WAVE OPTICS

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Lecture - 53: Circularly polarized light

Hello, students to the wave optics course. Today we have lecture number 53 and in today's lecture we are going to describe how circularly polarized light is produced. So we have lecture number 53 today and we are going to discuss the production of the circularly polarized light. In the last class we mentioned that the light E is the electric field that is showing a propagating wave Ex and Ey in a vectorial form and e x the x component which is essentially function of z and t can be defined as e 0 x cos of k z minus omega t and E y we have E 0 y cos of kz minus omega t plus a phase phi. However, if you remember that for circularly polarized light we put a condition that E0x is the magnitude rather than the amplitude of the x component and amplitude of the y component, peak amplitude they are the same. So, this is the condition we put and let us consider this as E naught. So, e x will be e naught cos of k z minus omega t, the wave is propagating along the z direction and e y is E naught cos of kz minus omega t plus phi. I can write this as E naught cos of kz minus omega t, cos of phi minus sine of kz minus omega t and sin of phi, just use the cos a plus b formula. Now what happened when phi is equal to pi by 2, that is the condition we consider in the previous class then my Ex will be E naught cos of kz minus omega t and ey is equal to if I put pi by 2 please note that then we have first term will 0 but second term will be there with a negative sign. So, we have minus of e naught sin of kz minus omega t. So, that is the value of the x and y component that turns out when I put the relative phase phi is equal to pi by 2 and if I do then this particular wave basically represents the left circularly polarized light that we described in the last class.

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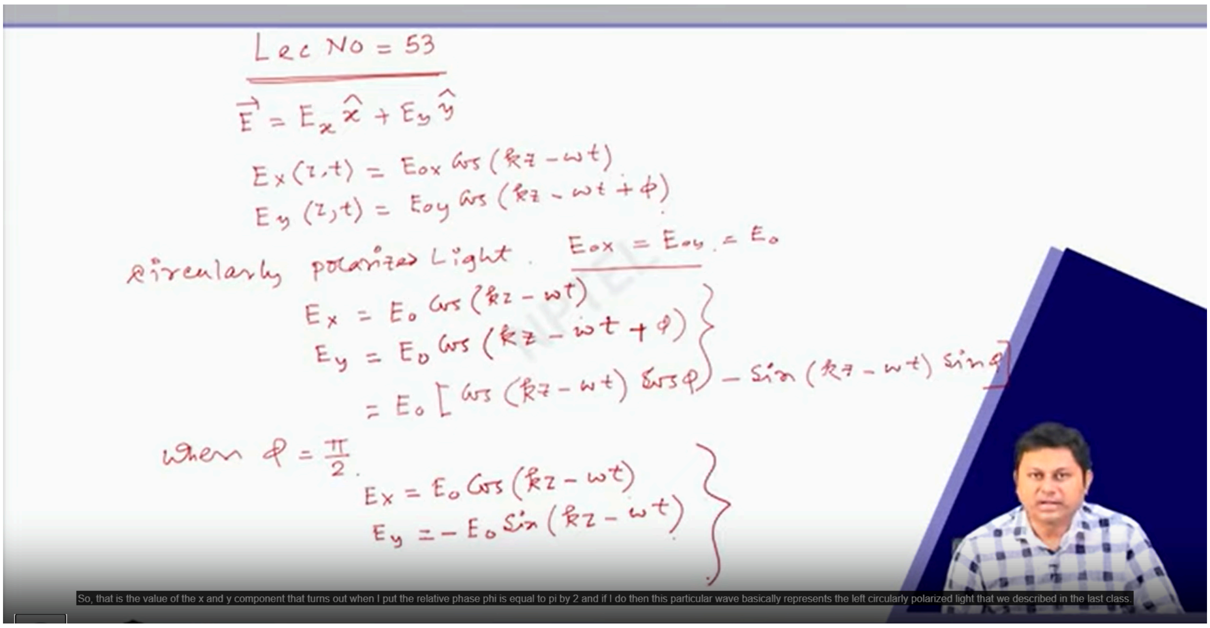

Now, we will expand this for another set of values and that value is for phi equal to minus pi by 2. If I do that then that basically results in the right circularly polarized light or RCP in shorthand notation. So, what happened here with pi equal to minus pi by 2, E x will be the same as E naught and then cos of k z minus omega t and e y will now become e naught sin if we have this. And now once we have the information of E x and E y we can frame it and total electric field we know that it is a vector quantity it is Ex x unit vector plus Ey y unit vector. Now what do we do? We will put z equal to 0, we want to see how this electric field will evolve over time at z equal to 0 0 plane . At z equal to 0 plane E x will be simply because it now does not depend on z anymore I put a specific value of z. So, it will be E 0 of cos of minus omega t which is cos of omega t and E y will be E naught sin of minus omega t, which is essentially minus of E naught sin omega t okay. Now, we change the value of t and c, how these things are happening, how the tip of the electric field changes? So, the first thing that we will do, this is my x axis and this is my y axis at t equal to 0, E x will be E 0 and E y will be 0. So, I will get the electric field along the x axis with this value, this will be the electric field here at t equal to 0. Now, I change the value of the t, this is the coordinate, this is the x axis and this is the y axis. I am just drawing the coordinate in just one quarter of the coordinate because I just required this part, then what happened when t equal to say pi by 6 omega. So, here at t equal to 0. Let me write the e x component will be e 0 and e y component will be 0 at t equal to pi by 6 omega that is when omega t is equal to pi by 6, e x will be root over of 3 by 2 E naught and E y will be minus of half E naught and if I put this value here I am going to get something like this, where this length is Ex and this one is Ey. The electric field will now shift here in this way. Now if I go on with other few values, say what happened when t equal to say pi by 3 omega then Ex will be half of E naught, E y will be minus of root over of 3 by 2 E naught and we will go to get something like this here. Finally when we put t equal to pi by 2 omega that is omega t is equal to pi by 2, my e x will be 0 and e y will be e naught, minus of E naught rather. So, I am going to get the electric field tip here. So, you can see the electric field is moving in this direction. It is going here and then here.

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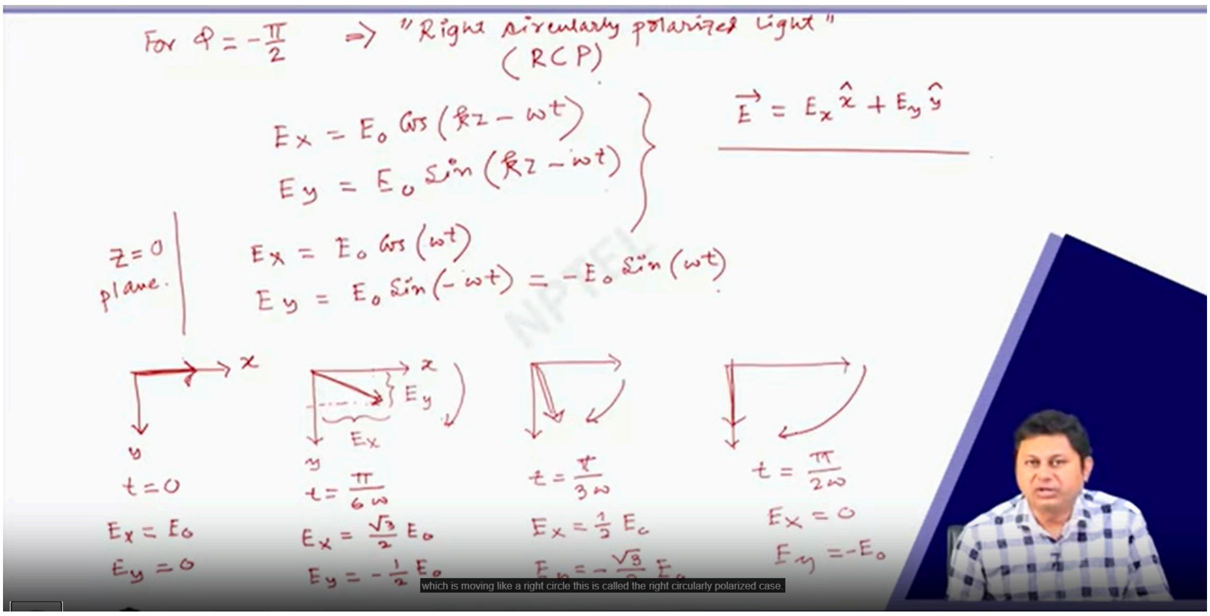

So, from here to here it moves which is moving like a right circle this is called the right circularly polarized case. So, if I draw that like we did in the last class that if I make a circle and if this is my x axis, this is my y axis the electric field starts from this point, this is the first point we have, then it moves to this value, this is at t equal to 0, this is at say t equal to pi by 4 omega, then it moves here this is at t equal to pi by 2 omega, then it moves here in this point, next time. Note that the time is gradually increasing and it is t equal to 3 by 4, pi divided by omega here then it moves at this point. So at t equal to pi by omega and then again it moves here which is at t equal to 5 pi by 4 omega then it reaches here which is t equal to 6 pi by 4 omega. So, I can write 3 pi by 2 omega then it comes here. So, that means, with the increment of time it is 7 pi by 4 omega. And, again at the time t equal to 2 pi divided by omega it comes back to its original position. So, that means, if at z equal to 0 plane, if I just note down how the tip of the electric field is moving under the condition when E x is equal to E naught cos of, I write the total field here k z minus, omega t and E y is E naught sin of k z minus, omega t if this is the way the electric fields are distributed E n, E x and E y component then we can see that there is a rotation in this direction, the electric field is rotating in this direction, the tip of the electric field and that is nothing, but the right circularly polarized light. t The condition here is E 0 x, e 0 y that should be equal to e naught, that is the amplitude is same for e x and e y and the relative phase should be minus of pi by 2 that is for right circularly, in left circularly polarized light that we described in the previous class, this value phi equal to pi by 2 should not have the minus sign, it should be simply pi by 2. So, we understand how this thing is happening, but from a very simple expression one can find it and that leads to an equation of circle because we say circularly polarized light how the coordinates of E x and E y change with respect to time. So, that we can do, for example, here E x is equal to E naught as we wrote here cos of kz minus omega t and ey is e naught sin of, I am writing several times about this thing. So that you are familiar with the notation I am using and how it is coming it should be clear to your mind. So, from this expression, I can have simply ex e0 square of that plus ey e0 square of that.

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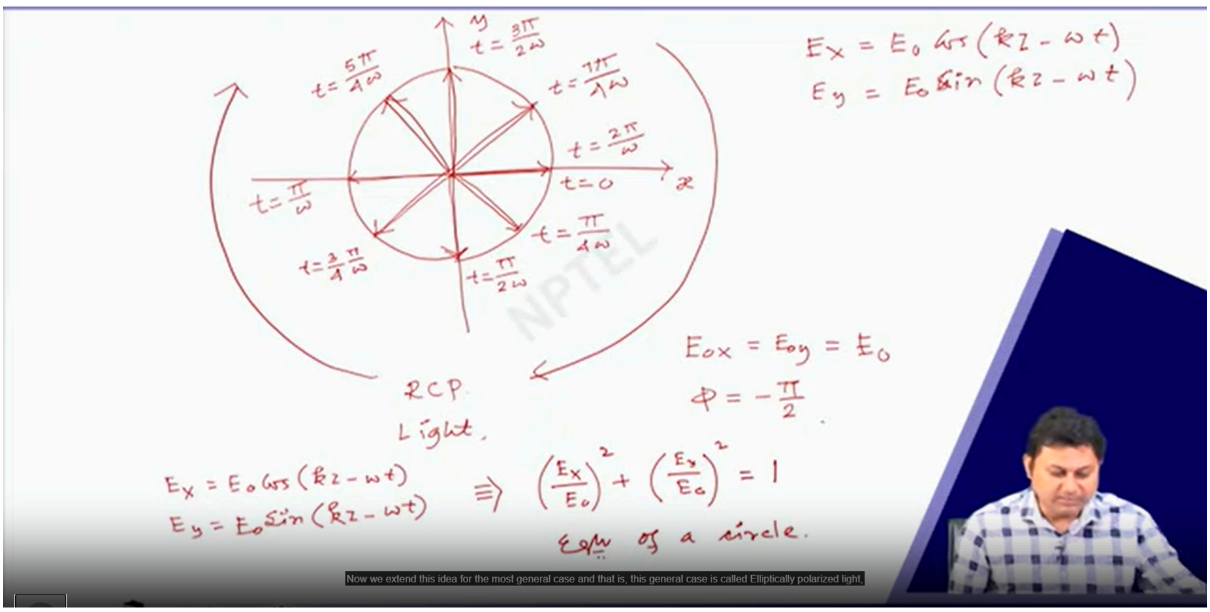

It is essentially 1 because it is simply cos square k z minus, omega t plus, sin squared k z minus, omega t and that is nothing but the equation of a circle whose radius is E naught that is all. Now we extend this idea for the most general case and that is, this general case is called Elliptically polarized light, in short it is EPL. This is the most general case because we will not put any condition over that. So, let me write down the most general expression of E x and E y. So, E total electric field as usual is distributed in x y plane moving in the z direction. So, I can write it as Ex x unit vector plus Ey y unit vector and E x will be E 0 x cos of k z minus omega t and E y will be E0y cos of kz minus omega t plus phi. This is the most general expression of the Ex and Ey one can have. Here also the condition that E 0 x and E 0 y is removed. We have E 0 x and E 0 y different here in general and phi is an arbitrary phase. There is no specific value of the phase we are putting. It is arbitrary. If that is the case then we can readily show that with this expression if I try to find out the relationship between E x and E y it should simply lead to the expression of an ellipse and that we are going to do here. So, Ex E0 x this is cos kz minus omega t and ey e0y that is I just split it, so I write cos of kz minus omega t cos of phi minus sin of kz minus omega t sin phi. So, now, what do I do? I replace this cos value whatever we have here and sin also. So, k z minus omega t, that parameter I will remove. So, and that we can do easily. So, let me do that. So, here we have E y E 0 y that is equal to this value cos, cos is again e of x and e of 0 x and then cos of phi and then minus of sin, I can write root over of 1 minus cos square kz minus, omega t is essentially 1 minus Ex E0x whole square and then we have sin phi. Then I can extend this more and I can write it as Ey by E0y minus Ex by E0x cos of phi whole square that is essentially a sine square. So what was there on the left hand side? Let me go back. It is 1 minus, this multiplied by sine. So it should be sine square minus e x e 0 x square, sine square phi. So, once we have this, then I can rearrange this stuff and essentially get this e y, e 0 y square then we have e x, e 0 y cos square and here we have e x, e 0 x square sin square. So, that I can merge together and essentially get a value like ex, e 0 x squared multiplied by cos square phi plus sin square phi. So, let me do that here which will be equal to 1 and then we have another term that is minus of 2 e x, e 0 x, e y, e 0 y and then we have cos of phi is the left hand side and the right hand side we have simply sine square phi.

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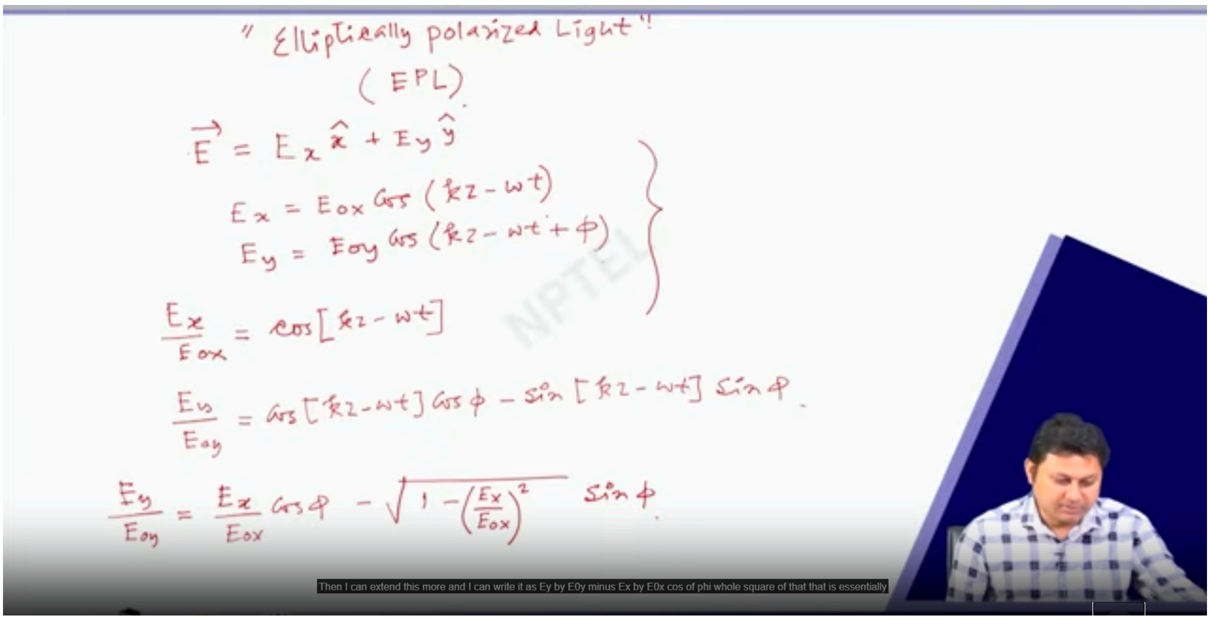

So, this equation essentially becomes e y square divided by e 0 y squared plus e x square divided by e 0 x squared minus 2 e x, e 0x, e y, e 0 y cos of phi that is equal to sin square phi. So, that is in general the equation of an ellipse making an angle alpha with the e x, e y axis. So, if I draw this ellipse it will be something like this, it will be a tilted kind of ellipse. Maybe we can draw in a different color. So, it will be like this, with this, where this angle will be alpha, this is Ex and this is Ey if I plot. So, this is the general equation if I plot this, where this alpha is related to phi in this way. So, it can be shown that tan of 2 alpha is equal to 2 of E0x, 2 of E0y divided by E0x square minus E0y square cos of phi. So, note that when we have phi equal to say plus minus pi by 2, then alpha is equal to 0, alpha 0 means the ellipse is no longer tilt, it will come to is normal with synchronize with the original coordinate and then the equation here if I put this, the equation will be very very recognizable and it should be e y square divided by e 0 y squared plus e x square divided by e 0 x square is equal to 1. This is an equation of an ellipse. So, that is a recognizable form and that is the equation of an ellipse. So, light will be the tip of the magnetic tip of the electric field will circulate either in this direction, in the opposite direction depending on whether the relative phase between these E x and E y is minus or plus. And in that way we can find whether it is elliptically polarized or right elliptically polarized the same footing that we have done in the circular polarized light case. So, here we do not have much time to discuss. So, we will like to conclude and in today's class we learnt how to recognize an electric field by just showing the E x and E y component. If the amplitudes are different and if they have an arbitrary phase relationship between them in E x and E y components. So, the polarization of the result in the electric field will be simply the ellipse which is the most general form one can have. Also we discussed in today's class how the circularly polarized lights are there, how the left and right circularly polarized lights emerged and which is a special case of elliptically polarized light. So, with that note I conclude here. Thank you very much for your attention. In the next class we will learn a matrix method to understand all these things which will be much simpler to realize the polarization problem. Thank you very much and see you in the next class.

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