WAVE OPTICS

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Lecture - 56: Jones Matrix for polarization (Cont.)

Hello, student, welcome to our wave optics course. Today we have lecture number 56 and in today's lecture we will continue on the understanding of the Jones matrix, which represents different polarizers . So, we have lecture number 56 today and essentially we will try to understand the Jones matrix of different polarization. So in the last class we mentioned, let me do it once again. Suppose this is a polarizer that can polarised and unpolarize light and this is a linear polarizer. So if I launch an unpolarized light the outcome will be a linearly polarized light. So in the input we have unpolarized light and this is supposed to be the pass axis then at the output we have a polarized light which is parallel to this. We get this output which is if I put the x and y axis here also, so, this is the way the polarized light is there. So this is a linear polarizer. So, this linear polarizer can be represented by the matrix a-b-c-d. Now the question is how to get this value a b c d once the angle of this pass axis or transmission axis is given. So if let us start with the most simple case that is the pass axis of a system having vertical, this is my polarizer and the pass axis is vertical. So here we have the pass axis. So, if I want to represent this polarizer in terms of the Jones matrix how to represent that? So, that is the question. So, it is for sure that when the unpolarized light will pass here only the light which is parallel to this pass axis will go to pass through and the light which is perpendicular whose polarization is perpendicular to this, for example this one, this will not come in this side that is a two condition we get, one is this e h, this is a vertical polarized light it will pass through but e horizontal will not going to pass through which is exactly perpendicular to EV. If that is the case we know what is EV mathematically.

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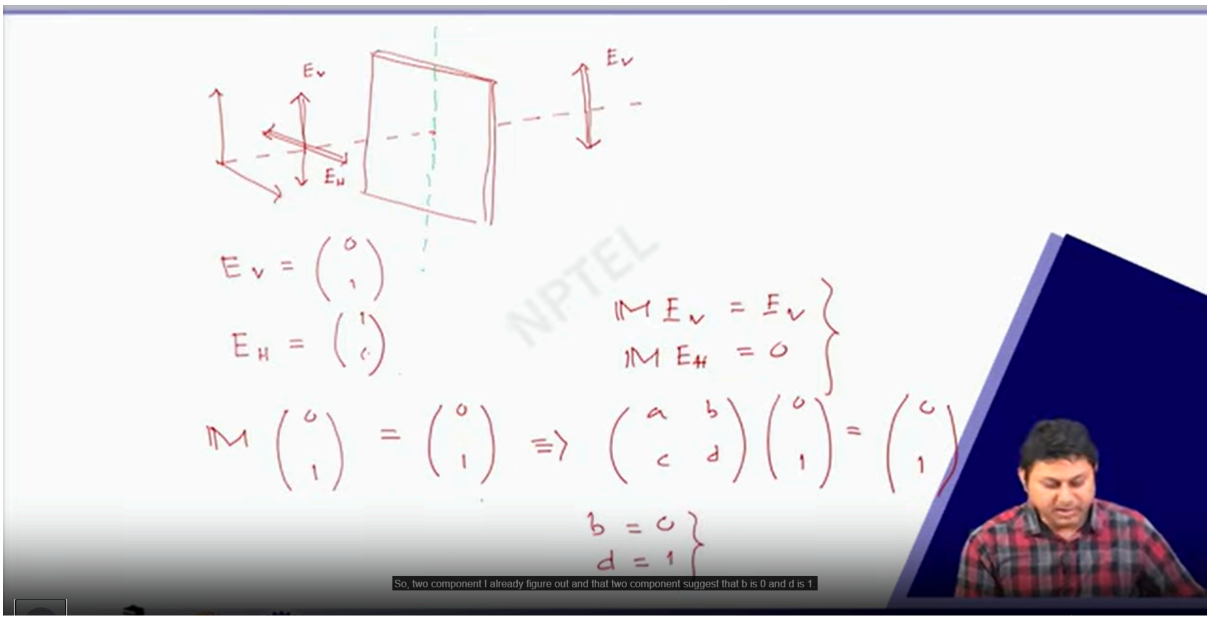

So, EV that is a polarized light which is vibrating along y axis is 0-1. And EH, a light which is polarized along the x-axis, so I can write 1-0. Now when this light is passing through the Jones matrix the output can be represented by this operation. So the matrix that corresponds to this polarizer is going to operate over this vertical polarization state and we know that since this line is vertical. So, I will get the output of this. So, that means, M that will operate over E v will give you E v, but M that is operating over e h, no e h will be this side, so, I get 0. These are the two equations we have and those two equations we need to solve. These are two matrix equations. So, we just need to solve these two matrix equations. So, it essentially gives us a-b-c-d, 0-1 is 0 1 that means b is equal to 0, from this equation I can readily see that and d is equal to 1 these two I know. So, two components I already figure out and that two components suggest that b is 0 and d is 1. If I put another condition that is a-b-c-d it will be over the horizontal and I will get a matrix here 0-0 because nothing will come outside. So, that means, here from this equation I can get a equal to 0 and c equal to 0. So already I get b and the value of b and d where b is 0 and d is 1. So, that means, my matrix representation for the polarizer whose pass axis is situated along y axis, that matrix can be represented by 0-0-0-1, that is the matrix representation of a linear polarizer t along y axis. So M represents the linear polarizer whose transmission axis or pass axis is along y axis. It is easy to show that if I have a polarizer, in the same way I will not do this problem, rather I will give this problem as homework to the students. Suppose this is my polarizer and the pass axis is along x direction this is the path axis and if I draw the XY then for this the value of m will emerge as 1-0-0-0 this is representation of linear polarizer with pass axis along x axis. So, in the same way one can prove it, but also it is shown that what happens when we have a different orientation of the path axis. For example, a polarizer like this so we can draw this polarizer and we have an x and y axis. Suppose this is the x axis, this is the y axis and the pass axis is making an angle theta. Suppose in any arbitrary direction we have the pass axis, so, this is pass axis or transmission axis and that is making an angle theta for example with x axis, so, if that is the case the question is how to represent M matrix for such polarizer in a general case. So that is the question how one can represent this polarizer.

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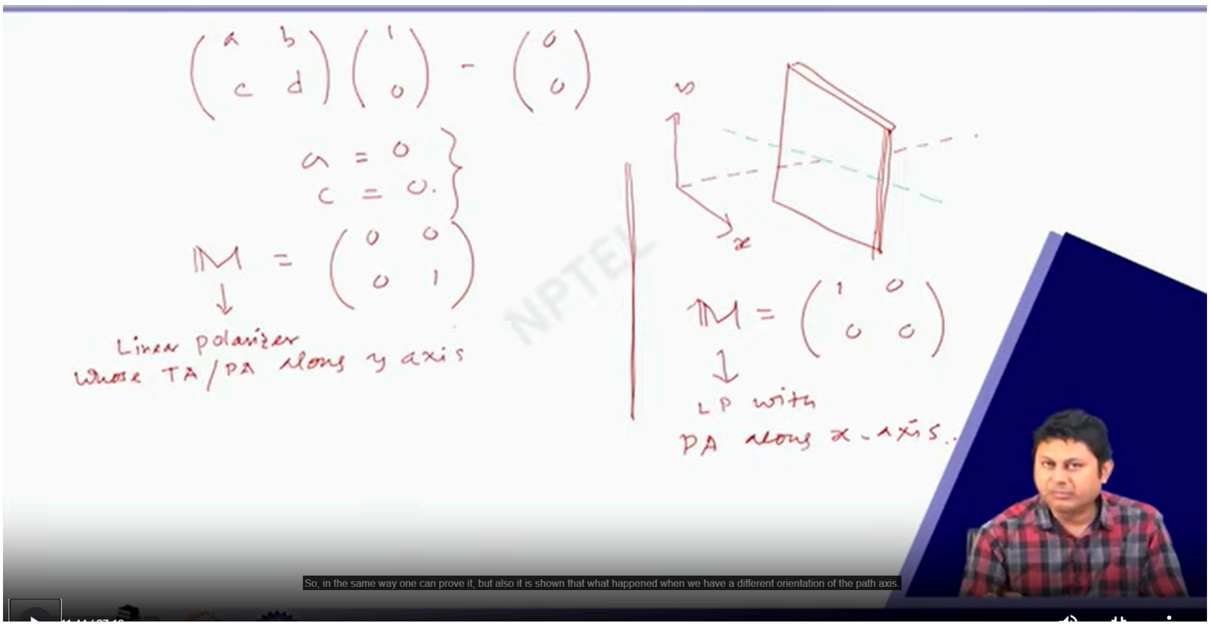

So if that is the way the pass axis is distributed then we know that the vector e theta that can generate this side is simply cos theta, sin theta, so that means m axis, this m vector if I operate this m over e theta then I will going to get e theta back. And also if we have e theta plus pi by 2 ,suppose, we have another vector which is rotated by 90 degrees. So, I can have cos of theta plus pi by 2 and sin of theta plus pi by 2. So, that is essentially minus sin theta and cos theta. So, I have another equation, this is my equation 1 and I can get another equation which is e theta plus pi by 2 this will be 0 because no light will pass because the pass axis is exactly perpendicular to this Jones vector. This is the orientation of the Jones vector for a polarized light which is exactly perpendicular to the pass axis. So, this light will never pass through to this polarizer. So, with these two equations 1 and 2 we can find out the component a-b-c-d. So, I can have my first equation that a b from equation 1, ABCD if I operate over cos theta, sine theta I will get cos theta-sine theta. So I can have one equation here and that is a cos theta plus B sine theta, that is cos theta and C cos theta plus D sine theta. That is 2 equations, but we have 4 variables a, b, c, d. So, I need to have another equation and from equation 2, we have a, b, c, d and then we have minus of sine theta cos of theta that will be 0-0 because this is a perpendicular I should not have anything. So I have a minus of a sine theta plus b cos theta is equal to 0 minus of c sine theta plus d cos theta is 0. These are the other two equations. So from exploiting these two I can get for example here, if I replace a sine theta to b cos theta I will get something. So, what should I do here? For example, here we have b cos theta. So, what I do, a sin theta we have. So, what I do is multiply with this equation with sin theta. So, I get a cos theta, from this equation this is say my equation 1, this is equation 2, this is equation 3 and this is equation 4. So from 1 and 3 I can get, I multiply equation 1 with sin theta. So, a cos phi, cos theta, sin theta not cos phi plus b sin squared theta is equal to sin theta cos theta and from 3 if I multiply cos theta so I have minus of a cos theta sine theta plus, b cos squared theta is equal to 0. So from here if i just add these two equations then I simply have b is equal to sine theta cos theta. So I already get b in terms of theta. Once we know the b then I can make use of another equation, so I can get a sine theta is equal to, so what I get from equation 3?

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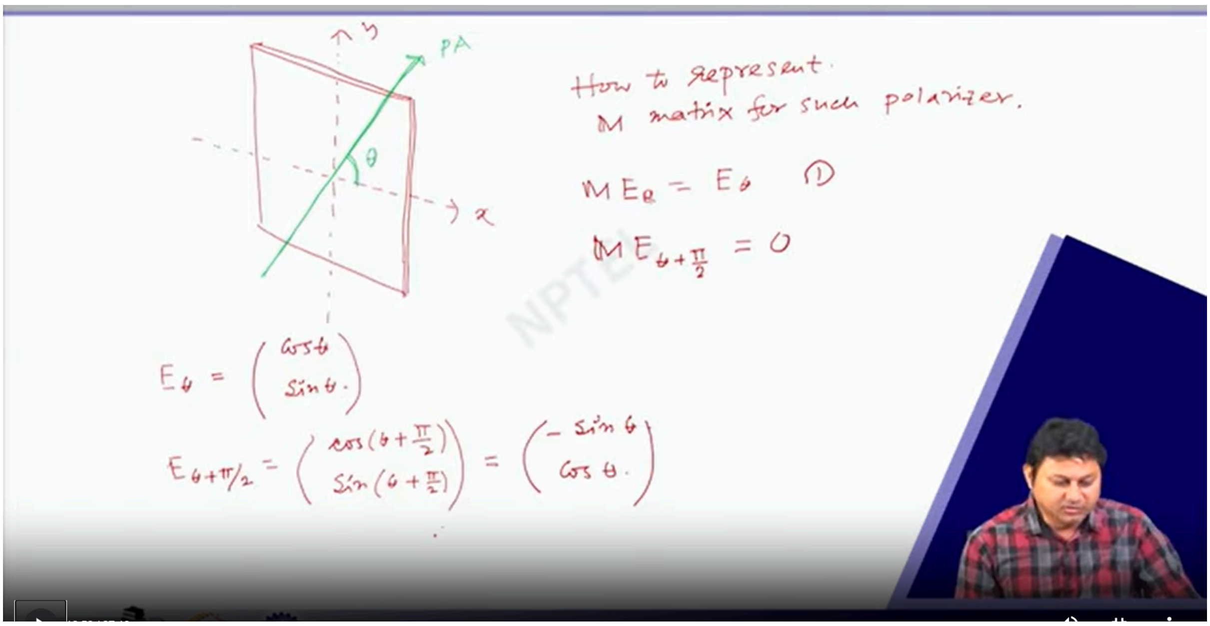

Let me go here, so equation 3 I get a sine theta is equal to b cos theta but b is already I find it is sine theta cos theta. So a is essentially sine theta cos squared theta divided by sine theta, so I get b, I get a. Similarly if you do the process, I am not going to do that. Exactly in the same way one can get c equal to sine theta cos theta because identical equations are there and d is equal to sin squared theta. So my m which is the representation of a polarizer having a general pass axis making an angle arbitrary angle theta is this cos square theta-sin theta cos theta, sin theta cos theta and sine squared theta, this is a very important and interesting result. Now if I put the value of theta I can get all the cases. So for example when the pass axis is along y axis then theta will be pi by 2. So, the value of m will be cos pi by 2 which is 0 here it will be 0, it will be 0, it will be 0, it will be 1 and exactly this we calculated. When the axis is along the x-axis then what we have the theta value will be 0. So, the value of the M matrix will be 1, it will be 0, it will be 0, it will be 0 and that is exactly what we find. In a similar way one can find what happens when theta equals pi by 4 that is 45 degree angle and one can calculate these results. So, with that note I would like to conclude today because I do not have much time. So, in the next class we are going to extend this idea of how a polarizer is represented by the Jones matrix. Now, instead of polarizers we have wave plates which can change a linearly polarized circularly polarized phase modulator rather. So, it will change the phase. So, how one can represent this in the form of a matrix we are going to discuss in the next class. So, with that note I would like to conclude today. Thank you very much for your attention and see you in the next class.

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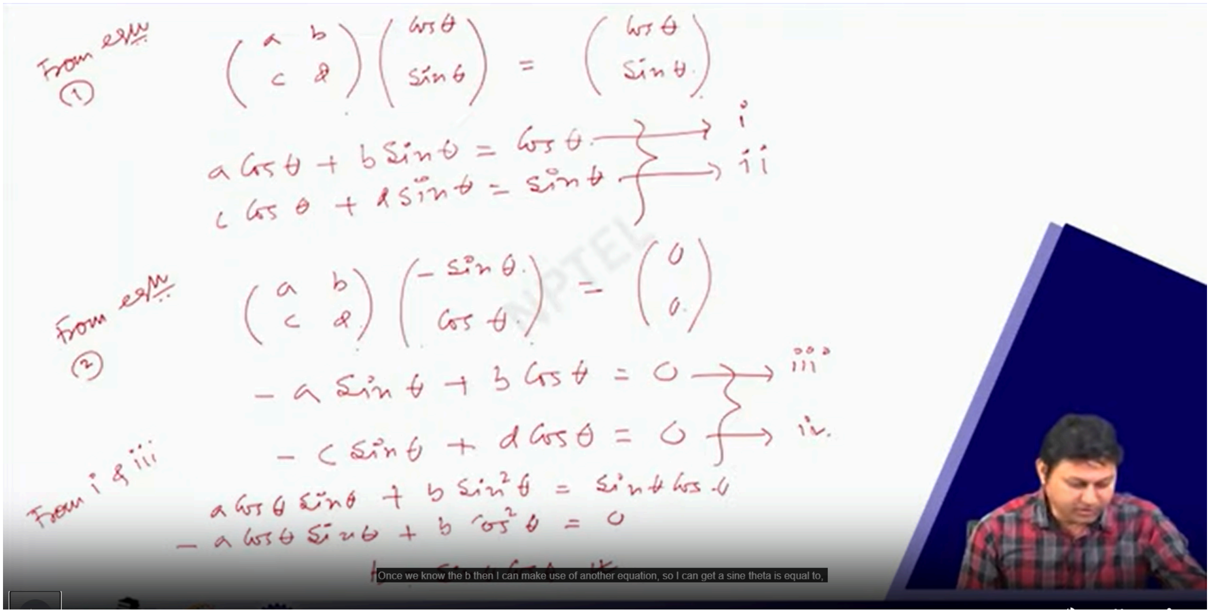

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