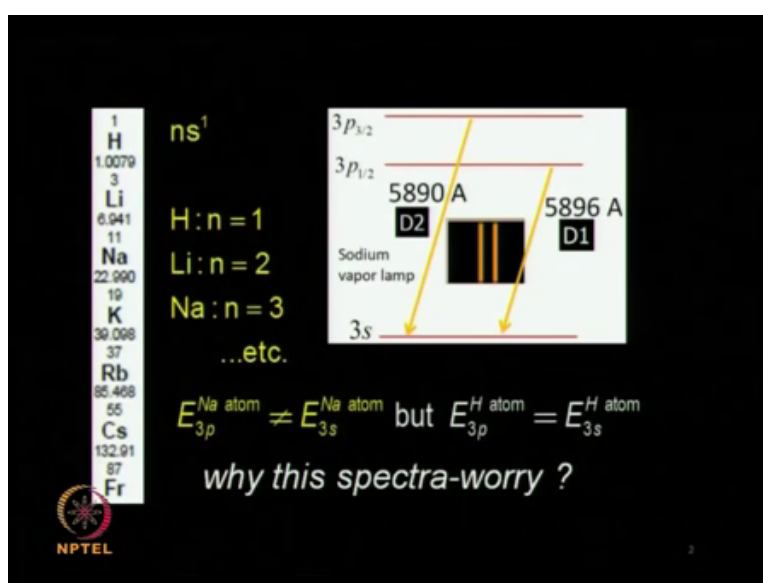


Select/Special Topics in Atomic Physics  
 Prof. P.C. Deshmukh  
 Department of Physics  
 Indian Institute of Technology, Madras

Lecture - 1  
 Quantum Mechanics and Symmetry of the Hydrogen Atom

Greetings and a very hearty welcome to this course, on mechanics and symmetry of the hydrogen atoms is the topic of the our discussion for the first unit.

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And what we will do in this is to addresses question, which perhaps some of you are at least familiar with, have we look at the elements in first groups of the periodic tapes. Hydrogen, lithium, sodium, potassium, rubidium, you are all familiar with this, these are all first groups elements. And there all in the first group, because there are some thing common about them, what common about them is the electronic configuration, all of the have got one electron in the outer most cell.

So, the hydrogen atom there is nothing like an outer and inner, because there is only one cell, and each atom in the first group of the periodic table has got this  $n s 1$  configuration in the ground state. Of course, the electrons can be exited to difference state, that is a different matters, but the ground state the configuration is  $1 s 1$   $n s 1$  and all of you have done some experiments in the optics lab and work with the sodium vapor lamp.

So, you are familiar with the d 1, d 2 lines of sodium, these are the 5890 and 5896 Angstrom wave length, yellow lines of the sodium vapor lamp, and as an I show your familiar from your first force and quantum mechanics. These transitions are from the 3 p level of the sodium to the 3 s, now all of you are away, and it is obvious if these energy levels were degenerate.

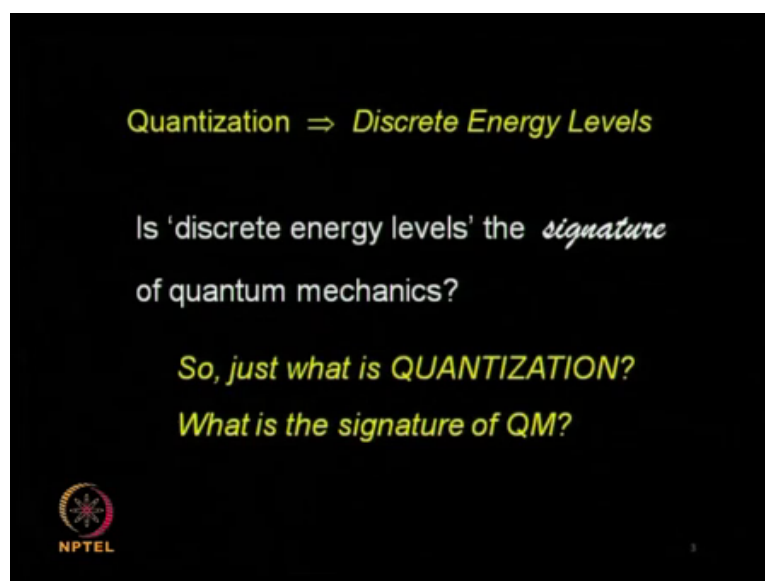
And you know what ((Refer Time: 02:09)) if you have linearly independent function, which belong to the same Eigen value, then you have degenerative. So, if 3 p and 3 s wave functions were to be degenerative, they were to have the same managings, then 3 p which is here and 3 s which is here there is a transition from 3 p to 3 s. If they were degenerate, they would be at the same level, and they could be no transition from 3 p to 3 s, and they would be no emission of light, and they would be no absorption, as you find in front of a lines.

So, it is obvious that the energy of 3 p is different from that of 3 s in the sodium atom, but in the same group if you go to the hydrogen atom, which is the simplest of all of these atoms in the first group. The energy of 3 p and 3 s for the hydrogen atom is the exactly the same, the energy Eigen values of the hydrogen atom, they goes as  $1/n^2$ , they do not depend on the l quantum number.

So, the question that, one would ask is this that the energy of 3 p is different that of 3 s for the sodium atom, but it is not the case for the hydrogen, and this; obviously, has got series implications is spectroscopy as is; obviously. So, the question here is why this spectra ((Refer Time: 03:50)) as you can see this slide was made when that, why this kolaveri song was it has gone viral, but I think most of the people have happily unfortunately forgot a that song now.

But, we do have this worry, as to why is this spectrum of the hydrogen atom, which is really simple, this is the simplest atom that you can think of. And this is the question that we are going to address in the first unit, we are going to study the quantum mechanics of the hydrogen atom, and we will try to get an answer to this.

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Now, whether it is 3 s 3 p and so on, essentially we are talking about this discrete energy levels, and we often say that quantization means having discrete energy's, do we all agree with that. Now, is having discrete energy levels the signature of fundamentals, it is not, what is the signature of quantum mechanics, do you understand this question what is the signature of quantum mechanics, because here we are dealing with discrete spectrum.

And we have to ask this question is what is the signature of quantum mechanics is this always the case, and just what is the signature of quantum mechanics, and what is really meant by quantization. So, many often it is suggested that quantization means having discrete energy levels, anybody has a comment or a question in this, you agree with this or you do not agree with this, what is quantization. I need you to start talking well anybody like to suggest a comment on this.

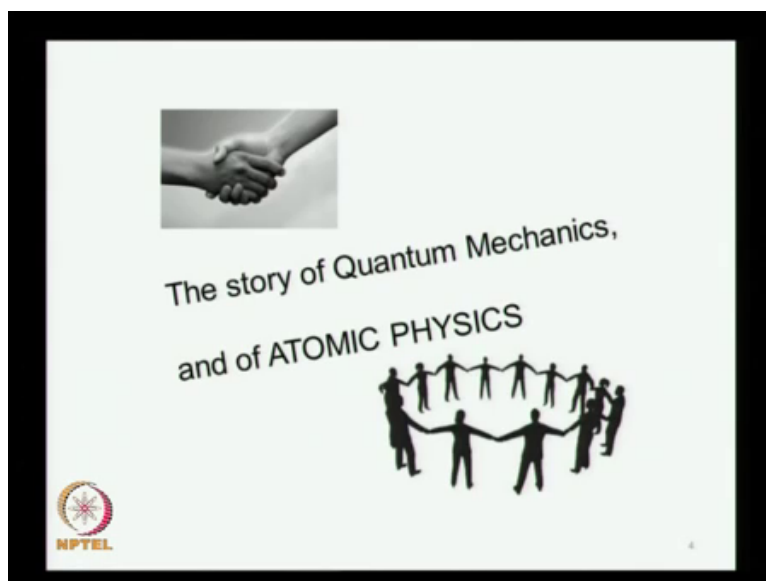
Student: On the particular values of some property shall out. So, that is quantization.

That is quantization anything any other answer, what is quantization, that is one way of looking at what else any other. Let me take this answers, and that will come here somebody else any other, what is your name.

Student: Pranay

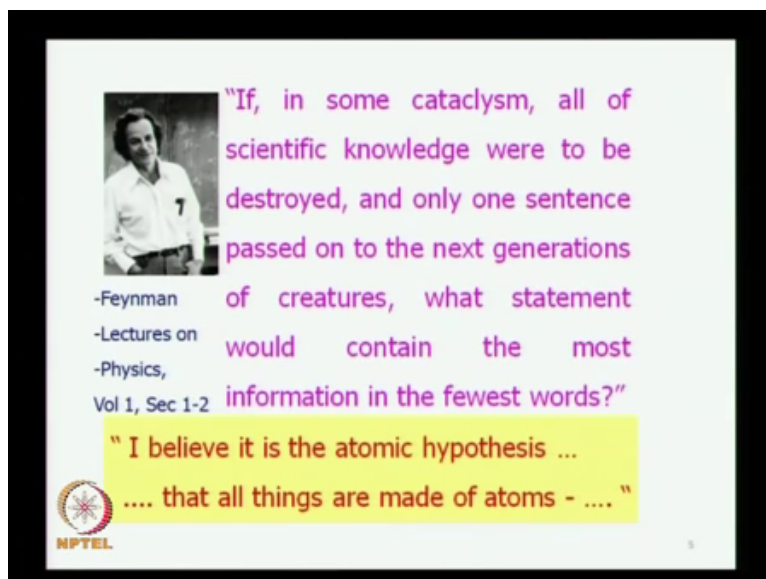
We will get the correct answer your scheme predict rows, but not exact and all the other answers and something lacking and you will see what was missing these answers.

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So, it is important to us that we discuss quantum mechanics, because quantum mechanics atomic physics have develop hand and hand. And what we are going to study means this courses about atomic physics primarily, but you know quantum mechanics will be developed along with it.

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And I will like to quote Feynman here, who says in his volume one section, 1 dash 2, that if in some cataclysm, which is the catastrophe, I had to consult the dictionary, I have no idea. If in some cataclysm all of scientific knowledge were to be destroyed, and only one

sentence were to be passed on to the next generation of creatures, what statement would contain the most information in the fewest words. This is the question which Feynman raised, and he goes to answer it saying that I believe, it is atomic hypothesis that all things are made of atoms. So, obviously, it is extremely important to learn atomic physics in a very thorough and regress manner.

(Refer Slide Time: 08:04)

AP Physics Nobel Prizes – last 3 decades: <span style="border: 1px solid black; padding: 2px;">2/decade</span>		
Year	Name of the Awardees	Citation
2005	Roy J. Glauber	For his contribution to the quantum theory of optical coherence
	John L. Hall Theodor W. Hänsch	For their contributions to the development of laser-based precision spectroscopy, including the optical frequency comb technique
2001	Eric A. Cornell Wolfgang Ketterle Carl E. Wieman	For the achievement of Bose-Einstein condensation in dilute gases of alkali atoms, and for early fundamental studies of the properties of the condensates
1997	Steven Chu, Claude Cohen-Tannoudji, William D. Phillips	For development of methods to cool and trap atoms with laser light
1989	Norman F. Ramsey	For the invention of the separated oscillatory fields method and its use in the hydrogen maser and other atomic clocks
	Hans G. Dehmelt Wolfgang Paul	For the development of the ion trap technique
1981	Kai M. Siegbahn	For his contribution to the development of high-resolution electron spectroscopy
	Nicolaas Bloembergen Arthur Leonard Schawlow	For their contribution to the development of laser spectroscopy

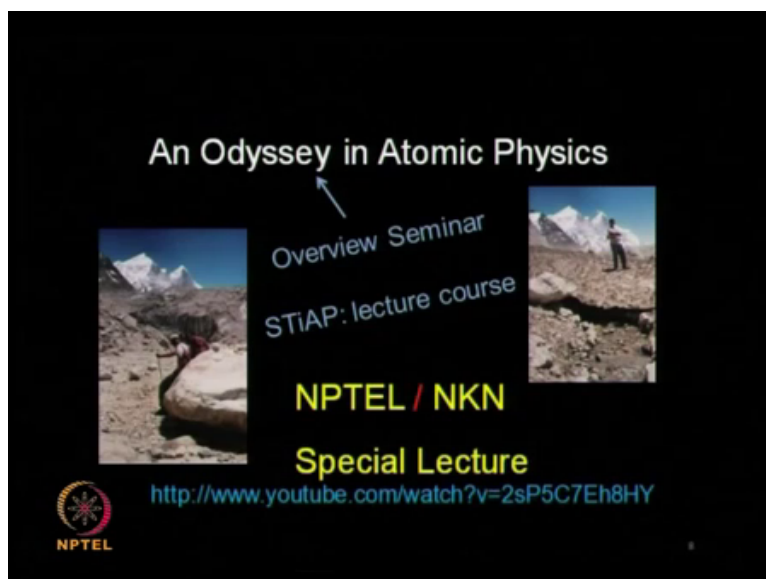
So, this is the many exciting subject, if you just see the kind of Nobel prizes, which go to this field, it averages almost 2 per decade, in the last 3 decades. And most of you must would be familiar with some of these big names, who have done similar in atomic physics.

(Refer Slide Time: 08:26)




And of course, they have been many contributors right from Dalton, or even higher to Dalton perhaps, how large number of scientist have contributed to this field, so we will touch upon the contributions of some of them as the horse develops.

(Refer Slide Time: 08:44)



Now, I have the review talks sometime back, which is on the you tube and it would give you an overall survey of atomic physics. And you will find the link to the, this from my webpage, but this is about the to our talk which is about, it reviews the developments and atomic physics, and what we have set out to do.


(Refer Slide Time: 09:09)




Hydrogen:

Atomic Number  $Z=1$

$q = 1.602176487 \times 10^{-19} \text{ C}$

  $m_p = 1.672621637 \times 10^{-27} \text{ kg}$


  $m_e = 9.10938215 \times 10^{-31} \text{ kg}$

Unit 1:

H atom : Complete Set of Compatible Observables

Quantum Mechanics

Spectrum & Symmetry of the Hydrogen Atom



Now, is a full lecture course on atomic physics, so we begin with the simplest of all the atoms the hydrogen atom, one electron, one proton. This is the simplest thing that, you can think of it is two body system and essentially we will discuss the symmetry and quantum mechanics of the hydrogen atom in first unit.

(Refer Slide Time: 09:29)

Who is this course for?

It is for students


*who do have some,*

*but not a whole lot,*

exposure to


quantum mechanics

of atoms & molecules.



Now, this course is meant for those of you who have some exposure to quantum mechanics not necessarily hold on, which is why we are doing the hydrogen atom again, but some exposure to quantum mechanics is certainly assumed.

(Refer Slide Time: 09:45)



Unit 1: Learning Goals:

- Understand the Hydrogen atom spectrum,
  - its eigen-spectrum, eigen-values, eigen-functions
- Complete set of 'good quantum numbers', associated constants of motion and associated symmetries.
- The Laplace-Runge-Lenz vector : Dynamical Symmetry and the Fock  $SO(4)$  symmetry of the Hydrogen atom.
- Contrast between the spectra of H atom and other atoms in the I Group of the Periodic Table, such as Na.

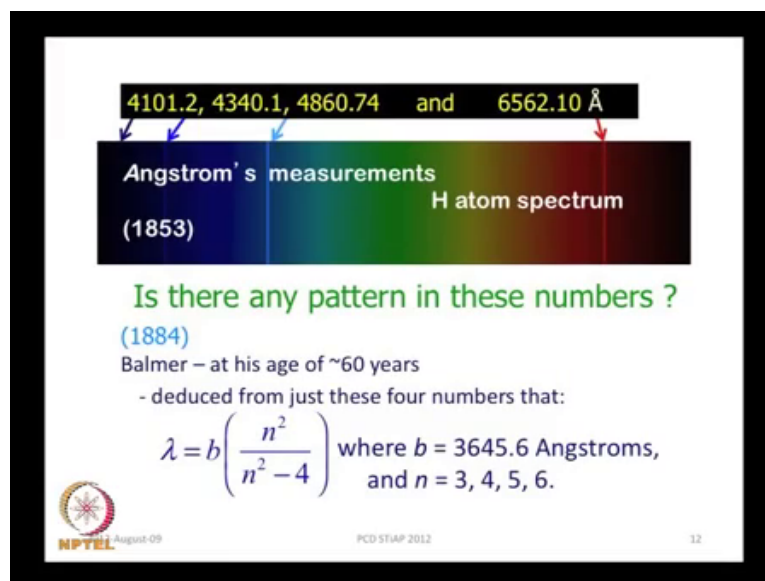
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We will learn about the complete set of good quantum numbers for the hydrogen atom, we will discuss what is meant by good quantum number, what are the associated symmetries constant of motion all of these things. We will discuss in this units, and in particular we will discuss what is known as the  $so(4)$  symmetry of the hydrogen atom. And how the properties of the  $so(4)$  symmetry lead to the Eigen spectrum of the hydrogen atom, which is different from that of the sodium atom with regard in the question that we raised.

So, then answer finally, comes from an understanding of the  $so(4)$  symmetry of the hydrogen atoms, so that is what this unit is going to address. So, it will helps us understand the different between the Eigen spectrum of the hydrogen atom, with that of the other elements in the first group of the periodic table.



(Refer Slide Time: 10:43)



So, let us begin with this spectrum of the hydrogen atom, these are 4 lines of the hydrogen atoms very precise measurements done by Angstrom's in 1853, he develops one of the first spectra meters. So, the wave length unit is named after Angstrom. Do you see any pattern in these numbers, do you have these numbers 4101.2, 4340.1, 4860.74. There is a certain four numbers in front of you, and if you want to be given some 4 numbers like 3 6 9 12, you would majorly tell me what the pattern is.

And when you look at it set of number like this, do you recognize any patterns, does it offer some clue do the numbers suggest some pattern. If you knew it may be you do, if you did not know at it is not something, which is very easy to discover, would you agree, it is not easy to discover that matter. Now, Balmer at his age of 60, which is good news for me in 1884, he actually found the pattern in these numbers.

And this is the pattern, that all of these 4 lines have got wave lengths, which fit exactly the formula, which you have on the screen the lambda must be into n square b square minus 4, with n going from 3 4 5 6, for these four lines. But, also the proportionality b is again not a very simple number to come up there, it has got a very peculiar value which is 3645.6 Angstroms. And Balmer figured this out, and as matter of fact I am sure all of you know this Balmer, you poly study with in your high school, and then in your college.

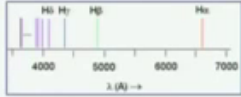

Despite that it did not occur to you that may be, I already know the pattern and this is the pattern which is Balmer discover, I think it is to me, it is very impressive in sight an arithmetic that Balmer code figure this out.

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**4101.2, 4340.1, 4860.74 and 6562.10 Å**

June 25, 1884:  
Johann Jacob Balmer's  
lecture to the  
*Naturforschende  
Gesellschaft in Basel.*

Balmer taught at Basel.  
From 1859 until his death  
in 1898, he was a school  
teacher of mathematics  
at a secondary school for  
girls in the city.

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Not only that Balmer was of course, able to do it, because he was a teacher in a girl school, and he was teaching mathematics, so may be girls are very sharp arithmetic scales, and that might of pushed Balmer skills.

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**4101.2, 4340.1, 4860.74 and 6562.10 Å**

Angstrom's measurements  
(1853)

H atom spectrum

Furthermore, Balmer predicted:

$$\lambda = b \left( \frac{n^2}{n^2 - 4} \right)$$

- There would be other lines
  - in the infrared
  - corresponding to  $n = 7, 8, 9, \dots$  etc.,
- The number 4 could be replaced by 9, 16, 25, ...

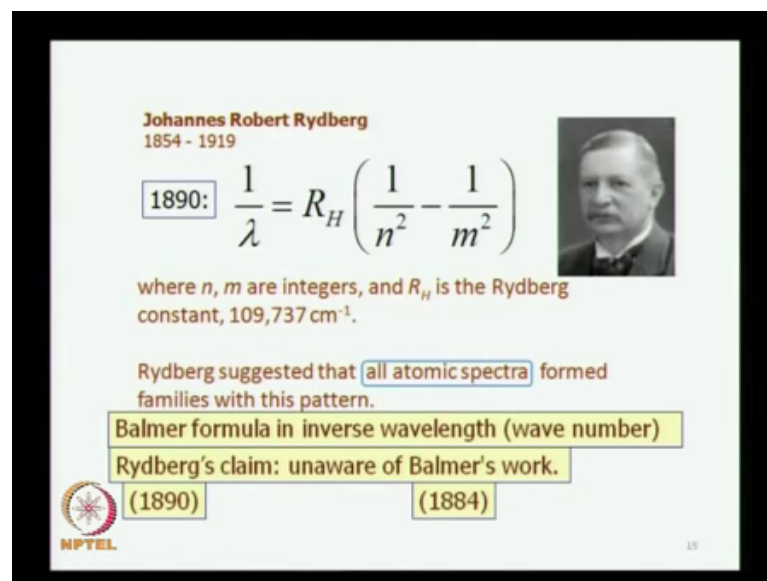
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But, Balmer also predicted that there would be other lines in the infrared corresponding to  $n$  equal to 7, 8, 9, etcetera, these lines were not seen at that time, there were no detectors in the respective region, but he predicted that these lines exist. He also predicted that the number 4 that you see in the denominator  $4 - 4$ , could be replaced by the square of 3 and 4 and 5. And then you would get other spectral lines, and all of these lines do exist, so Balmer not only found the formula, but he also predicted and these predictions turned out to be correct.

(Refer Slide Time: 14:34)



**Johannes Robert Rydberg**  
1854 - 1919

1890: 
$$\frac{1}{\lambda} = R_H \left( \frac{1}{n^2} - \frac{1}{m^2} \right)$$

where  $n, m$  are integers, and  $R_H$  is the Rydberg constant,  $109,737 \text{ cm}^{-1}$ .

Rydberg suggested that all atomic spectra formed families with this pattern.

Balmer formula in inverse wavelength (wave number)

Rydberg's claim: unaware of Balmer's work.

(1890) (1884)

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So, this is really amazing Rydberg also discovered that these lines correspond to a certain formula, which is a Rydberg formula which, in fact, is the same as Balmer's formula except that this is for the reciprocal wavelength. And Rydberg got this formula quite independent of Balmer, although it came 6 years later, that great men's often think alike, and Rydberg discovered this relationship quite independently and on his own.

(Refer Slide Time: 15:09)

**Johannes Robert Rydberg**  
1854 - 1919

1890: 
$$\frac{1}{\lambda} = R_H \left( \frac{1}{n^2} - \frac{1}{m^2} \right)$$

where  $n, m$  are integers  
constant  
Rydberg's atomic spectra formed  
this pattern.

**semi – empirical**

Balmer formula in inverse wavelength (wave number)  
Rydberg's claim: unaware of Balmer's work.

(1890) (1884)

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Now, this is a semi empirical form and you know what is mean by semi empirical form, semi empirical means, that you have got some exponential observations. You do not really have a theoretical model, but you do some mathematical jagulars, and you figure out that all of these numbers fit this particular form, so this is semi empirical, it is not fundamental it is not theoretical this action.

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1913

Quantum Theory of 'atom'  
The atom of Niels Bohr  
Bohr-Sommerfeld model

-Set of postulates  
-Orbits!  
-like Kepler Orbits

Niels Bohr  
(1885 - 1962)

Arnold Johannes  
Wilhelm Sommerfeld  
(1868 - 1951)

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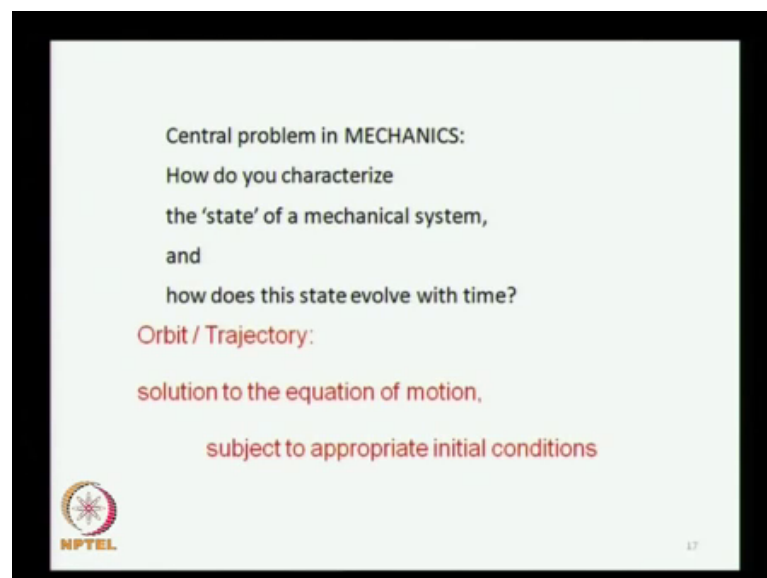
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Then later that was to at end of the 19 century, and then in the beginning of the 20th century Niels Bohr came with, what we called as Bohr's model of the hydrogen atom.

And this model which I am sure all of you have studied, also gives you the  $1/n^2$  term, and this model is based on this idea, that you have got a central nucleolus then electron revolve. It around this nucleation orbits, but the electrons do not radiate during this acceleration, and with a set of postulate that we call bolds postulates, quantization of angle of momentum and everything.

You are unable to get the one over  $n^2$  formula, so all of you have studied that, now this is certainly based on a set of postulate. So, it is not exactly same empirical, it is postulator, and it what the orbits that more invoked or like Kepler orbits, so there is a very close parallel between the Kepler model of the planetary motion around the sun and the ((Refer Time: 16:53)) of the hydrogen atom.

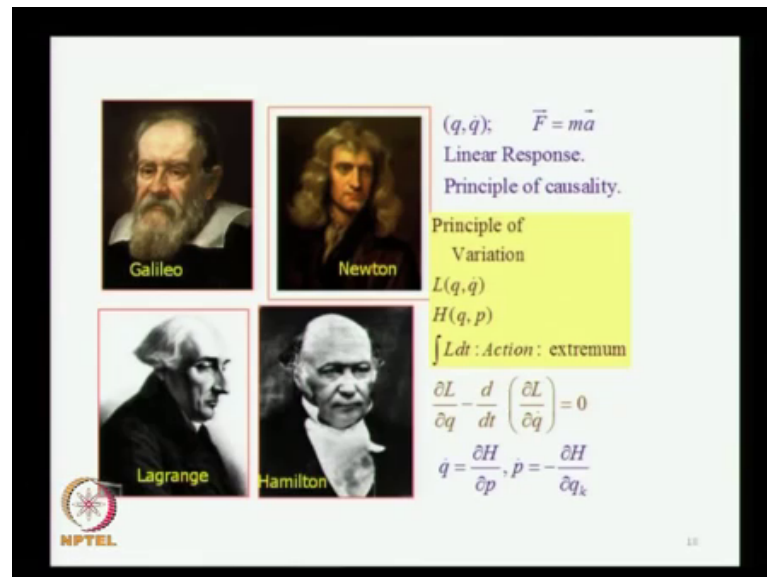
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So, this brings us back to the central problem in mechanics, and it is the same whether we are dealing with classical mechanics or quantum mechanics, the central problem in the mechanics is how do you characterize the state of a mechanical system. And how does the system evolve a time, this is the fundamental question. Now, in classical mechanics, you spouse the mechanical problem solve it, and the solution appears as a trajectory, the locals of the moment of the particle, that is the solution to your mechanical problem.

And that tells you how the state of the system involves with time, because a local of the particle of the trajectory of the particle is a time path of the particle in the phase space, so that is the solution in classical mechanics.

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Of course, subject to appropriate initial conditions, now this comes from our understanding of classical mechanics, and of course the foundation is released by Galileo, who discovered the law of inertia. What we called as Newton's law of inertia actually discover by the Newton by Galileo, the credit goes to Newton because he integrated this into his scheme of dynamics, but Galileo was able to discover the law of inertia.

And then Newton figure out, that what is that needed to disturb equilibrium of a system, and he invented the idea of a force, that force is the agency, which disturbs equilibrium, and that is the second law  $F$  equal to  $m$ . The disturbance from equilibrium manifest itself, as acceleration of the object this is what Newton formulated in his scheme of mechanics, and this is really the heart of the principle of causality, and the linear response theory in Newton dynamics.

Now, alternative formation of classical mechanics exist, in the form of Lagrange or Hamiltonian and this is based on the principle of extremum action. That action is an extremum, any variation and that would vanish, and the condition that must be satisfied,

such that action is an extremum that condition is what gives you the equation of the motion.

That is Lagrange's equation, you can also get Hamilton equations corresponding, but in these scheme in classical mechanics. This central idea is a following that the state of the system is describe by position and velocity  $q$  and  $\dot{q}$  or alternatively by position and momentum  $q$  and  $p$ , this is the heart of classical mechanics.

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State of the system:  $(q, \dot{q})$ ;  $(q, p)$

Observables:  $L(q, \dot{q})$ ;  $H(q, p)$   
 $F_L(q, \dot{q})$ ;  $F_H(q, p)$

Do repeated measurements of  $(q, p)$  give same result?

**There can be *no* orbits!**

**QUANTIZATION cannot follow Niels Bohr's path!**

**The atom of Niels Bohr *kneels*!**

The strange theory of the quantum – Banesh Hoffman

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And then it is evolution is this described by how the state evolves a time, so dynamical variables are functions of the state of the system, position and velocity or position and momentum. So, you can describe the system either by  $q$  and  $\dot{q}$ , or equivalently by a function of  $q$  and  $\dot{q}$ , which is a Lagrange or a function of position momentum, which is the Hamiltonian.

And the observables are then various functions of these independent variables, which characterized the state of the system, and this is how classical mechanism works. That the question is what would happen if you measure position and momentum, you carry out the measurement of position record, your answer, storage. Then you carry out measurement of the momentum, record there answer, your happy you can go ahead, but if you decide to repeat your measurement on the position.

It would not be guarantee that you will get, the first answer that you got at the first measurement you would get a spread. So, you cannot get same answer to repeated measurement position, if you enter in these measurement by measurements of momentum, this is the principal of uncertainty. And the reason you cannot get consistently the same answers to position momentum of measurement is not, because of the principle of uncertainty, it is the other way round it is, because you cannot get same answer constantly, that the uncertainty principiplies, so do not put upon the car upon the way.

So, now, we have a problem, that if repeated measurements of position momentum do not give you the same answer, if a measurement of momentum is enter wind by a measurement of position and vice versa. You do not get the same answer, then it would be possible to characterized state of a system by position and momentum, our contrition classical mechanics is that a state of system is characterized by position and momentum.

You cannot do it, because repeated measurement do not give you same answers, so you cannot do it, you cannot describe the state of a system by position momentum or my position velocity. And if you cannot do it then how do you do mechanics, because a central problem in mechanics is how do you characterized the state of a system, and how does the system involve a time. And if you had agreed to characterized state of system by position and momentum, you cannot do it anymore, so you have to find a different mechanism to do it, and that is quantization.

You have to abundant this approach, that the state of a system is described by  $q$  and  $p$  is something that you cannot carry on any further, because it does not take you anywhere. So, you have to find a new theory, and this new theory is quantum mechanism, whatever you do to get into this theory is quantization, it is not just having energy spectrum, which is discrete. That energy spectrum is discrete is one of the consequences of quantization, but it is not the signature of quantum mechanics.

You have to do something else, and if you cannot describe the state of a system by position and momentum, then you cannot set up Newton's equations. You cannot set up Langrage or Hamiltonians equations, you cannot get solution as trajectories and locals, and then trajectories do not exist in quantum mechanism, so what is Bohr of orbit does that exist in quantum mechanics. So, the Bohr Sommerfeld module of the hydrogen atom

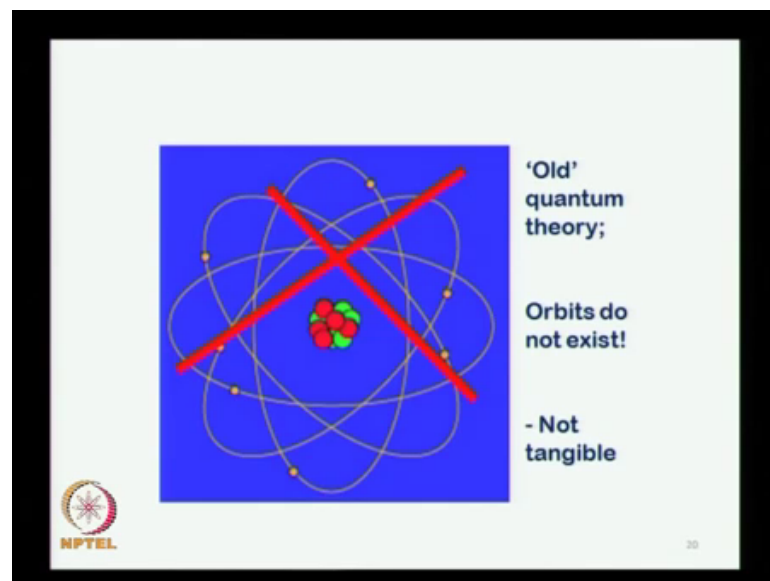


is again, certainly it gives you the one over n square formula, it explain the spectrum of the hydrogen atom, but it is really not quite quantum of mechanics.

Although, it is quantum mechanics is certain extend, because one of the postulates of board, may use of the trans constant is one of our friend suggested, that it has to another trans constant, and action is quantized in the Bohr model. So, these things are there, but it not really quantum mechanics, it is what we now call as old quantum theory it is not formal quantum theory section, there can be no orbits in term mechanics.

So, any model of the hydrogen atom, which is based on orbits really is not tangible, so quantization cannot follow the path of the Niels Bohr and Banesh Hoffman called it by saying that the atom of Niels Bohr Neils, it does not what. So, in her books strange story of the quantum, she is got a chapter who slight less the atom of Niels Bohr, the next chapter is the atom of Bohr Niels it does not what enough, so this model is wrong.

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All this picture of orbits is not correct.

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State of the system:  $(q, \dot{q}); (q, p);$   
 $L(q, \dot{q}); H(q, p)$   
"OBSERVABLES":  $F(q, p)$   
ABANDON the classical description of  
'state of the system'.

---

state vector:  $|\rangle$   
dynamical variables: operators  
 $|\text{ labels? designation? } \rangle$

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So, you have to abandon the classical description completely, you need to get into quantization, and now we can really answer what quantization, in quantization suggest that you do not describe the state of a system by  $q$  and  $p$  any more. What you described it by a state vector, dynamical variables you represent by operators, and this is something which can I suggest. and then these look likes some abstract quantities, but then has direct as a books principles of quantum mechanism which I saw you recommend.

You begin with this, and as you develop the algebra further, gain more experience with that you are then able to figure out, how to connect these things to observes. State vectors by themselves or mathematical creatures, which exist in the halberd space, these are vectors in mathematical space, which were name by John ((Refer Time: 27:33)) after his Guru David Hilbert, so these or the Hilbert state vectors.

And then these operators are operate on the state vectors, and then you need to develop the algebra further the moment, you do that you have quantize the system, this is quantization. That you dispense with classical description in terms of position and momentum, replace this  $q$  and  $p$ , which are dynamical variables and classical mechanics by the position operator and the momentum operator, you have quantize the system.

We have not turned about discrete energy spectrum, that is not the signature of quantum mechanics, this is the signature of quantum mechanics. So, you have a state vector, but

you are going to label this state vector in some manner, the meant of final mechanism the basic needs the state vector, so the next question is how do you label these statement.

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Quantization! state vector:  $|\ \rangle$   
dynamical variables: operators

$A|\ \rangle \rightarrow |\ \text{label?} \rangle$

$|\ \text{new vector} \rangle \propto |\ \text{old vector} \rangle$   
eigenvalue equation

$A|\ \rangle = a|\ \text{label?} \rangle$

label

$A|\ a \rangle = a|\ a \rangle$   
eigenvalue equation

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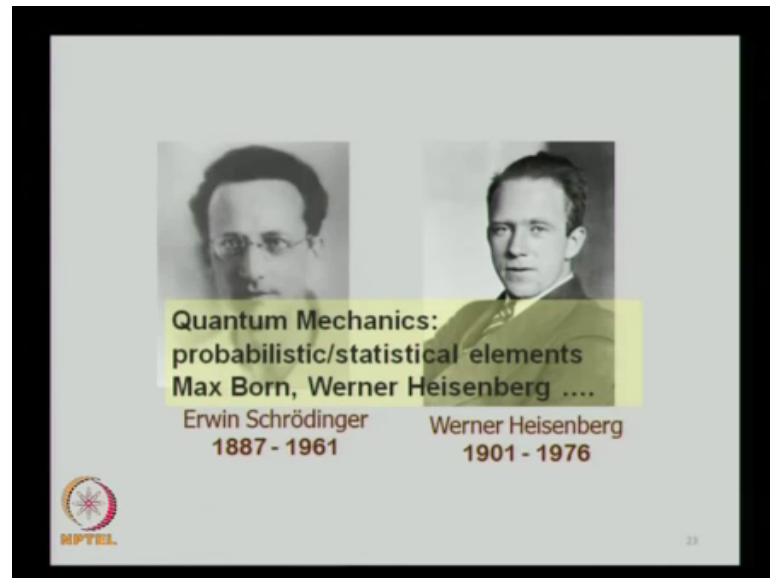
So, what you are going to do is to figure out a mechanism to label these state vector, so let us discuss that very briefly, if you have an operator, which operates on a state vector. You get a new vector, you have not put labels anywhere; however, if the new vector is proportional to the old vector, you just have a proportionality between the 2, then you have an Eigen value equation. Then you pick the Eigen value, and use that Eigen value to label the vector, so insert this Eigen value into the vector, this is how you label the state vector.

See how we have written the Eigen value equation, we had an operator operating on a vector, and then when this I do not what is happening here. So, you get a proportionality, you recognize this proportionality put this into the vector as its label, and then you have a complete Eigen value equation. So, we are not written this Eigen value equations of left to right, we let it operate got a new proportional vector, then pick the proportionality and insert at that into the vector.

So, we hopped a little bit, the way you play the night of on board of trust, and then you have the complete equation write, so it does not go completely from left or right. So, this is how you write the Eigen value equation, and this is what measurement does to a

system, that may be perform measurement on a system the system collapse is into an Eigen state, and this is the Eigen value that we use to describe the state of a system.

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Now, for enough this is the approach, which was developed of course, by Schrödinger and Heisenberg, as you have learned in your course quantum mechanics, this is the probabilistic theory. It has got statistical elements and Heisenberg, Born many others contributed to it, I will statistical and the probabilistic elements in quantum mechanics is something that one of the persons, who was the originator of quantum mechanics was not comfortable.

That was Einstein, Einstein contributed to quantum mechanism, and a certain sense quantum mechanics has it is beginnings not quite in the work of max plane, because max plane did it just to fit the black body radiation formula. You never agreed with it in some, but only when Einstein explain the photo electrical fact using that hypothesis, this people actually grass the meaning of energy quantization. So, Einstein actually contributed to the development of quantum mechanics, but he was not happy with the probabilistic, he was very trouble by statistics by probabilistic things.

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And he felt that all this probabilistic things and so on, do not fit into his mind is in his imagination of how god what with our created the universe. So, he Benton to say that god does not play dice, and there are this famous 1927, the base between Einstein and Bohr, which Bohr always one hands down. And he found ((Refer Time: 32:33)) in Einstein's arguments and convince them that he aspect probabilistic nature is physics of contain mechanics is nothing wrong about it, so even own to tell Einstein that is not your business you have got what to do.

So, quantum theory does have a probabilistic element, and it really came stay at the Solvay conference in the debates of Einstein and Bohr. And we will designated by a label, which will come from the Eigen value of the measure, and there can be not just one Eigen value, there can be more operators for, which you can have an Eigen value equation satisfied, because you want to get as much information about the system as you possible as you can.

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$| \text{label(s)}? \rangle \xrightarrow{\text{label}} A | a \rangle = a | a \rangle$   
 $A | \quad \rangle = a | a \rangle$  eigenvalue equation  
 Measurement: system 'collapses' into an eigenstate  
 What *else* can be measured?  
 C.S.C.O.  
 Complete Set of Compatible Observables  
 Complete Set of Commuting Operators  
 $B | a, b \rangle = b | a, b \rangle$   $[A, B]_- = AB - BA$   
 CSCO:  $\{A, B, C, \dots\}$   
 NPTEL

So, what you like to do is find out the operator, corresponding to a measurement carry out that measurement, which is a same thing operating on the state vector by the operator. So, you have to have a one two can connection in mind between the experiment which is being done, and the corresponding quantum mechanical simulation. So, do not separate experiments and theory in your mind.

It is exactly that they are completely synonymous in this description, the system collapse into an Eigen state, but then you ask what else can be measured. And if you can find everything else that can be measured, but not all measurements have compatible, some of the are, and some of them are not, you already know that position and momentum and not compatible, which also mean that attentional energy and kinetic energy will not be compatible.

But, some measurements will be compatible, and you stack as many measurement as possible, so that you get the complete set of compatible observables or completes sets of comminuting operates. And this is directs vocabulary this is very nice, because of the CSCO stands for the completes set of compatible observables, as well as complete set of commuting operate. And you can then find another measurement b, you carryout this measurement this is system will now collapse into an Eigen instead of b, but it remains in the Eigen state of a, because a and b are compatible in this case.

So, if you go back and repeat the measurement of  $a$ , you will recover the earlier value, which was  $a$  which was the lower case  $a$ . So, you now stack all the compatible measurements, and then you get a complete set of commuting operators, which commute with each other, so that their commutator vanishes. And then there is a simple theorem, that they can be simultaneously diagonalizable etcetera, and some of these things you have learned into  $a$ .

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Quantization! state vector:  $|\psi\rangle$

dynamical variables: operators

Which considerations govern the 'choice' of the operators?

$q \rightarrow q_{\text{operator}}$        $\vec{p} \rightarrow -i\hbar \vec{\nabla}$

$\vec{p} \rightarrow -i\hbar \vec{\nabla}$  Generator of translational displacement in homogeneous space

$\frac{1}{r}$  Radial symmetry of the potential

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So, this is quantization and we have to ask what are the considerations, which govern the choice of operators, and you have to study gain in your first course in quantum mechanics. I will not spend any time discussing this, but I will quickly remind you that you represent the position operator by  $q$  operator, which is  $q$  times a unit operator, and the momentum by the gradient operators, which is the minus  $i\hbar$  times gradients. So, this is an appropriate operator for momentum, and the reason this is, so because momentum is the generator of translational displacement in homogeneous space.

This is the fundamental reason that momentum is represented by the gradient operator, because when you find out what is the displacement operator, you get this form of the momentum operators. It is not just a postulate, that  $\vec{p}$  is minus  $i\hbar$  times gradients comes very naturally, from the fact the momentum is the generator of displacement in homogeneous space. So, the hydrogen atom you have got radial symmetry, so you will look for the

generator for the rotations and isotropic space, so that will lead you to the angular momentum, so for the hydrogen atom this could be an important operator.


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Two expressions for the infinitesimal translational displacement operator

$$\tau(ds) = 1 - i \frac{p}{\hbar} ds \qquad \tau(ds) = 1 - ds \frac{d}{dx}$$

$$p = -i\hbar \frac{d}{dx} \qquad \vec{p} = -i\hbar \vec{\nabla}$$

**MOMENTUM: Generator of Translation**

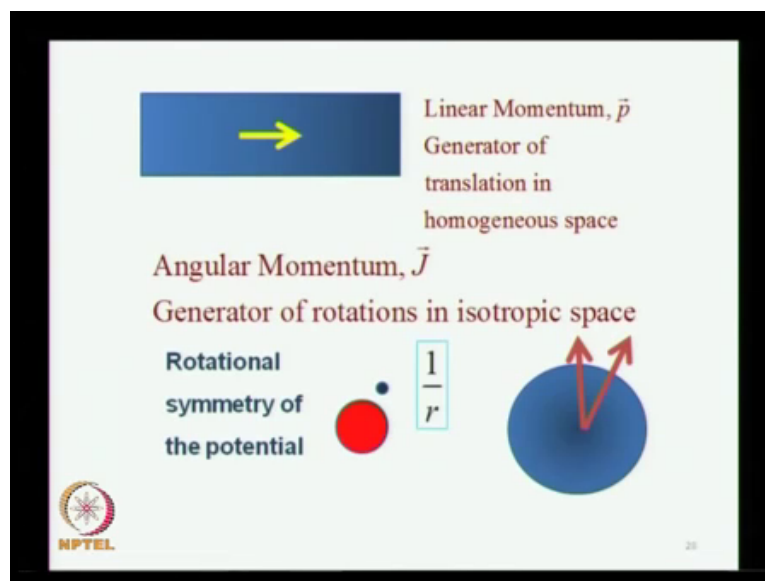


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What you get as you find in circles book modern quantum mechanics is that you get two alternate, and for the ((Refer Time: 37:30)) operator for infinitesimal displacement translational displacement. And if these two are to be equivalent, then you must have the momentum to be describe by minus isotropic. So, you carry out the similar analysis for the angle of momentum, and you will get the corresponding operator for angle of momentum.

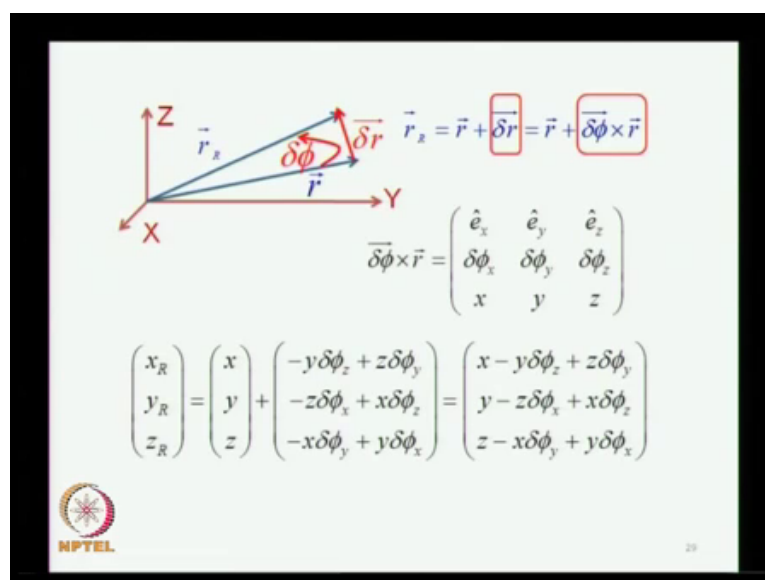


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So, you have displacement translational displacement, and homogenous space, for the linear momentum operator, and an isotropic h space, where you carry out infinitesimal rotation find out what is the generator for this. And you will get the operator for the angel momentum, this comes from the rotational symmetry of the hydrogen atom, which is 1 over potential.

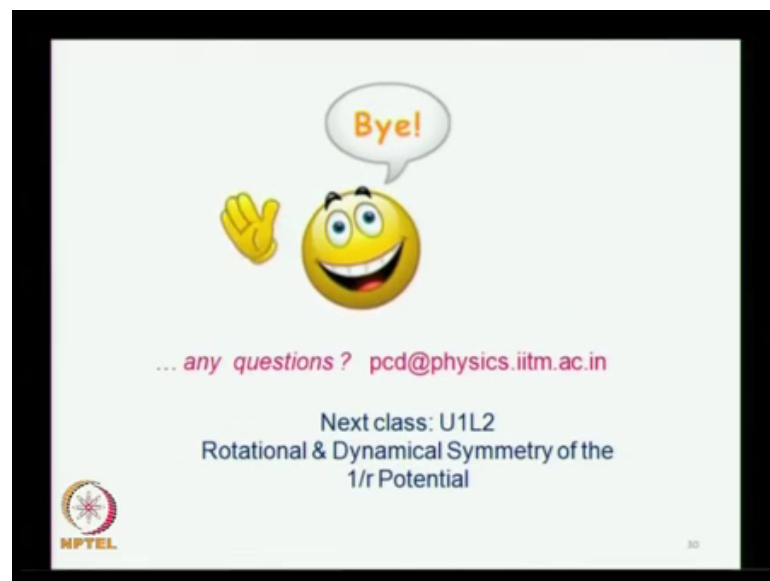
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So, consider these rotations, this diagram is self explanatory, you have got the rotated vector with r subscript capital R, which is the original vector r, plus this infinitesimal tiny

displacement,  $\Delta r$  which is nothing but the cross product of  $\Delta \phi$  and  $r$ .  $\Delta \phi$  is infinite symbol rotation vector, finite rotation are not vector, but infinitesimal rotation are or you can write this as a determinant for simplicity. And write the new vector as a column vector, and express this result of a rotation by a matrix transformation, so when you do that you get the matrix, for the rotational operator and i will carry on from this point in the next class.

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If there any questions for now, I will happy to take.

Student: Since, the previous slide not that just before that that b e operator on, sir here actually we are doing the experiments measurements, we are doing for one particular property at a time when you are doing a measurement, we are doing either a or b. So, one we do the measurement a and a sample whatever, if the system is collapse to one particular state. So, I mean if we are doing that a again on that system, it is already addition just particular, but there was just I mean, I can we do again that b on that particular.

Yes, what is your name Pranav, you are Pranav,

Student: Yes,

See, because what is really happening is you perform the measurement, the system collapse into Eigen state of that measure, now you perform some other measurement

which is a measurement of  $b$ . Now, when you perform a measurement of  $b$ , the system will collapse into an Eigen state of  $b$ , now if you repeat measurement of  $b$ , you will always get the same answer. No matter, how many times you carry out, because the system remains in an Eigen state, no matter how many measurements are repeated, so it remains in Eigen state of  $b$ .

So, let say you perform 13 measurements of  $d$ , and now you come back and make a measurement of  $a$ , now you will get the same answer as you had got earlier. If and only if  $a$  and  $b$  are compatible not otherwise, if  $a$  and  $b$  are compatible measurement of  $a$  will recover the original value, the Eigen value of the measurement of  $a$ . And you the system will then remain in Eigen state of  $a$ , but it will continue to remain in the Eigen state of  $b$ , because it is the simultaneous Eigen state of both  $a$  and  $b$ , which is what you really means, when you say that operators, which commute have got simultaneous Eigen states.

I am sure you have done the theorem, and then you perform measurements of  $a$  you can perform another 17 measurement of  $a$ , and then come back and carryout the measure of  $b$ , you will recover the original value, but this will not happen. If  $a$  and  $b$  are not compatible matter measurements, and the correspond operators do not come, that is exactly what happens to position and momentum. Is it clear, any other question, this is quantum mechanics, this is really the uncertain principle, any other question measurement of observation is not allowing by system to revolve it.

Well the system continues to evolve, but the temper evolution can be only by a face factor, so it is not that it is stagnates, there is the time evolution, but the time evolution of the stationary state will be only by the face pack. Which, is the  $e$  to the minus  $i$   $\omega$   $t$  or you know  $e$  or extra  $t$ , but if it is not an Eigen state of that measurement, what will happen is that the operator will through the system into a different state, and there the new state will then be a linear combination of the original Eigen basis.

So, it will be not pure state anymore, it will be in a mixed state it will be state in a super position, is that clear any other question. Complete set of comminuting observables is correspond to a complete set up experiments, which can be carried out different experiments, because you cannot have any additional measurement, whatever can be known about the system.

It has to be compatible, but whatever can be known about the system, we want to know something about our friend over here, and we want to know everything about him. What is his name what is his address, who is girlfriend is everything, once you get all the information you done with it. You cannot get any further information it, when you catalog all of this information together, you have a complete set of observation. And that is what is physic like to do to a in object, because given the hydrogen atom, or given any quantum system, any physical system.

All physical systems are quantum systems, that give an any physical system you like to get information about the physical system, and you like to get as much information about it as possible, that is reason that you do not stop at one measurement. You carry out the next measurement, and the interesting thing is that it is not guarantee, then after the performing the next measurement, you come back and repeat the first measurement, you will get the same answer.

That is the reason, you have to look to compatible measurements, and once you have exhausted everything that can be measured, you get a complete set of compatible observables tables, and correspondingly complete set of comminuting operators CSCO. So, I guess we will stop here, I will be happy to take questions by emails also.