WAVE OPTICS
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Lecture - 58: Jones Matrix for polarization (Cont.)

Hello student, welcome to the wave optics course. So, today we will discuss the Jones matrix. We have lecture number 58 today and in this lecture we are going to continue the formation of the Jones matrix for different systems. So, we have lecture number 58 today. So we were discussing the Jones matrix for the quarter wave plate. Now we are going to discuss this for a half wave plate. And in the half wave plate we know that the relative phase difference delta phi will be pi. So, this is phi x minus phi y with a mod sign that is equal to delta phi and it is pi. So, the relative phase difference between x and y components will be phi. The Jones matrix formalism for this case again very straight forward if let me first write down it should be e to the power of minus i pi by 2 0 and 0 e to the power of i pi by 2, when the slow axis is horizontal, sorry slow axis is vertical. So we can get as e to the power of minus i pi by 2 then it should be 1-0-0 minus 1 or it is equivalent to 1- 0-0 minus 1 because e to the power of i pi by 2 minus i pi by 2 will give that. So we give a minus 1 sign. So here will be a minus 1 and then m is essentially minus 1-0-0-1 and that is for the half wave plate. When we have a half wave plate and the Jones matrix is representing that half wave plate, not only is the slow axis vertical in the direction. In a similar way, if the slow axis is horizontal then this M matrix will be e to the power of I pi by 2 then 0 and then we have, so, let me go back and check once again. So here we have e to the power i pi by 2 0, I take e to the power i pi by 2 common then this quantity will be e to the power of pi and cos pi is 1 and this is 1 and this quantity e to the power i pi by 2 is cos pi by 2 minus i sin pi by 2, so, it should be minus i.
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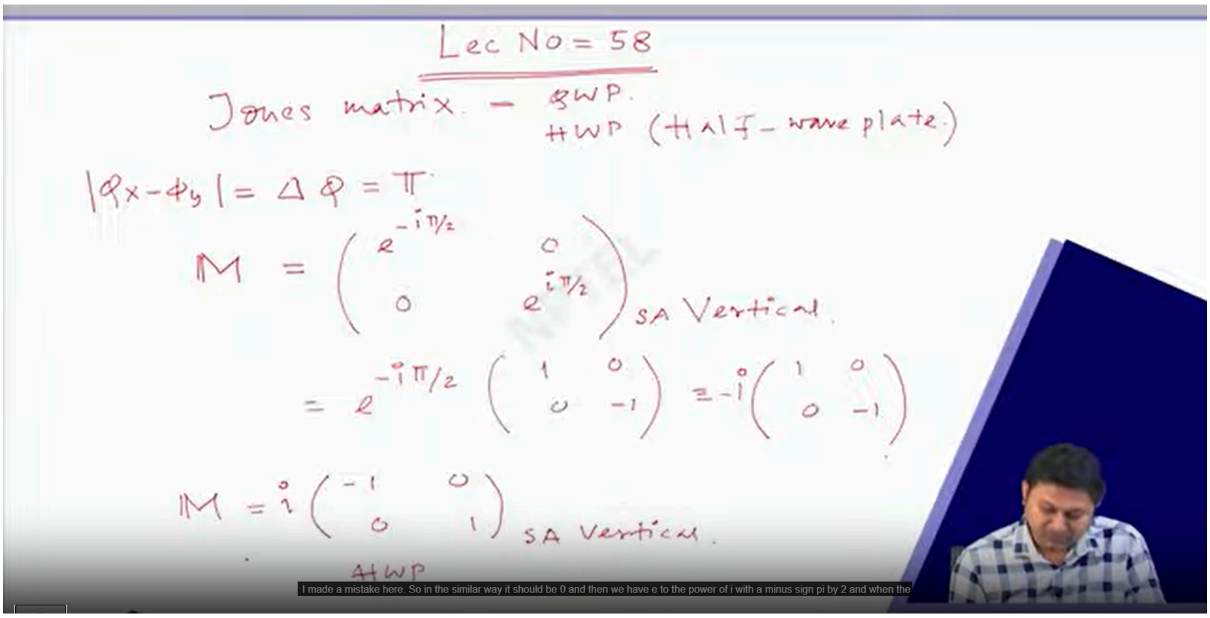

So here we should have a minus i present here not 1 and here we should have a i sitting here representing that quantity. I made a mistake here. So in the similar way it should be 0 and then we have e to the power of i with a minus sign pi by 2 and when the the slow axis is horizontal in that case so if I take e to the power of i pi by 2 common then we have 1-0-0 and then this quantity will be e to the power of minus i pi, so, e to the power of i it should be 1-0-0 and that quantity e to the power of i pi is cos pi minus, i sin pi. So it should be minus 1 and this quantity is e to the power i pi by 2 is simply i because e to the power of i theta is cos theta plus i sine theta. Let me write it clearly b to the power of i pi by 2 is cos pi by 2 plus i sine pi by 2 which is cos pi by 2 is 0, sin pi by 2 is 1. So, it should be i. That I am going to put here and I will get 1-0-0- minus 1. Previously, if I go and check, this will be i actually. It will be minus 1-0-0-1, with I multiplier and here it is this and this is when the slow axis is horizontal we will come back to this point. What is the meaning of slow axis horizontal etcetera but let us first formulate this and then now let us examine that what happened when a linearly polarized light with electric field vibrating at 45 degree angle with the horizontal axis is incident on a quarter wave plate with a slow axis horizontal that is the case. So, let us quickly write what the condition is. So, here we have electric field E and what is vibrating along, vibrating at 45 degree with the x-axis or horizontal x-axis and slow axis is also along x-axis or horizontal axis that is the condition along. So here m is e to the power i pi by 4, 1-0-0-minus i that we calculated for the quarter wave plate. So this is for the quarter wave plate. So this is passing through a quarter wave plate that I need to mention. So the quarter wave plate and E are passing through the quarter wave plate. So pictorially if I do then this is a quarter wave plate and this is the slow axis and this is the x axis and y axis and the electric field that is passing through this system is making an angle. So this is the electric field actually, which is making an angle 45 degrees. So this is the electric field that is launched, that is passing through this system. So m we know for what is my e, then before passing through this e will be 1 by root over of 2, according to the Jones matrix this is a linearly polarized light. So, it should be 1 by root to 1-1, 1 by root is for normalization. Now, what do we have here?

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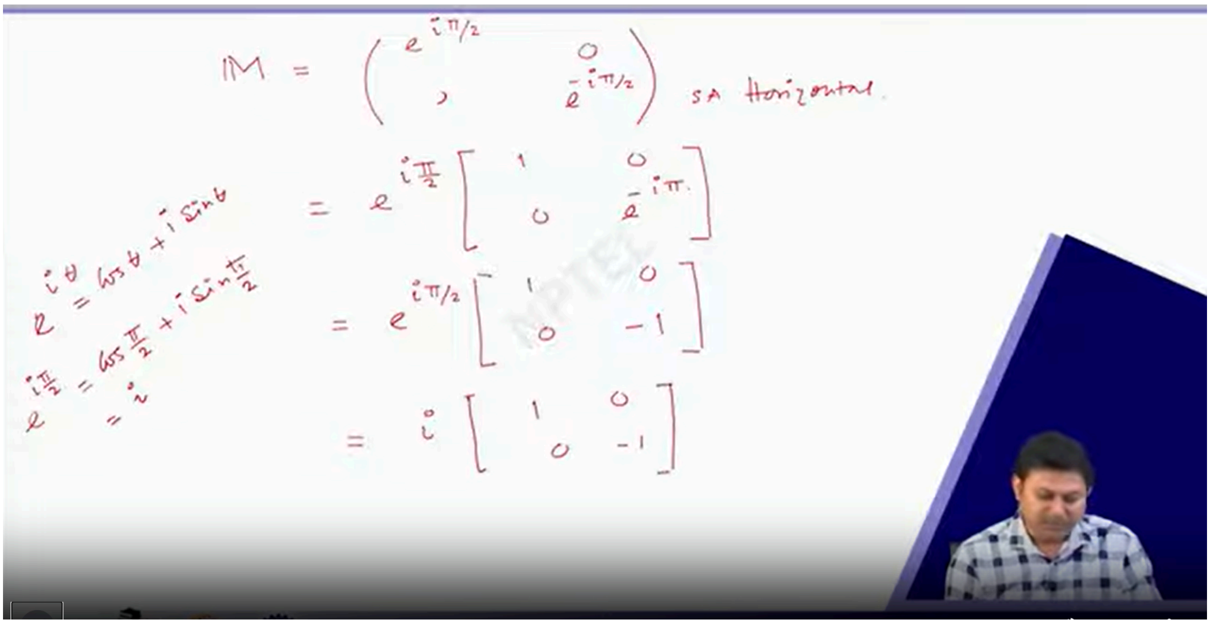

So, this is my E, this is my M and I find what should be my E prime here at the output. So e prime will simply m operate over e. So we have e to the power of i pi by 4, 1-0-0 minus i e vector 1 by root 2, 1-1. Let's see what we get here. So here we get e to the power of i pi by 4. Let us put it multiplied by 1 by root 2 that will be there and then this into this, this into this, so this component will be 1, then this into this, this will be minus i. So if I just ignore this only, this quantity suggests that this is a right circularly polarized light. So, the output we get, we will get a polarized light definitely and this is right circularly. So, we will get a polarized light like this and if I see from that direction, it should be a right circularly polarized light like this. So that is a beauty, here I launch a linearly polarized light and I allow the system to propagate through a quarter wave plate. What is a quarter wave plate, how does it build? We will want to discuss in detail in the later part of this course. Maybe in a couple of classes we're going to discuss this but the important thing is we can manage to get a circularly polarized light here precisely the right circularly polarized light from a linearly polarized light. So, from linear polarized light, I allowed it to pass through the quarter wave plate and by exploiting the Jones matrix technique, we get it as a right circularly polarized light. Now, if I change, it is easy to show that I am not going to do that, but I ask the student to do it. Everything is the same. What happens if I just change the slow axis orientation? Now instead of horizontal if the slow axis is vertical for the same quarter wave plate which produces pi by two phase difference on two components then what happened for the same incidence? And you will see that instead of getting a right circularly polarized light we are going to get a left circularly polarized light. This is straightforward using the same technique, I suggest you do and check by yourself that really it is happening or not. In the same way, you can also do what happens when a half wave plate is there. So, suppose the same problem, but now I place a half wave plate. So, what is the difference between a quarter wave plate and half wave plate? Half wave plate is a system where it accumulates a phase difference of pi. So this is the system and now I say this is a halfway plate. And I also define what is the direction of the slow axis. Based on that we need to write the matrix. So maybe the slow axis is in the vertical direction. So this is the direction of the slow axis.

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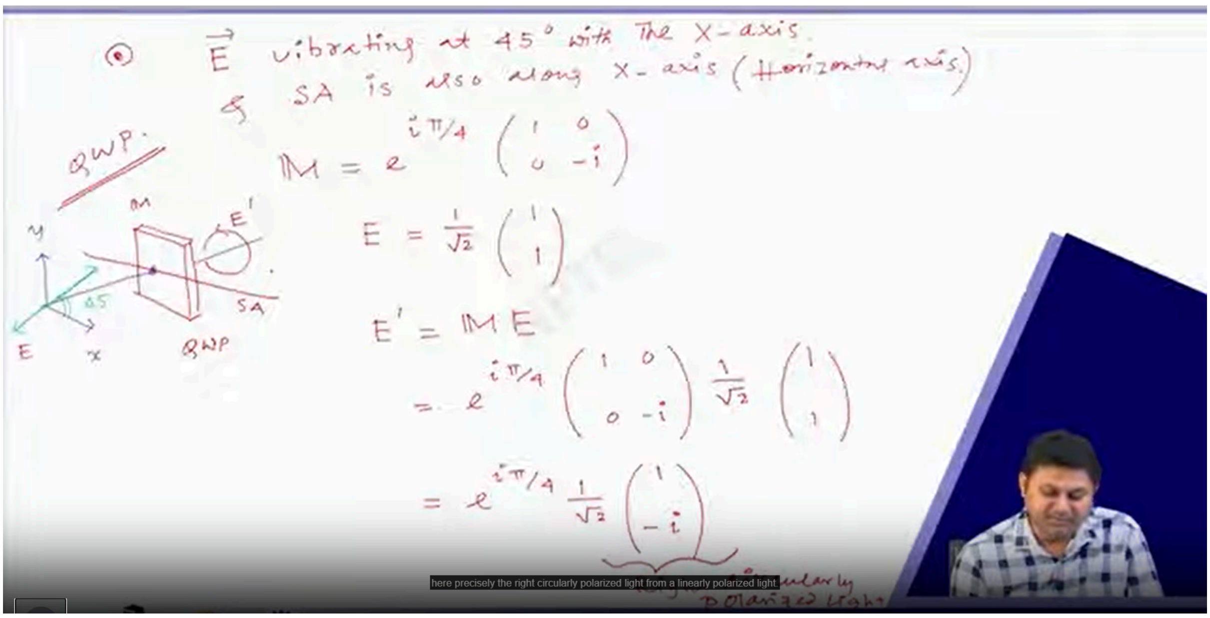

Now the m for this case will be simply e to the power of minus i pi by 2, 1-0-0 minus 1 when SA is vertical and it is operating over this e, which is a plane polarized light that is having an angle 45 degree with the horizontal axis. So, m into e, so e prime that will be m operate over e. So, that means e to the power of i pi by 2. So, this and we have 1 minus 1. So, this 1 minus 1 suggests that there will be a rotation. So, whatever the plane. So, this is a plane polarized right by the way. This is a linearly polarized light but its orientation is going to change, its orientation will be 90 degree, it will be a pi by 2 rotation. So what half wave plate will do? It will change. So, suppose let me draw it. Then how does it happen? So the electric field was like this 45 degree making and 45 degree angle with this one and when it goes outside so there will be a change in the orientation and it will move this direction making an angle 45 degree here. So it will just rotate 90 degrees by the placement of this halfway plate. Now in the similar way, we can draw, we can write down a general thing that, if incident light is passed through a series of optical elements, these are called optical elements, this half wave plate, quarter wave plate, they are essentially the optical element represented by M1, M2, etc. So, the final light, so, let me write it clearly, this is the way it should work that if the incident light is passed through series of optical elements represented by represented by the Jones matrix M1, M2, M3 like Mn, then the state of the polarization of the emergent light, so the light that is coming out through the system after passing through n such optical elements then it is e prime will be mn minus 1, m 3, m 2, m 1, E that is in general. So every time it passes through this system we need to multiply this matrix and then it should act over e and as a final result we are going to get the state of the output light with this formula. Well that is fine. Now what do we like to do? We would like to find out a few more such optical systems and one important system is the rotator. So what do rotators do? It will simply rotate the light to a certain angle. So this optical element, if I draw schematically that this is my optical element or system and what it does is rotate the light to a certain angle. So this is the z direction along which light is propagating and I have a polarized light. Suppose vibrating in this what happened that we need passing through this system these things will go to rotate to a certain angle it will rotate like this and suppose it rotates an angle beta here, so these things is called the rotator, that is going to rotate.

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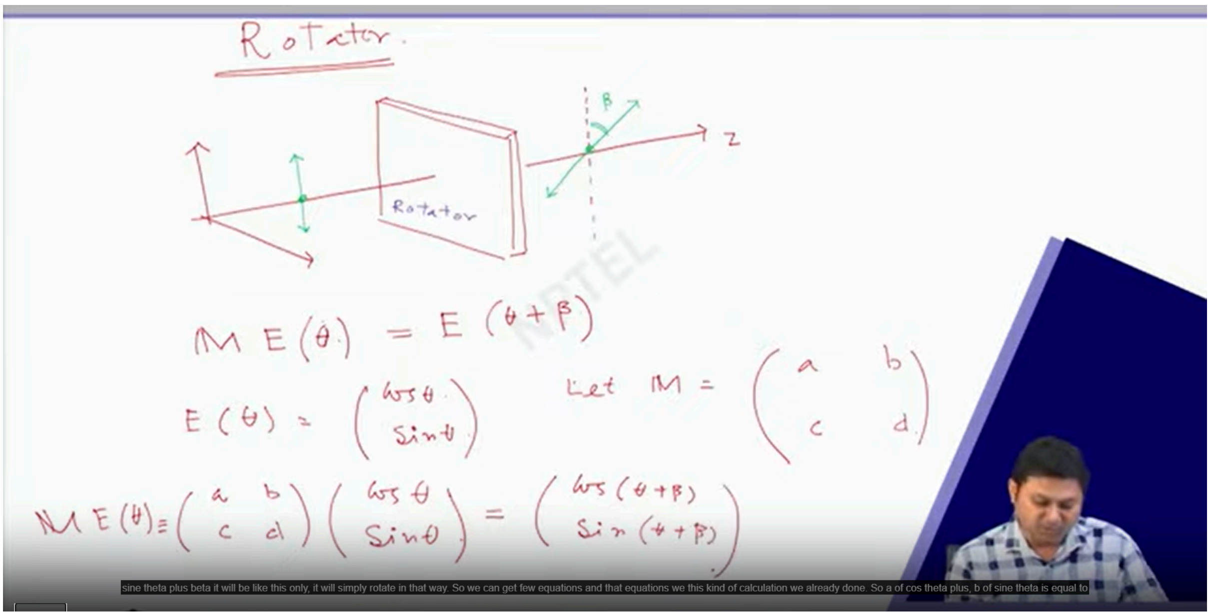

The polarized light if that is the case then what happens is that in general whatever the M we have it will operate over the electric field in this case it is horizontal but if it is having an angle theta then outcome will be E theta plus this angle because it is now rotated to this angle beta. So, with this formalism we can find out what should be the form of the matrix because in general it is nothing but cos theta and sin theta. These are the two components having angle theta and on top of that there is a beta rotation. Let M is equal to A, B, C, D and M will go to operate over E. So, A, B, C, D multiplied by cos theta sine theta. What is this quantity? M is operating over e theta that is equivalent to this one. So here what we get? We get simply cos theta plus beta and sine theta plus beta it will be like this only, it will simply rotate in that way. So we can get a few equations and that equations are the kind of calculation we already did. So a of cos theta plus, b of sine theta is equal to cos of theta plus beta and C of cos theta plus D of sine theta is equal to sine of theta plus beta. Now cos theta plus beta is cos a cos b minus sine a, just putting the formula and sine is sine a cos b plus, cos A sin B that we have. So, from writing these two side by side one can simply get A is equivalent to cos of beta, B is equivalent to minus of sine of beta, C is equivalent to sine of beta and D is equivalent to cos of beta. So, the M matrix for rotator simply comes up to be cos of beta, minus of sine beta, cos of beta, so, that is the value of the rotator. That is the value of the M through which one can get this rotation. Now, today I finalize this Jones matrix. So, I can make a summary of what we have done. So, the summary of Jones matrix, so for linear polarization, for linear polarization means for linear polarizer rather, this lp is linear polarizer. So, M was defined as 1-0-0-0 when T A, that is transmission axis or pass axis, was horizontal. M in the similar way was 0-0-0-1 when TA was vertical, when TA is having 45 degree angle with horizontal axis or x axis then m was that quantity. Next we discussed the phase rotator or right phase retarder, rotator we did later. In phase retarder the general M is represented by this. Now when a quarter wave plate with SA vertical, then M was defined as this. When a quarter wave plate is horizontal then M was defined like this, for a half wave plate with a slow axis vertical M matrix was given like this half wave plate slow axis horizontal M matrix was this one and that was the some and then rotator was there.

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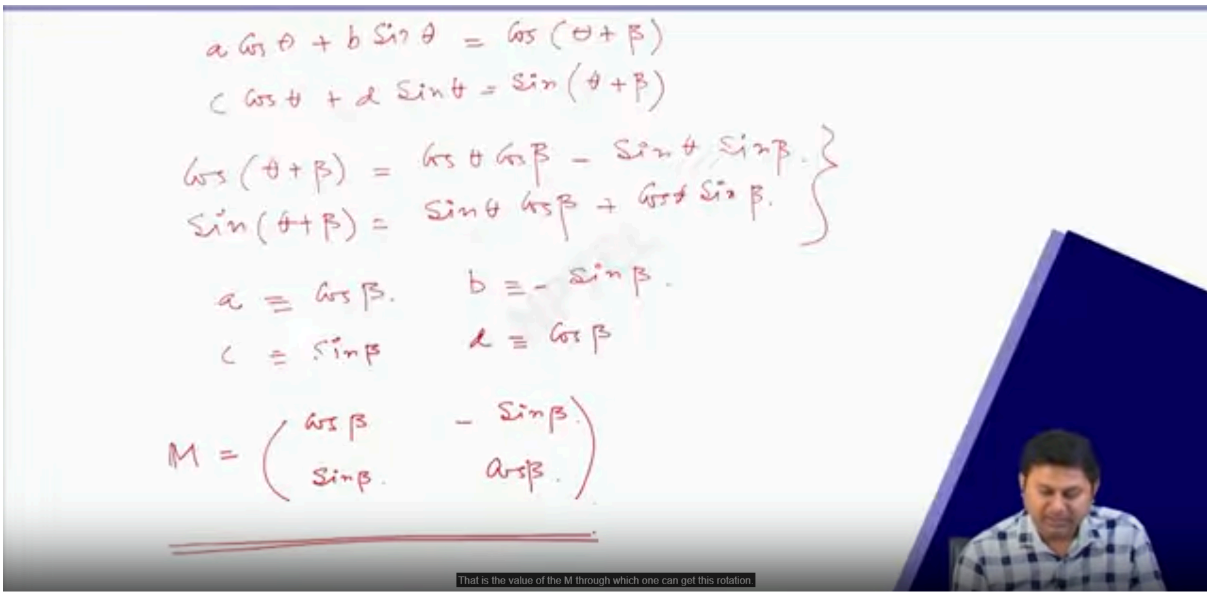

So in the third case rotator where theta is rotated to theta plus beta, so m was defined by a matrix cos beta minus sine beta, sine beta cos beta is like a rotation matrix. So with this note I would like to conclude. So I just write down in the summary to all the matrix that we develop by jones matrix and these are the optical elements and in first case the optical element is simply linear polarizer, the next case the optical elements are phase retarder, where the phase is shifting for two different components because of the orientation of the slow axis and finally also there is some sort of rotation. So it is not changing the polarization but if it is a linearly polarized light rotates the linearly polarized light to its rotation there is a rotation of this vibrational angle, these three we demonstrate. So with that note I would like to conclude here. So in the next class we will discuss a few aspects of the Jones matrix. We list down what happened for linearly polarized light, circularly polarized and make a list of that. So that it will be useful for you and then we try to find out how polarization can be possible and what is the way to polarize the light and that we are going to discuss in detail. So thank you very much for your attention. See you in the next class.

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