WAVE OPTICS

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Lecture - 57: Jones Matrix for polarization (Cont.)

Hello, student, welcome to the wave optics course. Today we have lecture number 57 and today we will discuss the Jones matrix that we have already started in our earlier classes. So lecture number 57 today and as a continuation of the previous topic that Jones matrix. We will start from that problem quickly. We look that if we have a system like this a polarizer having this pass axis such that it is first. Let me draw the axis and the pass axis by making an angle say theta with x axis, this is x-axis, this is y-axis then this particular system can be represented by the Jones matrix as this. So the Jones matrix for this system is M equal to cos squared theta in general sine theta cos theta then sine theta cos theta and sine squared theta. So that means if I launch an unpolarized light when it passes through this thing then this operator will be operated with this initial Jones matrix and we're going to get a final result that we derived in the last class. So we are going to expand this idea more to different kinds of systems. So the next system that we are going to discuss is the phase retarder. So we will calculate the Jones matrix for a phase retarder. What is the meaning of phase retarder we are going to discuss. So, the phase retarder does not what it does not remove either the orthogonal components of the vibration but rather introduces a phase difference between them. So what it does is it will introduce a supposed one component that has a phase e to the power of I Phi X, this is the X components having a phase like this and Y component having a phase like this. So, this phase retarder is going to add some phase like e to the power i whatever the original phase was there and add something like this and in this case it will do like this.

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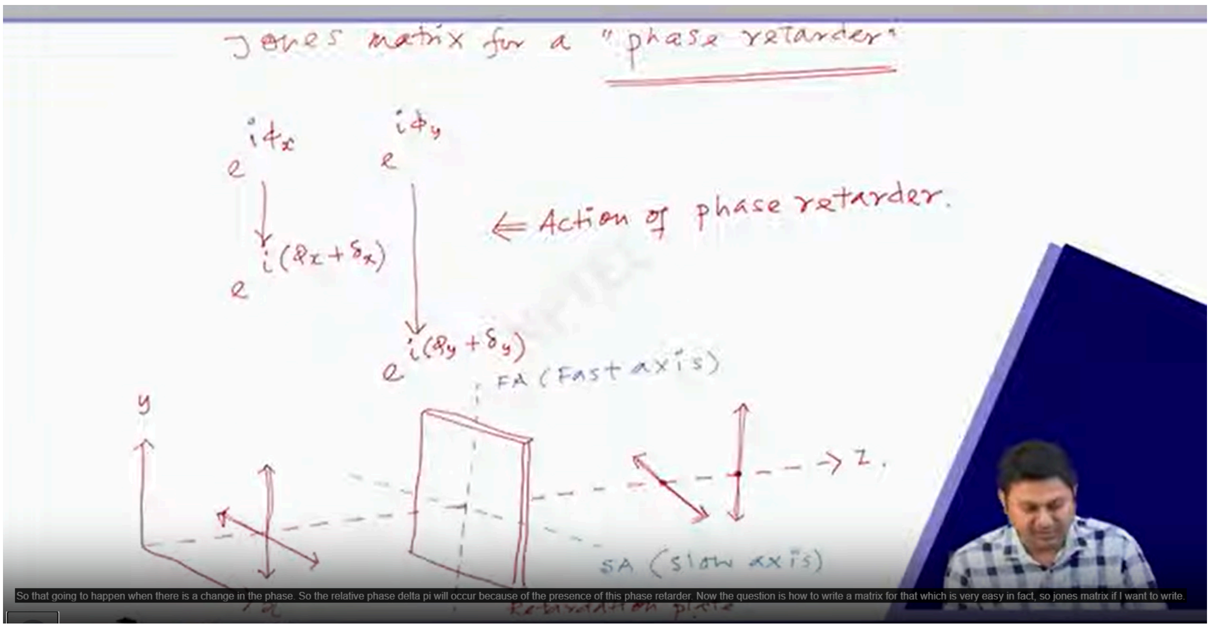

So, this delta x, delta y that is the additional phase that will be added actually by this action of phase retarder. So, this is the action of phase retarder. So, it adds phase to the component. So, let us see how I mean pictorially. So, suppose we have this is my x axis, this is my y axis and here I place the phase retarder. This is the phase retarder. Let me place it first. This is my z axis and it has two different axis and that is interesting this we call first axis FA that is when the light is perpendicular to this axis is going through the system, it will travel faster and another axis is there the horizontal one and this axis we call slow axis as the name suggest when the light component will pass through this axis parallel to this axis then there will be this component will travel in a slower manner I mean the velocity will be less compared to the first axis. So obviously there will be a phase lag between these two and exactly this phase lag will be incorporated by the system as mentioned as phase retarder. So if I now draw the light having x and y components. So after passing through this system the horizontal component which is proportional to the first axis will travel faster compared to the vertical axis which is proportional to which is parallel to the first axis will travel faster compared to the horizontal axis. So there will be a phase lag because these two are traveling at different speeds. These things are retardation plates. This is the retardation plate. Here these two orthogonal vibrations travel with different speed resulting in a phase difference as I mentioned and I just demonstrate this pictorially that how it should happen. So that is going to happen when there is a change in the phase. So the relative phase delta pi will occur because of the presence of this phase retarder. Now the question is how to write a matrix for that which is very easy in fact, so jones matrix if I want to write. So, for a phase retarder as I mentioned that one component e 0 x that is the x component suppose amplitude and it is having a phase e to the power i phi x. So, that component will go to this value e 0 x e to the power of i phi x plus delta x this additional phase will be there and another component e 0 y e to the power of i phi y it will also be replaced by this. So delta x and delta y represent the advance in phase that is created due to this.

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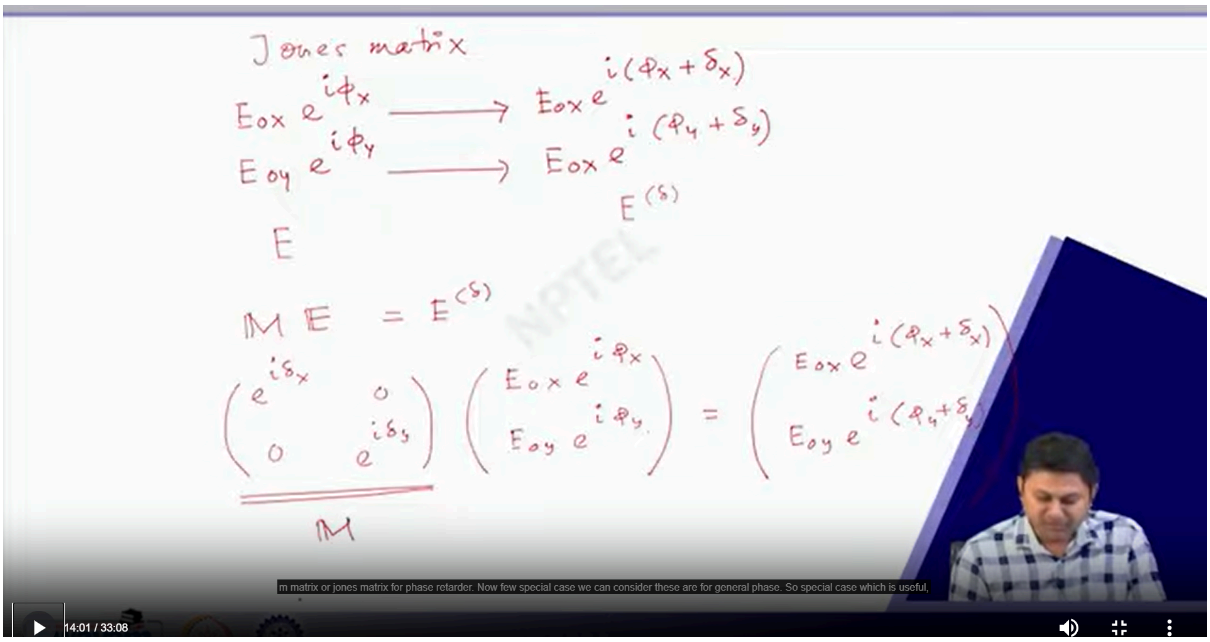

And now delta x, delta y can be negative quantities also. In one case it is advanced, in another case it is delayed. So, in general I write plus, but there is if I consider that in one case it is advanced and in one case it is delayed. So, this sign will be minus also depending on the sign of delta. So, how to get this is very straightforward. So, these are the two components. So, I can write this matrix as say E matrix and then this is the e 1 matrix or I write the e delta because the delta phase is there. So the equation is m it will operate over e and it will produce e delta. So, if I write this thing, it will be very easy to show what is the value of the m. Let me first write about it. It will be i phi x, phi y that is equal to this quantity which is e delta which is e 0 x e to the power of i phi x plus, delta x and e 0 y e to the power of i phi y plus delta y with bracket close. So, that means this quantity m can be intuitively written as i delta x 0, 0 e to the power of i delta y. So, that is my m matrix or jones matrix for phase retarder. Now few special cases we can consider are for the general phase. So a special case which is useful, a special case is something where the phase retarder is called the quarter wave plate. We will discuss this quarter wave plate in detail in the later classes. So, for the quarter wave plate, what happened was that this delta x minus delta y, which is the relative phase difference between these two components, became pi by 2. Why the name is quarter wave plate? Because if there is a path difference of lambda, this is the path difference will get a phase difference of 2 pi, if the path difference is lambda by 2 then the phase difference will be pi and if the path difference is lambda by 4 the phase difference will be pi by 2. So since the path difference is lambda by 2. So that means the system will introduce a path difference of lambda by 2, which is a quarter of lambda. Then we are going to get a phase difference of pi by 2. That is why it is called the quarter wave plate. In the same way, the half wave plate is the plate where we have the phase difference of pi. And this is the way the names are there. Now, these delta x minus delta y equal to pi can have different combinations and one can achieve these things in a different way. So, for example, if I write simply delta x minus delta y it is pi by 2. This is one possibility minus delta x plus delta y that can also be pi by 2 other possibilities. So, in the first case, for case 1 say, so delta x can be pi by 4 and delta y can be minus pi by 4. And if I do this, I am going to get 1 in that case, it will give the equation 1. Also, another combination is delta x is simply pi by 2 and delta y is 0.

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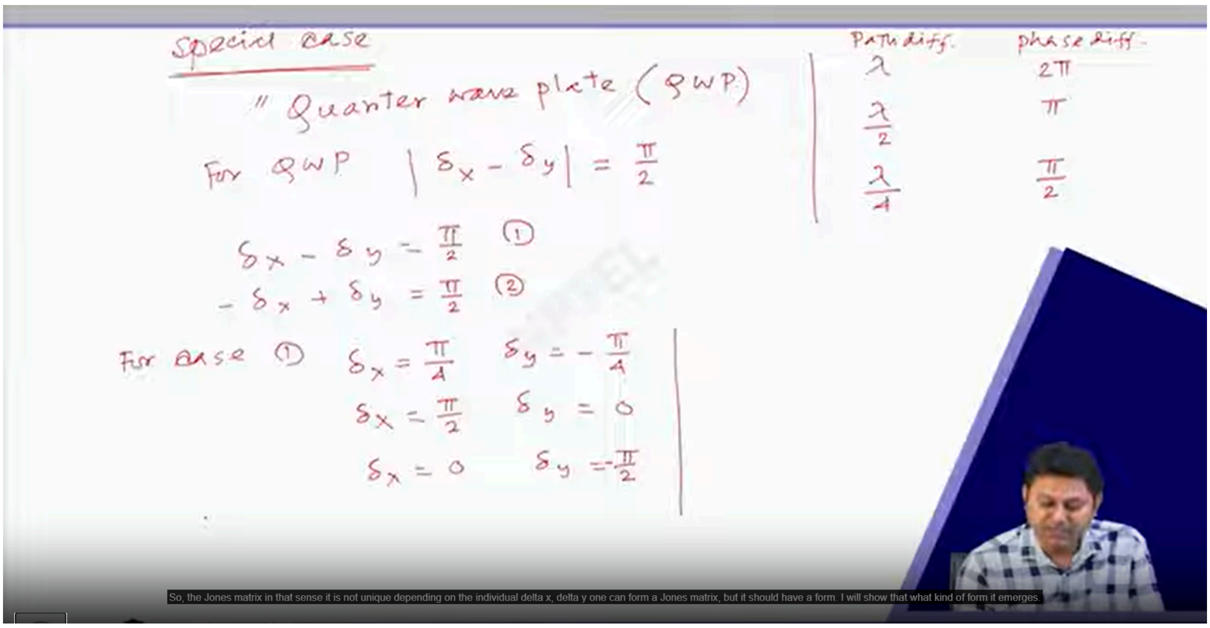

Also, delta x is 0 and delta y is pi by 2 minus pi by 2 rather that should be another

combination. So, there will be many many combinations that one can think of if I write this. So, the Jones matrix in that sense is not unique depending on the individual delta x, delta y one can form a Jones matrix, but it should have a form. I will show what kind of form it emerges from. So let us consider a specific case because if you remember we mentioned the fast axis and slow axis. So I am doing it once again. So suppose the slow axis SA I place this as a horizontal axis. This is equal to the horizontal axis. So let me draw it. This is my plate and let me draw the x and y axis. T So this is basically along the x axis we have the slow axis. So obviously along y axis we have the first axis and then I launch a light having vertical and horizontal components like this and it is passing through the system since the first axis is here. So this component will travel faster compared to this component. So, this is x, this is y, this is along z direction and this is the retardation plate. So, if I want to find out how it happens, in this picture, let me draw it that this is one component and this is another component that was traveling together before it encounters the phase retarder. So they are in phase. Actually these two waves were in phase x and y. So here my ex is e x naught sine of kz minus omega t and ey is e y0 sine of kz minus or negative that is the way we define. Now after propagating through this phase retarder what happens that there will be a phase lag and if I draw it, so, we are drawing this way. So, x will travel. So, the x component, the y component will travel faster. So, the y component may be like this but the x component will have a phase lag and it will be like this sorry it will be a mistake it will be like this. So, there will be a phase lag between this x and this y component. So, when we have a maxima here, this wave has a minima. When this wave has a maxima, this wave has a minima. When this is a minima then we have a maxima here. When this is a minima, we have a maxima here. So it will follow like that and this is my z. So here what happened e y will be e y naught then y naught sine of kz minus omega t but e X will be now e X 0 there will be a phase lag. So it will be sine of k z minus omega t plus pi by 2. A phase lag will be there which we can represent in this way. So here delta x is pi by 2 and delta y will be 0 this combination is there.
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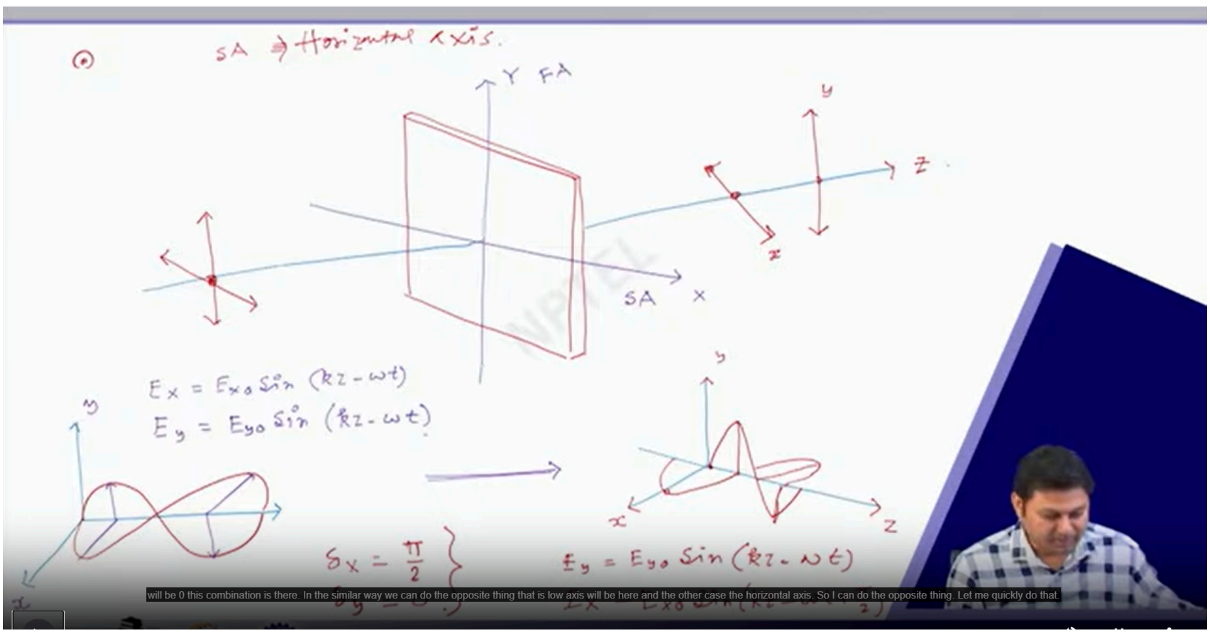

In the similar way we can do the opposite thing: the low axis will be here and the other case the horizontal axis. So I can do the opposite thing. Let me quickly do that. Now I can have these two axes. This is my first axis now and this is my slow axis. And in that case, what is the outcome? The outcome will be ex, not going to change, it will be ex0, then sin of k z minus omega t and E y is sin of this is the way one can represent. Now delta x here is 0 but delta y is pi by 2 when SA is vertical. So, also we can have delta x is equal to pi by 2 and delta y is equal to minus of pi by 4. So, that combination is also there, but before going to that combination let us now form the you know these matrix Jones matrix. So, for the slow axis in the vertical case for the quarter wave plate where the phase difference is pi by 2. So if I write delta x is minus of pi by 4 and delta y is pi by 4 plus pi by 4 then the M matrix one can form in this way which is e to the power of i delta x 0 then 0 e to the power of i delta y. So, this will be equivalent to the one combination I am just showing this will be e to the power of minus i pi by 4 0 0 e to the power of i pi by 4 and that essentially gives e to the power of minus i pi by 4. If I take this common, then things will be easy. It will be 1, 0, 0, i. So this condition is for when the slow axis is vertical. So this is when SA is vertical. I just wrote this symbol that the slow axis is vertical. Now if slow axis is horizontal for quarter wave plate delta x then will be pi by 2 and delta y will be minus pi by sorry pi by 4, if I divide into this way pi by 4, then the matrix M in the similar way one can show that it will be e to the power i pi by 4 and 1-0-0 and minus i. So, this e to the power i pi by 4 comes because we consider this pi by 4 minus pi by 4. If you consider this pi by 4 to be 0 and for delta y it is pi by 2 then you will get a similar structure except this term. So, the main thing is what you get in this matrix that will remain the same for all the cases. For a half wave plate we will get a similar kind of structure. But today we don't have that much time to discuss the halfway plate. So maybe in the next class we will start from here and discuss in detail about the halfway plate and how these things will be confined etc. So with that note I would like to conclude today's class. In the next class what we do is we will discuss more about the formation of the Jones matrix and for the halfway plate where the phase difference will be pi then we will see what happened in the form of the Jones matrix. Thank you very much for your attention and see you in the next class.

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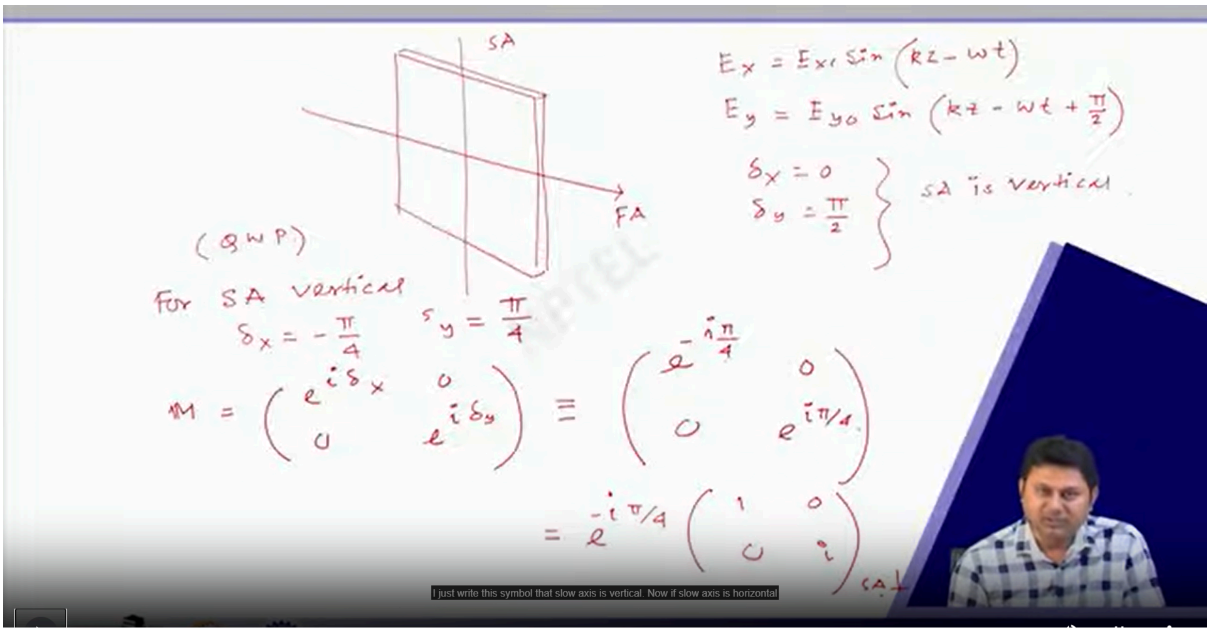

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