Select/Special Topics in Atomic Physics
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Lecture - 12
Angular Momentum in Quantum Mechanics

Greetings, today we will introduce to the Wigner-Eckert theorem, and also conclude the second unit.

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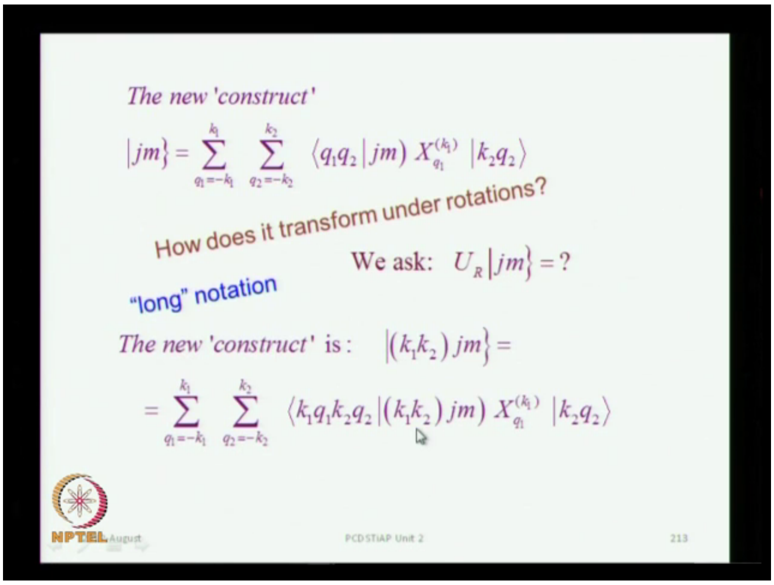

Now, we introduce in our last class certain vectors, which were composed of the irreducible tensor operators, and angular momentum of vectors. Now, we were encouraged to do this by recognizing that angular momentum vectors, and irreducible tensor operators both respond, similarly to rotations, fact one. Second fact that angular momentum vectors are coupled using the Clebsch-Gordan coefficient, so we asked if we could coupled the irreducible tensor, operators could not be coupled using the same law of addition. Have we discovered that indeed they can be coupled, and you get a new vector, which is also an irreducible tensor operator.

A new construct by combining two irreducible tensor operators, we get a new tensor operator again. So, now we have a mixed creature, which we know is a vector, because it is a result of an operator X operating on a vector, so the result of course, will be a vector and then you are summing over q 1 and q 2. So, it is a linear super position of various vectors, on this will give you a new vector which I have denoted by a this beautiful bracket on the left hand side, it could be any bracket it does not matter.

And our question is, what kind of a vector is it and we expect it to be an angle of momentum vectors and if it is so it will need to satisfy the defining criterion for angel of momentum vectors, and it should have similar response to rotations. So, we ask how does it respond to the rotation operator, so the same vector written with all the labels with the full notation. Because, this is the vector which is made up of k 1 and k 2; k 1 is the rank of the tensor, and k 2 is the angular momentum of this, so k 1 and k 2 are like j 1 and j 2 in dices, so those are the quantum numbers. But, one corresponds to the angle of momentum vectors, the other corresponds to the irreducible tensor operator. So, this the full notation for the same expression at the top and the Clebsch-Gordan coefficient is now written fully over here.

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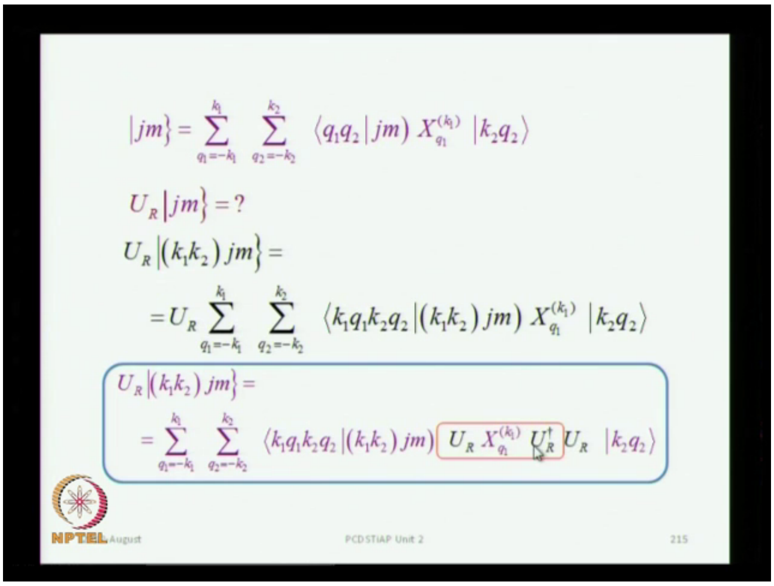

And we are asking how does this vector respond to rotations, so what we will do is consider the response to rotations. So, the rotation operator is going to operate on whatever it certainly does nothing to the scalar, which will just come out as scaling factor multiplier. And then it would operate on this vector here, which is the result of the vector obtain by operating on this vector by the irreducible tensor operator X. So, this operator U is positioned over here and in between over here I have inserted a unit operator.

The rotation operator being a unitary operator U dagger U is unity and I have sandwiched a unit operator in between, so there is no loss of generality there. And the advantage of doing this is that, you can factor this multiplication because operator multiplication is associated with multiplies both to the left and right depending on how you look at this association. As long as you do not play with the order, the order you can play with only for commuting operators, but otherwise you can always have the association. And I can associate this unit operator which was inserted over here, and look at what is inside this red loop. And this is nothing but the response of the irreducible tensor operator X to the rotation operator, and we know what the law is.

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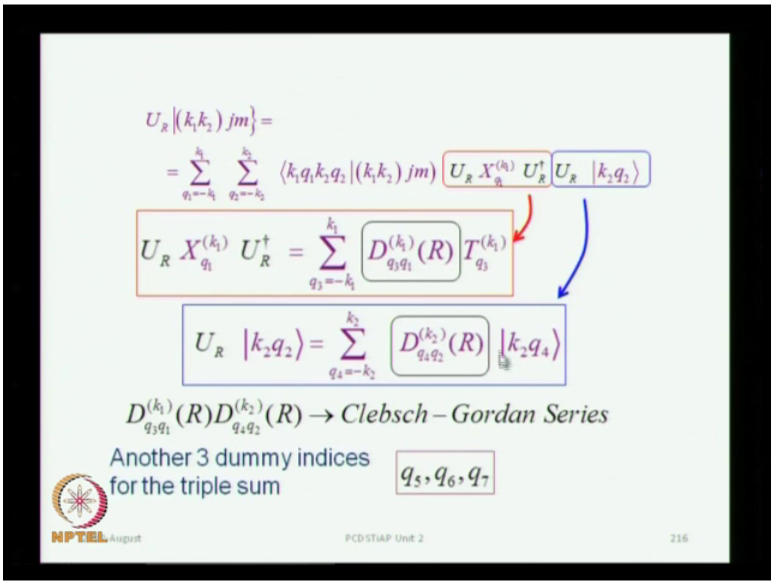

So, we find that we have over here in this red loop, the response of the irreducible tensor operator to rotations, and in this blue loop we have the response of the angular momentum vectors to rotations, and both have similar responses. And we are able to get it by inserting this unit operator, factor as U dagger U of which this UV associate on the right and this U dagger we associate in this block. Now, we know what this response of the irreducible tensor operator to rotations is, you get a linear super position of all the members of the family of irreducible tensor operators.

The weight factors are nothing but the matrix elements of the Wigner D matrix and likewise this also has got a similar response. And now you already have summation over four indices, there is a summation over q 1, there is a summation over q 2 over here, now you have got the summation over q 3 and a summation over q 4, so there already are four summations. Now, what do you have over here, you have a product of this D matrix element with this D matrix element, and you can look at this product using the Clebsch-Gordan series which we have studied earlier.

And if you remember the result of the Clebsch-Gordan series, it expresses this product in a triple sum and now there will be three more summation indices, q 5 q 6 and q 7, so there will be total of 7 dummy indices over which summations are carried out. Although you can exploit orthonormality condition and so on, and contract them, what the orthogonality relations are. So, I will leave it as an exercise, and you can work out these contractions very easily by using the orthogonality relations. And it turns out that all of these 7 summation contract very nicely exploiting the properties of Clebsch-Gordan coefficient, and the orthogonality conditions. And you are left with only one sum, and a single sum all those 7 through the chronicle deltas, they contract was single sum and the result is this.

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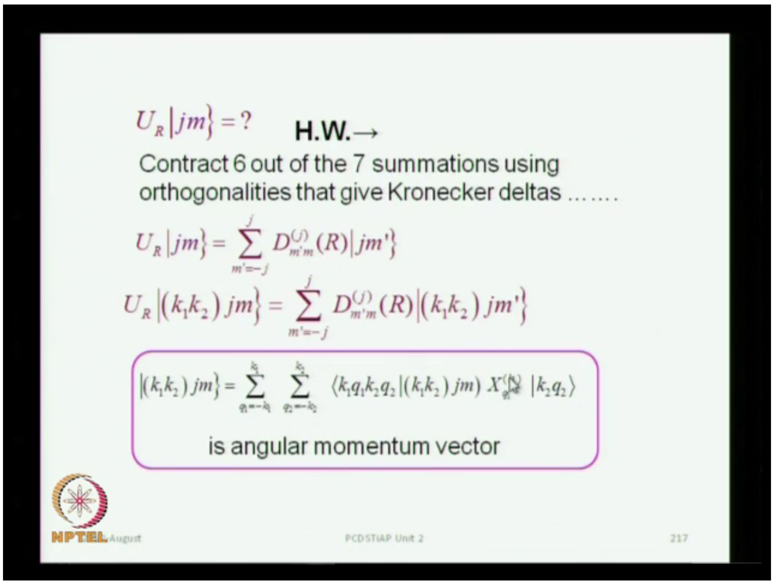

That the response of this vector to rotation, is that you get a super position of similar vectors with m prime going from minus j to plus j, and the weight factor the coefficient are nothing but the Wigner D matrix elements. What does it mean that, this beautiful bracket, this construct in the beautiful bracket is also an angle of momentum vectors. It is not something that we were willing to take for granted, because we had not introduced it using angle of momentum vectors, we had constructed it by coming up with super position of the irreducible tensor operating on an angle of momentum vectors, waited by certain Clebsch-Gordan coefficient.

And then carrying out some according to a law, which we know combined two angular momentum states, so we use the same law to compose this vector, and we find from the fact that it has the same response to rotations, it is actually an angle of momentum vectors and you can write it in the full notation with all the dices plugged in. So, k 1 and k 2 which were suppressed earlier can be put inside as well, and this is our conclusion that this combination, this method, this prescription of con constructing a new vector, actually gives us an angle of momentum vectors.

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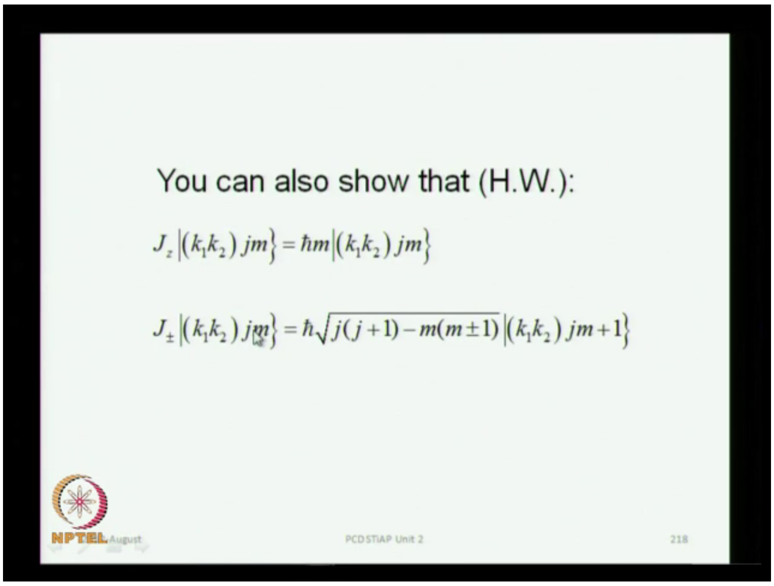

You can also show and this is also left as an exercise that this vector is an Eigen ket of J z belonging to an Eigen value m h cross, and that the ladder operators operate on it just the way they do on angle of momentum vectors. So, these properties are not surprising, but it is good to work them out just to get confidence in this.

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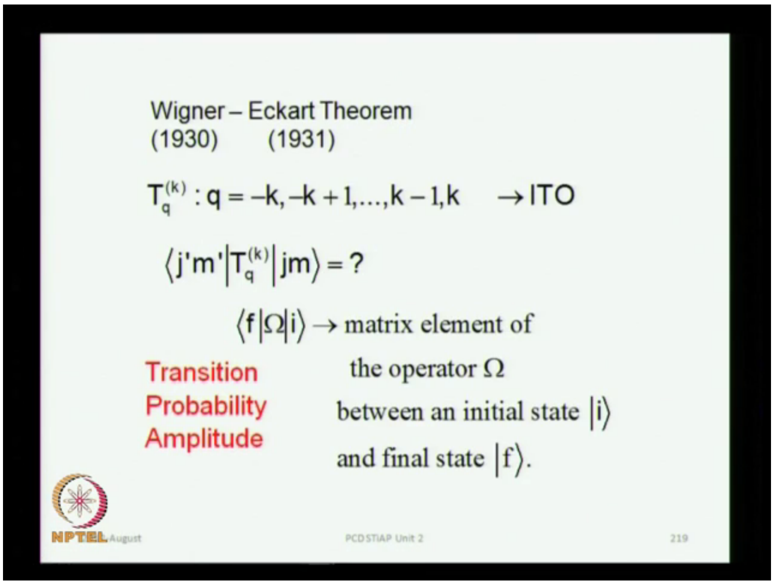

Now, we are ready to work with the Wigner-Eckert theorem, this was introduced by Wigner-Eckert independently around the same time 1930 and 31. And what it does is, it deals with the matrix element irreducible tensor operator, now this is obviously, important, because physical interactions are represented by operators in quantum mechanics. It is physical interaction which caused the transition of a quantum system, from a certain initial state, to a certain final state, and this is denoted by a matrix element of this kind, that if omega is a certain operator which corresponds to physical interaction, which induces a transition from an initial state to a final state.

That this matrix element is a major of the probability amplitude for transition from the state i to the state f. And this physical interaction which for ask is now a certain irreducible tensor operator, we have now to look at the matrix elements of this tensor operator. Now, the state i and the state f will be labeled by a appropriate quantum numbers, it will be label by what we call as good quantum numbers, these quantum numbers would typically consist of the angular momentum quantum numbers. The quantum numbers of j square and J z, they will also include quantum numbers of some other operators, such as the Hamiltonian.

So, there will be also be an energy label and essentially, you will have a complete set of commuting operators or complete set of compatible observables, which will give you the right set of good quantum numbers for the complete system. And the label i and the label f represent the complete set of good quantum numbers, for the initial state and the final state respectively. And we are interested in finding what is the transition probability amplitude for the transition from i to f and essentially we have to look at this matrix element.

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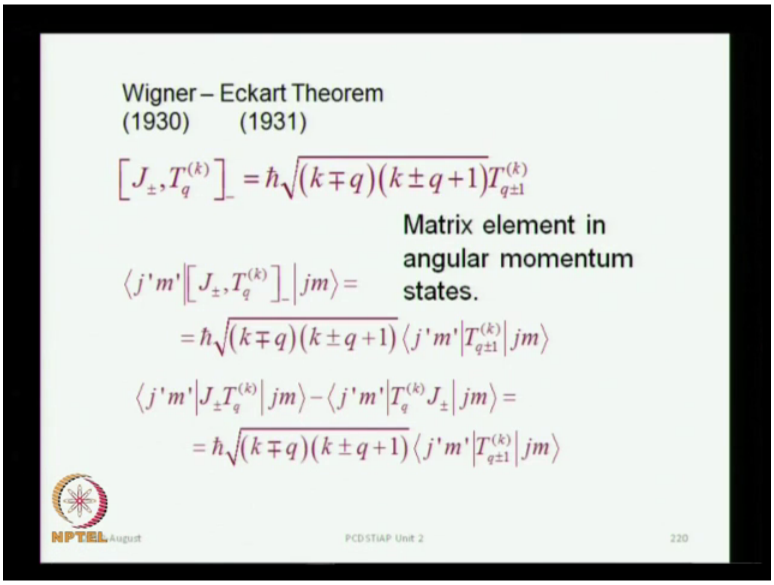

Now, let us do a little bit of angular momentum of algebra here, because we know from the defining relation that, the irreducible tensor operator have got very specific well defined computation relation with angular momentum operator; that is an equivalent definition, that we have already learned earlier. So, we take this expression which we are already familiar with, and we determine the matrix element of the operators on both the left and the right in angular momentum states. So, we take this operator and take it is matrix element in the state j m and j prime m prime and do the same on the right hand side, of which this factor comes out as a multiplier.

And then you have to take the matrix element of this T k q plus or minus 1 in the states, so we begin with the relation which is an identity, which comes from the definition of the irreducible tensor operator. And take its matrix element in angular momentum states, now on the left hand side you have got a commutator of J plus with T k and J minus with T k, so you get two terms J T minus T J. And these are the two terms I have written on the left side, which is J T over here minus T J, but J subscribe are plus or minus, so I am dealing with two equation at a time, one corresponding to the plus sign and the other corresponding to the minus sign.

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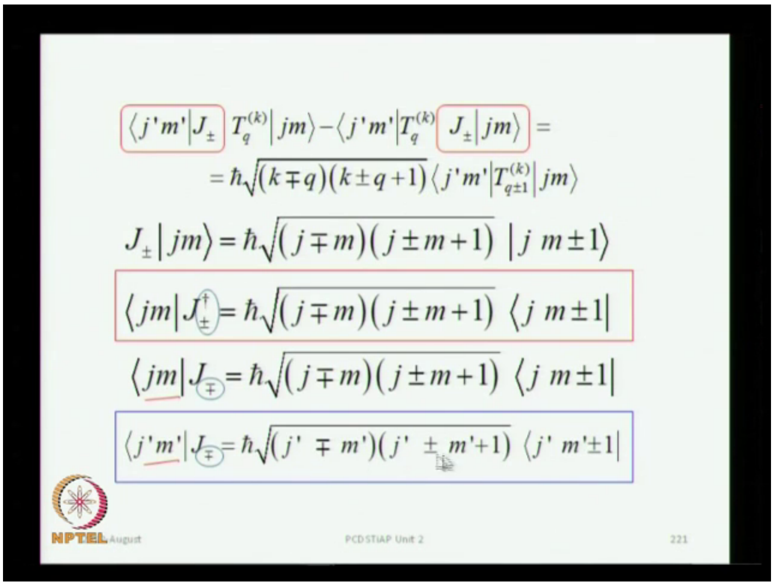

So, you have two terms at the left hand side here they are, now this is something that we know, because we know how an angle momentum state responds to the ladder operators, and this is the same thing happening in the joints space. So, we know the results of these, and we use those results and plug in that expression, so how does j m respond to J plus and J minus you get the m index switched by unity. It goes either up or down depending on this ladder operator being J plus or J minus, and you know what the corresponding coefficient over here is.

Now, you can take the joint of this relation, so this is just that joint of this relation ((Refer Time: 14:16)) and since this is an adjoin, the joint of J plus and J minus and the joint of J minus is J plus, they being a joints of each other; you switch the sub switch over here minus and plus and you have this relation here. So, this J dagger plus minus become J minus or plus at the right hand side is the same, because the left hand side are the same. So, the right hand side are essentially the same, but here we need the matrix element of J plus minus, so the plus is on the top and minus at the bottom.

So, we have to aim to change these two signs, so let us do that, so first before I do that I drop the label j m, I change this label from j m to j prime m prime. So, all the j and m are replaced by the corresponding primed labels, because that is what I am going to insert over here.

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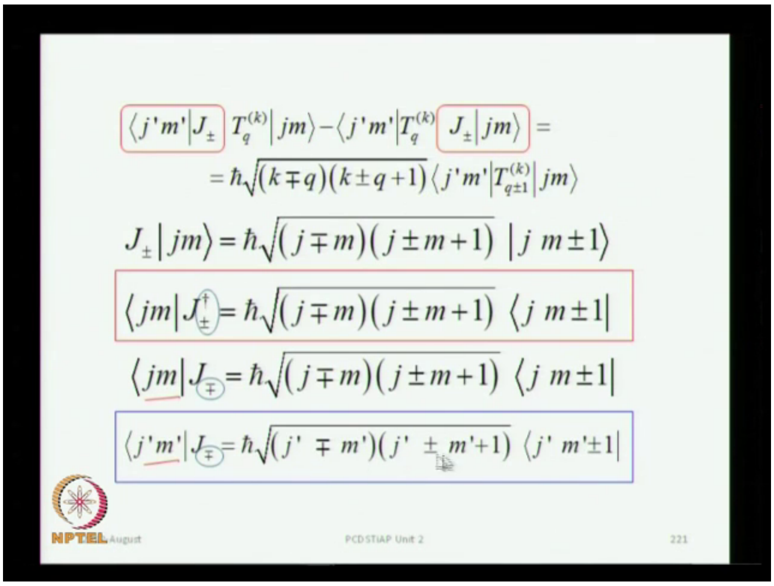

And now I have this relation for J minus and J plus from which by interchanging these two signs, I have the relation for J plus and minus which is what I need over here, and these symbols this was minus or plus. So, this is changed to plus or minus and you have the same change in sign in every location, so you have to do it consistently. It is very easy, but it is important to keep track of these details otherwise, it is very easy to make a careless mistake.

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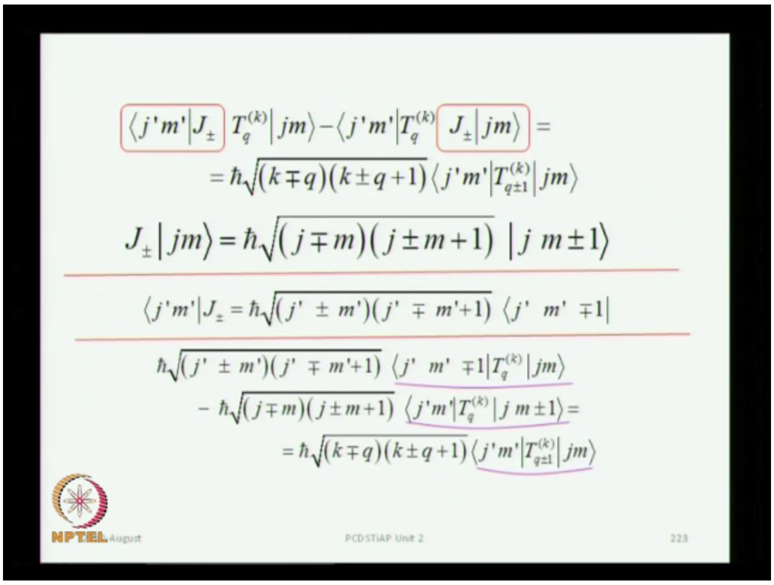

So, now we have everything that we really need, we have both the relations J plus operating on j m and this one in the joint space, so this is the relation for the joint space, we use both the results. And now you have matrix element of this operator T in angular momentum state, because this result is plugged in from here, this result is plugged in from the first relation here. And now you have these multipliers h cross times the square root terms with the appropriate plus minus signs, you remember how we got them; and then what you are left with are one term coming from here, second coming from here.

So, this is the matrix element of T k q which is this operator, so there are two of these on the left hand side and one of this on the right hand side, whose index is different. Now, is this relation a little familiar, does it look like something you have seen before, that looks like recursion relations Clebsch-Gordan coefficient. It is not the same, but it is similar, it is obviously, not the same, so this is what we have got.

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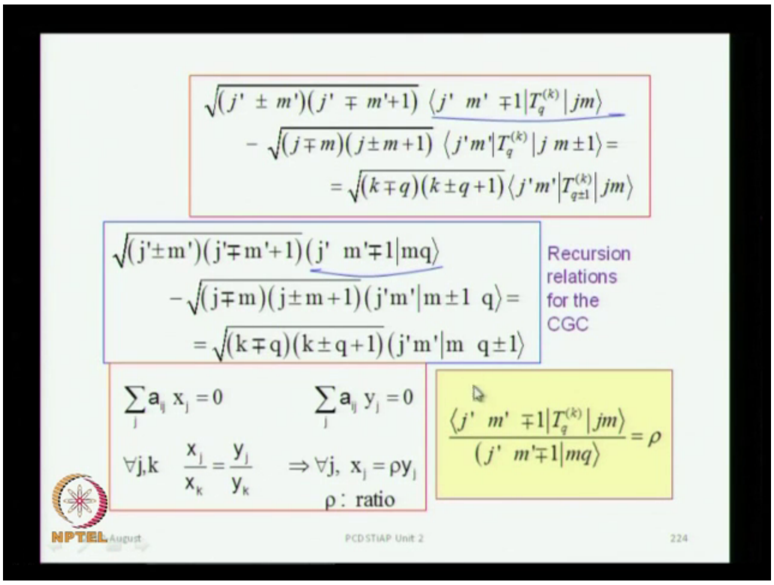

These are the matrix element of the irreducible tensor operators, and the recursion relation for the Clebsch-Gordan coefficient have an exactly identical form, and you can study term by term very carefully. And you discover that it is actually exactly the same with the difference that the matrix element of the irreducible tensor operator are replaced by the Clebsch-Gordan coefficient, in the expression for the recursion relations, that is only difference.

Otherwise they are absolutely isomorphic term by term, what it means that you have got a family of equations, these are linear equations one in x and the other in y, the coefficient being the same. Because, the coefficience are exactly the same, these square root terms with these are the coefficient, the square root terms this one ((Refer Time: 18:37)), then this one and this third one, and they are corresponding coefficience in the expression for the recursion relation for the Clebsch-Gordan coefficient.

So, these square root terms are exactly the same, and when you have such a system of equations, then from your knowledge of linear equations, that the ratios of the corresponding term must be exactly identical, that is something that you know from your knowledge of linear equations. Essentially what it means, that x j is proportional to y j and x j over y j is a certain constant which is a ratio, so there is no very involve mathematics, which we have to invoke in getting this relation.

And what it tells us that, if you take the ratio of the matrix element of the irreducible tensor operator, though this corresponding term over here which is nothing but a Clebsch-Gordan coefficient and all of you are now expert on Clebsch-Gordon coefficient. So, this ratio is what I have called as rho and this remains the same, no matter what the indexes.

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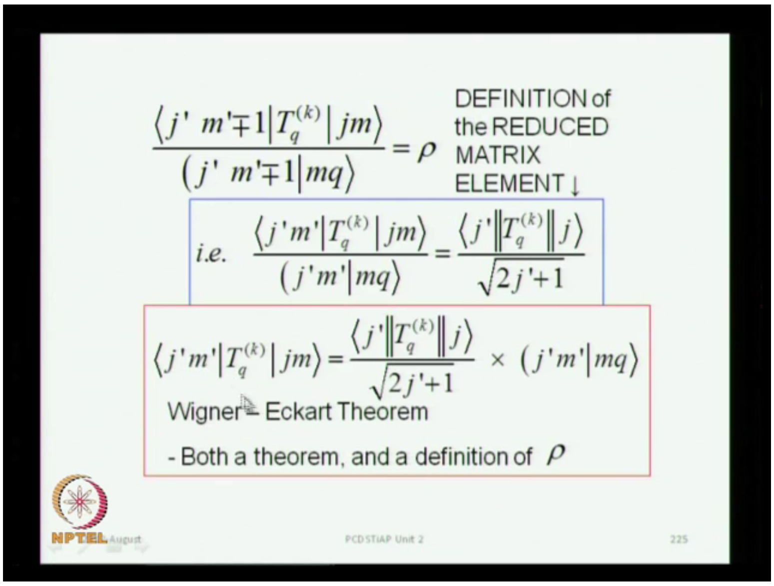

So, this is our relation, and this ratio allows us to introduce what is called as a reduce matrix element, so this is just a math of notation the same ratio. So, you have got a matrix element of the irreducible tensor operator in two angular momentum states, one is j m the other is j prime m prime. Or you can use m double prime if you like, because here you have used m prime minus or plus 1, it is some index. So, you have got the matrix element of irreducible tensor operator, it bears a ratio to the angular momentum coefficient, which is denoted by this rho.

And you can introduce a new notation which is rather convenient, because this ratio is independent of the azimuthal quantum number m, it does not appear anywhere, but it does depend on the angular momentum quantum numbers and the rank of the tensors. So, it does depend on j prime, it depends on j, it depends on k, so this rho has a certain dependence on j j prime and k. And then, you can write it as a function or introduce them as parameters or introduce some notation, which explicitly tells us that, this ratio is going to depend on j j prime and k.

And this is a notation which is introduce, this is called as a doubled barred matrix element or reduce matrix element, this is the definition of the reduce matrix element. And you can take this coefficient in the denominator to the right by cross multiplying, and you have got an expression for the matrix element of the irreducible tensor operator, which is a product of this reduce matrix element of the double barred matrix element times the Clebsch-Gordan coefficient, this is the just cross multiplication.

And some books tell us that this is a theorem as well as a definition, because what it does is a defines the reduce matrix element, it gives us the definition of the reduce matrix element. We know that it came from the system of the linear equations that we solved, by comparing the recursion relation for the Clebsch-Gordan coefficient with the relationship we get, for the matrix element of the irreducible tensor operators. And this essentially is a statement of the Wigner-Eckert theorem, we have actually proved it as you have just seen.

And what is very nice about it is that, it tells us that the matrix element of a physical interaction represented by an irreducible tensor operator, can be actually factor into two pieces, one is this piece ((Refer Time: 23:17)) and the second piece is this.

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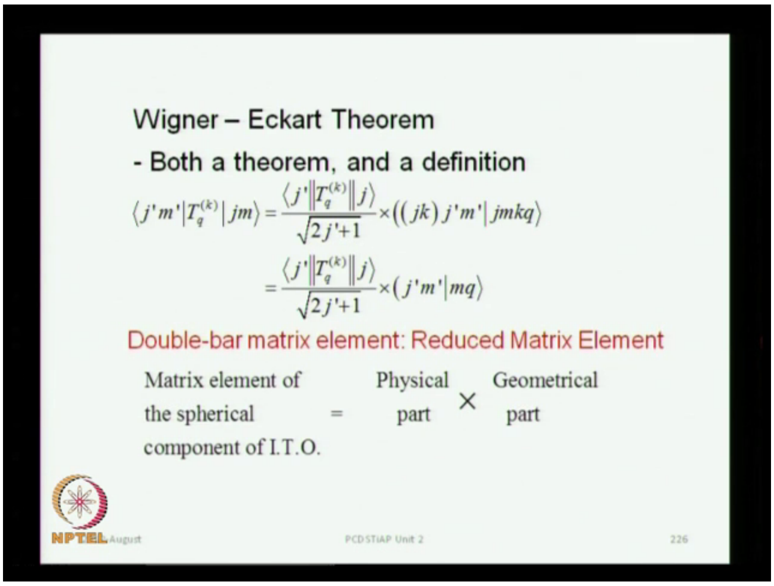

So, whenever you have a certain quantity which you can factor into two pieces, then you can look at each factor and get some independent physics out of it. So, this factor is the geometry that is where the geometry is not work, because the excess of quantization is along, let us say the z axis J dot U, is what can be simultaneously diagonalize with j square. So, J square and component of an angular momentum along some direction whatever, it is some unit vector U or you can call it as z if you like, that is how you choose the z axis.

But, that is where the geometry is, and those quantum numbers which are coming from the Eigen values of j z or they are coming from the Eigen value of J dot U. Those are the only quantum numbers which appear in the geometrical part, because that has to do with the access of quantization which is got a certain orientation in space. The remaining part what is call is a physical part, so the matrix element of the irreducible tensor operator is now expressible as a product of two pieces, one is a physical part and other is a geometrical part.

And this off is great convenience and you will see some of the application as I will discuss now, so this is the statement and the proof of the Wigner-Eckert theorem, along with the definition of the reduce matrix element. So, the geometrical part consist essentially of the Clebsch-Gordan coefficient.

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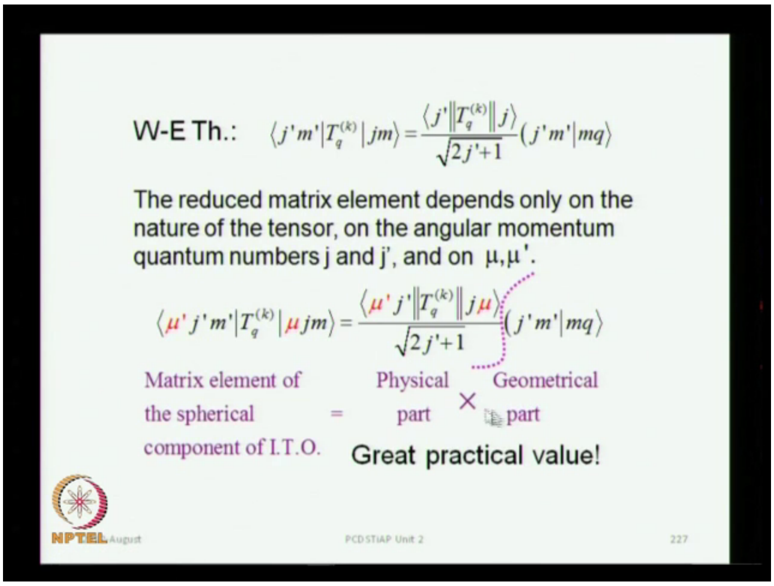

And it is just a matter of notation, because there may be some additional quantum numbers which I have indicated by mu, because after all the quantum system is described by a complete set off good quantum numbers, which come from Eigen values of a complete set off compatible observation. So, there may be some additional quantum numbers, which you can indicate by mu and mu prime, this root 2 j prime plus 1 again is the fact it depends on the value of j prime.

And the left hand side this physical part already has a dependence on j prime, so some authors this root 2 j prime plus 1 in the definition of the reduce matrix element, it is just a matter of convention, so that is not a such a big things. So, you will find some books in which this root 2 j prime plus 1 is not written whereas, in some books you will find that it is written, and it is just a matter of some convenience you can always factor it out. And the essential ingredient of the Wigner-Eckert theorem is the factorization of the matrix element into a physical part and a geometrical part, so it is got very important applications.

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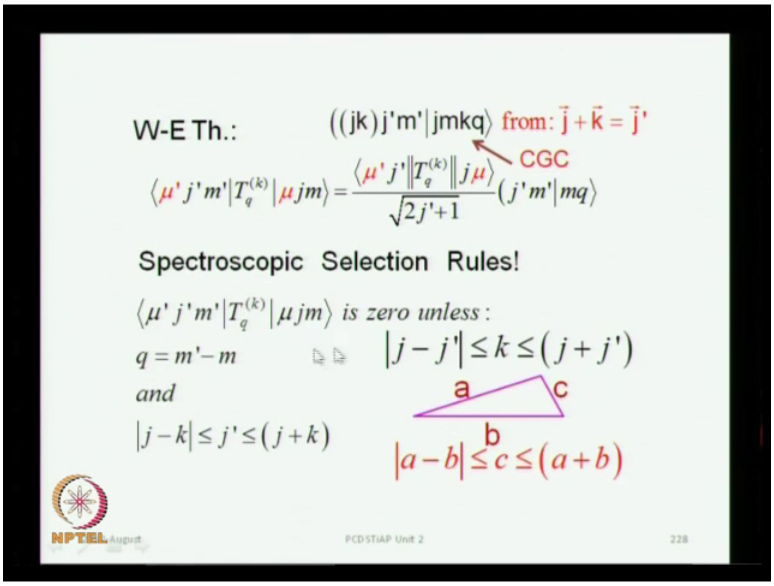

And the first one is the, one of the most important ones that I will discuss are the spectroscopic selection rules, which I am sure you are familiar with. Everybody has some familiarity with what are known as the dipole selection rules, and you will see how they come into play. So, now you know that because a transition from a initial state to a final state by any physical interaction is affected through this mediator, which is the operator, sandwich between the angular momentum state, at this matrix element is factor into a physical part and a geometrical part.

If the geometrical part is 0 then of course, the transition probability amplitude is 0 and Clebsch-Gordan coefficient goes to 0, if this q is not the equal to the m 1 plus m 2, so the selection rules. The triangle law of equality, because here in combine these Clebsch-Gordan coefficient when you combine j and k to get a j prime, then j prime will belong to this range to a minimum to a maximum, where in the minimum is given by a modulus of j minus k and the maximum is j plus k.

This is the triangle law of an equality, which is involved in the coupling of two angular the momentum and if this is not satisfied the angular momentum coupling does not take place, it goes with the angular momentum coupling. Now, this especially gives you a selection rule, because unless this condition satisfied, the transition will not take place; what does it mean, this is triangle law of an equality which we have discussed earlier.

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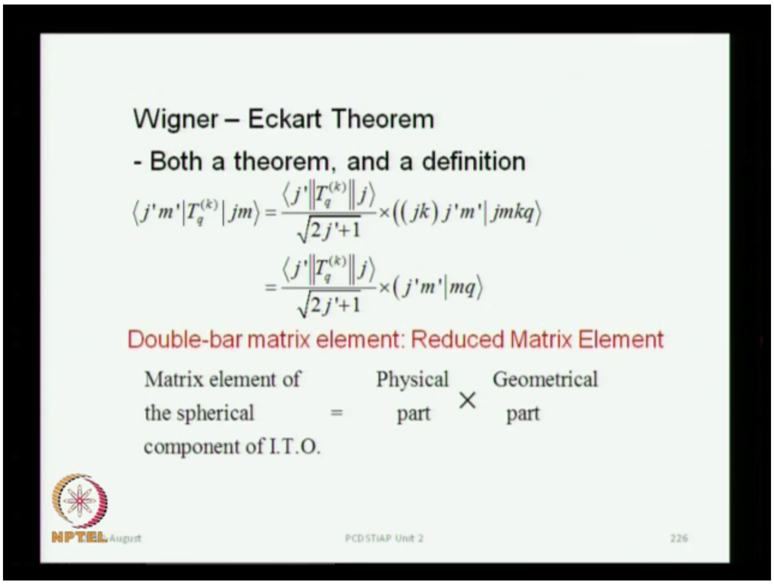

Now, if this interaction is a vector interaction, and let me use the result which you might have studied in time dependent perturbation theory, now we will be independently dealing with this when we do some spectroscopic, and we will redevelop the Fermi's golden rule, But, some of you possibly have an acquaintance with the Fermi's golden rule from your earlier course and quantum mechanics. And you will remember that the transition probability to a state f is given by the square of the matrix element and then, there are delta there is a square of the coupling vector potential A and so on.

So, this relation we will redevelop again independently when we do spectroscopic, but for the time being I will use it to illustrate the application of the Wigner-Eckert theorem, because here you see that the transition is affected by this operator. You see the gradient over here which is of course, the momentum operator, this e to the i k dot r is coming from the vector potential. And you can expand this e to the i k dot r in powers of r over lambda, lambda being the de Broglie wavelength of the electron, and if lambda is large compare to r, compare to the atomic size.

So, you have what is called is a long wavelength approximation, which is equivalently a low energy approximation, and when you expand empowers of r over lambda the low energy approximation, the first low energy approximation is what you get, when you set e to the i k dot r equal to 1, take the leading term. Now, this is what is called as a dipole approximation, and the reason it is call as a dipole approximation is because when you put this e to the i k dot r equal to 1, then you get essentially the matrix element of a momentum operator. And you will see that you can write this also as the matrix element of a dipole, now the first step is to recognize that this matrix element is the matrix element of a momentum operator. You see that, because of the gradient operator, so the i over h cross takes care of that.

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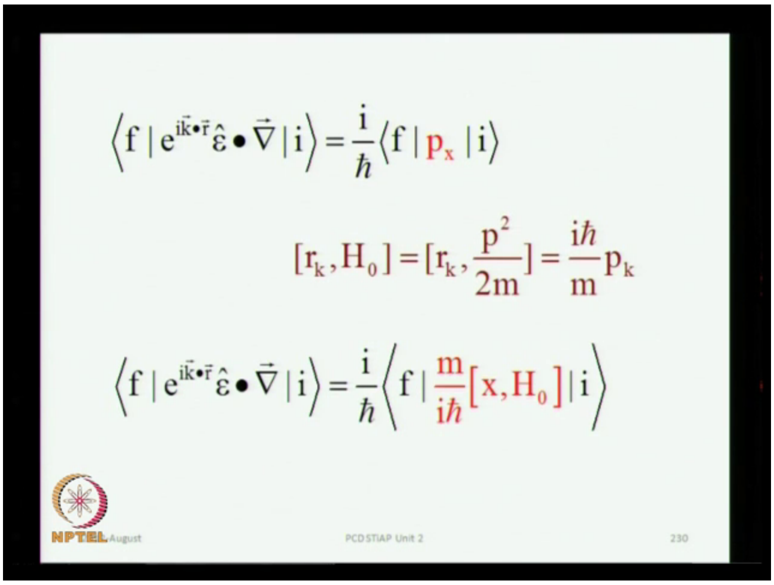

And then, that if you write the commutation between the position operator and the Hamiltonian, then this competitor is equal to i h cross over p time the momentum, which means that the matrix element of the momentum operator can be written as a matrix element of the r H minus H r. The matrix element of the momentum operator here, can be written equivalently as a matrix element of X H minus H X, now H 0 operating on i either on the right or H 0 operating on left on f, because it is a hermitian operator we know that anyway.

So, that will give you the corresponding Eigen values and then, you have to take the matrix element of the position operator. And the matrix element of position operator, the position times charge is the dipole, the position displacement times the charge is the dipole, which is why these are known as dipole transition. So, now we know what is involve in the dipole transition the dipole is a vector.

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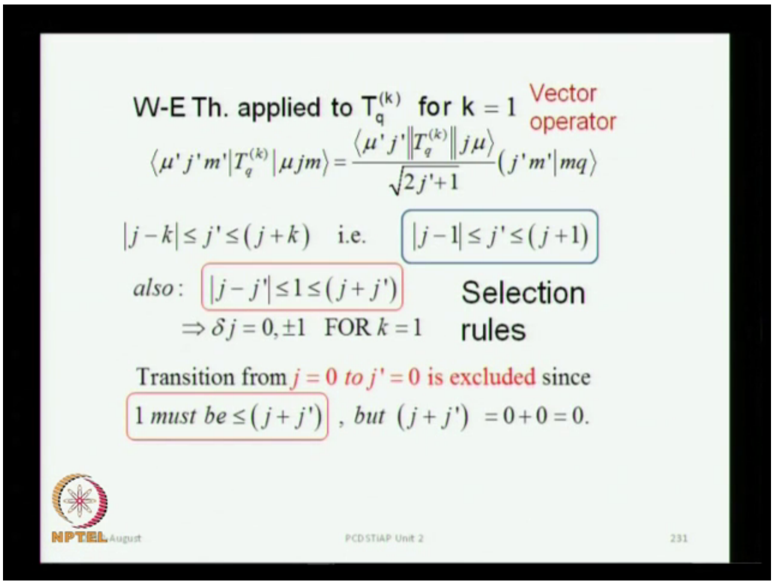

So, you have to take the matrix element of an irreducible tensor operator of rank 1, a q vector operator is a tensor of rank 1, so k is equal to 1; the triangle of an equality must be satisfied with k equal to 1. So, j minus 1 must be less than or equal to j prime and this must be less than or equal to j plus 1, all have done is to recognize that I have got a dipole, which is a vector operator tensor of rank 1. And this essentially tells us that delta will have to be either 0 or plus or minus 1, so that is a selection rule, that a dipole transition can take place, if and only f delta j is either 0 or plus or minus 1.

Now, so far so good, but you cannot have a transition from j equal to 0 to j prime equal to 0, the reason this is excluded is very simple, because one must be less than or equal to j plus j prime. But, if both of these are 0, then one cannot be less than 0 even in atomic physics, so delta j equal to 0 or plus or minus 1 is the selection rules, with the exclusion. So, j equal to 1 to j equal to 1 is allowed, because delta j is 0, but j equal to 0 to j equal to

0 is not allowed, so these are the dipole selection rules. And essentially you see that they come straight out of the Wigner-Eckert theorem in a very simple paschan.

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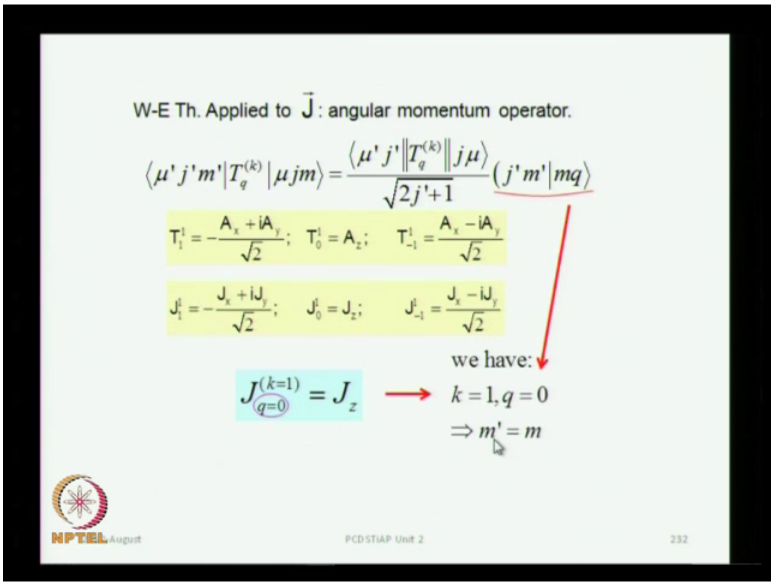

So, there are some specific applications of the Wigner-Eckert theorem, when you deal with the angular momentum operator itself, now this relationship is valid for any irreducible tensor operator and therefore, it will be valid also for the angular momentum operator. So, if you write the angular momentum, you can write it an Cartesian components or you can write it in corresponding spherical components. So, you have got spherical components of the angle of the momentum as j 1 1 j 1 0 and j 1 minus 1. And this is nothing but the J z component and because that this clebsch-gordon coefficient must necessarily have q equal to 0 for this particular components, it means that m and m prime must be equal.

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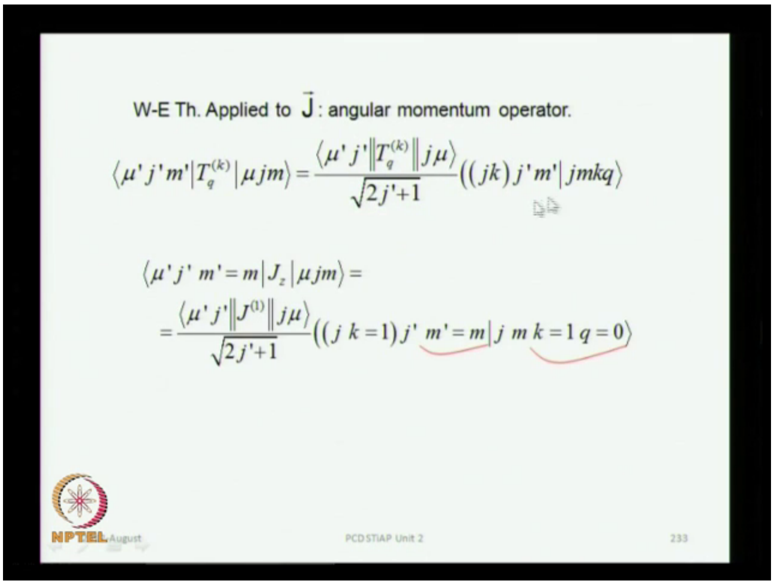

So, that is you get that and m must be equal to m prime, so that is what I put in expression for the Wigner-Eckert theorem. I have k equal to 1 which is the tensor operator rank 1, I have got q equal to 0 m prime equal to m.

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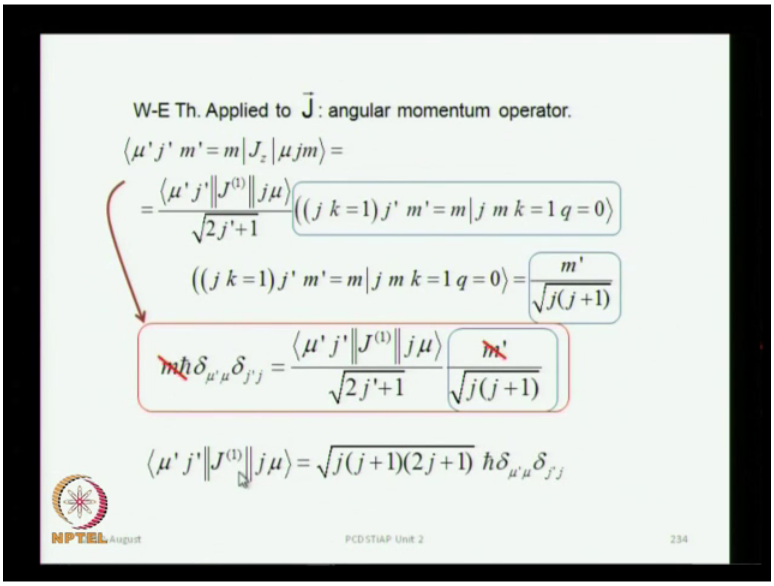

And now on the left hand side, I can actually solve this relationship, because this is and Eigen ket of J z, so this is an Eigen cite of J z and then, you just have the projection of j m on j prime m prime, which will give me delta j prime j and also delta mu prime mu. Because, of the orthonormality between the mu and indices, so this is a very simple relationship it has many applications and spectroscopy, you can strike out the common terms that the m prime must be equal to m.

So, this m 1 the left hand side must be equal to this m prime on the right hand side, so this m 1 the left hand side must be equal to this m prime on the right hand side. And you get the double barred matrix element of the angular momentum given by this square root factor and bunch of chronicle deltas.

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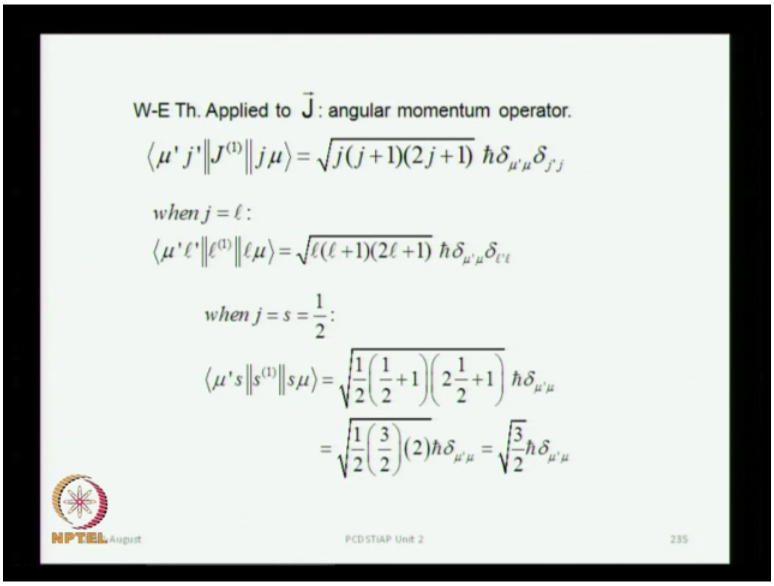

Now, this angular momentum could be anything, it could be the orbital angular momentum l, in which case the square root of j into j plus 1 into 2 j plus 1 becomes square root of l into l plus of 1 into 2 l plus 1 whereas, if it is spin and the value of span which is half. So, if you just put half for these values of j, you get root 3 by 2 times h cross, and the chronicle delta between mu and mu prime.

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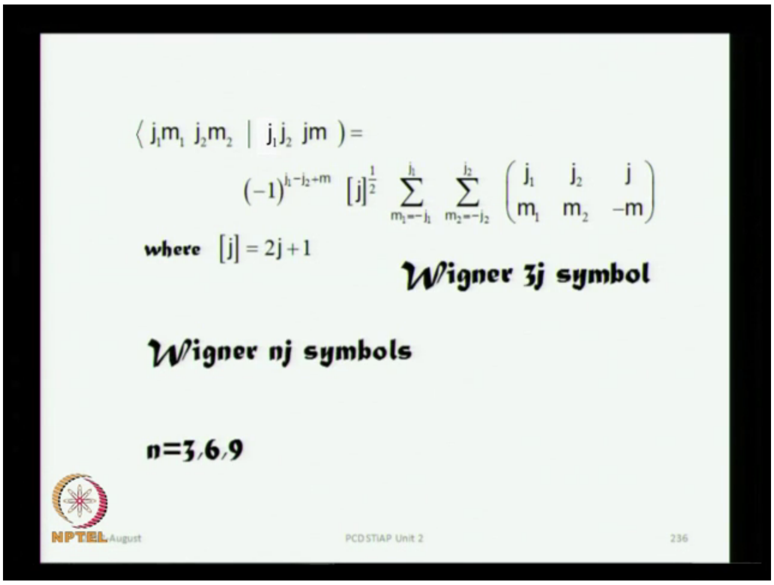

So, the only thing I will like to mention here is a matter of notation, otherwise we have pretty much done with the angular momentum algebra and the wigner-eckert theorem, is a math of notation that in many books, you will find that the wigner 3 j symbols are used. Wigner 3 j is just defined in terms of clebsch-gordon coefficient, clebsch-gordon coefficient you have handled quite extensively by now. And by carrying out these double sum you can express this in terms of the clebsch-gordon coefficient, but you can also have in inverse relation, because of the orthonormality conditions.

So, that is the simple exercise for you to work out, so I will not spend time on that and there is a certain, this is again a math of notation, essentially this is square root of 2 j plus 1. But, some books write it as j in a square bracket, in a rectangular bracket raise to half, so this is again a math of notation. And here you find that, these are the quantum numbers which go into the angular momentum coupling, the two angular momentum j 1 m 1 and j 1 m 2 coupled to give you j m.

But, in the wigner 3 j the index here is minus m, so that the sum of these 3 m indices vanishes, so there is some advantage in having a notation of this kind, this called is a wigner 3 j symbol and depending on how complex angular momentum algebra gets. Sometimes you work with coupling of more than two angular momenta, coupling of three angular momenta, coupling of four angular momenta, because you may have multiple sources, when you have an the angular momentum for a whole atomic system.

Then there are number of electrons each having it is own orbital's angular momentum, each having it is own spin angular momentum, and you need to compose the addition of all of them to get that net angular momentum. We discussed this yesterday, when we argued that it is important to define the angular momentum of a dynamical system correctly. Because, the corresponding angular momentum Eigen states must transform together jointly in the product space, so you often have to combined more than two angular momenta.

And when you have to do that you introduce what are known as Wigner 6 j symbols or 9 j symbols, so the algebra where comes little more complex, but not difficult it is little laborious it takes time. But, there is no new physics which is involved in it, and the basic elements of physics are essentially the coupling of two angular momenta, so with that I believe we are ready to conclude this second unit.

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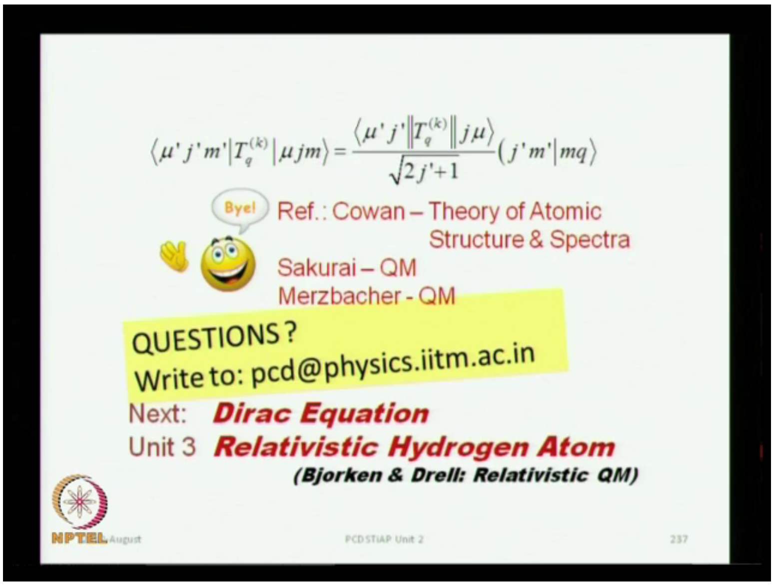

And I will like to take up some applications as spectroscopy, which is the heart of atomic physics in a certain sense, and I will be taking a slightly unconventional root to this. Because, very often even when I have thought this course earlier, I have introduced spectroscopy, before I did the Dirac equation other relativistic hydrogen atom. I have chosen to reverse the order, I will first do the Dirac equation and the relativistic quantum mechanics before I do spectroscopy.

Because, essential of factor which is of interest spectroscopy are when you do things like Zeeman effect, you must know what are the electrons spinners. And you can plug in electrons spin as an ad hog property, you begin with the Schrodinger equation and then, say that you postulate that the electron has got an additional internal degree of freedom which is the electrons spin. And it has got a angular momentum, and it has got a corresponding magnetic moment, but then it is like a partiality that you have to insert on an ad hog basis.

Instead I would rather do the Dirac equation first, because when you do relativistic quantum mechanics spin comes out naturally, so you do not have to make any ad hog assumption, spin comes out naturally. And it is appropriated do the relativistic quantum mechanics, because the Schrodinger equation is not covariant under Lawrence transformation, it cannot be. Because, you have got the potential energy term in the Hamiltonian, which is space dependent and space does not, it is not a space in trivial is not invariant in the Lawrence transformation.

It is the interval in the mean course keys space, in the space time continuum which in invariant, so the Schrodinger equation is not the correct equation of quantum mechanics it is a very good approximation. And the reason you need to improve on it, is because the speed of flight is not infinite, the speed of flight happens to be finite and that leads to various consequences. The major consequence is that of the special there of relativity, and we will then, find in appropriate quantum equation which is relativistic correct, and that is what we get from Dirac.

So, that will be our subject for unit 3, I will be using Bjorken and Drell book the relativistic quantum mechanics for this purpose. And it is a very good book, and many of you might be able to get a handle of that; now if there any question on unit 2, I will be happy to take or else good bye for now.