WAVE OPTICS
Prof. Samudra Roy
Department of Physics
Indian Institute of Technology Kharagpur
Lecture - 55: Jones Matrix for polarization

Hello, student, welcome to the wave optics course. Today we have lecture number 55 and in today's lecture we will discuss the Jones matrix that is the representation of polarization. So today we have lecture number 55. Today we are going to study the Jones matrix, before going to study the Jones matrix let me recap what we have done. So, we know that for linear polarization I write it LP, LP stands for linear polarization where the electric field is x coordinate and y coordinate. The electric field vibration will be in general in a plane and if this is an angle alpha. So, this is the way the electric field will going to vibrate and if I want to present this electric field with this complex notation a matrix form E0 x, e to the power i phi x, this is the general form to represent the matrix for linear polarization, the representation will be simply two numbers a b, where a and b are real numbers. For circularly polarized light, F what happened the tip of the electric field will move in a circular path like this. So, this is the electric field that will move either this direction or this direction depending on the phase. So, here we can also state the light as if I write in terms of these numbers then these numbers will be simply some amplitude then 1 I or some amplitude E0 tilde then 1 minus i, 1i is the representation of left circularly polarized light because it goes in this direction. So this is a representation of left circular polarization and this is for right circular polarization, this is a representation for these two cases. A general case can be considered also where we have the elliptic polarization and the ellipse may not be the x and y axis is defined like this is the most general form and the major axis and minor axis of the ellipse may not coincide with x and y it may be but this is a general form I am writing then we have e tilde with a general form a b, plus minus i c, this is the way we can represent,
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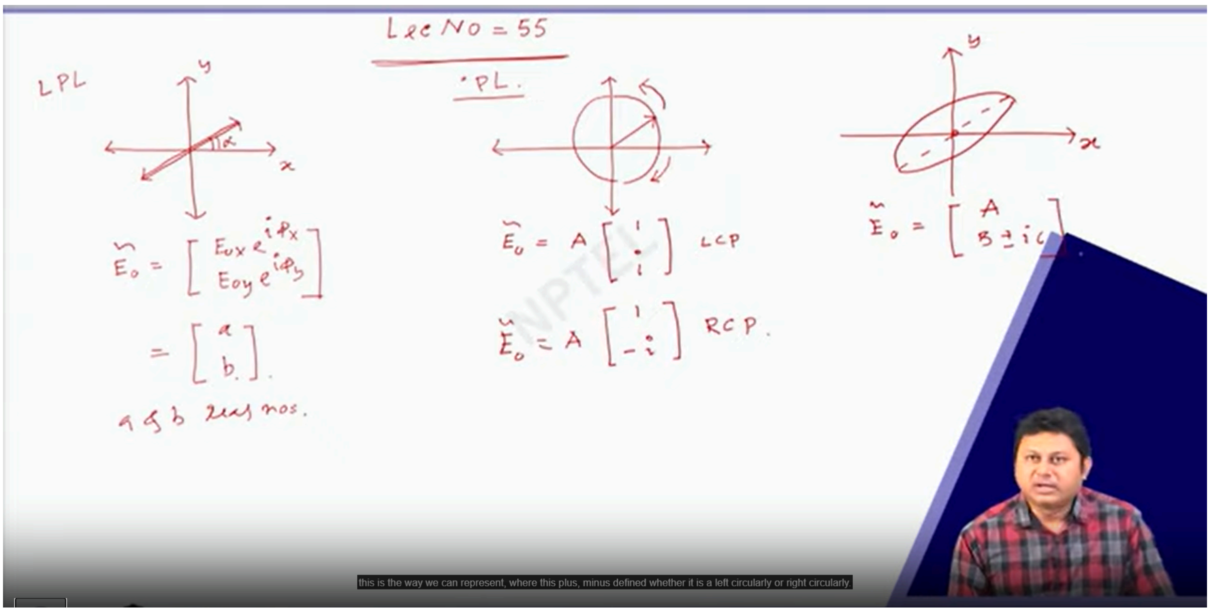

where this plus, minus defined whether it is a left circularly or right circularly. So, that is the general form we had and we discussed in the last class and how it is formed is discussed in detail. So, today we will go to extend this idea to form the Jones matrix. So, this matrix that is generated here which is representing the state of the polarization is called the Jones vector. So this jones vector has certain properties. So let me write down one by one. So how do these things actually work? So one thing important: the sum of any two jones vectors is also a jones vector, so the superposition of different states of polarization always gives you another superposition, another state of polarization. So that is the meaning of any number of so-called Jones vectors is also a Jones vector. So that means if I add two Jones vectors, I'm going to get another. So let me simply put an example that I just add two linearly polarized light. One is X polarized and another is Y polarized. And if I add these two, I'm going to get a simple matrix 1 1, so this is X polarized light, this is Y polarized. When I add these two, the resultant which is this one gives a linearly polarized light with an angle 45 degree with axis. So here this represents X polarized. So the electric field was vibrating in this way. So that is this electric field, which is X polarized and now I have Y polarized electric field, which is this one and when the superimposed to each other that is if I add these two polarized light then the resulted polarized light is simply this superposition of two polarization can leads to another polarization I can extend these two for circularly polarized light also. So what happens when we superimpose two circularly polarized lights? So this is the superposition of two circularly polarized light. Let us add these two Jones matrices for these are the two polarized light representations of two polarized light. So in one case it is left circularly polarized light and this is right circularly polarized light and when we add these two polarized light we get like this which is a linearly polarized light. So how it works x axis, y axis and this is the direction of the polarization which is left circularly polarized light and we add this with right circularly polarized light. So this is right circularly polarized light and when I add these two I get a linearly polarized light and in this case the linearly polarized light is along x direction. So, I get the resultant polarized light as linearly polarized light. So, this is linearly polarized light along x direction.

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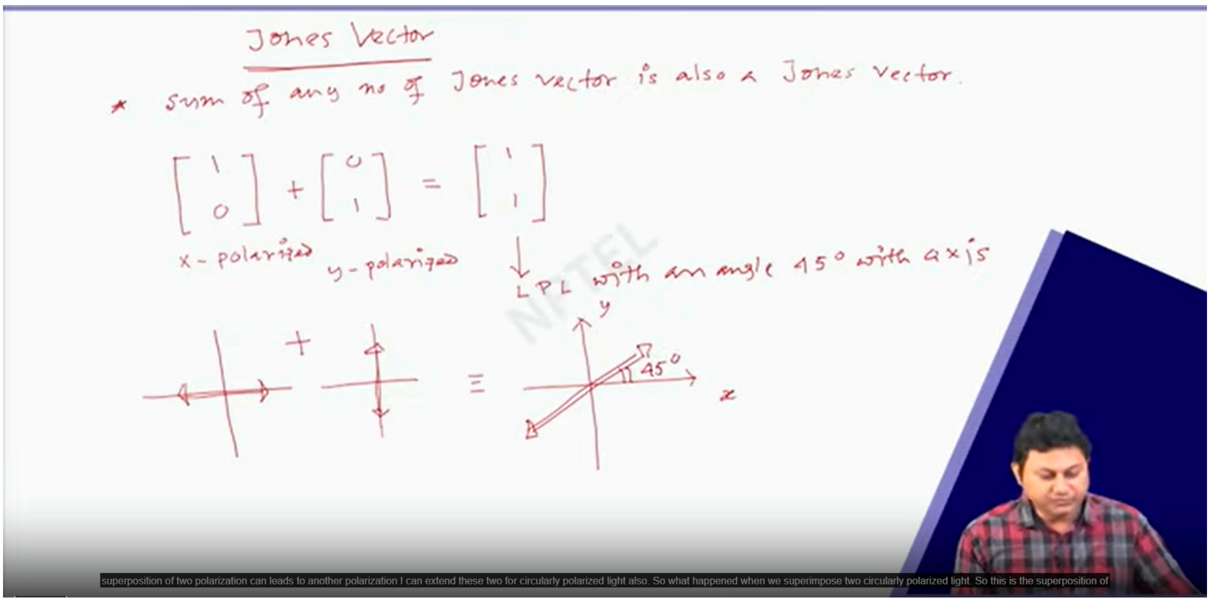

So, I add these two and get this also there are few important properties of the Jones matrix.

So like the orthogonal Jones vectors, I can have the linearly polarized light which is horizontally polarized. For example one polarization is along this. So this is X and this is Y, so, this is horizontally polarized light. So I write EH which is horizontally polarized light I can write it as 1 0 and vertically polarized light which is polarized along the y-axis I write EV, which is 0 1. Now EH and EV they are orthogonal, how they are orthogonal and what is the condition in order to find the orthogonality I need to do this operation, this star transpose multiplied by this v uv that is first I make a transpose and then make a complex conjugate. So, if I do that for this column matrix it will be 1 0 like this and it will be simply 0 1 which gives us null vectors like this. So when this is the condition then that means they are orthogonal to each other. So horizontally polarized light and vertically polarized light are in this jones matrix method, they are orthogonal to each other. So they are orthogonal not only for that the orthogonal polarization is also one can realize by circularly polarized light. So let me draw that. So x and y and this is left circularly polarized light and if I write, e l which corresponds to left circularly polar or E lcp which corresponds to left circularly polarized light I should write it as 1-i. Similarly the right circularly polarized light will be this right circularly polarized light and I write it as ERCP this is 1-i. So when you try to find out the orthogonality between these two, what I do is that I multiply E LCP star and then I make a transpose that is essentially 1 minus i. And if I multiply with E LCP star transpose multiplied with ERCP, that is right circularly polarized light, it will be essentially 1 and then minus i multiplied by 1 minus i and this matrix multiplication gives me 0, so that means these 2, this left circularly polarized and right circularly polarized they are also orthogonal to each other. The horizontally polarized and vertically polarized light are orthogonal to each other. The left circularly polarized and right circularly polarized light, they are orthogonal as well. So if we have an orthogonal set, then an orthogonal set forms a complete set and behaves like a basis, so that means any arbitrary polarization can be represented by this set of orthogonal pairs.

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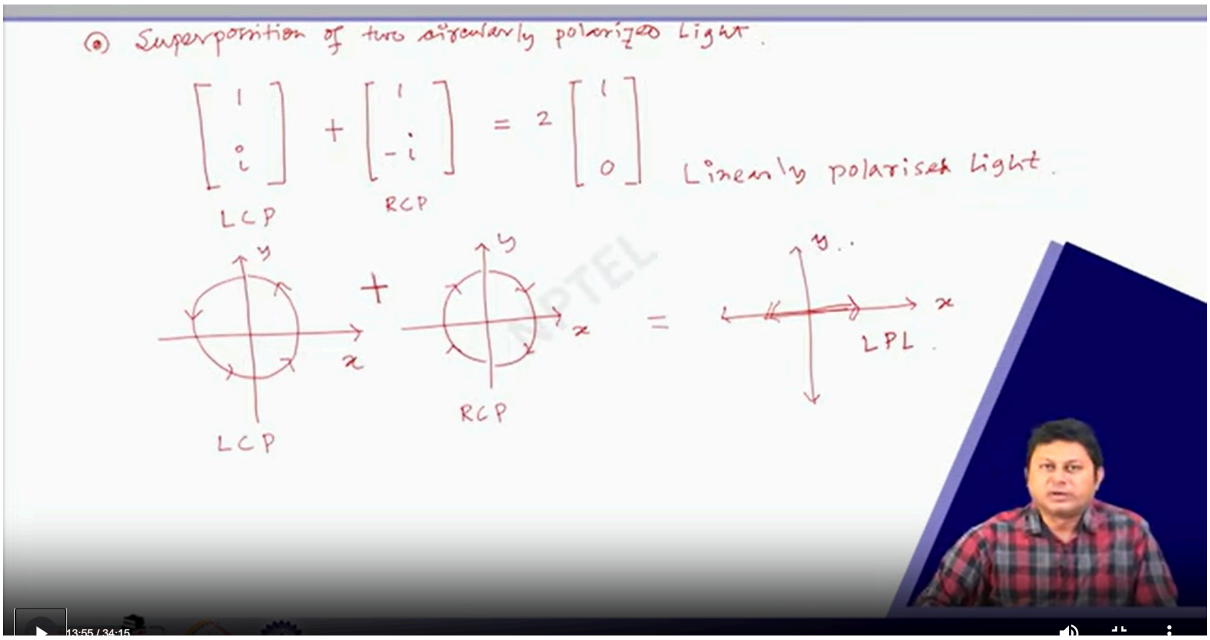

So for an example any so I can write for example 1 by root over of 2 i, see, if I write this, so that represents a circularly polarized light x, y and it shows a left circularly polarized light with this magnitude, one by this magnitude one actually and this can be divided with the basis vector. So I can write it as plus this combination or 1 by 2, 1-0 plus i, 0-1. So, this can be represented by adding these two polarizations. These are represented by the horizontal and vertical polarized light. This is horizontal polarized light and this is vertically polarized light. Both are linearly polarized light and a combination of these two can give me the left circularly polarized slide. In the similar way the right circularly polarized slide one can find with the combination with just putting 0 minus 1. So that means or here if I put minus 1, so this combination can give any arbitrary polarization with these combinations. So more examples can be given like these linearly polarized light with arbitrary angle say alpha, it can be present simply by numbers. So I can have E which is cos alpha, sine alpha, this is the way it is represented. Now it can be divided into two parts and I can write it as cos of alpha 1-0 plus sine of alpha 0-1. So, this is essentially cos of alpha, then the polarization which is vertical plus sine of alpha polarization in this case it is horizontal sorry and the vertical. So, E h and E v they are orthogonal sets and with this set any arbitrary orientation can be represented and that is the way it works. The next thing that we are going to discuss when we go to the next page is the mathematical representation of polarizers. So, how can a polarizer be represented mathematically? The mathematical representation of polarizers is interesting. So what is the meaning of polarizer? Let us try to first understand, so in general polarizer is a substance which can polarize the light. Suppose, we have a system like this, this is a polarizer I place and a unpolarized light, for example let me put the x and y axis ,also this is x, this is y and unpolarized light is allowed to pass through this polarizer and what we have in the polarizer. This is supposed to be an unpolarized light and in the polarizer we have a pass axis. Let me draw this pass axis. Suppose we are having a pass axis or transmission axis like this. So the meaning is the light which is unpolarized. The portion of the light which is parallel to these lines is only allowed to pass and the rest of the light will cut out. So, at the output, what we see is a light parallel to this pass axis.

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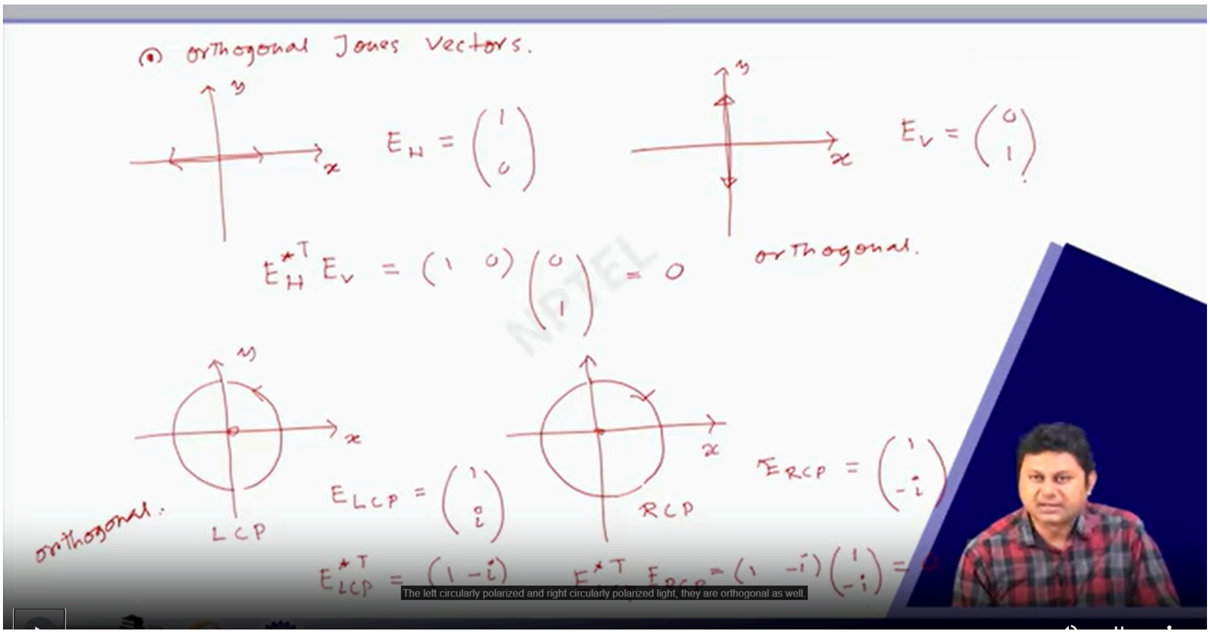

So, unpolarized light means essentially that all directions of polarization are there. But when it is passing through this polarizer having the pass axis here, so this is the pass axis or transmission axis. This is the pass axis or sometimes it is called transmission axis TA transmission axis. So, this will allow only the light that is parallel to this and we are going to get a polarized light here. So, this is unpolarized light and this is polarized light. So, the system through which we can see an unpolarized light is called the polarizer. So, this is a linear polarizer. Now, the next question is how to make a matrix form of this linear polarizer? So, there are various devices that can act like an optical element, that transmit the light, but modified state like this and they are called the polarizer. In this case for example, this is a linear polarizer. So, how we represent this polarizer and this representation of the polarizer can be done mathematically and this mathematical representation is a matrix. So, this polarizer can be represented by a 2 cross 2 Jones matrix and the representation will be simply like this. This is a matrix a, b, c, d are few numbers. So, this is the way we can represent an optical element that can change the state of a polarization. And today I do not have much time to discuss more about that. So, in the next class I will discuss how using these matrices one can represent different kinds of polarizer and half wave plate etcetera. So, first we try to find out what kind of mathematical form is there for the representation of this Jones matrix, for linear polarization and we calculate that if the pass axis is given based on the angle of the pass axis this value a, b, c, d will change. So, that is what we will do in the next class. So, with that note I would like to conclude here. So in the next class I will start from this point and discuss more about the representatives of the Jones matrix of these polarizers. Thank you very much for your attention and see you in the next class.

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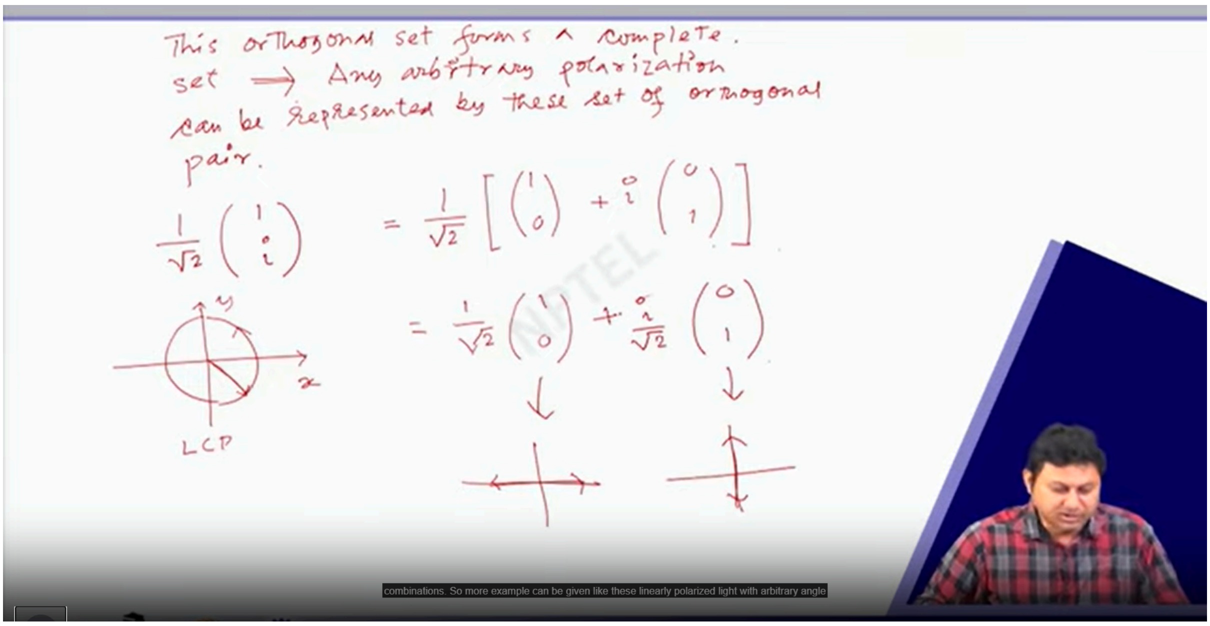

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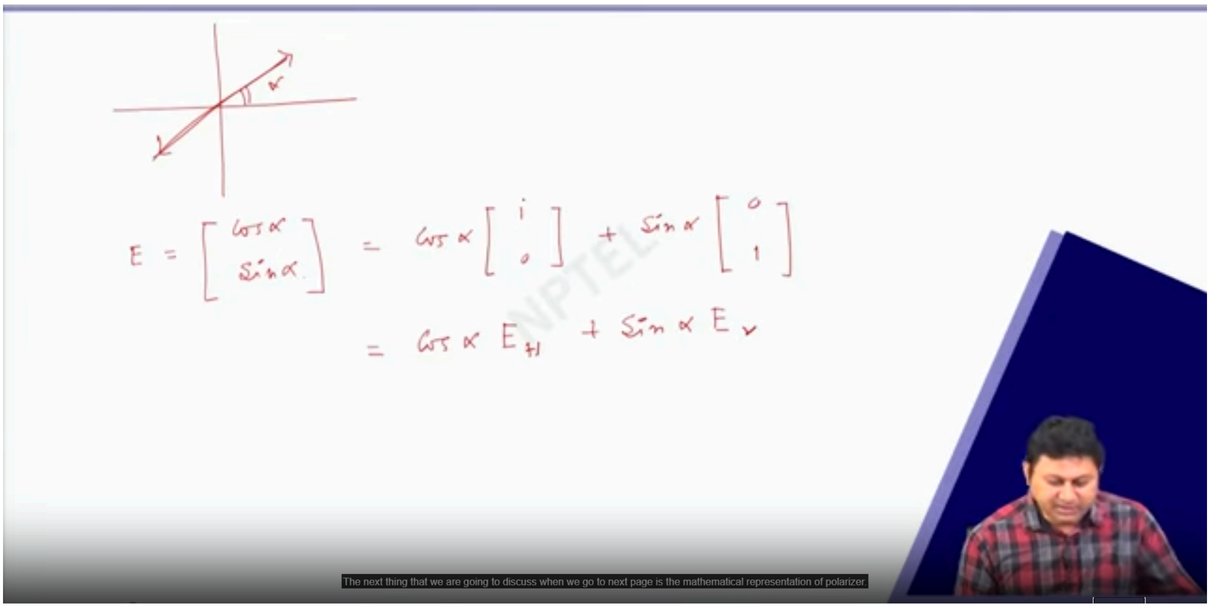

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