Select/Special Topics in Atomic Physics
Prof. P.C. Deshmukh
Department of Physics
Indian Institute of Technology, Madras

Lecture - 11
Angular Momentum in Quantum Mechanics
More on ITO, and the Wigner - Eckart Theorem

Greetings, so we intent to introduce the Wigner Eckart Theorem we will be studying it and also proving it. But, before that we are sought of laying the foundation for the winger eckart theorem, which as I mention is an extremely important theorem in quantum mechanics. And the foundation requires a very strong you know, background in angular momentum algebra, so we do various things with the angular momentum, operators and the irreducible tensor operators.

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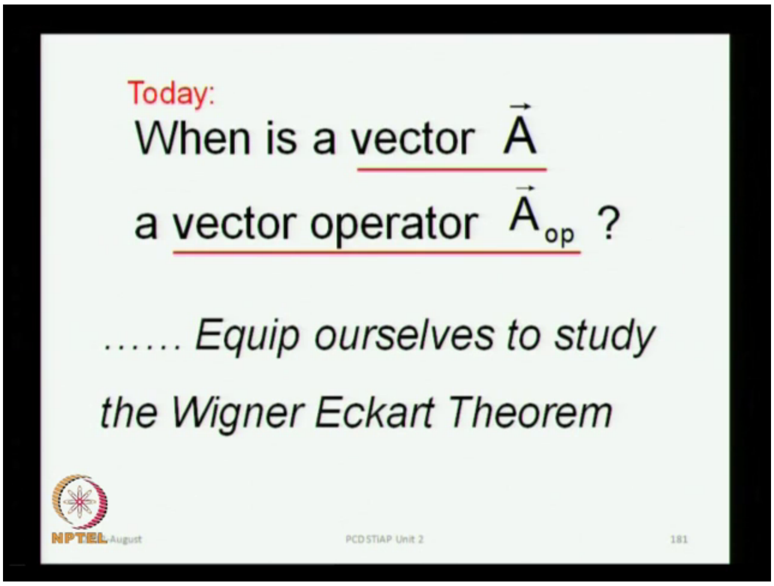

So, one of the question that I am going to discuss today is that, if you have a certain vector, what is the condition and a rich it becomes a vector operator is it automatic or is there more to it. So, there are certain consideration which I will be discussing about this, and these are some of the things that we need to equip ourselves to deal with the winger eckart theorem, these are some of the questions that I want to discuss in today's class.

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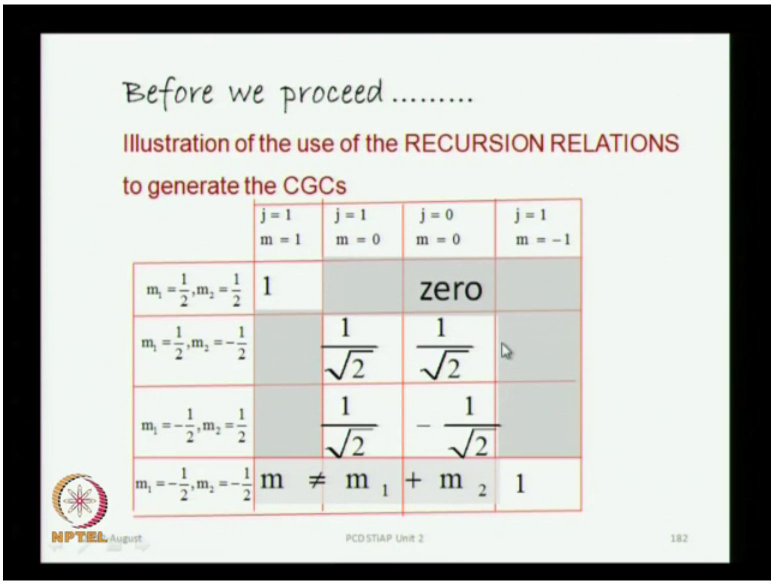

But, before I do that I will illustrate how the recursion relations are used, so that you go from one coefficient to another one, and let us take this example over here, we have already discussed this.

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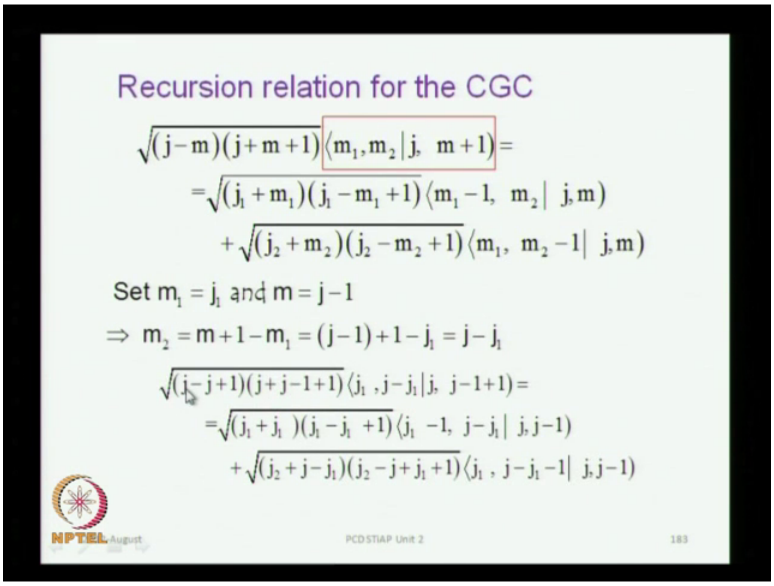

So, what you do is begin with a recursion relation, we have already established the recursion relation there are two of them. let us take this, and let us take a special case with m 1 equal to j 1 and m equal to j minus 1, m can take all values from minus j to plus j instead of 1. So, one of the value is j minus 1 now corresponding to this choice in this coefficient here, this quantum number m plus 1 must be equal to the some of this chow otherwise clebsch gordan coefficient is 0.

So, m 2 in this case must be the difference between this m plus 1 and m 1, so it is m plus 1 minus m 1. And since, we have chosen m to be j minus 1 it is j minus 1 plus 1 and then you subtract j 1, so this is j minus j 1, so that is what you get for m 2. Now, rest of it is very straightforward because you just substitute all of this quantum numbers by the specific values for this choice. So, wherever you have m you have j minus 1, so this is the minus m. So, you have minus j plus 1 over here, here you have m, so it is replaced by j minus 1.

Wherever you have m 2 like here, m 2 we found m 2 is j minus j 1, so m 2 takes a value j minus j 1 over here. So, in this step all I have done is to use a recursion relation and only substituted the quantum numbers appropriately, now let us have look at this further because we know the some of these terms will get simplified. For example, you already see that the first factor under the square root here is j minus j plus 1. So, this is just one multiplying the remaining factor right, and the remaining factor is minus 1 plus 1 cancel each other and then you have j plus j. So, this whole thing will simplify to just square root of 2 j.

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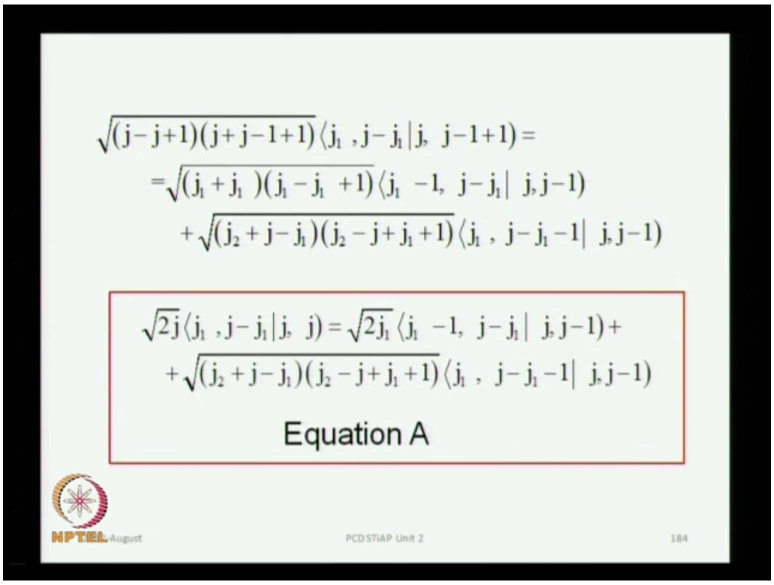

So, let us do that, so this the same expression which I have brought to the top of the slide and now I get square root of 2 j coming from this term. And likewise I simplify the remaining terms, this is just mere substitution nothing very big about it, now once you do that you see that the recursion relations are giving you a relationship between three coefficients. So, 1, 2 and 3 and quite; obviously, if you write this for this term it will involve the difference between this term and this term, and that difference is what we let the coefficient go either positive or negative. So, there is nothing very strange about a coefficient going negative it is just the difference between those jobs. So, there is nothing very strange about it, so let me call this is equation A.

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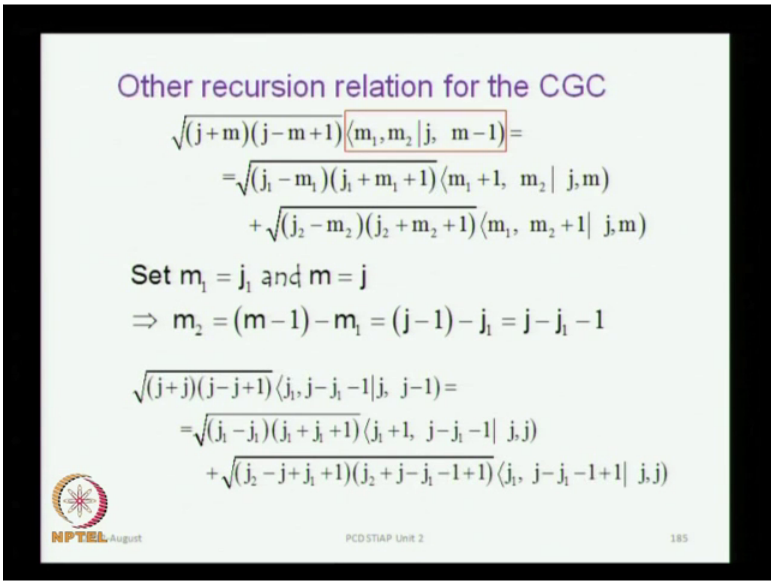

Now, let us take the other recursion relation, now I am just demonstrating how you play with these terms, you take the other recursion relation which also we have proved. And in this you set m 1 equals to j 1 and m equal to j, but in this recursion in this coefficient you must have this m minus 1 must be equal to m 1 plus m 2. So, m 2 must be m minus 1 minus m 1, but m minus 1 will be j minus 1 because we have chosen m to be equal to j, and then you subtract m 1 from it and m 1 has been set to j 1.

So, m 2 now becomes j minus j 1 minus 1 now you do the same thing again, just go ahead and substitute these terms in the recursion relation. So, here you get j plus m, so that will become j plus j and here you have j 1 minus m 1, but m 1 has been chosen equal to j 1. So, you get j 1 minus j 1 that will give you 0, so this term would vanish and you will get some simplifications of this kind. So, this is just a matter of doing this carefully and I let you workout the detail for yourself, I will only illustrate the technique just, so that you know how this algebra is done.

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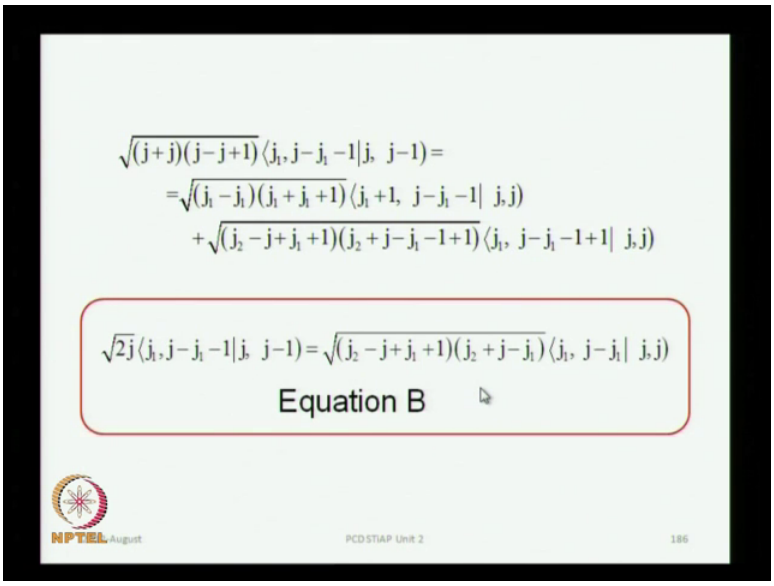

So, now, you simplify this term and this j 1 minus j 1 this first term vanishes, and now you have relationship between the remaining two coefficients, which is a most simply relationship and I will call this as equation B. So, now, you have got two results one that we got from the first recursion relation and the second that we got from the second recursion relation, and these are the two relations that I have called respectively as equation A and equation B.

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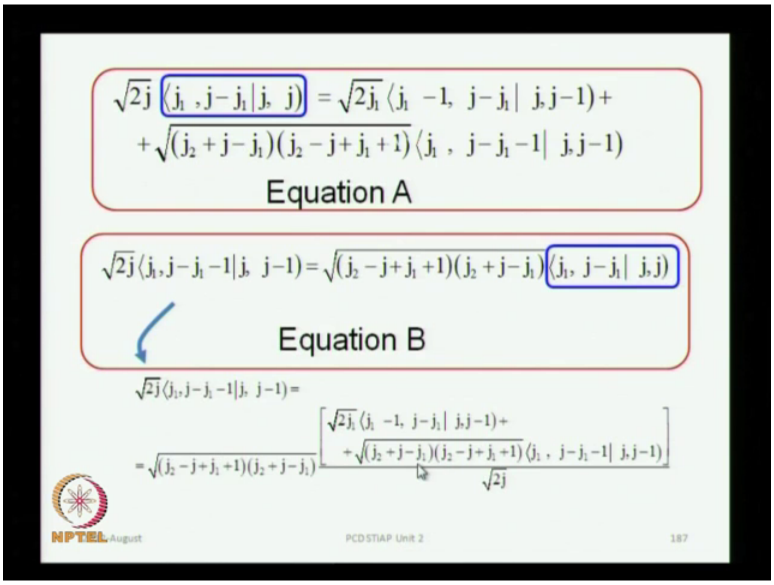

Now, notice that this term over here in equation A is exactly the same as what you have over here in equation B. And you can there for borrow this result from equation 1 from equation A and plug it over here a vise versa or either way right because these two terms which are enclosed in this blue loop there exactly the same. So, once you do that you have this left hand side, then you have this square root factor which is here, and then this term is replaced by the right hand side of this divided by this square root of 2 j which comes over here, so this very straightforward substitution.

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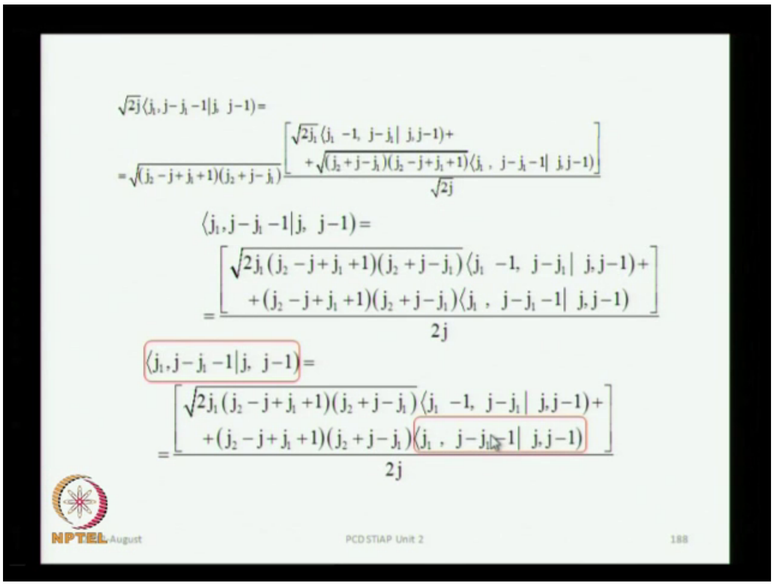

Now, that you have this result and I write it for this coefficient, so this square root of 2 j i again move to the right hand side. So, this root 2 j and this root 2 j give me a 2 j over here, and now this is the relationship that you get right and now you can simplify this for specific values of j 1 and j 2. So, this is the coefficient that you get and you have the same term over here, so you can actually combine these terms and get the residual relationship which will come out of it.

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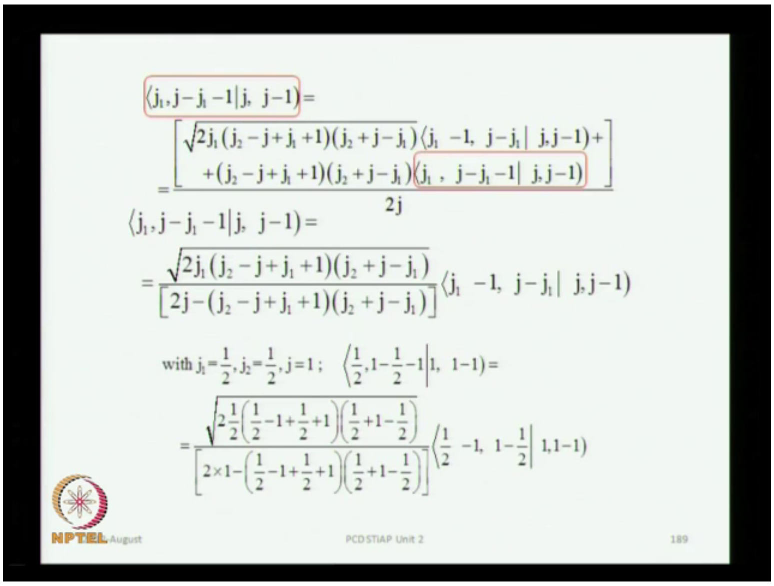

So, you have the same term coming in over here j 1 comma j minus j 1 minus 1 is same over here, and this is j comma j minus 1 which is the same thing over here. So, when you transpose and rearrange this term simplify this, and then take specific values let us put j 1 equals to half j 2 equal to half and j equal to 1. And just substitute these numbers, put the specific quantum numbers in that relationship, and you can get a much simplified relationship. And then this is just arithmetic, this is half minus 1 plus half plus 1, so minus 1 and plus 1 will cancel half into half will give you 1, save got 1 multiplying the remaining factors, so you can do this arithmetic in a very simple manner.

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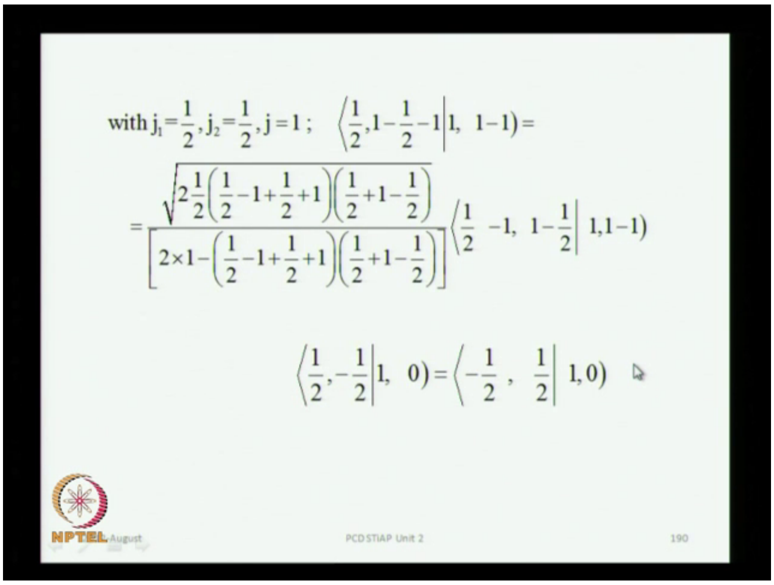

But, you have to do it carefully and once you do that what you find is that this two clebsch gordan coefficients, which is half minus half 1, 0 must be equal to minus half, half 1 0 that is the result that you get. So, you get some handle on the relationship between the clebsch gordan coefficients by using this.

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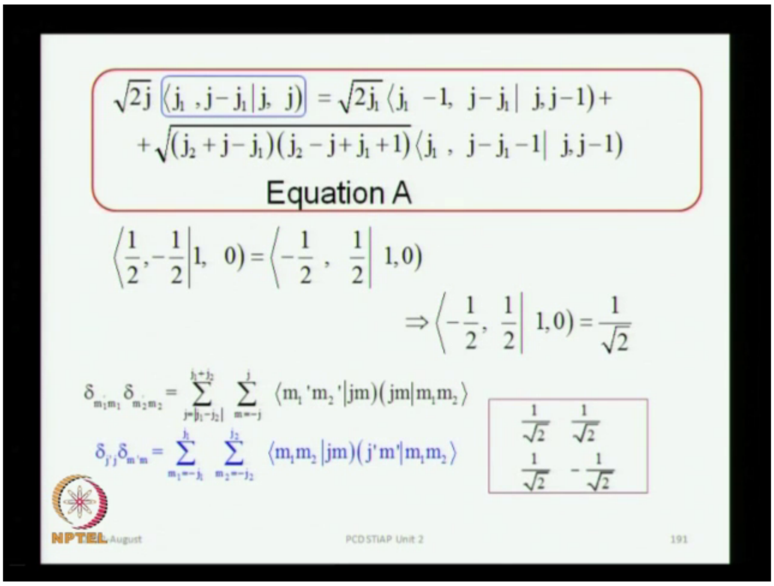

Now, let us extent this further let us use this result that we have just got in equation A, when you plug it in and substitute all the numbers, you find the this coefficient turns out to be 1 over root 2. So, that is it this how you get specific values and then of course, you also have the orthogonality relationships, and by making these orthogonality relationships, you can get this particular block that we were interested in. So, using the orthogonality relations and the recursion relations, you can easily get the rest of the terms. So, this is just a demonstration of the technique and I will let you, you know work out the details you know you can get some of the other coefficients just to get some practice.

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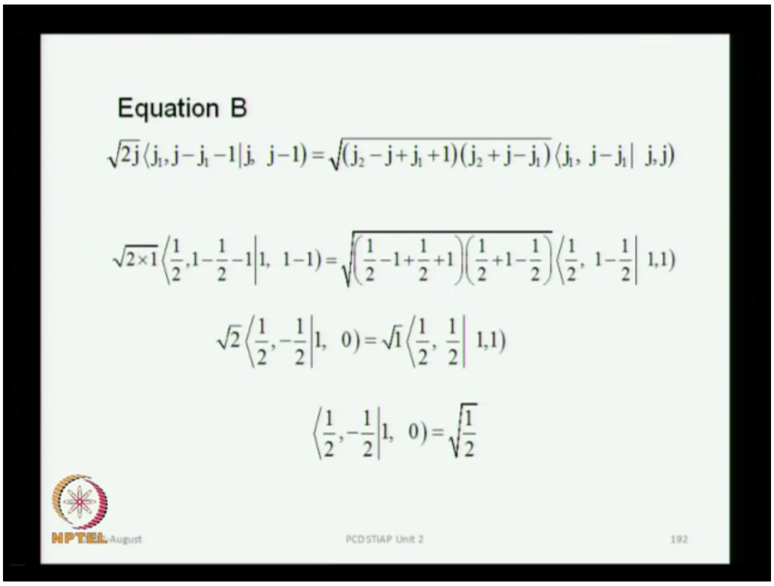

Now, you can also see the same result coming from thIS equation B, if you handle this equation B in which you had this relationship. You can put j equal to 1 j 1 equal to half and j 2 also equal to half and just plug in the actual values of these quantum numbers, and simplify these terms and you find that this coefficient, which we know has got a value equal to plus 1 right it has to be because that is the only one which can generate the state 1, 1 that is the only one. So, you immediately get this coefficient to be 1 over root 2.

So, there are, so many different ways different roots to get certain result, and you know essentially what you doing is to play with the recursion relation, use orthogonality relationship. And you will be able to get all the clebsch gordan coefficients just from the seed value, which is this which is m 1 taking it is maximum value, m 2 taking it is maximum value, j taking it is maximum value and m taking it is maximum value that coefficient must be unity. So, that is a seed coefficient and from this you can get everything else by using these recursion relations.

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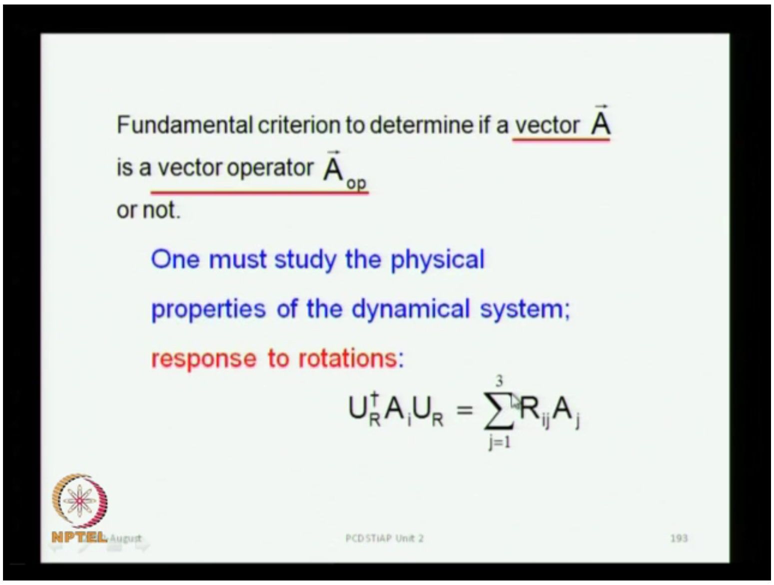

So, now I really take up the question that I really wanted to take up for today's discussion, as to when is a vector A vector operator. And we know what vectors are we know what vector operators are and we need to address this question, and for a vector A to b treated as a vector operator. It must subscribe to the properties for the vector operators, it is not enough there is side dissatisfied just the vector algebra, but it has to side dissatisfied the algebra of the vector operators. And the vector operators, we know they transform under rotations according to this rule. So, we have discuss this role and we have seen that the wignor demetrics, elements, appear in these transformations.

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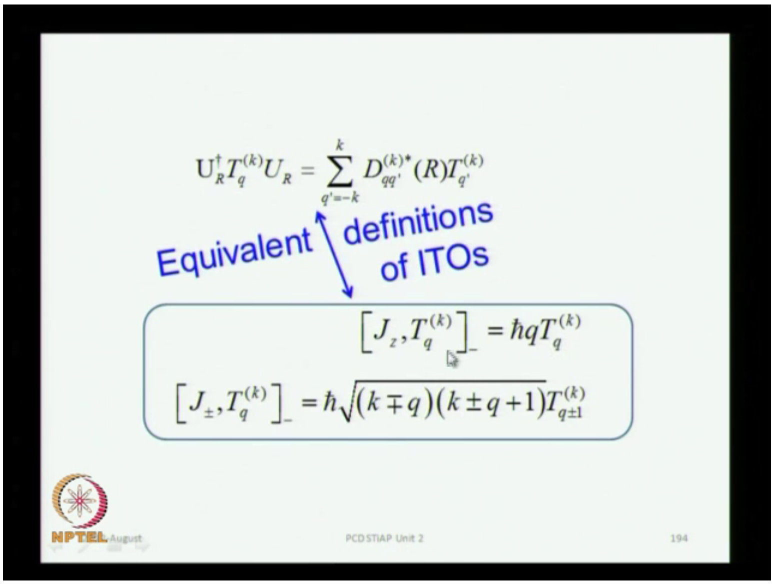

So, this is the transformation of a irreducible tensor operator of frank k, you are taking the q with component, and this is the defining relationship. So, this will be our guideline this is the basic definition of the irreducible tensor operator, and this is completely equivalent to these commutation relations. So, you can define the irreducible tensor operator family, by this relation at the top or equivalently by this commutation relation, the commutation with the generator of angular the momentum.

So, it is not surprising that they should be involved because when you deal with vectors you always see how their components transform into the rotation of a coordinate access right. The generator of rotation is the angular momentum, which is why the angular momentum operators get involved in the commutation relations, which define the tensor operators of appropriate ranks.

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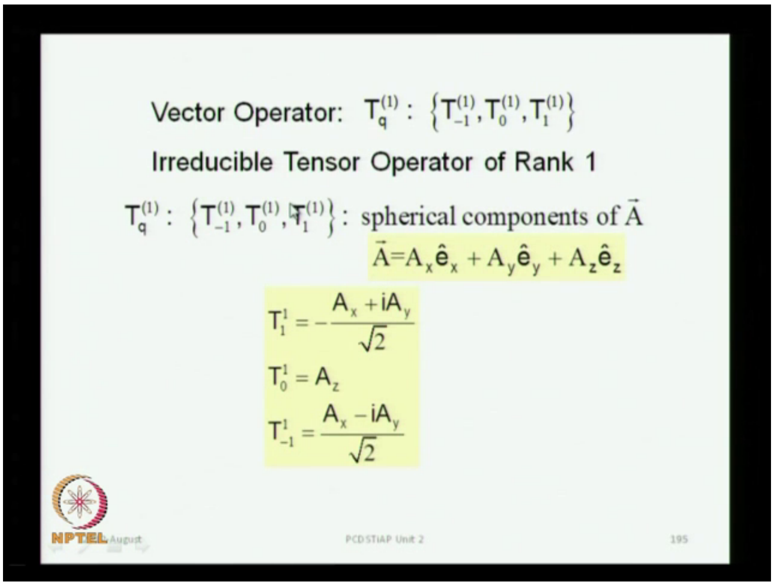

So, let us take a vector operator and these are the spherical components, this is the irreducible tensor operator of rank 1. And you can also express the Cartesian components, as you always do and then you have these relationships between the Cartesian components on the spherical components. So, there are the transformation which tell you how these tensor operators are related to the Cartesian components, they both define essentially the same vector operators. So, it is just a matter of expressing them either in Cartesian components or in spherical components, is much more much nice say to express them spherical terms because that is how the algebra of angular momentum is developed.

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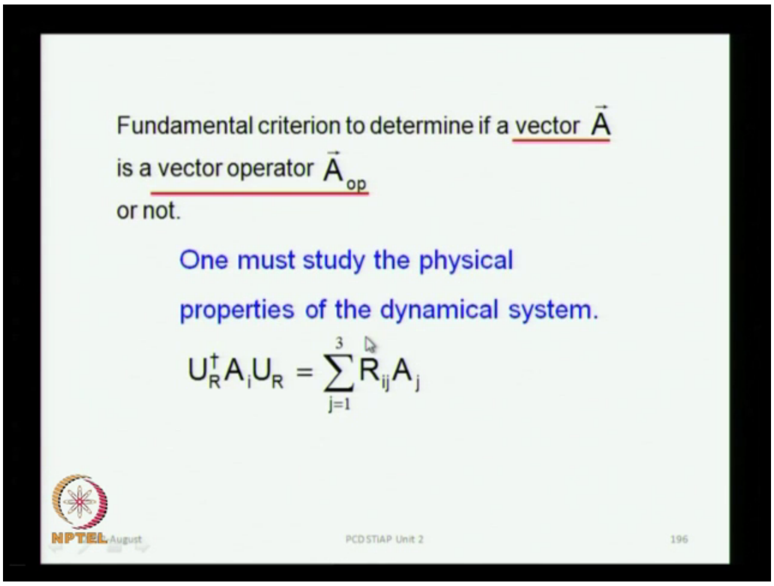

And to study the fundamental criterion of when a vector A is a vector operator one must study this response to the rotation.

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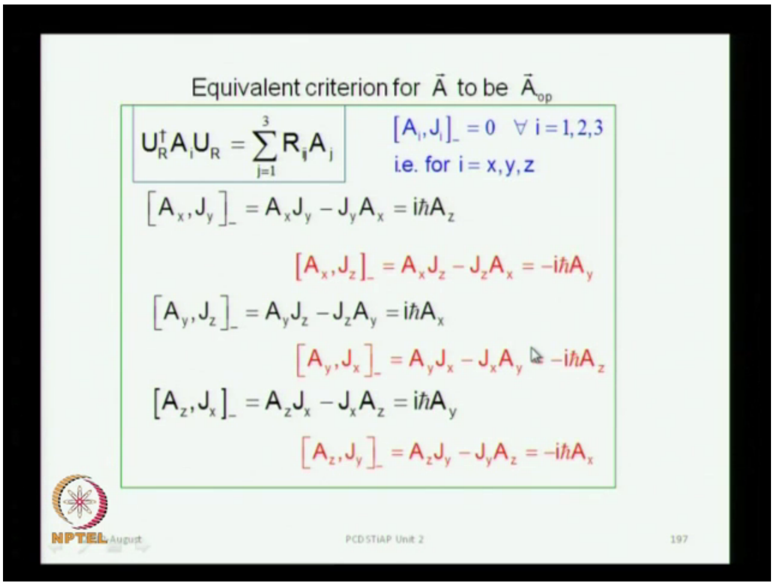

So, let us find an equivalent criterion for this, so this is the response of a component to rotations, and the corresponding equivalence with Cartesian components is given over here. That if this relation holds, then the commutation of A i with J i would vanish when both are the same components, if you take different components A x with J y, then this is A x J y minus J y A x then this must be i h cross A z, this very similar to the angular momentum commutation relation.

So, the angular momentum itself is an angular momentum vector operator, so this natural that you should expect that relation to appear. Likewise it is commutation with the third component J z will be minus i h cross A y, you can write this is J z A x then the b plus, but this A x comma J z. So, this is minus i h cross A y and then you have similar relationships for A y with the other two components, and also for A z with the other two components.

So, they are completely you know you just change the indices x to y, y to z and z to x you will get the remaining relationships. So, you get the corresponding relationships by simply shifting the subtract x to y, and y to z and z to x by cyclically rotating them, and you get these two equivalent criterion one is what is denoted in this in set box, which is a response to rotation. The second is in terms of commutation with respect to the angular momentum generators.

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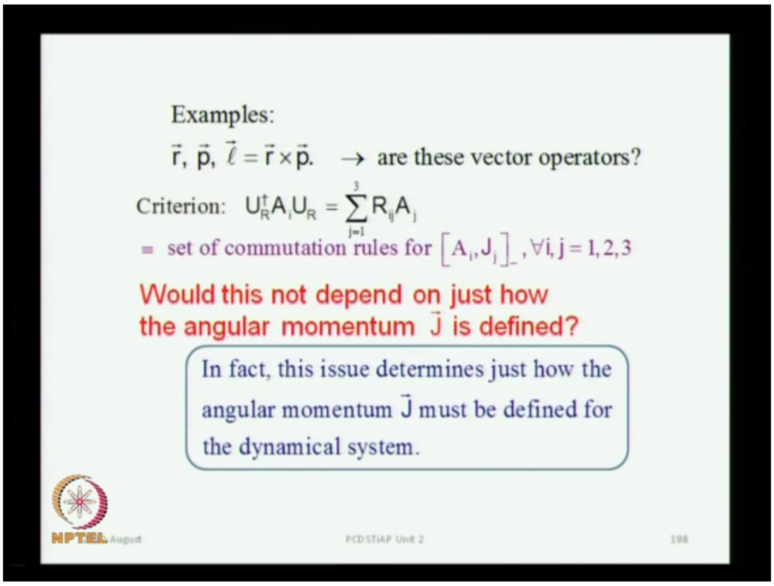

Now, let us take some examples the position of vector, the momentum vector, and the angular momentum vector I take this example. And we has the question are these vector operators, we know they are vectors because we know how their components transform, and we ask if they are vector operators. And what you will do is to subject these operators to the fundamental criterion that we have defined, and if the criterion is fulfilled either in terms of this relationship or equivalently in terms of the set of commutation relations, which we wrote down on the previous slide.

So, either this relationship or this and this for completely equivalent to each other, we have seen that earlier by subjecting the these to the criteria that we have chosen to define vector operators. We can ensure that these are vector operators, but mind you this certainly depends on how angular momentum J itself is defined because it appears explicitly in this commutation relation. So, if you have not defined angular momentum appropriately.

This will not work and in a few minutes you will see an example, where it will become quite obvious to you as to what is meant by defining the angular momentum appropriately. But, at this point you recognized that it definitely has to be defined appropriately because it is appearing explicitly in the commutation relations. So, this particular feature as I mention over here, it actually determined how the angular momentum J must be defined for the dynamical system. What exactly is the definition of angular momentum of a dynamical system. So, that is actually determined by this consideration and you will see why this is, so important.

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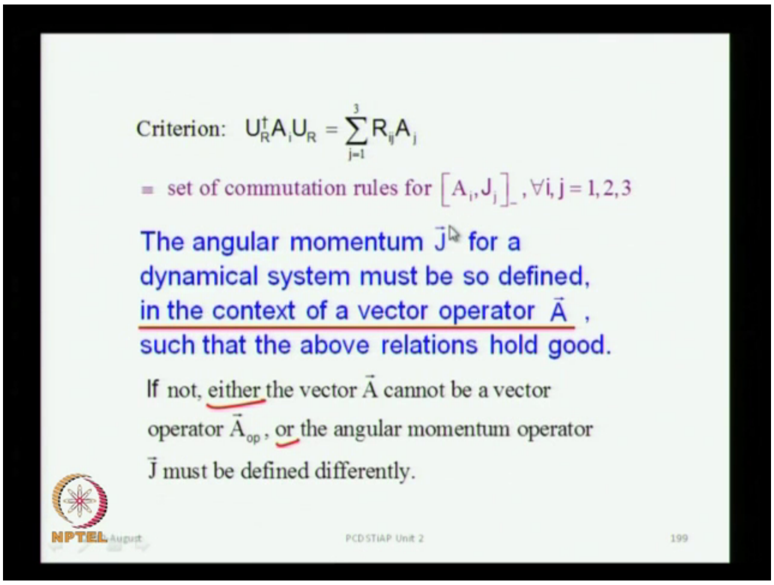

So, let us refer to the criterion over here and it is important to remember that this criterion is extremely important that they would determine, how the angular momentum is appropriately defined. If this does not hold for the definition of angular momentum that you have chosen, then it would mean either that A is not a vector operator or else it could mean that angular momentum has not been defines appropriately that is the consequence. Now, this is the necessary and sufficient condition that if this criterion breaks down, then it would either mean that this vector A is not a vector operator or else angular momentum has not been appropriately defined.

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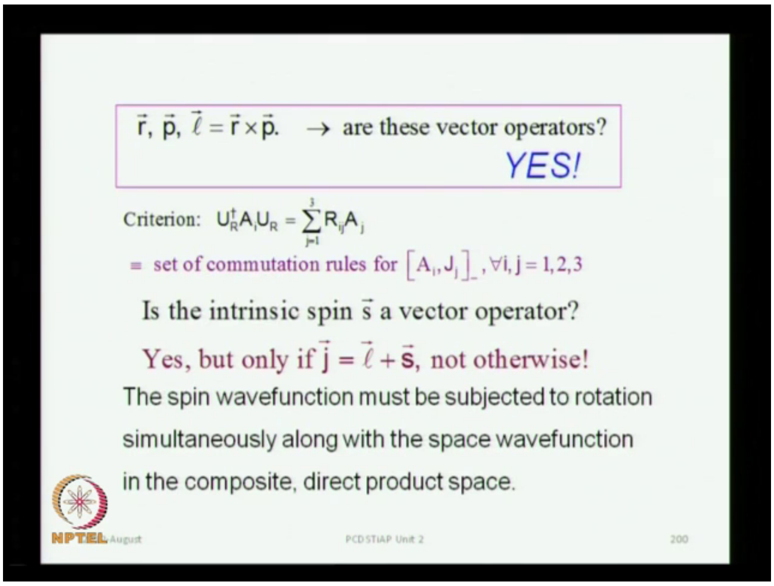

So, let us take an example here that these operators that I took for the first consideration the position momentum, angel of momentum. And we has this question that do they satisfy the criteria or this is the matter of you know working of the algebra you can discover for yourself that yes, they are vector operators. What about the spin is it a vector operator, now here comes the nice answer that it is a vector operator, but only if angel of momentum is defined, if the system already has orbital angular momentum.

Then the net angular momentum must be defined as l plus s, which means that the rotation operator whenever you subject this component to rotations, you must subject the components A i must include the components of l as well as s. If you do not do that it would be that the angular momentum has not been appropriately defined, so spin would be an angular of momentum operator.

But, only for a system which if it already has a orbital angular momentum, then the net angular momentum will have to be defined as a some of the two. And this is where the combination of angular moment are j 1 and j 2 have you coupled j 1 with j 2 becomes important. Because, you have got now two sources of angular momentum, one is j 1 and other is j 2, and you combine them using the clebsch gordan coefficient right. And the two angular momentum in this case, one is orbital angular momentum l, the other is the spin angular momentum s. So, the spin wave function must be subjected to rotation simultaneously along with the space wave function, you cannot do just one and not the other.

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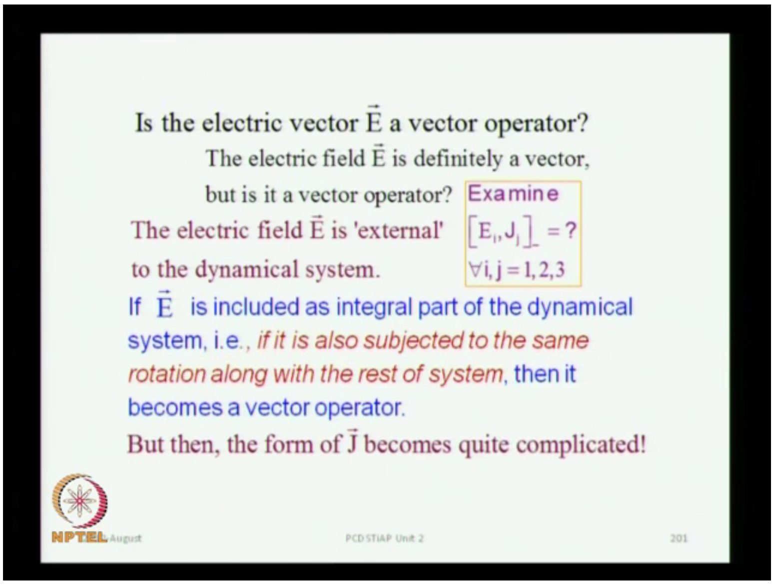

Now, let us see another example here, how are the electric vector, electric field we know it is a vector. Now, we let us ask the question is it a vector operator, now we know what to do what we are going to is to subject it to the same criteria right, now this electric field we definitely know as a vector, there is no doubt about it we have been using it in vector algebra, and vector calculus. And it is external to the dynamical system, like if you have got the electron or an atomic system, which you place in an electric field.

So, now, the electric field is external to the dynamical system that you are really dealing with, which is the atomic system or the electron angular momentum. And if you have not included that in the definition of the angular momentum, it will not be a vector operator. So, what you have to do is show examine the this commutation relation, these are the equivalent defining criteria right, so these are the if equivalent defining criteria, and you must subject the components of the electric field to these commutation relations and ask yourself, if they satisfy the same relationship as you use to define what a vector operator is.

Now, it turns out that if the electric field is included as an integral part of the dynamical system. So, you define your system not to be the atom, but the atom pass the electric field, and then define an angular momentum you can certainly do that, but it will have an extremely complicated form. And that is rarely done, but otherwise you treat it just electric vector, but not quite as vector operator because we have two option over there that if the criterion satisfied, then you know how to define a vector to be a vector operator.

And when the criterion is not satisfied, we know that either it is not vector operator or angular momentum is not appropriately defined. So, you can certainly include this in the dynamical system, but then the form of the angular the momentum would be extremely complicated, then and many semi classical descriptions it is rarely done. So, electric field will be a vector alright, but not a vector operator, so these are some considerations which will be of significant to us.

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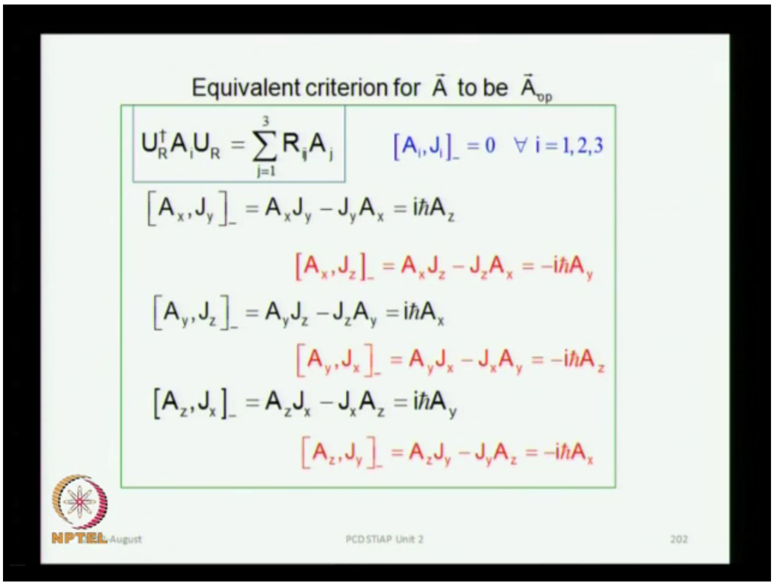

So, these are the set of relations one is this response of the spherical components to rotations, and the other is the commutation of the Cartesian components with the angular momentum components. And this is natural to expect because the angular momentum is the generator operations, so that is what defines what how a tensor components, how tensor components of tensor transform in a rotation of co ordinate system.

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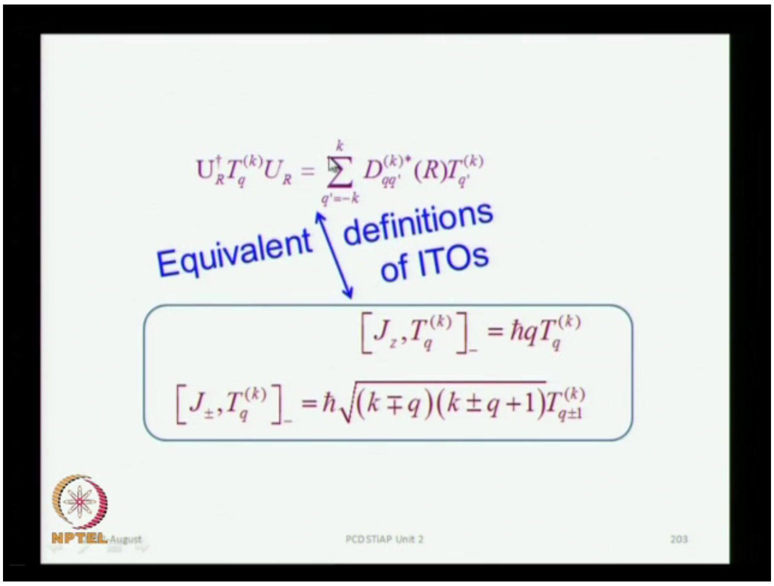

So, these are equivalent definitions you can use either one or the other, you can derive this from the first or vice versa there completely mathematically equivalent.

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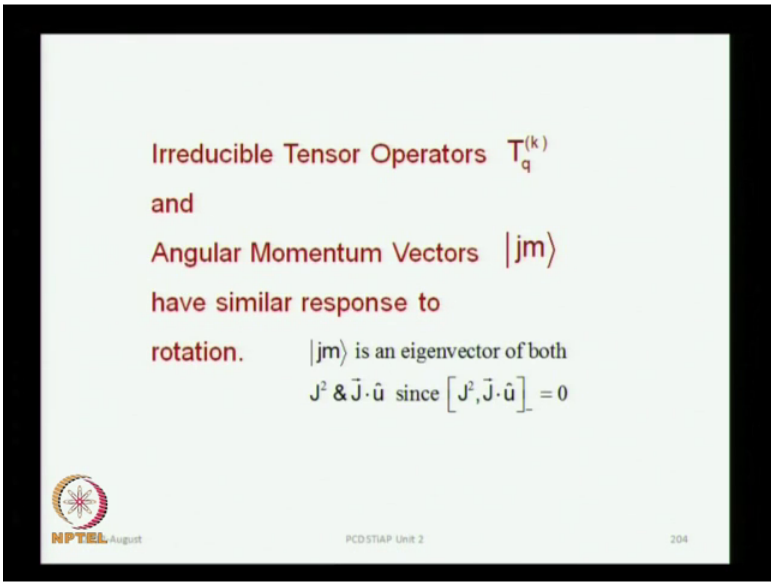

And we will now work further with the irreducible tensor operators, so now, we have some general similarities between angular momentum vectors, and the irreducible tensor operators. You would have notice them already, you know that both the angular momentum vectors, which are j m these are Eigen states of j square and j z, what is this, this is an irreducible tensor operator, which is define according to the definition as to how it is components transform under rotation, now both of them have very similar response to rotation.

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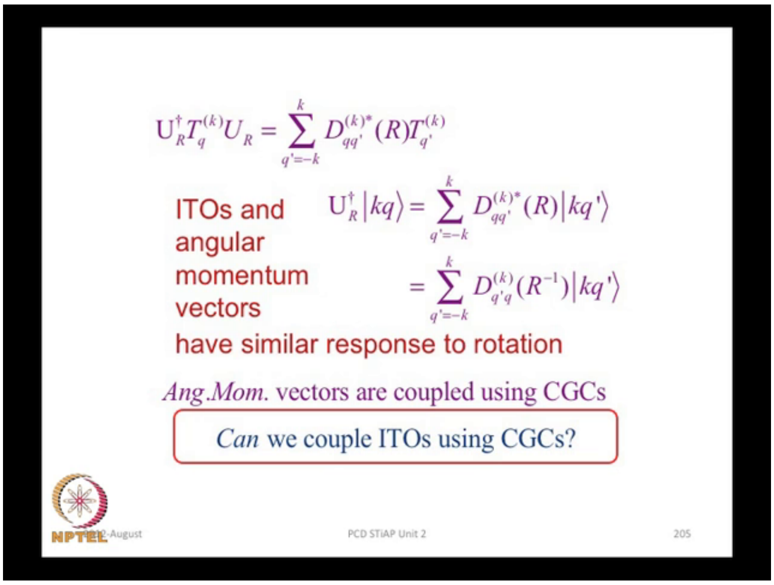

Now, this really interesting because if you look at this relation the response to rotation of these components is that you get a sum over all the remaining components, the coefficient being the elements of the Wigner D matrices. Likewise, if you see the response of an angular momentum vector to rotation, you have an exactly identical relation. The first relation over here is for irreducible tensor operators, this relation over here is for vector, how do vector, how do angular momentum of vectors respond to rotations.

So, this is the two different questions, but the answers a very similar the response of both to rotation is very similar, and they involve the liner super position of the elements waited by appropriate Wigner the metric elements that is the similarity. So, this is our conclusion that the irreducible tensor operator, and the angular momentum vector have got similar response to the rotation. Now, let us plug in another fact that we have learned, that angular momentum vectors are coupled using clebsch gordan coefficient we know that.

We have learned how to handle this, may be then ask if it is possible to couple irreducible tensor operator using the clebsch gordan coefficient. So, we are building on the analogy between the angular momentum vectors, and the irreducible tensor operators, we find the both of them have similar response to rotations, we know that one of them can be coupled using the clebsch gordan coefficients, we now ask can the other be coupled using the clebsch gordan coefficients right.

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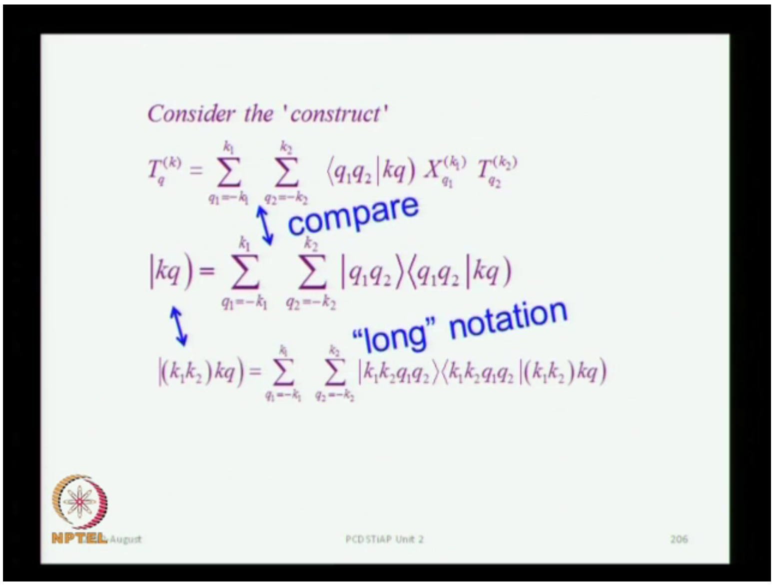

So, let us raise this question we take two irreducible tensor operators, one is X and the other is T, X is an operator of rank k 1 T is an operator of rank k 2. So, this will be a family of 2 k plus 1 operators, and this will be a family of 2 k 2 plus 1 operators, and you take one of this elements q 1 and q 2 element over here. And combine them using the clebsch gordan coefficients, and this is exactly what we did when we coupled two angular momentum.

See the comparison as I have written angular momentum of coupling this is the coupled angular momentum k q, which is a super position the you can recognize this as the resolution of unity of you like, this is similar to sum over m 1 m 2 m 1 m 2 right. [FL] But, I am now using dummy labels to be q rather than the m's, so it is exactly the same relationship, and I now have a construct, which has been composed using the prescription, what is a prescription that you have to combine two irreducible tensor operator X and T.

Like over here, you combined two angular moment k 1 and k 2, here you combine two kind of angular moment k 1 and k 2 or you can write it in the long notation. So, that will make it explicit because this k q is a shorten notation for coupling of k 1 k 2 leading to k and q, k would go from more or less of k 1 minus k 2 to k 1 plus k 2 right. And for each value of k q would go from minus k to plus k in case of 1, so you know what we are talking about.

So, this is an exactly an identical relation in which you have constructed a coupled angular momentum stage from the factor states, using coefficients which are the clebsch gordan coefficient. And you use exactly the same clebsch gordan coefficient, namely q 1 q 2 k q which are precisely the once which are appearing here, this is q 1 q 2 k q clebsch gordan coefficient in the short notation, which is this right is exactly the same coefficient. And we are now going to ask that if, but coupling of k 1 and k 2 gives you a new angular momentum k.

Then, likewise would coupling of X and T using the same prescription, which is using the same law of combination, the law being combine them using the clebsch gordan coefficient. So, using the same law the construct that you get on the right hand side for which you generate a symbol T k q is this then an irreducible tensor operator that is the question. How we know how to answer it because whether or not it is an irreducible tensor operator is determined, by whether or not it satisfies the defining relationship for an irreducible tensor operator and we know what it is.

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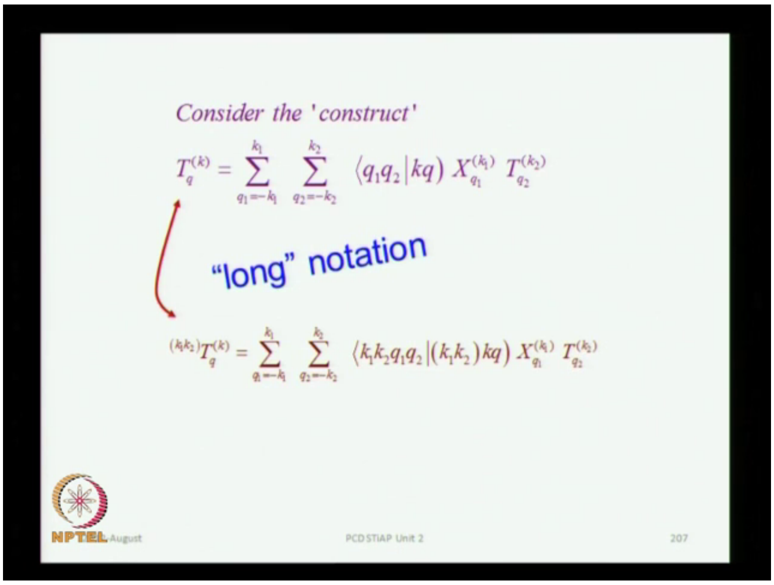

So, what we are going to do is to take this construct, which I have written in the long notation over here just for clarity. But, you know the writing k 1 and k 2 is not important they or specifically in this context, the ranks of these two tensors over here you know in the context of the angular momentum of coupling, those who are the two angular moment j 1 and j 2 that we are talking about.

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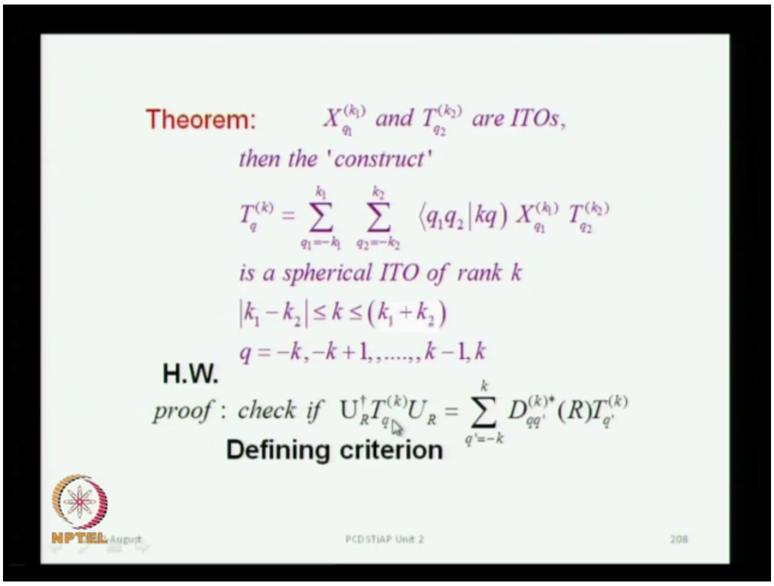

And what we do is to state this theorem that if X n t are two irreducible tensor operators, and if they are combined using the clebsch gordan coefficient, which is a same law that you use to combine angular momentum spheres right. Then the result that you get is also an irreducible tensor operator of rank k, such that k belongs to this range from a certain minimum value to a certain maximum value. And you notice that, you once again see the triangle law popping up.

And then for each value of k the q component, which is the member index in that family of irreducible tensor operators would go from minus k to plus k. So, this is the theorem that one can prove and all you have to do is to subject it to this criterion because this is our definition of an irreducible tensor and operator. Now, this is your homework that you subject this operator to this criterion.

And if the response to rotation turns out to be given by an expression this kind, then you can conclude that it is, in fact an irreducible tensor or operator. So, that would prove the theorem I am not going to prove it over here in this class, but you have to manipulate this terms, and see it for yourself.

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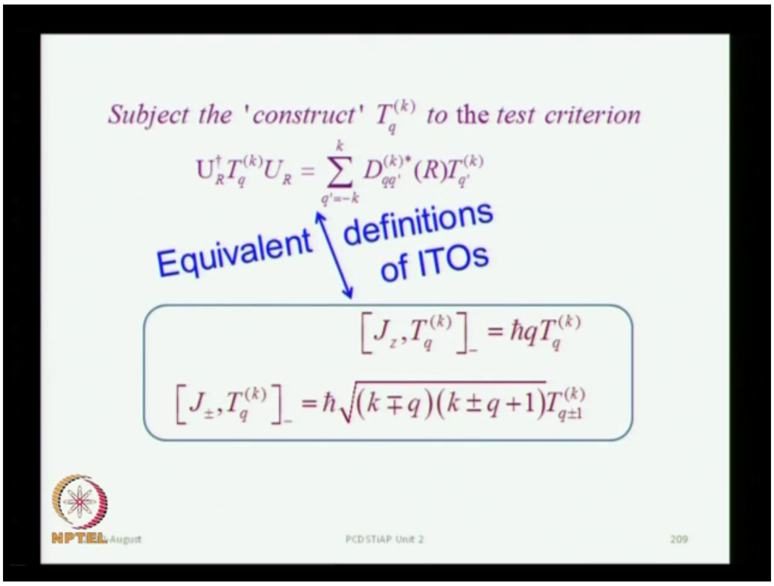

So, subject this construct to this criterion actually you can see that the proof is really not very difficult, you can subject it to the equivalent criteria of the commutation relation as well do one or the other.

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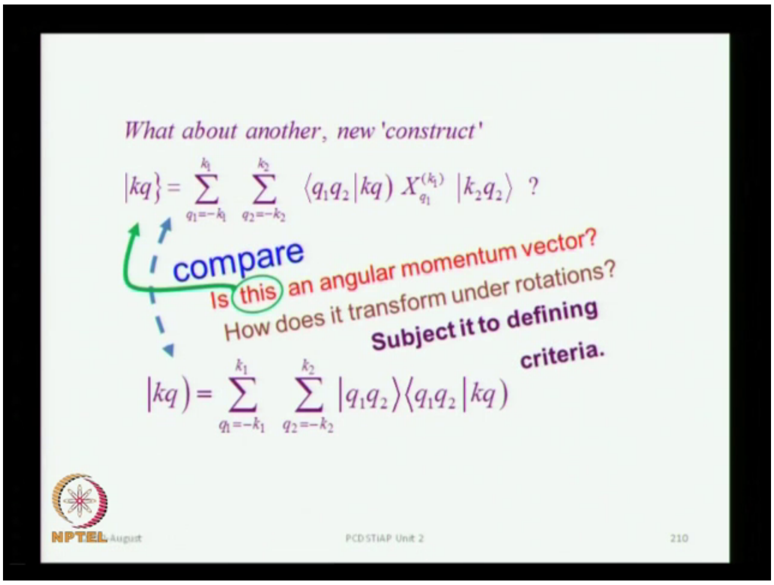

And now we ask another question, we had combine an angular momentum vector with another angular momentum. Then we combined two irreducible tensor operators, using the same law, now we ask if you take one of this to be an operator and the other to be vector. And construct a new creature, which is some sort of a hybrid creature alright because it is coming from the combination of irreducible tensor or operator, and an angular momentum state.

But, this is an operator, so this operator operating on a vector you know will give you another vector. So, the right hand side is a super position of vector, so you know that it will turn out to be a net vector, and we ask if this net vector will be an angular momentum vector right. And since, this has been composed form a tensor operator and angular momentum of vector I have denoted it this time by a beautiful bracket, once we find that it is, in fact an angular momentum of state, we could use the same notation use an angular bracket or recycle or bracket or a beautiful bracket it does not matter.

But, just keep tracker what we are really doing I have used different notations, and they are I think quite useful. Because, when you look at angular momentum coefficient you an especially when you put in numbers q 1 equal to half q 2 equal to half k equal to 1 q equal to 0, then it is useful to keep track of which is the side which corresponds to the coupled state, and which is the side which corresponds to the uncoupled state. So, that is part of reason I like this notation in which I use angular bracket on one side and circle of bracket on one side.

But, now we have constructed using the same prescription a completely new kind of vector, in which you are super posses super posing the result of an operation by an irreducible tensor operator X on this angular momentum of vector. So, this operating on a vector will give you a new vector, then you carry out this double sum and have each term waited by a clebsch gordan coefficient. So, this is the exactly the same kind of sum as we used, when we composed two angular momentum j 1 and j 2 right.

So, our question now is what kind of a vector is this, and notice that this relationship is exactly identical to the composition of two angular momentum of vectors, using clebsch gordan coefficient. So, exactly an identical relationship and the question is, is this beautiful bracket is it an angular momentum vector that is the question that we ask, and to answer this we must examine how it transforms on the rotation. So, we you must subject this to rotations subjected to defining criterion, find out how the responses we will discuss it in the next class. We will see that you can subject it to the defining criterion, and it turns out the it is also angular momentum vector.

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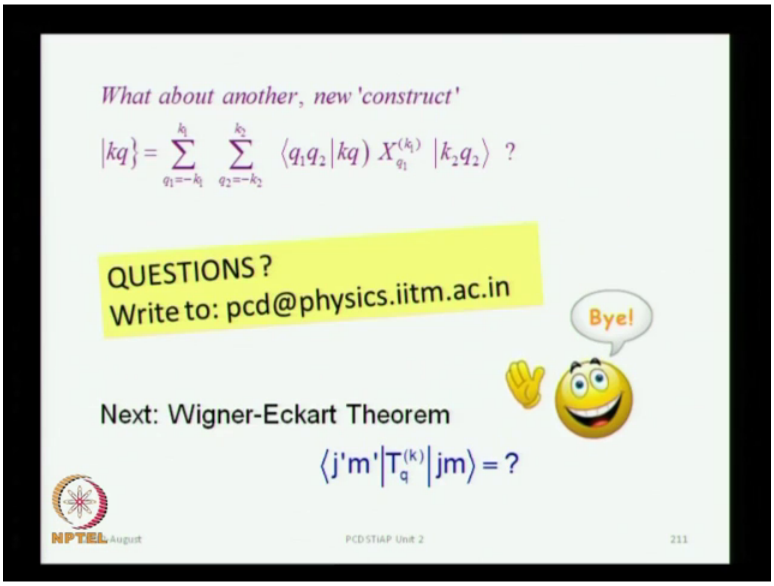

And that is very nice feature because what you find is that what you have composed over here is an angular momentum vector, which comes from the result of an operating on a vector. And the left hand side is an angular momentum vector, now just ask yourself what would happen if you take a projection of this vector on an adjoin vector, if you take a projection, so this is now eckart you construct a scalar which is a bracket that is how you get the projection of eckart right.

So, you come construct a bracket scalar and that is what you get on the left hand right side. And the right hand side you are going to take the projection of this entire right hand side, you going to take the projection of this entire right hand side on the same adjoin vector. This will factor out as a multiplier coefficient, and you will get essentially the metrics elements of an operating, which is this X into angular momentum states.

Now, this is the quantitative of interested in periscope because in physics you are always interested in doing quanta mechanics. So, that at some point you can relate to the observations, and what are the observations, what kind of observable do you really have, you have got the probability density for example, the size are side or you can have transition probability amplitudes. What is the probability amplitude that an operator which is going to represent a physical interaction that you have got physical interaction.

And is this physical interaction going to new transition from a certain initial state on the right, to ascertain a final state on the left. So, this metric element will actually give you a major of the probability amplitude, it is module square will give you the probability and then, when you put the right numbers this will give you a major of the intensity of transition from an initial state to a final state. That is what you doing in spectrum, when you look at a spectrum right.

You see various lines with various intensities, and these intensities are proportional to the transition probabilities, which are the modulus square of the transition metrics element, which is a creature of this kind. And the Wigner eckart theorem addresses the metrics element, what the Wigner eckart theorem does and that is our topic for discussion tomorrow that it will factor this metrics element, into two parts of physical part and a geometrical part. So, that will be our topic for discussion in the next class, if there any questions today I will be happy to take questions.

Student: ((Refer Time: 44:13))

So, the question is whenever you have an interaction of the electric vector, when you place an atom an electric field for example. Then electric field vector will not be treated as a part of the angular momentum of the dynamical system, so you have to define it correctly, it will generate a perturbation, but it will not be a part of the angular momentum. And therefore, it will not be an operator.

Student: ((Refer Time: 45:06))

You can do it, but that is not particularly advantageous in this case because you will you can generate an angular momentum which will include that. And have an angular momentum which is extremely complicated mathematical form, and it is not of much interest there is no need to do that. But, in principle one can do that and if you are keen that it should be defined as an operator, then you will have to include it into the definition of the angular momentum, any other question.

Thank you very much.