

**WAVE OPTICS**  
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**Lecture - 65: Analyzing Polarised Light**

Hello, student, welcome to the wave optics course. Today we have lecture number 65 and in today's lecture we are going to understand how a polarized light can be analyzed. So we have lecture number 65 today, before going to understand that how a polarized light can be analyzed I like to recap what we have done in the last class that we calculated this expression for index ellipsoid and let me write down the equation here  $1/n^2(\theta)$  was something like  $\cos^2 \theta / n_o^2 + \sin^2 \theta / n_e^2$ ,  $n_o$  stands for ordinary light and  $n_e$  stands for extraordinary light. So, if this is the axis along which the optic axis is placed then we know that for ordinary rays we have the index will be equal to all the directions; it should be simply a circle. So, in all directions the value will be  $n_o$ . On the other hand if I look for extraordinary ray the value the index value for extraordinary ray depends on the direction, for example here if I draw a small ellipse inside this circle then this small ellipse will be measure of suppose, this is the direction we are talking about  $\theta$ . So this value over this ellipse is the value of  $n(\theta)$  and at this point when  $\theta$  equal to 0, so at this point you can see  $\theta$  is equal to 0, so when  $\theta$  is equal to 0, I have refractive index  $n$  which is essentially function of  $\theta$  is equal to,  $\theta = 0$  means this portion will not be there and  $\cos \theta$ . So, it suggests that  $\cos \theta$  is 1 and  $\sin \theta$  is 0. So, I can write that  $1/n^2(\theta)$  is  $1/n_o^2$ , so simply  $n(\theta)$  is equal to  $n_o$ . Now we increase the value of the  $\theta$  and when  $\theta$  is equal to  $\pi/2$  then we have here this value, this one, then  $\cos \theta$  will be 0, but  $\sin \theta$  will be 1.

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Lec No = 65

$$\frac{1}{n_{\theta}^2} = \frac{\cos^2 \theta}{n_o^2} + \frac{\sin^2 \theta}{n_e^2}$$

When  $\theta = 0$   
 $\cos \theta = 1$   
 $\sin \theta = 0$

$$\frac{1}{n^2(\theta)} = \frac{1}{n_o^2} \rightarrow n_{\theta} = n_o$$

When  $\theta = \frac{\pi}{2}$   
 $\cos \theta = 0$   
 $\sin \theta = 1$

$$\frac{1}{n^2(\theta)} = \frac{1}{n_e^2}$$

So, in that case we can have the equation  $1/n^2(\theta)$  which is a function of  $\theta$  for that

theta pi by 2 case we can get this value is equal to 1 divided by n e square. So, this value is n e and this value is n o for this particular system you can see that let me draw here. So, in this particular system, what we have is the direction of the optic axis, x and y on the crystal. So this value is n o and this value is n e, note that here this value is n o either. So, here n o is greater than n e that is the refractive index of the ordinary ray is greater than the refractive index of the extraordinary ray. So the velocity of the ordinary ray will be less than the velocity of the extraordinary ray. So this kind of crystal is called the negative crystal. On the other hand if I have a figure like this suppose here this is the direction of optic axis and we know that along the optic axis the refractive index of the ordinary and extraordinary they are same and if I draw a figure like this, then this value is n o, this value is also n o but this value is n e. So now n e is greater than n o and velocity of extraordinary ray in that case less than velocity of ordinary ray and this kind of crystal is called the positive crystal. So sometimes this difference whatever the difference we have in the refractive index  $\Delta n$  which is this quantity is often referred as the birefringence, the value of the birefringence of the crystal. Birefringence of the crystal means that if the value of  $\Delta n$  is high we have high birefringence, if the value is low we have low birefringence it just determines the birefringence of the crystal. Well after that, that was the thing we did in the last class. After that we will understand how polarized light can be analyzed. So to understand this technique and how to analyze a polarized light we need to understand a system called a Nicol Prism. Nicol Prism is essentially an optical device designed from this is an optical device designed from calcite crystal which is used for the production as well as to analyze the plane polarized light production and analyze the plane polarized light. So let me draw how this crystal is made. So this is the structure of the crystal. Let me draw it first. Then this is the plane we are interested in. Let me set this plane. We have a plane that is of our interest. So let me name this crystal A then B, C, D, E, F, this is G, this is H and this angle is 78.5 degree 5 minute typical angle and this angle is 101 degree 55 minute something like this. So now what we do with this plane is called the principal plane.

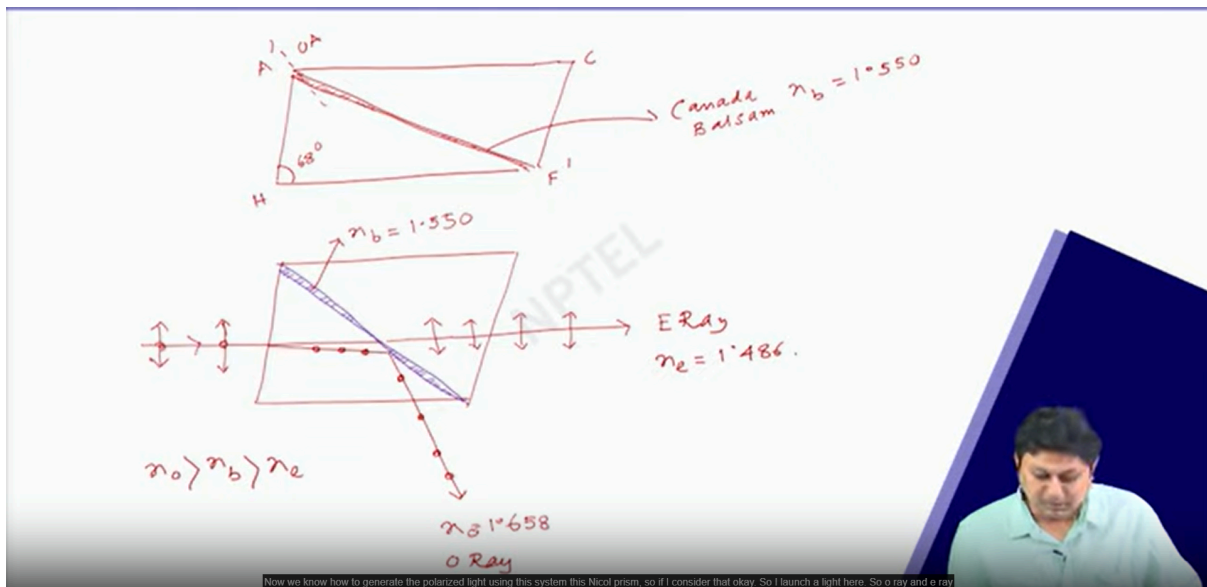
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The image contains two hand-drawn diagrams of crystal refractive index surfaces. The left diagram shows a prolate spheroid with the vertical axis labeled 'y' and the horizontal axis labeled 'x' (also labeled 'OA'). The vertical axis has a point labeled  $n_e$  and the horizontal axis has a point labeled  $n_o$ . Below it, the text reads:  $n_o > n_e$ ,  $v_o < v_e$ , and "Negative crystal". The right diagram shows a prolate spheroid with the vertical axis labeled 'y' and the horizontal axis labeled 'OA'. The vertical axis has a point labeled  $n_e$  and the horizontal axis has a point labeled  $n_o$ . Below it, the text reads:  $n_e > n_o$ ,  $v_e < v_o$ , and "Positive crystal". At the bottom center, the equation  $\Delta n = |n_e - n_o| \rightarrow$  "Birefringence" is written. A small inset video of a man is visible in the bottom right corner of the slide.

So let me draw this principal plane here. So a then, this point is c and this point is h and this

point is  $f$  and  $af$  is the point if I draw a dotted line through this, this is the optic axis and this angle is 71 degrees and this angle comes out to be 109 degrees. This is the principal plane that we are talking about. Well, after having the principal plane now what normally we do? We cut this plane in a particular angle and that is in this way. So this is the plane we are talking about and we essentially cut this plane that was dotted, one was the original one. Let me draw it properly. So we cut this thing in such a way that this angle previously was 109 degree, after cutting this angle becomes 112 degree, this angle previously was 71 degree. Now, after cutting this angle it has now become 68 degrees. So, we cut the principal plane with these angles and then we use this particular section as a generation of polarized light. So, we are going to show it here. So what we do here, we cut this plane along the optic axis, this principal plane. So we have two sections here. One is this and another section is this cut from here actually and this is my new  $a$  prime because this is  $C$  usual, this is my  $F$  prime and this is  $H$ . So if I go back what is prime, let me draw it because this was originally  $H$ , this was  $A$ , this was  $C$  and this was  $F$ , after cutting that this become  $F$  prime and this become a prime. So these two new points emerge after cutting this thing this prism. So now this is the structure we have and what we do that this region we glue with a specific glue there is a reason behind that and that glue we call this glue we call Canada Balsam for which the refractive index is 1.550, well this is 68 degrees and along this direction we have an optic axis. So I have the ordinary and extraordinary ray and the refractive index of the ordinary and extraordinary ray should be different, so this is the structure. So now what happens if I launch a light here? Let us see a unpolarized light I launch and when we launch unpolarized light here both the component will be there perpendicular to the optic axis and parallel to the optic axis, so one ray which is perpendicular to the optic axis which is the ordinary ray will get reflected from this interface and the reason is the refractive index of the ordinary ray is around 1.658 and now it is launched from a high refractive index and the refractive index of this quantity which is the glue  $n_B$  Canada Balsam is 1.550.

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So there will be a total internal reflection from this point for ordinary rays. However for extraordinary ray it will simply pass through because the refractive index of  $e$  ray, this is  $e$  ray

is around 1.486 here, so for this system we can have  $n_o$  greater than  $n_b$  greater than  $n_e$ . So, I can separate out the ordinary ray and extraordinary ray with this structure because the total internal reflection will be experienced by ordinary ray only. But the extraordinary ray will simply pass through because this refractive index is less than. There will be refraction obviously but it will pass through. So whatever the ray I get here on this side is O ray which is polarized along a particular direction and here E ray which is also polarized with a particular direction. Note that the angle of the incidence of the balsam layer must be greater than the critical angle for which this total internal reflection can take place and accordingly we need to launch the light. Normally it is around less than 14 degrees so that one can have the condition for this total internal reflection at this particular point. Now we know how to generate polarized light using this system, this Nicol prism, so if I consider that okay. So I launch a light here. So o ray and e ray will generate here in this way and here we get this light and here we get this one. Now, if I place a prism, Nicol prism again, which behaves like a polarizer, right. So now I place a Nicol prism again and what happened the light will pass through without any issue this light will pass through. So I can get light here however for the same system unpolarized light launched this light will go this direction, extraordinary light will pass through and now we place the Nicol prism but we rotate it such that looks like this make a 90 degree rotation in fact, so this is a rotated Nicol prism. So what happened? For this case this light will not pass through because it will now behave as ordinary light for this. So no light, with the simple arrangements again this is a polarizer which polarize the light but Nicol prisms behave here like an analyzer which analyzes the light. In this case we can have light but if I rotate these things I don't have any light. Well now the production of different light which we discussed earlier. Let me quickly discuss this as well as the production of how to produce circularly polarized light. So how we produce circularly polarized light exploiting this matrix tool. So it is a superposition so we can write down the circularly polarized light. The Jones matrix for this we know we did several times is circularly polarized light. Now this is a left circularly polarized light, I can write this left left circularly polarized light as a combination of linearly polarized light in this way, so this is a linearly polarized light with x vibration plus if I introduce a phase here which we can do by introducing half a plate then I can have this this is one and this is a lip this is this is again a linearly polarized light but y vibration and by combining these two one can generate the linearly circularly polarized light, this left circularly polarized light. In a similar way the right circularly polarized light I can generate as a combination of two lights in this way here my phase will be negative  $\pi$  by 2. And again, this is x vibration, this is y vibration. And the phase difference here between the two components is minus  $\pi$  by 2. So in this way, we can produce left circularly and right circularly polarized light using the linearly polarized light and also using a structural light Nicol prism. We can analyze the light and this is the way we can generate and analyze the polarized light. So today I don't have much time to discuss more about how to analyze this polarized light. In the next class I'm going to discuss how with different combinations of the light one can generate the polarized light. And also how we can analyze other systems. How we can put the phase difference. I just show how we put the phase difference. How one can put a systematic phase difference by using a system called Babinet's compensator. So that we are going to discuss. With that note I would like to conclude here. Thank you very much for your attention and see you in the next class.

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⊙ Production of circularly polarized light.  
(CPL)

$$\frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} \equiv \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 0 \end{pmatrix} + \frac{1}{\sqrt{2}} e^{i\pi/2} \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

LCP                      LPL                      LPL  
(x-vib)                      (y-vib)

$$\frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ -i \end{pmatrix} \equiv \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 0 \end{pmatrix} + \frac{1}{\sqrt{2}} e^{-i\pi/2} \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

