((Refer Time: 10:53)) the function representation psi alpha r is nothing but coordinate representation of state vector alpha. So, when you have the argument r prime minus delta r which is what you have in the integrant right, in this integrant the argument r prime minus delta r.

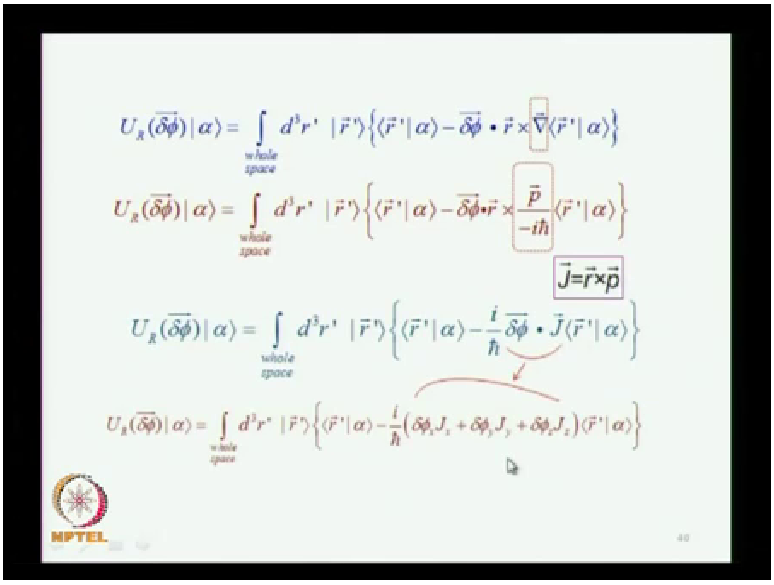
So, you looking at the functions psi at a coordinate r prime minus delta r. So, this is like looking at the value of the function neighboring point, which is the equal to the value of the function at the original point and you have to add the difference coming from the displacement, but you have to take the only component of the gradient directions right. So, this is elementary first order differential calculus. See plug in that result the original function is psi alpha r prime which you have written here the second term is the component of the gradient along the displacement and it will also depend on the size of the displacement and now you can write this expression is said these 2 terms in the integrant over here. So, let us do that we have these 2 terms inserted in the integrant and now you have 2 integrals to worker.

(Refer Slide Time: 12:24)



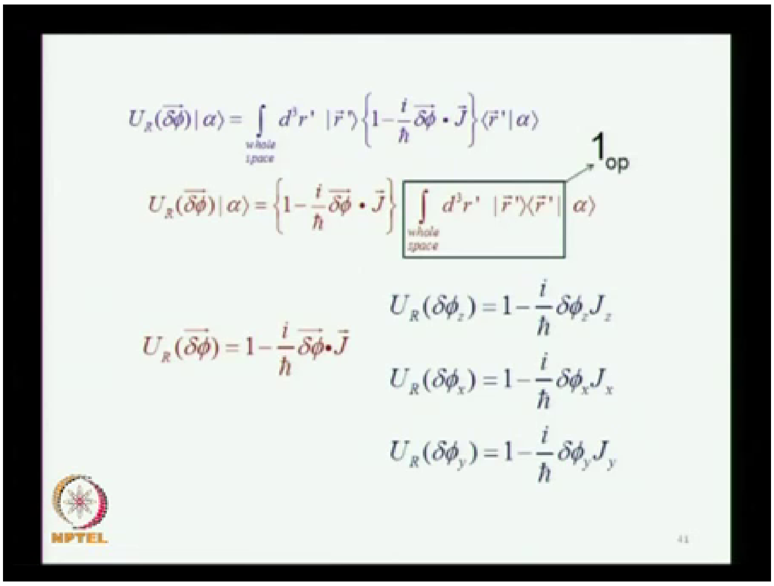
So, let us see what term r these are the 2 integrals at the top, In which delta r, i recognize as the cross product of delta pi r. We have already seen that. So, we insert this equivalent term delta pi cross r over here and here you see scale of triple product this is the triple product of delta pi r in del, and gradient of this functions right and in a scale of triple product you can always interchange the dot of the cross.

(Refer Slide Time: 13:10)



So, now you have delta pi dot r cross the gradients of this functions. Now what is essentially it means, this is the gradient operate is nothing but the momentum operator it is momentum divided by minus I cross. you recognize that you have the cross product of r with p, that is the angle of moment. So, give the angle of momentum operator and you have the projection of angle of momentum along the infinitesimal displacement, vector angular delta pi. This is the scale of product of 2 vector delta pi is the vector as an infinitesimal rotational angular vector.

(Refer Slide Time: 14:02)

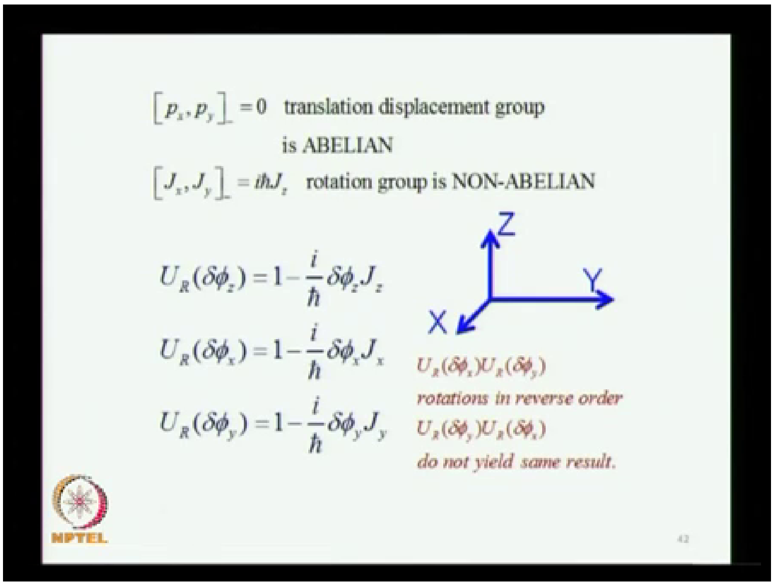


So, I have just expanded it in terms of the component of delta pi and j. Let us bring it to the top of the next idea and I now have expression for the rotation operator, because the operator in this beautiful bracket in this parentheses right. The operator in the beautiful bracket does to alpha, what the rotation operator does to alpha. So, it essentially gives you the exact form of the rotation operator.

Because, in the integral you see that you have got a unit operator you can inserted you can take it off. So, essentially what you looking at is the rotation operator operating an alpha. So, this gives an exact identity of the rotation operator, the rotation operator as much as there are linear momentum of operator, was expressed by minus phi cross gradient.

The rotation operator is generated by the angular of momentum rotation are that is what we have seen over here. So, 1 minus I over h cross delta pi dot k, which is the operator sitting in that beautiful bracket, that is a rotation operator and you can write it for different components depending on the rotation, being about the x axis or y axis or z axis are about any arbiter excess in place you can always, find the corresponding form. So, you have these 3 components for j x, j y and j z.

(Refer Slide Time: 15:47)

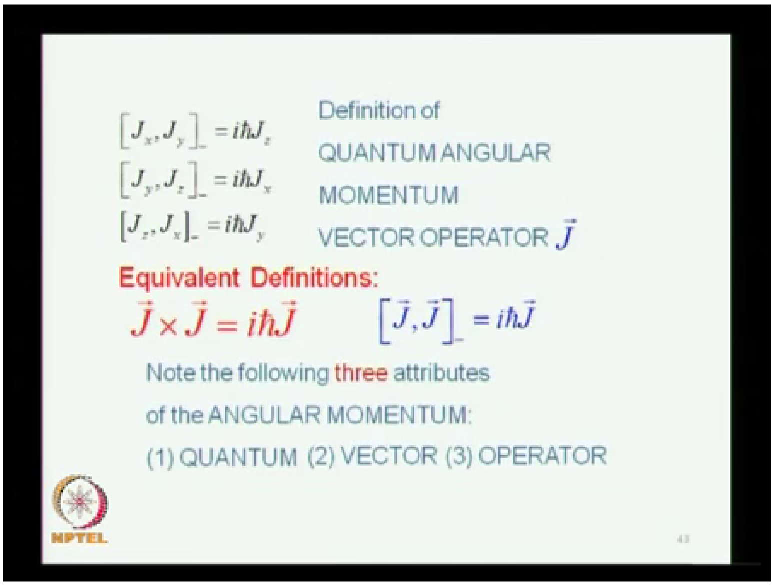


And if you just consider this rotation 1 after another. So, you take any shape it can be absolutely arbitrary shape it can be this shape, it can be this bottle or whatever you want right and subjected to rotation about the x y and z axis. rotated about the z axis do it like this, then rotated about the x axis or do it in the reverse way. first about the x axis and then about the z axis.

Now you know how to do it, but what I did to exerted to make a pile I show you pi rotation for 90 degrees, I show you or rotation through 90 degrees like this and another 90 degrees rotation like this do not do like that, do this rotation to infinite similar angular, but do exactly the same first your rotation about x axis often arbitrary shape it could be this bottle, it could be this pieces of stone, it could be any and do this exercise you have to draw this diagram in your note book.

Take some arbitrary shape, subjected to about rotation of z axis, do an infinitesimal angel not a finite, about the z axis then do the same about the x axis and then carry out the rotation the same to rotations, but in the reverse order. What you will find that the result of these 2 in the reverse order, will give you the rotation about the third axis. Just do this we have to actually do carry out joggies picture in your note book this in fact, a problem from ship quantum mechanics and when you do that you will find this rotation no need to commute. They do not give you the same result, linear displacements they commute p x commute p y, but j is not commute j 1.

(Refer Slide Time: 18:20)



So, the group of the displacement operator is abelian the 1 for rotation is not abelian, and Essentially what you find that the commutation relation of angular momentum automatically come out of this property, that rotations about orthogonal axis through finite through infinitesimal angle do not come, you get it different result. So, these are the commutation relation you get the angular of momentum and you except this to define angular momentum, this is now the definition of angular of momentum we are not going to refer anymore to r cross p. We have use that idea all right never the less we are not ever going to refer to angular of momentum to r cross p, about definition of angular of momentum.

Now is the following that angular of momentum is set an operating in quantum mechanics, it is a vector operator in quantum mechanics whose component follow the commutation rules, which are in front of us the j x comma j y is equal to i h cross j z and corresponding relation for j y, j z and j z, j x.

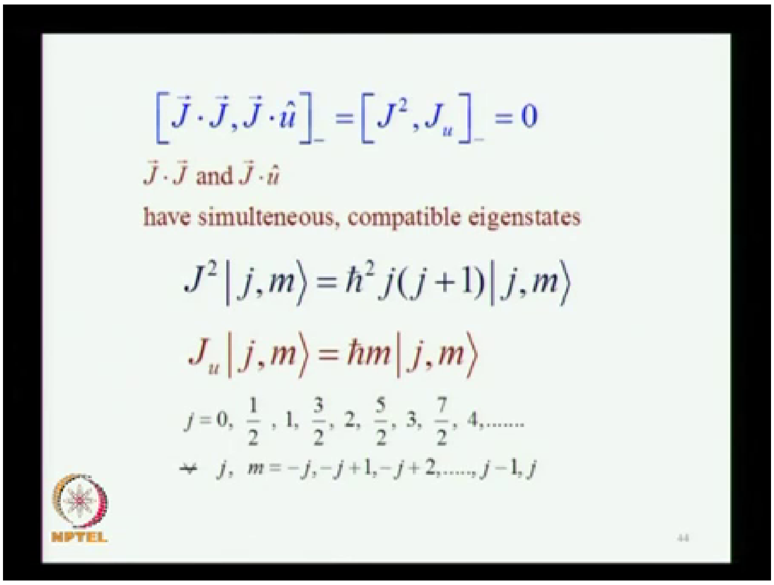
This is the definition there are other equivalent definitions, that the cross product of these operator with this term is does not vanish. This is the completely non classical result in classical, if you are dealing with classical vector. The cross product of any 2 colonial vectors led the loan itself would vanish right, but angular momentum is peculiar, at this definition is completely equivalent definition to the commutation rules definition you can get 1 from the other vice versa. you can also define it.

So, this commutator that the commutator of j with j is equal to I th cross j classical operator would always operate will always commute with any other operator which is proposed to the original operator right. A comma any constant multiply by the operator a would vanish, that does not happen with the angular of momentum and all of this 3 definition the committed definition, the cross product definition or the commutation of the components these are completely equivalent.

You can get one from the other is just a matter of you know doing some algebra to get 1 from the other these are all equivalent definition of angular of momentum in quantum mechanics are essentially one must remember that there are 3 attributes of angular of momentum that you must carry one is that it is a quantum creature second that it is a vector and third that it is a operator.

So, all the 3 attributes that it is a quantum operator or it is a vector operator and that it is essentially operator and therefore, it will meet the requirement of operator algebra it will operate on a appropriate operand given you new operands. So, on right give you a new vector. So, all that algebra of operator has to be invoked when you work with this.

(Refer Slide Time: 21:37)



So, we now have define angular of momentum in quantum mechanics, again it will be a matter of striate forward algebra to show that the commutated of j square, but commuted with any 1 component of the angular of momentum, but not with the other. So, it could commute with j x it could also commute with j y or with j z with does not matter what you call it x y and z.

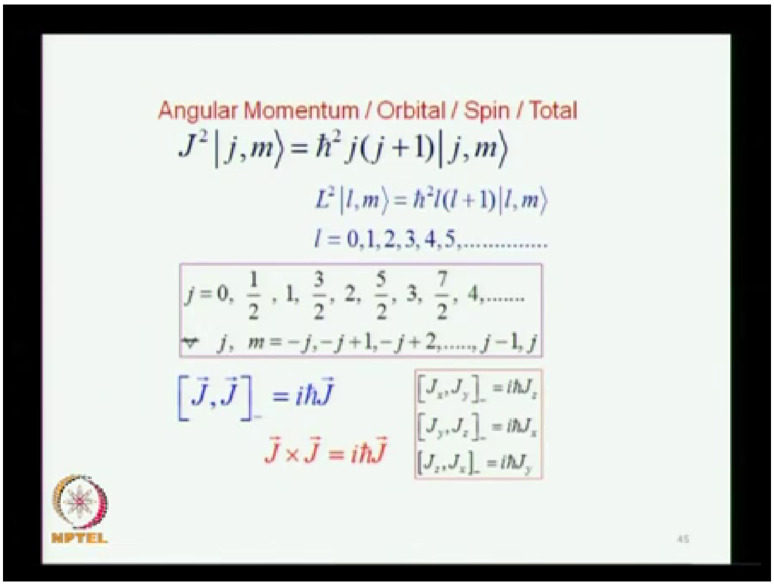
I can always choose a Cartesian coordinate system in which the z axis is pointing toward the heaven, but you can find it toward hell or east or west or any interaction it does not matter, any 1 direction in space and therefore, they have simultaneous Eigen states and what we discuss in our first lecture in our previous class was that in our quantum mechanics looking for a complete description of the state you are looking for as many observation as many measurement that you can carry out.

So, that you can describe a state completely, but the fundamental constraint is that the measurement half to be comprisable with each other. So, here you have compactable operator because they commute with each other. So, you can carry out this measurement simultaneously and you can get that much information at the same time, some you looking about the information of hydrogen atom.

You can then gets simultaneous information about j square as well as j a, along any given direction does not matter. So, you can develop the Eigen value equation you have the Eigen value of j square, you have the Eigen value of 1 compound that would be j z. It is any unit vector u, it could be z or x or y whatever insists it is simultaneous Eigen states of 2 operator which commute with each other the 2 good quantum numbers which come out. the 1 is j and other is m.

So, these 2 quantum numbers will now describe the hydrogen atom and from the algebra of angular of momentum some of it we will discuss separately, it will turn out that the Eigen values j can take any value which is either 0 or half or 3 half or 2 and so on, any half or integers, half r integers or integers. The corresponding value of m will go minus j to plus j. So, these are the possibility.

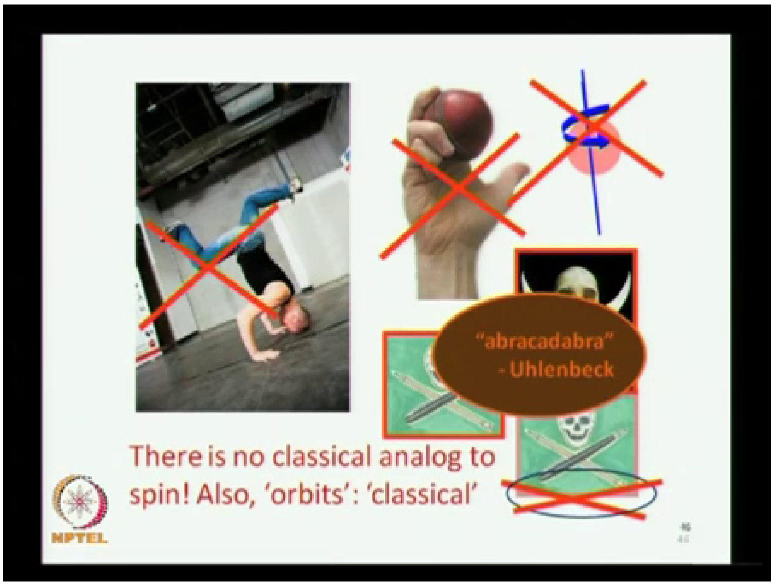
(Refer Slide Time: 24:58)



Now this angular of momentum, it could be the orbital angular of momentum, it could be the spin angular of momentum, it could be the total of angular of momentum or it could be the resulted of angular of momentum coming from the addition of the large number of sources of angular of momentum. So, whatever be the sources angular of momentum. it will all integrate to give you net angular of momentum and in every case if it has to be angular of momentum at all then it must meet, the defining criteria of what angular of momentum.

So, now, our first consideration is reserved for the angular of momentum of the hydrogen atoms which is l square l will go in this case from 0 to any number that you can think of all integer numbers, half integers will not appear in this, when you dealing with the orbital angular of momentum you will never have half integer quantum numbers they come from a different source of angular of momentum which is spin. So, spin will contribute half integer quantum numbers, but not angular of momentum and both spin as well as angular of momentum will satisfy the defining criteria angular of momentum which are these 3 equivalent definition of angular of momentum.

(Refer Slide Time: 26:36)

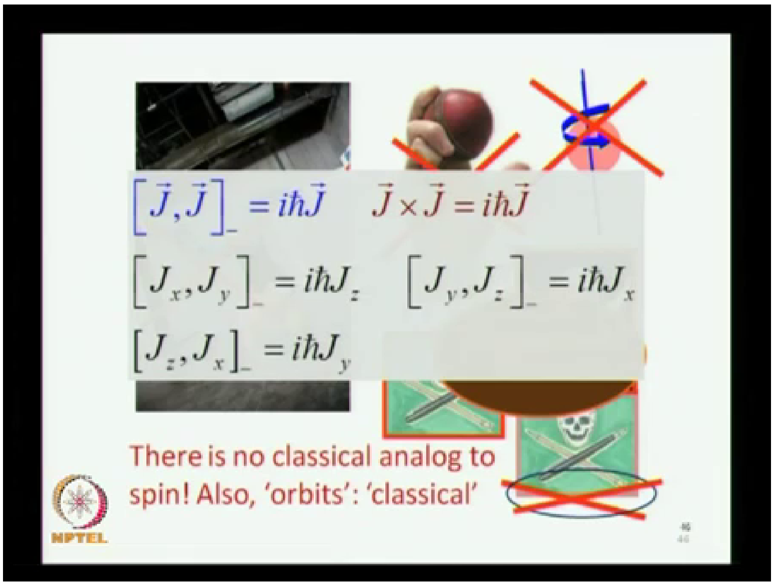


When you often when you think of spin these pictures come to your mind, this is something what you wide. because you know spin is something always think about these are the pictures that come to your mind on that is very important. because that what spin is if you see any of these picture associated with the spin angular of momentum, these are absolutely wrong and there will be 3 devils 3 sculls coming after you 3 sculls minimum 3 sculls.

So beware, so do not use this pictures whenever you talk about electron spin I am sure you recognize the sculls whose they are do not you 1 might be Polly, where the other true I do not know which 1 is Polly. So, there is no classical analog to spin absolutely none. It is a quantum creator there is no classical analog it comes straight on the quantum mechanics. So, we talk about spin angular of momentum which has got nothing to do with spin in the conventional sense you also talk about orbital of angular of momentum, which is absolutely nothing to without orbit. So, orbit also does not exist. We already discussed it for an orbit to exist in orbit is define as classical mechanics it is trajectory in face space of a point which is described by position and momentum, but these 2 cannot be simultaneously measured and therefore, orbit do not be exist.

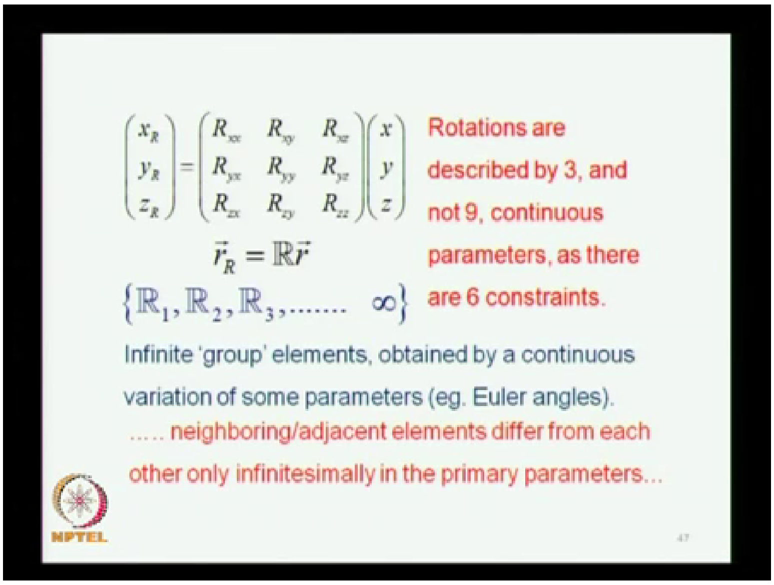
So, orbit angular of momentum has nothing to do with orbits spin angular of momentum nothing to do spin. What is it done first we have said what it is not, but we have also said what it is that it is a quantum vector operator, whose properties are such that j cross j is equal to i'th cross j. The j x comma j y is equal to i'th cross j these are the defining criteria of angular momentum.

(Refer Slide Time: 29:15)



So. In fact, when spin was introduced by Uhlenbeck back and gulls mate. it what they did not know why it was so. In fact, Uhlenbeck back call it was abracadabra. abracadabra u know some think like magic, it seems to work. But, the actual definition is just completely mathematical it words, but then it connects to observable it connects to observable, but to see these this connection you have to develop the algebra further they act as an invoke. So, these are mathematical definition now we are working with rotation over here.

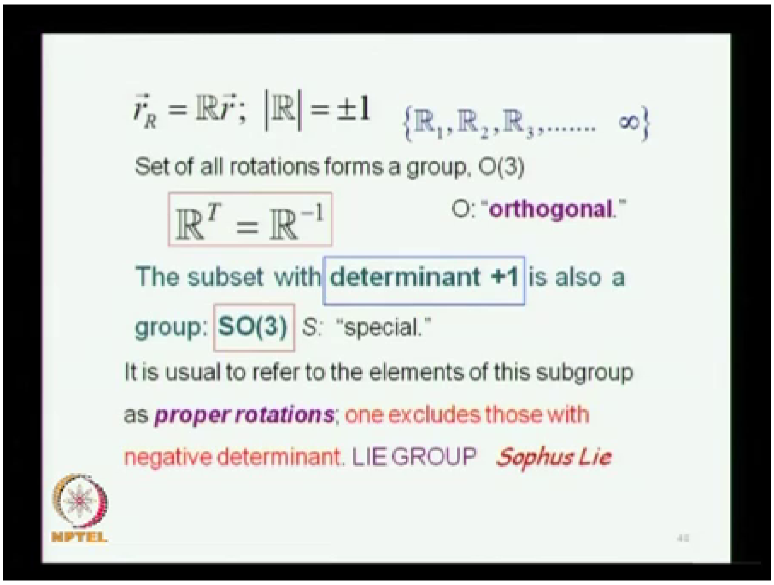
(Refer Slide Time: 29:40)



And the rotation matrix you can write with these 9 components, which is 3 by 3 matrix, but then there are 6 constraints. So, there really are 3 independent parameters which are the Euler angular consider. at this 3 parameters can be changed continuously to describe different rotation, so 1 rotation differs from the other rotation by an infinitesimal change in this parameters and because this change can be bought infinitesimal there are infinite number of this rotations.

So, you can stack them all together and they form what you call rotation group everybody knows what a group is. So, all the rotation operators you stack them in a set which is the mathematical group you have the existence of inwards existence of an entity you have got a binary operations that is define which is given as new element. there is a close of property and so on. So, these are you go from one to the next by variation of some parameter it could be the Euler angles and these changes can be absolutely infinitesimal, but you have the rotation group define by this now this place is very important role in the quantum mechanics of the hydrogen atom n.

(Refer Slide Time: 31:14)



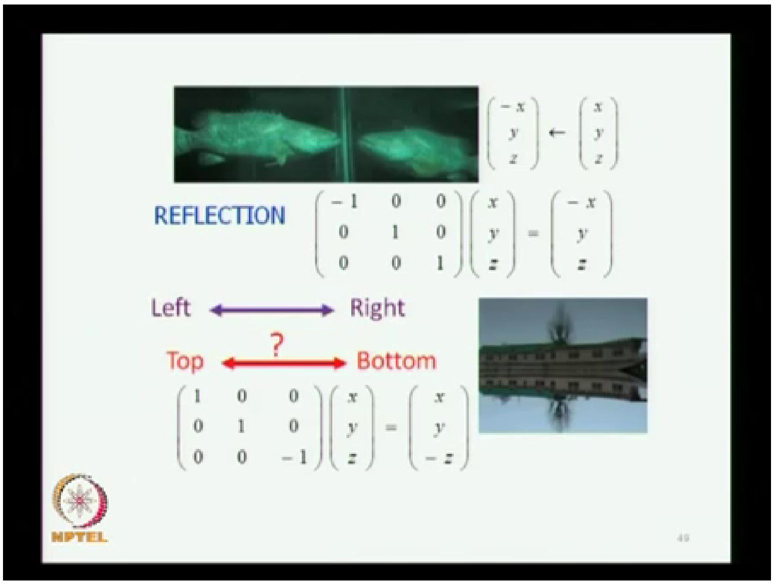
Now, typically when you go from 1 position vector to another, we agreed that you can represented by a matrix operation and if all this is done is to preserves its form then the determinate of the matrix are would be either plus 1 or minus 1 you have a choice you think of the plus 1 matrices where as the minus 1 matrices.

The 1 which has got determine which is plus 1 another got determine which is minus 1 and it is important to remember that in rotation you are dealing with this orthogonal matrices where the transpose is equal to the inwards. So, you always have operation of this kind, but then you take subset of those matrices, whose determinates is plus 1 and not minus 1 this is extremely important.

So, you're dealing with orthogonal matrices, but not all kinds of orthogonal matrices special orthogonal matrices, what is the special t that these are such orthogonal matrices whose determinates plus 1 those plus 1 and minus 1 are removed from your conservation. So, that is what makes it s o. s for special, o for orthogonal and then you have got the group is called SO 3, because the numbers that appears in parenthesis it is connected with the number of generator and I will tell you what that relationship is, but these are you know special case or you know some examples of lie groups and you excludes those with negative determinates, you are dealing with those rotations which are positives.

So, this is named after sophus lie and this would distinguishes from reflections, because when you look at a reflections you always see the right goes to left and left goes to right. We do not see the top going to the bottom and bottom going to the top. Is a same symmetry in a certain science right and the reason that there is happens when have heard all kinds of answers when I erase this question means you is begin to think about logs of reflection angles instead it could angles of reflection and so on all those answer are not correct.

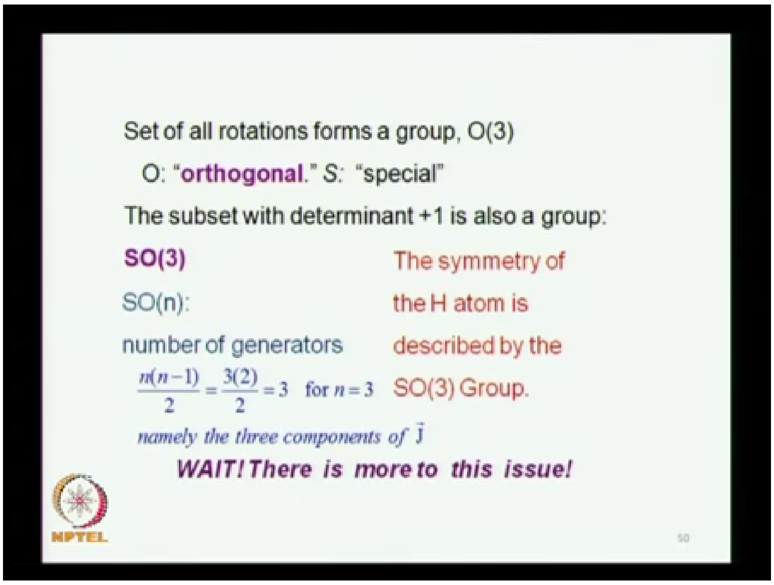
(Refer Slide Time: 33:47)



The real reason is the following, that when right goes to left then left goes to right, you have got a vector x y z. which goes to minus x y z for example, right and what you have is the operation is described by matrix whose determinates is minus 1 this is what happen in reflections.

So, this is an operator which is represented by a matrix whose determinates minus 1. So, it will not come in our consideration for rotations. That is what distinguishes reflection from rotations, the top would go to the bottom and bottom will go to the top, is just a matter of what is the plan of reflection. But then again the matrix will have determinates whose value is minus 1 as you can see and rotation you will always have a matrix. Whose determinates is plus 1. So, the reason top does not go to the bottom to the top, same time the right goes to left and left goes to right is because the matrix is different.

(Refer Slide Time: 35:05)



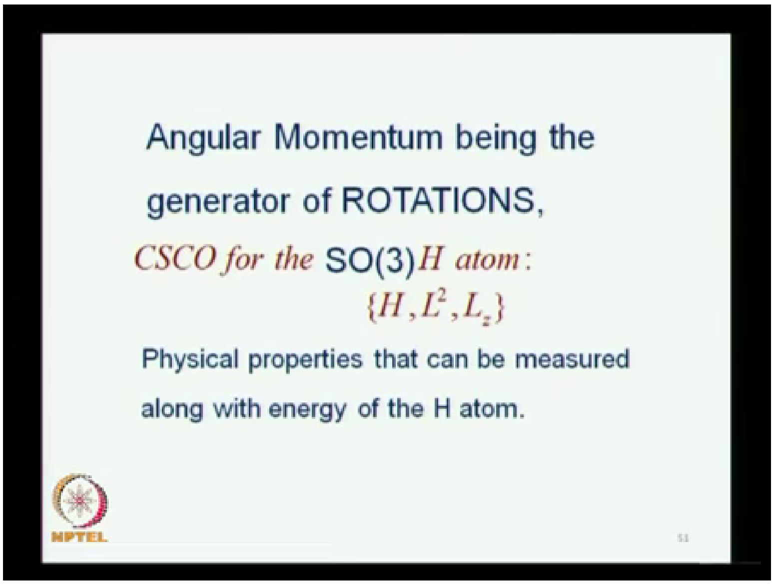
So, the different kind of operations it is a symmetry operations that, but it is the different symmetry is a reflection symmetry or rotation symmetry. So, this group which describe the hydrogen atoms is called as SO 3 the number 3 actually comes from it is you know generate, the family of groups called as SO n. it could be SO 3 SO 4 and so on where the argument of SO gives you the number of generators by the relation n in to n minus by 2. So, when n is equal to 3 into 2 by 2 gives you 3 and there are 3 generator for the rotation in nucleus space which are the 3 components of the angular momentum.

So, the constitute the SO 3 group and the symmetry of the hydrogen atom is described by the SO 3 group, but as we are going to learn in the next few classes there is more to this. So, this is our elemetric introduction to the symmetry of the hydrogen atom that this is described by the s o 3 groups. We know that j square can be measure along with 1 more component.

So, you can get information about 3 different characteristic of the hydrogen atom right, you want to describe the hydrogen atom the reason your doing quantum mechanics hydrogen atom that you are looking for a quantum mechanical description of the state of the hydrogen atom. At the state of the hydrogen atom is describe by a state vector which is to be label appropriately what are the designations, the designations will come from those measurement that you can carry out and you look for a complete set of comparable measurements and you have learn that the compatible measurement for the hydrogen atom are j square and j z or j square and j u where u is unit vector in something.

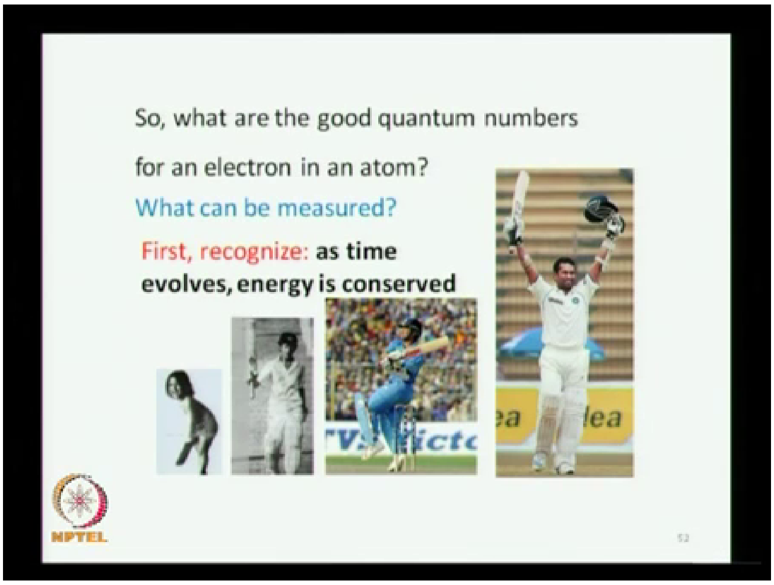
So, these 2 will give you some information about hydrogen atom and they both commute with the Hamiltonian. We describes the energy of the states, because energy is concerned if isolated hydrogen atom as time progresses from today to tomorrow day after you know left to itself energy concerned.

(Refer Slide Time: 37:57)



So, you can always measure n h. So, energy is concerned and these 3 the energy operator, which is the Hamiltonian the angular of momentum operator which is l square and 1 of its components which is z, these 3 will give you a complete set of compatible observables. The Eigen values of these operators will give a complete description of a state vector, that is the reason you characterize the state of the hydrogen atom assign n l m. n comes the Eigen value of the Hamiltonian, l comes the Eigen value of l square and m from the Eigen value of l z.

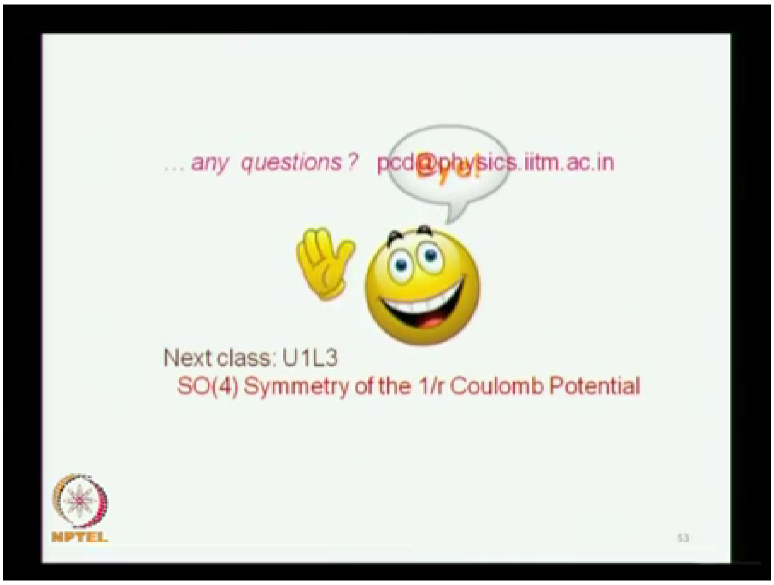
(Refer Slide Time: 38:50)



Because, these 3 operator commute with each other they correspond to measurement can be carry out simultaneously and these 3 together will describe the state of the hydrogen. Atom completely, they are therefore, called as good quantum numbers, this is what to you mean by a good quantum number quantum mechanics. You know you have numbers which have the word chosen these are good quantum numbers, because you can carry out corresponding measurement, these measurement are compatible with each other, first 1 of course, is the energy and you are basically asking the question. What is that you can measured.

So, since energy is conserved as time progresses and the example that are I give is this hero, whose energy to be conserved no matter what happens and energy of course, gives you 1 quantum number which is the energy label which comes on Eigen value of the Hamiltonian, but then you also get the Eigen values of j square and j u. So, n l m n these are the 3 quantum numbers that we have for the hydrogen atom. So, we are going to take some questions.

(Refer Slide Time: 40:04)



Now and any of the 2 classes that we had, so rest of it straight of algebra basic mole of the story is that you get you see as much information as you can, but the rider is that you cannot see whatever information that you want to see, that if you say I want to know the position of the subject I also want to know the momentum and I ask these trough question and this is something need is summaries very nicely, when he said that quantum mechanics will not give you all the answer, but quantum mechanics will tell you right question to be asked. So, it is not quite a right question to be ask to say that I want to know what is exact position, I want to what is exact momentum nothing wrong in asking this question means if you are not aware in quantum mechanics, but once you learn quantum mechanics you learn what are the right question to be asked you come to terms with the fact that certain measurements comparable some measurements are not comparable.

So obviously, what to do is stack all the measurements which are compatible and for the hydrogen atom the measurement, which are comparable with each are the energy the angular of momentum which is j square and one of its components of course, we all know that the forth quantum numbers the forth one will come relativistic quantum mechanics. It has no place as search with wrong relativistic quantum mechanics. So, we will of course, introduce that which are the spin, but in non relativistic quantum mechanics you have n l m n, n I am using tentevily.

Because it only represents the energy quantum number for discrete states, but you also have energy of continuous cases in which case quantum number is not n, but it still energy label. So, it will discuss in some further details, but tentevily available. So, thank you all very much any question.

Student: Sir j cross j equal j and j comma j equal to i j.

Can you say something more about that? Essentially it is a quantum feature.

Student: Sir j is not compatible with itself which means j and j cannot have complete set of simultaneous Eigen set of value.

Well it what it means is that you cannot get information about all the components of j let me give you very nice example here. Now, you know that magnetic movement is proportion to the angular of momentum. So, if you have a bar magnet let us say this is bar magnet and this appended with the spin or something it is oriented along the earth axis and you can say this is the z axis. Now you ask the question what is the momentum of this bar magnet and you will come up this value for m z and m x m i will be 0 because this bar magnet, if you told the classical bar magnet with will oriented self exactly along the z axis.

Now quantum bar magnet and nature is quantum mechanics. So, whatever we talk about this bar magnetic. In classical term is only appropriate description of magnet. Real magnet will have to be quantum mechanical, under real magnet cannot oriented itself like this.

This reason is if you want to do that you would then know all the 3 components are magnetic moment exactly at the z components, it m m x 0 and y 0 you get all the 3 and since, angular momentum is proportion to the magnetic moment you would get j x j y and j z exactly. That you cannot do, which means that quantum magnet cannot be oriented exactly like this and when you say that j does not commute with j is essentially is these that you cannot get all the 3 components.

So, the physical information in the commutation of j x comma j y equal to i th cross j z or j comma j is equal to I th cross j is essentially the same, which is the why the mathematical equivalent if you get from the other and the physical information is this that if you want to measure all the 3 components, then it is not a appropriate to ask the question is to what are the values of all the 3 components and I want to know them simultaneously.

Because, simultaneous measurement of these 3 components is these not compatible with each other for linear momentum there is no problem, p x commute with p y, p y commute with p z, p z commute with p x. That not is problem angular momentum it does not matter and that is not that property of angular of momentum it is. In fact, defining the criteria of angular of momentum this property is precisely, what define the angular of momentum in quantum mechanics.

It is a mathematical property, but then it connects to physical observation and that it important, it is not going to stay at that level. Because our interested physically is in connecting into observation it end of the day you want to explamentum the connected to what the theory that you develop and that is going to happen when you develop the algebra further.

How this actually show it that you know j comma j is equal to i is just say an algebra just work it. Out it will take in 5 minutes to do it. Begin now and in 5 minutes you will see write j comma j equal to j x e x plus j y e y plus j z e z comma j x e x plus j y y plus j z e z take it term by term, you will find that you have work with j x comma j y.

Student ((Refer time: 47:01))

Exactly right when you put it all together the answer will turn out to be i th cross j all right good, any other questions, yes sir you said that spin not have any physical analog. So, how does your response to the magnetic.

Student ((Refer time: 47:25))

Because there is magnetic moment association with angular of momentum, Sub magnetism is the property, which is not defined in terms of you know having a dipole of this term, magnetic moment is something which response to magnetic fields and how does it response. The reason it because you know when you keep a magnet having certain magnetic moment in a field there is certain energy which is given by this mew dot beta and it is this which will oriented in a certain directions, but this is precisely why a quantum mechanics does not oriented exactly like this and then in some of the modules are old quantum theory it is suggested as if you got an excess and this magnetic momentum vector precision about this.

The precision and Conway's idea that any set of time you do not have any component along the horizontal play, but this is really the old quantum theory, but in formal quantum theory, you have to describe it. Completely in terms of the components of the angular and there momentum and the commutations and that will give you exact answers and when you do the algebra develop it. Further get the expression values average values then you will connect them to observation. Those are the observable and this observable have to be define through Expectation values, transition forward it is and so on, That is very nicely said by Dirac says when you develop the algebra further you can connected to observations. So, you do not have the invoked wrong model, even if you get the some satisfactory answers.

So, thank you all very much.