WAVE OPTICS

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Lecture - 51: Fresnel's diffraction for semi-infinite opaque screen

Hello, student, welcome to the wave optics course. So today we have lecture number 51 and in this lecture, we are going to continue the understanding of Fresnel's diffraction in a semi-infinite opaque screen. So we have lecture number 51 today and before going to study the semi-infinite screen, let me remind you what we did in the last class. So the structure was this, we started with a rectangular aperture, and here is our aperture, this is the aperture through which light is passing this is x, and this is y. Then after doing a lengthy calculation, we eventually and this is the screen where we are supposed to get the fringe pattern due to Fresnel's diffraction, this is the Z axis. So any point P we find the intensity distribution here, let me write IP was I naught and then we had Fresnel's specific function this sum Y2 to y1 and then that is the intensity distribution we find. Note that the intensity distribution here comes through this function CNS, which was the Fresnel's integral and by definition, Fresnel's integral is C of nu that is 0 to nu then, cos function of pi by 2 variable x square dx and S was, that was by definition and based on that result, we find this is the case. Now, once we know that, then we are going to discuss the different cases. So, case 1 is an unobstructed wavefront. That is I extend this aperture in the y direction since this is a function of y to infinity in the positive direction and the negative direction. So that means my y1 lower limit can go to minus infinity and my upper limit y2 can go to plus infinity, that is the limit at which we are going to work. Then what happened that ip and under that limit ip from this formula whatever we get is simply i naught, and then we just need to put the value, and this value is c of infinity, minus c of minus of infinity bracket close, the square of that, plus s of infinity, minus s of minus infinity bracket close, square and third bracket close.

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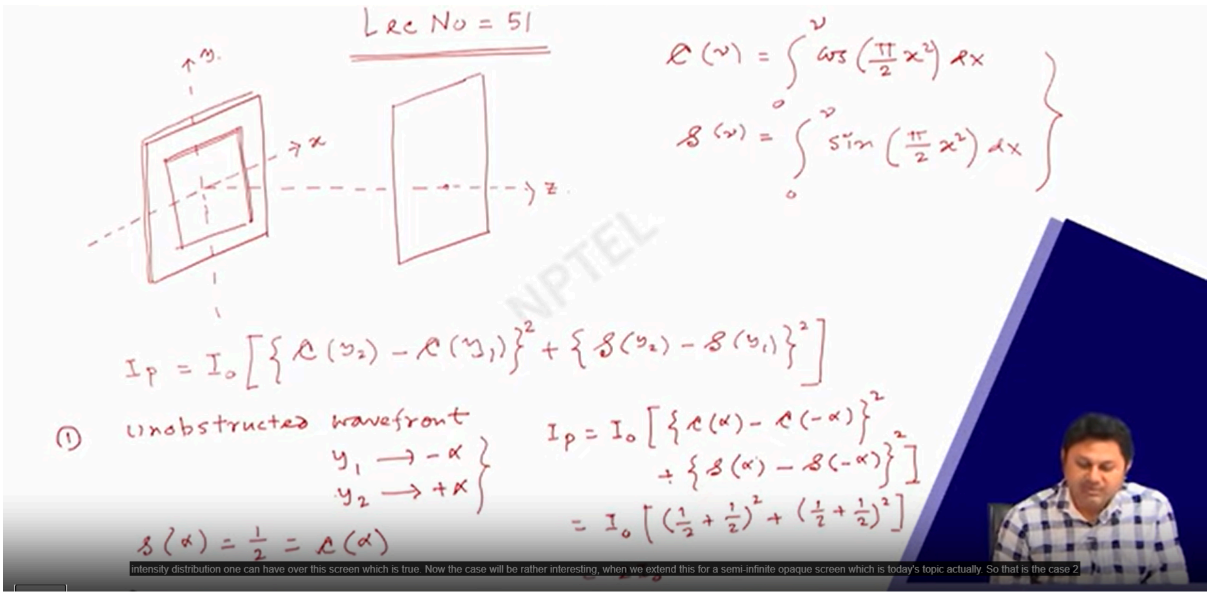

So, this value I know that, s of infinity that value is half, so as c of infinity. So if I put this

condition here, then I simply get a minus infinity. This function is an odd function. So it should be c of infinity with a negative sign. So then we are going to get a plus sign here. So essentially, what we get is this: that half plus, half square of it, plus half plus half square of it and the result will be 2 of I naught that is a uniform intensity distribution one can have over this screen which is true. Now the case will be rather interesting when we extend this to a semi-infinite opaque screen which is today's topic actually. So that is the case 2 and that is for a semi-infinite opaque screen. That means I have a screen like this, and the wavefront will hit this screen here, which is this portion is open, and only one it is diffracted from this edge, and here we have the usual screen. So, instead of having one rectangular kind of aperture, we have a semi-infinite opaque screen. That is, this is the opaque screen. One side is open, and the other side is blocked, so it is like an edge, this is opaque, I just placed it. So based on the general expression, I can write that IP, the old expression that is I naught, and then we have C of y2 minus c of y1 whole square of it, and then we have s of y2 minus s of this. Now for a straight edge, what is the boundary condition of y1. Now start from 0, so it tends to 0, lower limit and upper limit y2 will have some value y say this is a positive value along this direction, I just have up to a certain distance I have this. So, if that is the condition, then Ip, which should be a function of y, now will be I naught and c1 cy0 which means c of y1, y1 is 0 means it is 0, and s of y 1, which is 0, that is also 0. So essentially, we have i naught and then c of y square plus s of y square. So this is the expression we have. So now, in order to find out what is my intensity distribution along the y direction, we need to know the function cy and sy and how it changes. Before going to discuss in detail, let me draw what should be the form of the intensity distribution along y direction and it looks like this. Then, we discuss why it looks like this. So I am plotting for example IP divided by I naught and this dotted line is a point at half because this is normalized and this is 0 and this is y. So the plot will be something like this. There will be a sharp decay of the intensity because this is the region where we have this block. And then it gradually rises, goes to a minima and then goes to a maxima, then it will oscillate. And essentially saturates to this half value.
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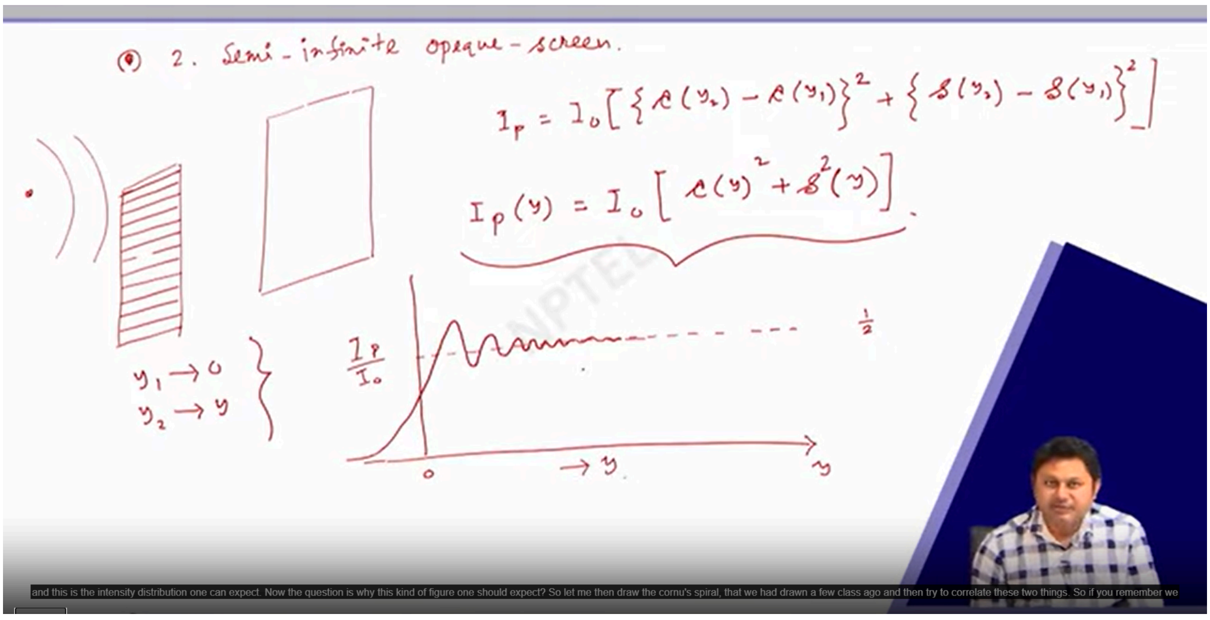
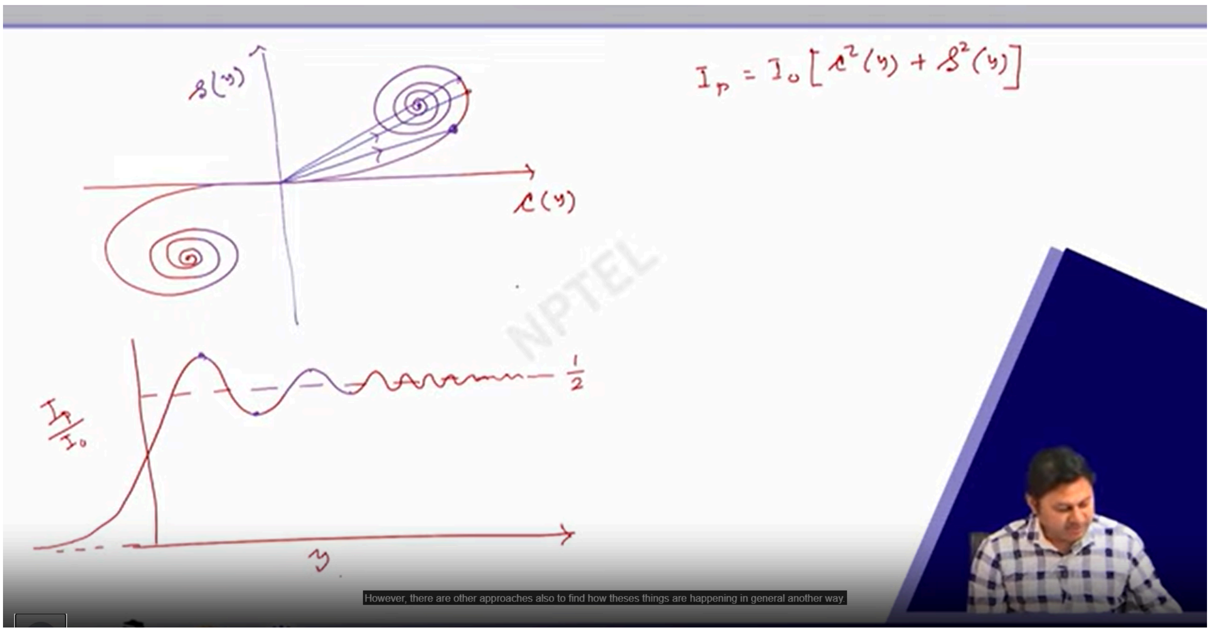


So along this direction it is y, and this is the intensity distribution one can expect. Now the question is why this kind of figure one should expect? So let me then draw the Cornu's spiral that we had drawn a few classes ago and then try to correlate these two things. So if you remember, that was the plot when we plotted C some variable Y and S some variable Y, that was the plot we had shown in a few classes ago where we demonstrated how S and C are distributed. Now if you look carefully if the value is increasing, y value is increasing. So what was our intensity here? So ip was i naught then c square of y plus s square of y. So that means essentially I am looking for all these points if I join from the center to any point here. For example, this is c square plus a square, then I join another point, so this is another point and so on. So, the length of these lines essentially tells us the value of C y square and S y square which is the length. So, how this length will change over the line, will give us how this intensity pattern is changing. So, now if I look carefully at the distribution here, let me draw the intensity distribution that I had drawn. So it is something like this, it goes maxima, then minima, and goes as it saturates to the value of half. So this is ip I naught, and this is y, so if I look carefully here, you can see that this value is changing. So first if y is changing this value is increasing over this line and goes to a maximum say, there is a maximum, and that point is essentially, this one is a maximum, but if I increase the y value more, then it starts reducing like here. So this is the point where we have maxima then it starts reducing again over this line, if I start to join then, it reduces again, and goes to a minimum here. Again it starts increasing and goes to a maxima and decreases and increases and decreases and eventually a square plus c square value goes to this saturated point which is exactly located at half. So this phenomena can quite easily be understood by drawing these two side by side and this is the way this intensity is distributed. However, there are other approaches also to find how these things are happening in general another way. So let me do that side by side, and this is another approach to address this problem. So in another approach, we have the source point here, and this is the wavefront that is emitting from the source point, and here from the source point, I draw a line here, and there is a screen. This is say point P naught, and here I place my opaque screen here. So this black line shows that I am putting an opaque screen here. So this portion of the wave is not coming here anymore. (Refer slide time: 19:09)



This portion of the wave is coming here. So this black region is the region of semi-infinite opaque screen. Now if I want to see what is happening at point p1, which is say a distance. So suppose from this direction it says y and this is y m this distance p to p1. So the light is the wave that is coming. So this is the source point from here to here say this is a and from this to this length is say p and ap is say q that this length is q. So the ray that is coming one is from here to here and another ray, if I join, is coming like this. So suppose this is point R. So the number of half periods in the region in AR depends on what is the path difference between AP and what is the path difference between AP1 and RP1. So that basically tells us whether we are going to get a maximum or minimum depending on how many half-period zones are there. So the number of T half periods in AR. depends on the path difference, and this path difference is AP1 minus AP0 ah AP1, not AP0, I am just looking at this. So there is a mistake, AP1 to RP1 so R erased everything. So, path difference, let me write it here, AP1 minus RP1, more of that, so that we can calculate. So let us do that, so ap1 is equal to q square plus y m square, and this appears once again. So let me erase this, and sp sp1 is similar I can write p plus q square of that plus ym square. So ap1 is nearly equal to because q is large compared to y. So I can write q plus ym square divided by 2 cubes, and sp1 is similarly nearly equal to p plus q plus ym square divided by 2 into p plus q. So r p1 with the simple geometry I can write this is s p1 minus p, so essentially it is q plus ym square divided by 2 of p plus q. So, the path difference is the delta which is AP1 minus RP1. I can write it as this is YM square 1 by 2 cube minus 1 divided by 2 of p plus q and then I can have Delta equal to, sorry this is y p m, so if I simplify it's a, 2q 2q will cancel out. So essentially you have this y m square, then 2 of p divided by 4 P plus q multiplied by q, which is square P divided by 2q and then P plus q. So I find the value of path difference in terms of this PQ, and then now, it is the condition for maxima. So at maxima at P1 to get the maxima at P1 or the maximum brightness at P1. So this path difference delta has to be 2m plus 1 into lambda by 2. From Fresnel's zone construction, we know that the number of half periods I mean if the path difference is lambda by 2, with an odd number of multiplication, then we have an odd number of zones and then we have a brightness that is the condition we are having here.

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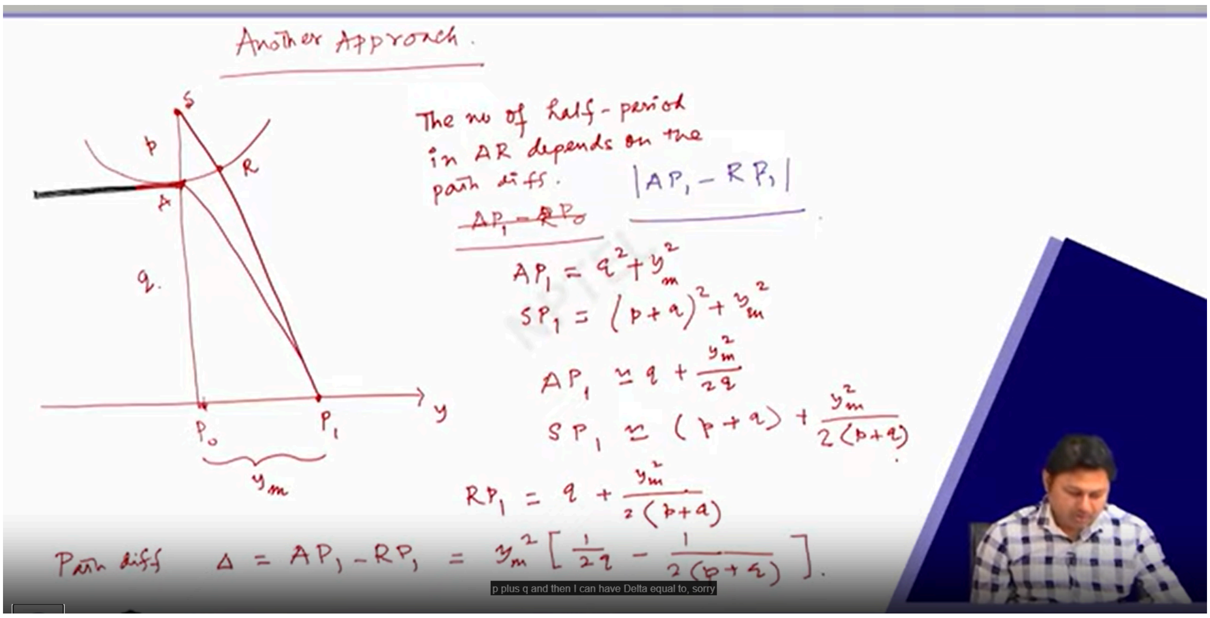

So that should be equal to y m square and then p divided by 2 q and then p plus q we have this. So from here, we can calculate what is the ym for getting the maximum. So, ym is the length at which I will get the maxima is essentially 2m Plus 1 multiplied by lambda into that quantity, which is Q multiplied by P plus Q divided by P and whole to the power half. This is the length T at which this is p1, this is p naught where we get the maximum, and now you can see that by increasing the value of m by taking other terms constant because that is you know what was our notation. Let me go, that is q and p that is constant. So this is q, so all these are constant, lambda constant. So if I increase the m then we will get this y m for which we will get maxima. Similarly for minima at say P1, if I want to have a minima, then Ym should be equal to 2 m into lambda q p plus q divided by p whole to the power half. So, again we will get a value for y m if that is a minimum. So, you can see that alternatively we can have a minima maxima etcetera over this length. Now if we see for mth minima, so this is the structure we had. So this is the point where we had the minima, and this is the point where we have the maxima, so for mth minima we have ym is root over of m. Okay let me go back and see y m is root over of m and then root over of 2 delta, where delta I write this quantity which we say constant, which is lambda multiplied by q then p plus q divided by p and m plus 1th minima, which is y m plus 1, it should be root over of m plus 1 into root over of 2 delta. So, I can have y m plus 1 squared minus y m square is equal to 2 of delta or ym plus 1 minus ym is nearly equal to 2 delta divided by 2 of ym because ym plus 1 is almost very close to 2 of ym and again ym is root over of that quantity, so I can have 2 of delta divided by 2 of root over of m and root over of 2 of delta. So it is simply the root over of the delta divided by the root over of 2m, delta is a constant, M is the order. And what is this quantity? This quantity is nothing but the fringe width beta. Beta is a fringe width and you can see that now I need to put a m suffix here because the fringe width that is from here to here this distance is no longer constant. It is a function of m. So if I calculate beta m which is equal to root over of delta divided by 2m, mth order fringe if I calculate then, from that calculation if p q, p is the distance between source to this, and q is the distance between the opaque screen and the screen where we try to find out the intensity distribution. If we know everything then from here you can in principle calculate lambda the wavelength. So, like a single-slit experiment or double-slit experiment by calculating the fringe width you can also calculate here the wavelength of the light that is diffracted by this opaque semi-infinite opaque screen. So, today my time is up. I like to conclude here. So we have almost completed all the topics related to diffraction. So all the topics have been covered. So in the next class, we will start a brand new topic, which is called polarization. Hope you will enjoy the polarization topic. That will be the last topic of this course. So with that note, I would like to conclude here. Thank you very much for your attention and see you in the next class.

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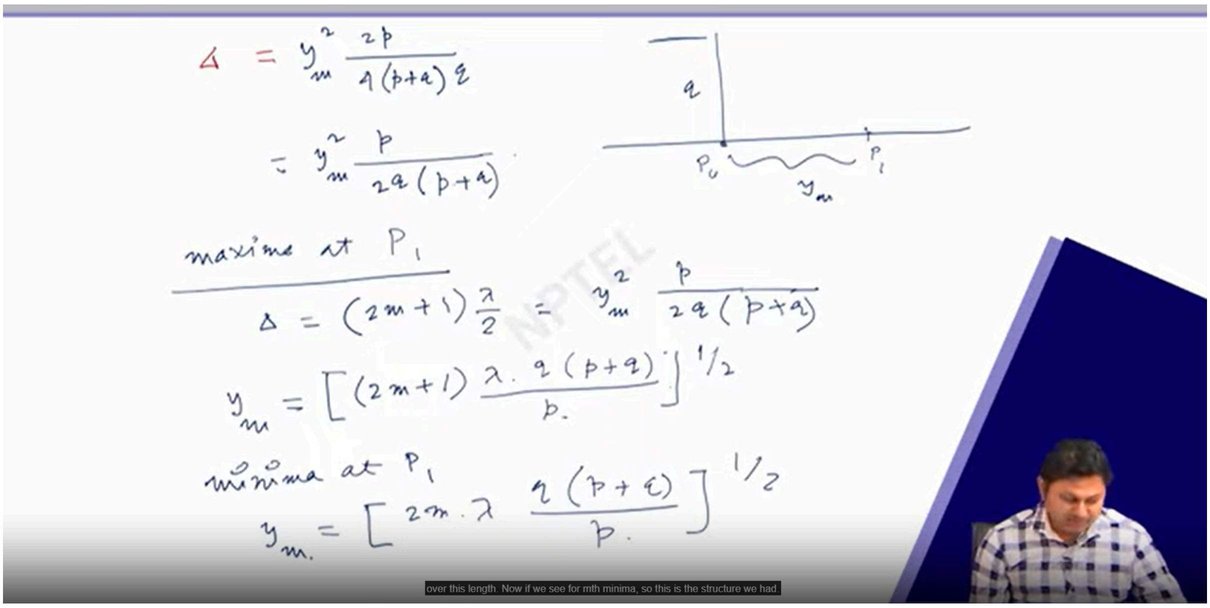

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