WAVE OPTICS

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Lecture - 08: Random and coherent source, standing wave formation

Hello, student to the next class of wave optics. Today we will have lecture number eight and we will discuss more about the nature of the superposition of waves and when the two waves randomly interfere then what should be the form and if two waves do not randomly interface they have a synchronization then what should we get at the output. Okay, so we should have today lecture number eight and we were discussing the superposition of waves. So In earlier class we find that if we have waves like E 1, E 2, E n in complex notation where, j 0 e to the power of i alpha j and if I add all these things, so, ER which is the resultant wave having the form E0, e to the power of i alpha then ER tilde is sum over j from 1 to n ej tilde and if we want to find out E naught from here with this then, E naught square essentially sum over E j0, j goes to 1 to n square of this quantity plus 2, j greater than i, sum of i that goes to 1 to n then, e j0, e i0 and then we have cos of alpha j minus alpha i that is the resultant amplitude one can have. If I add these widths now after having that we will describe two cases of random and coherent very important term source physically. What is the meaning of random and coherent source? In random sources what happens is that, the wave that is coming out from a source, suppose, this is the source S, the wave that is coming out from the source, so I mean for different sources rather, I should not say a single source all this for individual source all cases we have individual sources. Let me draw it properly. So in all cases we have individual sources, these are n number of source point which is emitting light or emitting wave, this wave are randomly faced that is a given time there is no phase relationship between these waves we have in number of so you can have for in randomly phased sources with equal amplitude. Let us consider the amplitude to be equal for the simplicity and frequency.

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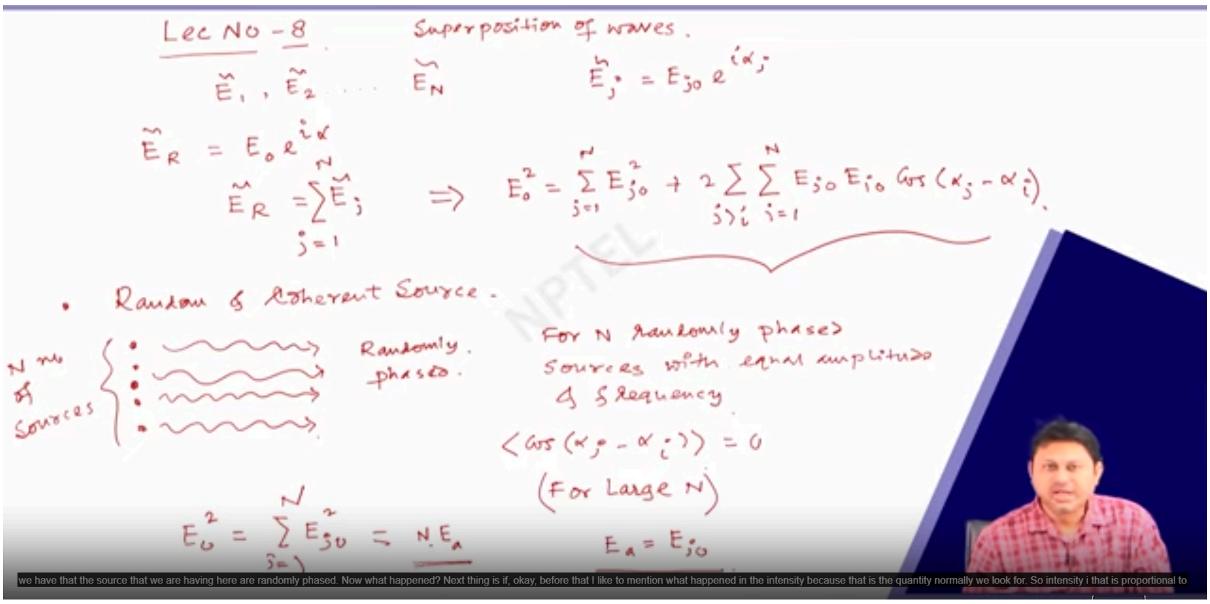

What happened is that we have a number of sources where their phases are randomly oriented and we also consider their frequency and their amplitudes are the same. In that case what happened that the average value of this quantity cos alpha j minus, alpha i should be 0 because this value will vary from minus 1 to 1 and since their phase relationship is random then the argument of the cost value will change in a random manner and if we try to find out what is the average value of this quantity, obviously for large n this is happening, not for small number of n 1 or 2 or 3 or 5 say, thousands of n if I do and then if I add the cos alpha j minus, i term then what essentially we get that this value will vary from minus 1 to plus 1 that is the cos value, that should vary from minus 1 to plus 1. And if I average that quantity then essentially we will want to get 0. So, if that is the case that if a randomly oriented source and source are there then emitting waves and if I add all the waves, so the value of e 0 square will simply be the sum of e j 0 square, where, j goes to 1 to n. Now if all the amplitudes are the same, then I am going to get n multiplied by that amplitude we say Ea. So, Ea is the amplitude for all the waves, same amplitude whatever that. So, j 1 0 equal to j 2 0 equal to j 3 0 is equal to E a. So, the amplitude will be multiplied by n factor if we have that the source that we are having here are randomly phased. Now what happened? Next thing is if, okay, before that I like to mention what happened in the intensity because that is the quantity normally we look for. So intensity i, that is proportional to E square and we get e is equal to n multiplied by the individual amplitude say EA which is same for all the waves. So in the case of light when the intensity is proposed proportional to the amplitude square the irradiance i is intensity or irradiance of a randomly phased source is equal to simply sum of the n individual irradiance. So, that means if we want to find out the irradiance for all the summation of all the waves randomly oriented then, that is simply n times of individual amplitude or individual irradiance. So, the irradiance will simply add up here. There will be all the individual irradiance that will simply add up if we have a number of sources that are not in phase. However, another case we can consider lead us to case number 2, where we have n coherent sources. So, for n coherent sources which are in phase such that alpha i is the same for all the waves. So essentially alpha is equal to alpha j, in that case. So let me draw the figure. There are n number of sources and these sources are emitting waves in form of light say with same phase their phase are same in sources with coherent sources, so they all are in same phase. In that case what happens if I add all the waves? All the waves are added up then, the value of the resultant amplitude square is e j 0, j goes to 1 to n that is the first term and second term since all the alpha are same. So we can have a cos of alpha j minus, alpha i this will be 1 as alpha j is equal to alpha i. So we have 2 of j greater than i, i go to 1 to n and then we have e j 0 and e i 0. Okay, so if we look very carefully this quantity is nothing but sum over j going to 1 to n e j0 whole square of that. So it is nothing but if I write this a1 plus, a2 plus, a3 square of that is essentially a1 square plus, a2 square plus, a3 square plus 2 of a1 a2 plus, a2 a3 plus, a3 a1 that's all this restriction j greater than i and i goes to 1 to n will give us these combinations. So if now all ej are same, so ej0 we mentioned that it is ea, the values are same for the same amplitude, then my e 0 square is essentially sum over n. Okay let me write down once again J goes to 1 to n and it is e j 0 whole square and that value is n multiplied by ea and whole square of that, that is a big difference between the previous expression where we get this e is equal to n of e. Now here we had an expression that, so the previous case e 0 square we get n into a. So it is Ee square that we get and here we get e 0 square is equal to n square, into e a square. So obviously this value e 0 square equal to n into ea, sorry, ea square I made a small mistake here in writing the previous case and here it is n into ea square but this case this is e0 and this case it is e0 into is equal to n square into ea square. So once n square is there if the number of n is too large then there is a huge difference between the two amplitudes of the two sources. In one case we have n multiplied by, so if I write this case for, so, let me do it here only. Then what is the meaning of that? The meaning is irradiance or irradiance of n coherent source and coherent sources is equal to n square multiplied by irradiance of single source. So, irradiance of a coherent source is equivalent to n square multiplied by the irradiance of the single source. Previously, we find irradiance of the randomly phased source is equal to sum of n individual irradiance, but here n individual, one individual source amplitude will be multiplied by n square if the sources are coherent. So obviously for coherent sources we have a much much higher value of irradiance. Okay let me go forward to the next concept regarding the superposition of waves and that is called the standing wave as the name suggests here the wave does not move. So, the superposition of the two harmonic waves of the same frequency propagates in the opposite direction. So, the same frequency superposition of the two harmonic waves means one wave is moving this direction and another wave is moving in this direction. If I add these two one is forward propagating and another is backward propagating, if I add these two with the same frequency, then they will form a standing wave. Suppose this is E1, which is an E0 harmonic wave, so I can write sine kx minus omega t, which is this direction and another wave which is moving in the opposite direction. So it should be e2 is equal to e0 say sine and then kx plus omega t because it is moving in another direction. So that's why if it is minus then it has to be plus with a phase. Let me add an additional phase, maybe whatever the wave that is moving in the opposite direction is having an additional phase and we write this phi r which is the relative phase, so phi r is the relative phase between these two wave phases. So if we you know put these two together then the resultant wave as a superposition of principle, using superposition of principle, we simply have E1 plus E2 and this amplitude is the same. So, it should be E naught. I write sine, say I write this is term beta 1.

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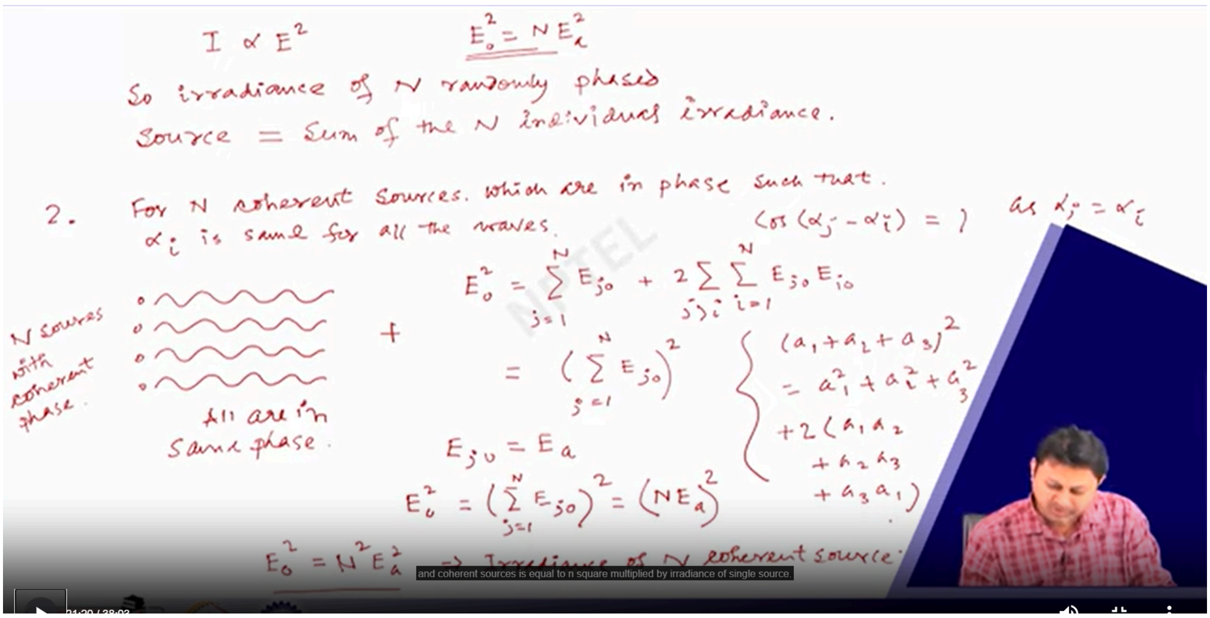

So, this is my beta 1 and this entire thing is my beta 2. S So, it should be sin beta 1 plus, sin beta 2. So, let me write it here beta 1 is equal to kx minus omega t and beta 2 is kx plus omega t plus phi r. So, this quantity is 2 E 0 sine of beta 1 plus, beta 2 whole divided by 2 and cos of beta 1 minus, beta 2 whole divided by 2 and essentially what we get is 2 e naught sine of beta 1 plus, beta 2 if I add these two and make it divided by 2 it should be simply kx plus phi r divided by 2 and it is, sorry, it should be sine term here sine of these things and cos of kx, not kx because when we make a negative it should be omega t cos of omega t minus, phi r by 2. So, essentially what we find is the resultant equation. So this is not in the form, if we look carefully, not in the form f of x plus minus vt. So, the resultant wave even though we start with two propagating waves, but when they are superimposed with each other I can separate out the x coordinate and t coordinate the space coordinate and time coordinate individually and it is not in the form of x plus minus v t. If it is not in the form of x plus minus v t this is essentially not a wave that is propagating. So whatever the resultant will we have, these are called the standing wave, they are standing in the same place. So, let us take phi r by 2 is some quantity, let us put some phase like pi by 2 then ER will be simply 2 of E0 then sin kx and cos of omega t. So, let me check so, this is sine a plus, b is sine a cos b and and then cos sine b. So, make it simple. This quantity pi by 2 should be cos and it should be sine. So, let us make life more simple actually, so let us take this as 0. So, there is the relative phase between these two zeros, the most simple case. Then maybe we go to another case. Then we simply have this in my hand that E r is 2 these things, sine of this and cos of this. So, this quantity is the amplitude and if we fix a point x equal to x 0 and look at what is happening for this wave we can see that this wave will change over time and with a frequency. So let me draw what is happening here and try to understand that. So this is the forward propagating wave and because of the backward propagating wave we have a standing wave and we get something like this. So this is my e2 and the other one is e1 and they are superimposing to form a resultant wave actually which is standing and which is changing over the time, its amplitude maybe and draw like this.

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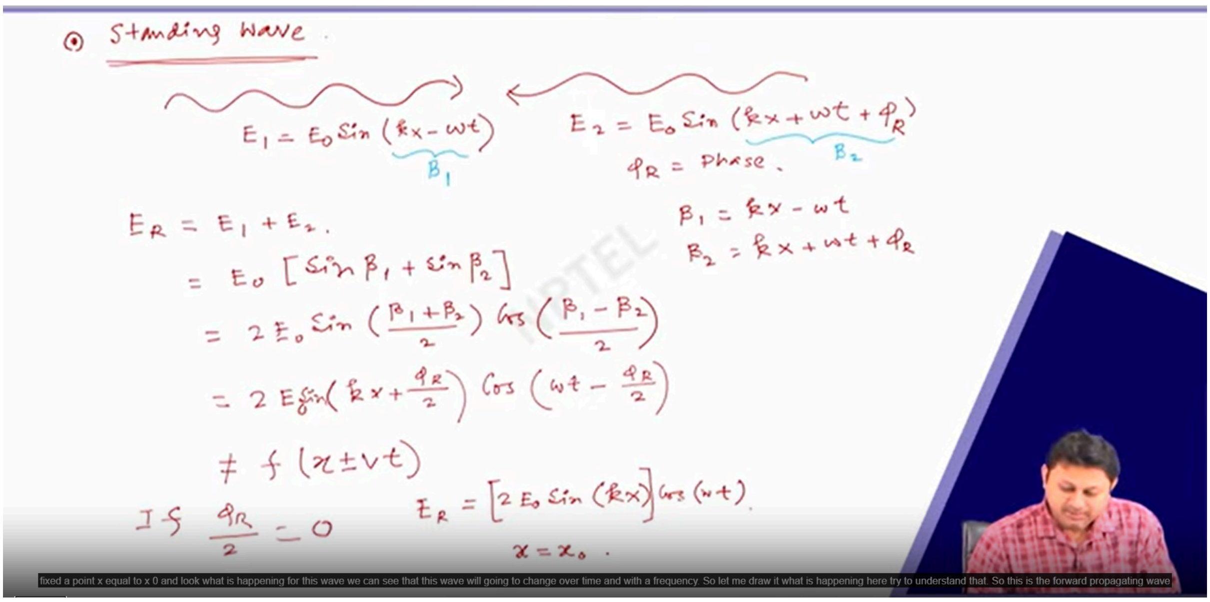

So its amplitude at some point is changing from maximum value to minimum value; sometimes it goes to 0, sometimes it goes over the time actually. So it vibrates like this. This is the point which we call the nodes. So the form is ER. The resultant waveform ER is equal to 2E0 sine of k x multiplied by a cross of omega t. So as I mentioned at a fixed place for example here this is some point at x equal to 0, the amplitude of the wave will change with this form 2a, 2e multiplied by cos of omega 2. So, over time this value will vibrate and we are going to get two values 2 e0 and minus of 2 e0 it will vibrate in these two values over the time. So, however at this node point that means at x equal to say lambda by 2, then 2 into lambda by 2, then 3 into lambda by 2 these are the points where the amplitude will always be 0. That is irrespective of the time, the amplitude of this point will always be 0. These are the points where it happens because at that point, the sine value will go to 0. So, similarly, E R will have maximum value at all x points when the cos omega t will be plus minus 1. So, for a fixed x if I want to find what as I mentioned what is the value of the amplitude then when cos omega t is plus minus 1 these points will get the maximum amplitude. So, cos omega t plus minus 1 means, omega t will be m pi or t will be m pi and that is omega is 2 pi divided by t and this is 2 pi divided by t multiplied by t. So that is the value. So, then t will be m multiplied by t by 2 these are the times. If we know the frequency of this omega then I can find out the time period and from that time period we can find out the time at which this goes to a maximum value. For example it happens at 0, then t by 2, then 2 of t by 2 these are the times, then 3 multiplied by t by 2, it is simply the integer multiple of t by 2 at which it will have the maximum frequency. So with this note I like to conclude my today's class because I don't have much time. So in the next class what we do is now we have the concept of propagating waves, the concept of the superposition of two waves. How these two propagating waves are interfering and forming a standing wave but all we have for a fixed frequency. Now in the next class we will try to understand if the frequency is slightly different then, in that case two waves are superimposing to each other then what will be the resultant wave. That means so far we are dealing with two waves having the same frequency, but in the next class we do that we superimpose two waves with slightly different frequency and check what happens if these two waves are superimposed, what should be the nature of the resultant wave. So, with this note I would like to conclude here. Thank you very much for your attention. See you in the next class.

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