WAVE OPTICS

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Lecture - 07: Superposition of wave (Complex method)

Hello, student to the course wave optics. Today we have lecture number seven and we're going to discuss the superposition of waves using the complex method. So let me start. So today we have lecture number seven and today we're going to discuss the superposition of two waves or many waves in general. So in last class we discussed that if we have two harmonic wave with same frequency but different amplitudes like these two waves E1, E2 then, if I add these two, this plus this, I can get a resultant wave ER, which is the linear superposition of these two waves E1 and E2. You may remember that the wave E1 was defined as a harmonic wave with the form having amplitude E10 and then cos phase alpha 1 minus the frequency term and E2 was E20 and then cos alpha 2 minus omega t, same frequency to both the case it will be omega that was the two wave we had and then we figure out what is ER. So, ER, if I add these two, then essentially we add this and then figure out what is the value of ER and we find there is a phase term and we calculate in detail in the last class. I'm not going to do all the calculations again. Here what we will do is, we'll do the same thing but using the complex notation that may be useful in a few cases. So, in complex notation, what happened? That I write this E1 or E2. In general, I write Ej with a tilde sign, That means I am using the complex notation 1. I have E0, E to the power of i, omega j minus, sorry, alpha j minus omega t, where j stands for 1 or 2, for one wave it is 1, for another wave it is 2. So that is the notation we're going to use. Now, to understand what is the superposition not only for two waves but for many waves it will make life simple actually. So if I write the resultant wave in the same form, I don't know what should be the value of the amplitude of the resultant wave but I can write an amplitude

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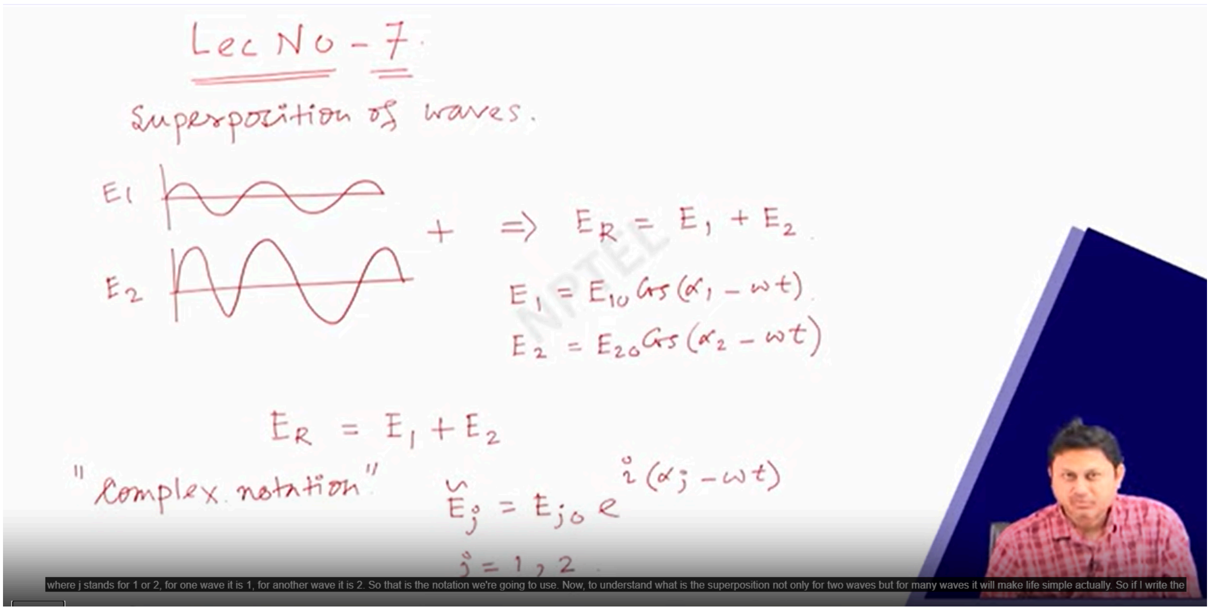

and also I can write the form of the resultant wave in the similar way that we have for E1 and

E2. So that is the form I have. Now this value ER is essentially, I should write ER tilde because I am using the complex notation. So ER tilde is essentially E1 tilde plus E2 tilde if there are two waves. But I can generalize that by adding more and more waves. So I can have E3 tilde and so on and I can add up to n number of harmonic waves with this given form where, ej tilde is Ej naught e to the power of i alpha j minus omega t. Okay, so when j is 1, 2, n, so that is the notation I'm going to use. So if I have this in the right hand side, or on the left hand side whatever the way we write ER, I can write it this way, I can use a summation sign, where j goes from 1 to n and then the waves that we have. So this is ej tilde, instead of writing plus i just use a summation sign and it should be this. Now again I can write this as the summation sign j goes to 1 to n and ej is this quantity. So I write e j 0 that is amplitude and then maybe I shouldn't put the tilde over amplitude because for the time being we consider this is a real quantity. So I can remove this for consistency. So then I have e to the power of i alpha j minus omega t, so omega t is common for all the waves. So what can we do? We can take e to the power of minus i omega t common and then we have sum over j, which is going for 1 to 10, then e j naught, e to the power of i alpha j. So, this is essentially the superposition of all the waves. Now, this quantity, already it is written that this quantity is equivalent to E naught e to the power of i minus i omega t e to the power of i alpha. So these two things are equivalent because I have already defined E naught here in the very first line in this way. So this is my equation 1 and this is my expression 2 then these two expressions if I compare then I simply have the superposition of n waves for superposition of n number of waves E naught e to the power of i alpha is equivalent to or equal to the sum over e j 0 e to the power of i alpha j, where j goes to 1 to n, that is the way these amplitudes will be related and by simple comparison I can have this. Now if someone wants to find out the value of e 0, maybe I can go to the next page. So if I want to find out what is the value of e naught, then I can do e naught square and e naught square is essentially E 0, e to the power of i alpha and then E0, e to the power of i alpha star of this quantity.

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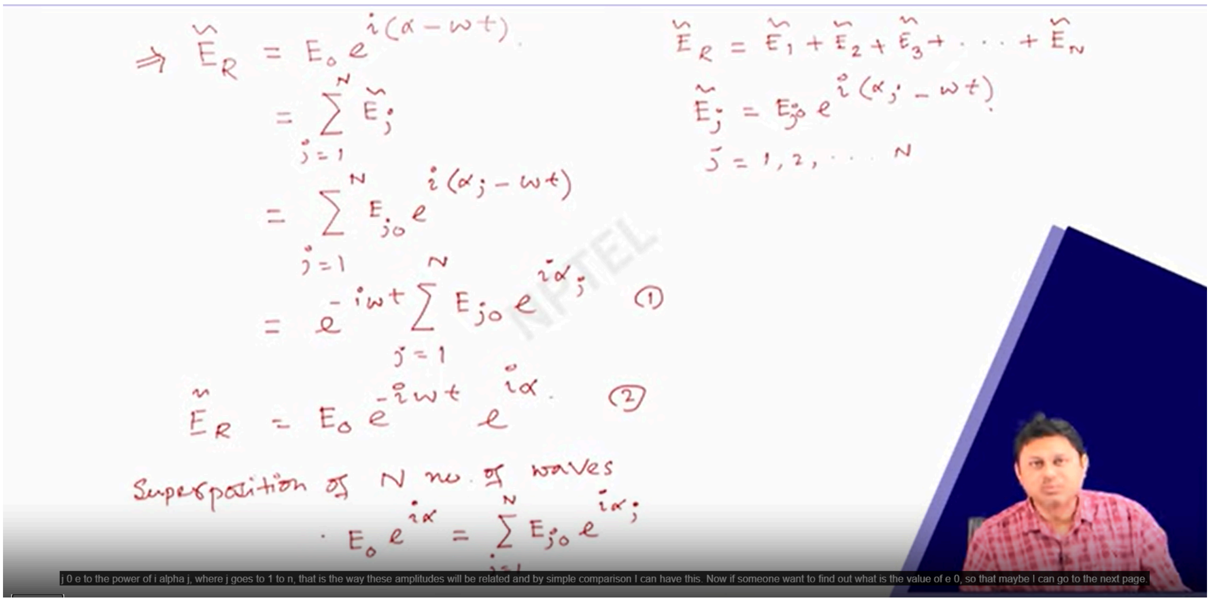

So that's in that way e to the power i alpha and e to the power i alpha these two phase terms will cancel out and essentially I'm going to get E0 square. So for two waves. Let us do it for two waves, for two waves we have n equal to 2, then, my E naught square, if I want to find out the amplitude square of the resultant wave, which is E0 square for n equal to 2 case, then we have two waves given by 10 e to the power of i alpha 1, plus E20 e to the power of i alpha 2 multiplied by e 1 0 star of this quantity, if I write e to the power of minus i alpha 1 plus e20, e to the power of minus of i alpha 2, this is the way I should write, if n equal to 2, this quantity multiplied by the star of another quantity and considering the fact that e10, e 20 or in general ej0 they are real. So from this expression it is easy to show that E0 square is equal to e 10 square plus e 20 square plus e 10, e20, e to the power of i alpha 1 minus alpha 2 plus, e to the power of minus i, alpha 1 minus alpha 2. It is a straightforward calculation. If I just multiply all these things and then collect all the terms it will come like this. It is exactly the same expression. Now we're going to get that we had in the calculation when we consider e in terms of cos function. So essentially we're going to get 10 square plus 20 square plus and if I execute this term, it will simply come out to be 2 of cos alpha 1, minus alpha 2. So I have 2 e 10 e20 cos of alpha 1, minus alpha 2. So that is the amplitude and if I make a root over basically that is the amplitude of the resultant wave and that is the exact expression that we had previously. So you may remember that if I put this as a delta then it should be e 10 square plus e 20 square plus 2 e10, e20 cos of delta and that is the interfering term we have, the value of delta determines the value of e 0 and cos delta value can varies from plus 1 to minus 1 and based on that we have the amplitude variation here but we can also understand this thing using something called, phasor addition that is the well-known stuff in complex analysis. So we're going to utilize this. So we have something called phasor addition and what we are going to do in phasor addition for two wave, I can write E0, e to the power of i alpha is equal to E10 e to the power of i alpha 1, plus e 20, e to the power of i alpha 2, that we have. Now if I add these two and make a phasor diagram it will look like this. So e 1 0 amplitude and e to the power i alpha 1, that is basically the angle. So it should be something like this. So I have the magnitude here, this line is e 1 0 and this angle whatever the angle it is making is alpha 1.

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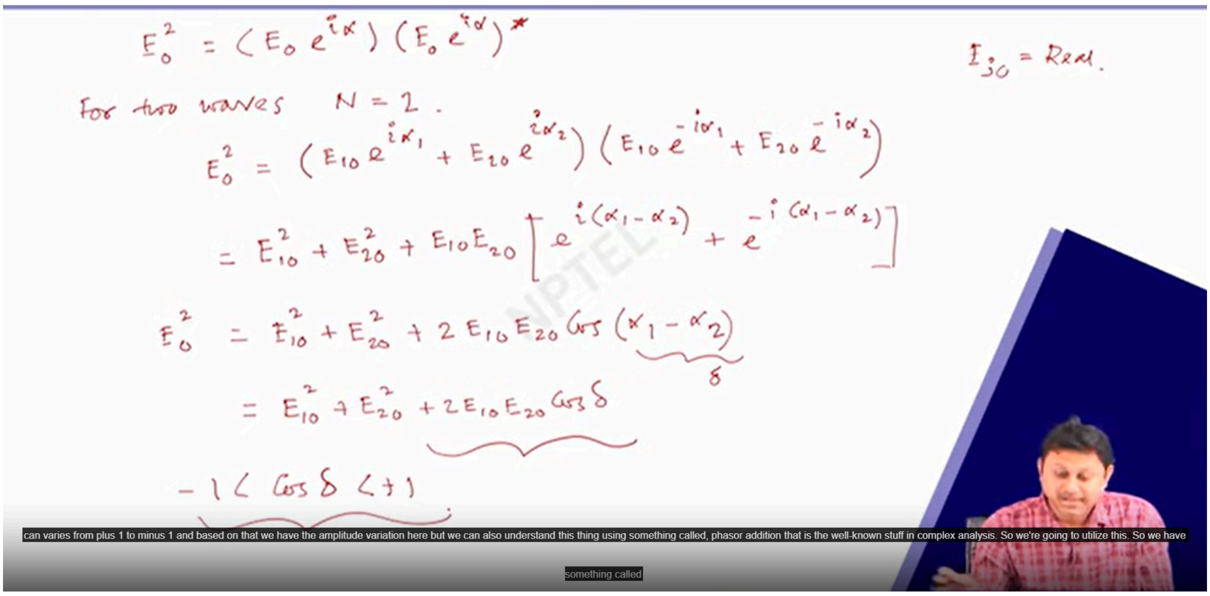

So this is my alpha 1 and this value is e 1 0 then we add the next one which is e 2 0, e to the

power of i alpha 2. So in order to add I make a horizontal line here and from the tip of these two, this line I add such that this alpha 2 this line should make an angle alpha 2 with this dotted line and I'm going to get something like this. so now my value is 20 and the angle it is making is alpha 2. Okay so alpha 1 and alpha 2 these are the angles and we get this. So when we add this we are going to get the resultant one and the resultant one if I draw maybe in a different colour I'm going to get this one, it's like some sort of vector addition we are doing here. So the green one is basically the resultant value and amplitude of this resultant value is e 0 and this angle that is making with this x axis is alpha okay and this axis normally this is a real axis and that is the imaginary axis and in real and imaginary axis we plot in this way. So the imaginary and real values are plotted accordingly and we're going to get two points and then we join these two points. Now try to understand component wise how these things are happening here. So if I quickly show them, we have one line here, another line here and this is my resultant line. So I have this structure, this angle is alpha 1, this angle is alpha 2 and this big angle is alpha. Okay, now component wise if I divide, so if I make a component here, then suppose, from here to here this component is say E 1 0 and then cos of alpha 1. In a similar way if I draw a component here, for X component of this line. So from here to here this is e to 0 cos of alpha 2. In a similar way the Y component I have from here to here and this is e 2 0 sine of alpha 2 and from here to here it is e 1 0 sine of alpha 1, well when we divide everything in component wise then in the left hand side of this part is simply, if I write E0, then cos of alpha it will simply come like E10 cos of alpha1 plus E20 cos of alpha2. In a similar way if I write E0 sin of alpha then it should be E10 sine of alpha 1, plus E20 sine of alpha 2. You can find it by making this component wise. Also, if you expand this expression, whatever the expression is given here, this one and if you expand this left and right side and if you equate the real and imaginary part you're going to get this two equation, also this angle, whatever the angle we have here, the tan value of this angle is this divided by this. So tan of alpha, this is this y divided by x and this y is summation of these things.
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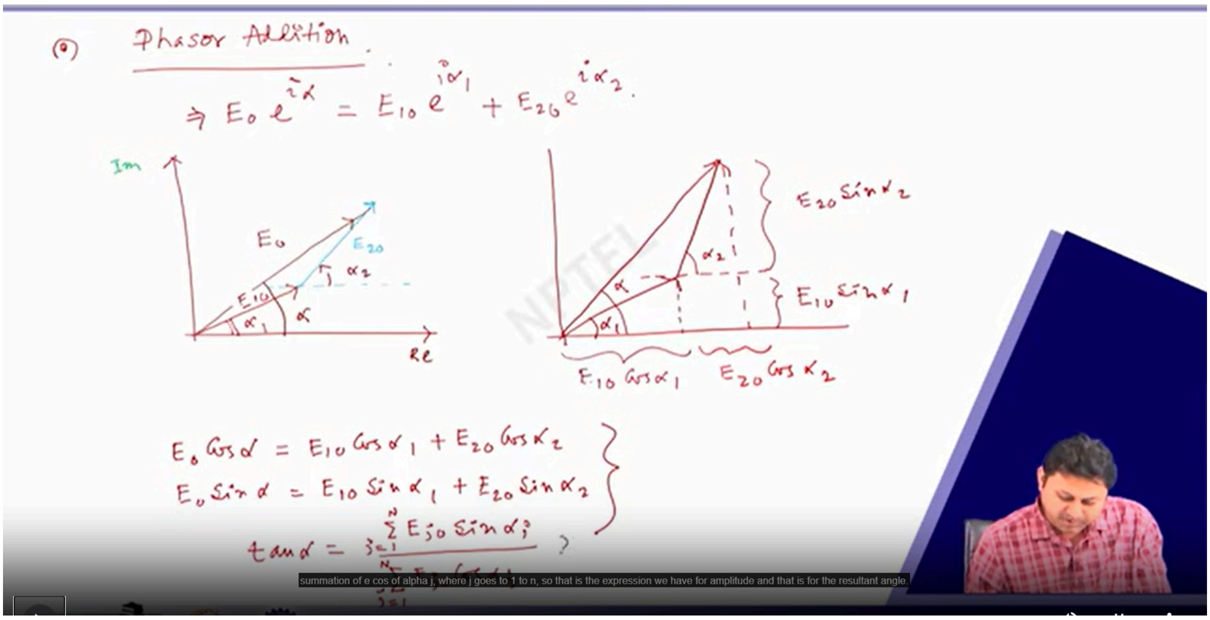

So in general, if instead of having two wave if I have multiple wave, we can have a general expression for that and that is this summation of e j 0 sine of alpha j divided by summation of e cos of alpha j, where j goes to 1 to n, so that is the expression we have for amplitude and that is for the resultant angle. If I use this phasor thing now, again resultant amplitude I can find. So from these two equation if I make a square and add both the side then I'm going to get the resultant amplitude and this resultant amplitude in general form one can write that e 0 square of that rather is equal to sum over j e j0 then sine of alpha j square of that plus e j0 cos of alpha j square of it. So that means many waves we are going to add and then I am going to get this. Now, here we should note one important fact : what is this left hand side value, right hand side value. Let us try to execute. So this E j naught sine alpha j, if I make a square of that then what value you are getting that is interesting because in that case you will want to get the value like a plus, b plus, c plus, d and the square of these things. So this is a square plus, b square plus, c square plus, d square plus, 2ab, bc, ca, cd and da. So, if I use that logic here, this formula here, it should be the addition of the square of this term j0 square and sin square alpha j, j goes to 1 to n and two of j greater than i because there will be two summation because we have the cross terms and i goes to 1,2, n but j will always be more than i, whatever the i you take and it should be Ej0, EI0, sine of alpha j, sine of alpha i. In the exactly similar way, I can have the cos term and let me write it down, that E j naught cos of alpha j and if somebody want to make square of this summation term then one can have exactly similarly the square of this, not sine, it will be cos, sorry, so it is cos alpha j and square of that plus two of this cross term. I suggest the student to please check it for two, three terms and check whatever the general expression is given is correct or not cos alpha j, so once we have this then we can write down what is my E naught square, then if i add these two, then you can see that there is a term e10 sine alpha 1 square and another term e 1 0 cos alpha square. So I can merge these two terms together and eventually we're going to get a sum over e j 0 square j equal to 1, 2, n because all sine and cos term will be added to sine square and cos square term and they will be 1. So, they should be multiplied by sine square alpha j plus, cos square alpha j will essentially give 1. So, I should not write that and the cross term that we have. In the cross term, if you look carefully, then this summation strategy will be fine, will be the same 1 to n and the j value, whatever the j value you took, that should be more than i. What is the meaning of that? Let me explain quickly if you take one, then this another term will be two. If you take i two, it should be three, if you take four, it should be five, something like that and more values, actually if n is there but the j will start more than the value of i, whatever the i you took. Now if I add these two we're going to get a term e i 0, e j 0 and cos of alpha j minus alpha i the old term that we already had in the previous calculation. So using the complex notation also I am going to get a similar kind of term where we have sum of all is if I take j in only two values, for n equal to 2 that is only two term, this general expression simply takes the form e 0 square is equal to e 1 0 square plus, e 2 0 square plus, 2 e 1 0, e 2 0 cos of alpha 2 minus, alpha 1, mind it when I take i equal to 1 and n is 2, then j has to be 2 and we will going to take in this way. So this is the term already we had previously, when the two waves are superimposed with each other. Now using the complex form what we get is that, instead of having one, two waves if I have more waves, then the superposition of these waves can be written down in a summation form. So I don't have much time today to discuss more about what is the consequence of these superposition of terms. In the next class we will start from this point assuming that the value of the e is essentially propagate all the waves having same frequency and if they have a random phase relationship

between them what should be the amplitude, if their phase relationship is not random, if their phase relation is some if they are following certain symmetry then what happened, that we are going to get the enhanced amplitude out of the superposition of all these waves. So that thing we're going to discuss in the next class which in general is called coherence, which is a very important term in wave optics. So with this note I'd like to conclude my lecture here, see you in the next class where we are going to discuss more about if two or more waves are superimposed, what should be the outcome if the nature of the source is either random or synchronized. So thank you very much for your attention. See you in the next class.
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