WAVE OPTICS
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Lecture - 50: Fresnel's diffraction for a rectangular aperture (cont.)

Hello, student, welcome to the wave optics course. Today we have lecture number 50, and in today's lecture, we are going to continue the understanding of Fresnel's diffraction for a rectangular aperture. So, today we have lecture number 50. And in this lecture, we are going to continue the understanding of Fresnel's integral. So, let me write down what we have done so far in the last class. So, by definition, Fresnel's integrals are like this, c of nu is equal to 0 to nu cos of pi by 2 x square dx and s nu is 0 to nu sine pi by 2 x square dx, by definition that is the thing. Then we find a few more stuff that is c of minus nu was minus of c nu, s of minus nu was minus of s nu, that is another and then we manage to get the limit out of that, and limit nu tends to infinity, c of nu is half and limit nu tends to infinity s of nu is also half, not only that, we also have c of 0 is 0 and nu s of 0 is 0. So these are the properties we tried to evaluate in the last class and we evaluated that and now we will do one thing today and that is to plot this function. So, this is called the parametric representation of this Fresnel's integral and we get a specific kind of structure which we will call the cornu's spiral. So, this is a parametric representation, in this parametric representation what we do, we plot the C nu
and s nu in the x and y axis for different values of nu. We just increase the value of nu and then we find what is the value of c and s and if we plot that we get an interesting-looking curve and this is called Cornu's spiral. So this is how this function will behave. Now after having the knowledge of this function, in the last class, we try to understand this. Now we are in a position to understand what happened. For a rectangular aperture, what should be the case for a rectangular aperture? So let us frame this problem in this way.
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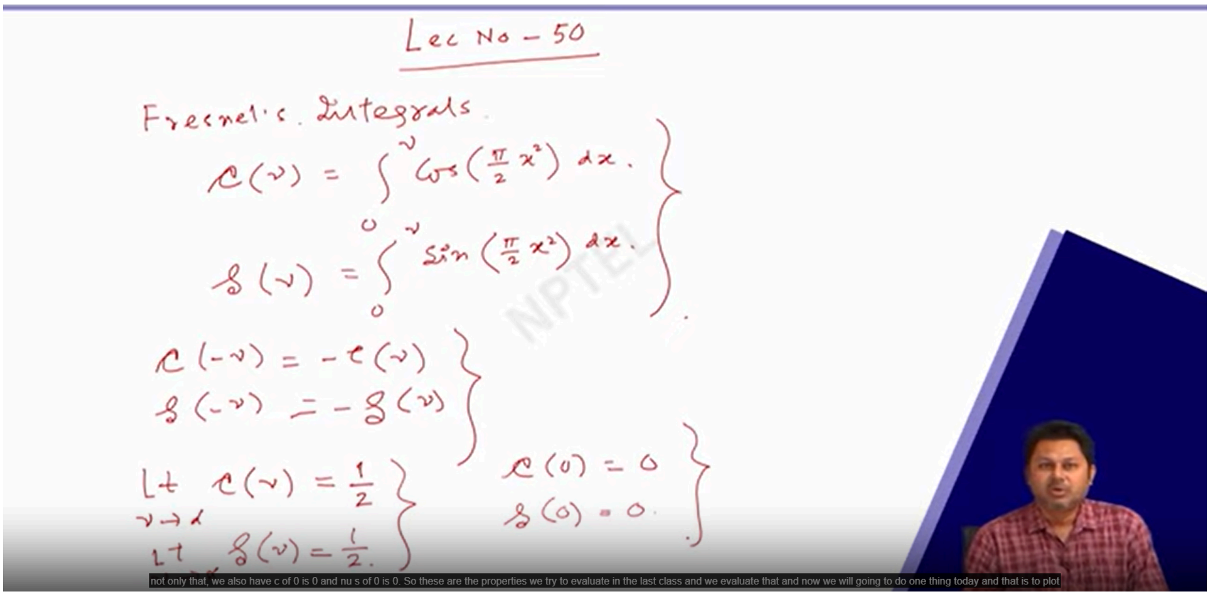

So this is the original problem that we aim for, x-axis y-axis and here we have this rectangular aperture like this is the origin, then we have a small strip here with a width.

Suppose this is width w and this is over a height, small h, and somewhere here we have the source and here the point P over the axis and from here to here I have a source here. So let me draw the source and here the point P and we draw the r here, this is r from here to here, this is, say, r1 because another r is there, this is r2, this is p, from s to this point o. If I write this at o and this is q, o to p this is q, so the equivalent structure I can have is this. So the wavefront will hit this aperture and S is here, P is here, this is P, this is S. Wavefront will go to hit this. So from here to here, we have R1 and from here to here, we have R2 say. And this is the height which is h, this is P, this point is o. So from here to here, this is p and from here to here, this is q, now the field at point p is due to this entire system that we already calculated for this ep that is, the famous proportionality constant that we calculated the intensity, the electric field at the source point s integral over whatever the aperture we have. Then, if theta is the obliquity factor, then, e to the power of i then it should be r1 plus r2 k minus omega t and divided by r1 r2 dA which is the form of Fresnel's expression through which you can get this. Now a few things we are going to do here. First, we neglect this obliquity factor of theta. So that is f theta we will consider as 1. That means if theta is equal to 1, the second thing that we do is R 1 square is essentially P square plus h square, so that is equivalent to P P square into 1 plus h square divided by 2 of p square. Now h is much much less than P; under that condition, I can write r1 to be P plus, h square divided by 2 P. Similarly, r2 will be q plus, h square divided by 2 q. So that is the condition when h by P is very less than 1 and h by q is, it is also having this condition, that simplification we have. Now after doing that, I can write R1 plus R2 is essentially P plus Q plus, h square by 2, then 1 by p plus, 1 by q. So that is my r1 plus r2 with this approximation. We make a few approximations here to make life simple, and those are the approximations we took. Now, let us rename a few parameters like, let D as P plus Q and 1 by L as 1 by P plus 1 by Q, that term was there. So essentially, R 1 plus R 2 essentially will be D plus H square divided by 2 of L, and R1 R2 is P plus H square divided by 2P multiplied by Q plus H square divided by 2 Q. So that is nearly equal to PQ, neglecting the higher order term of H, which is small okay.

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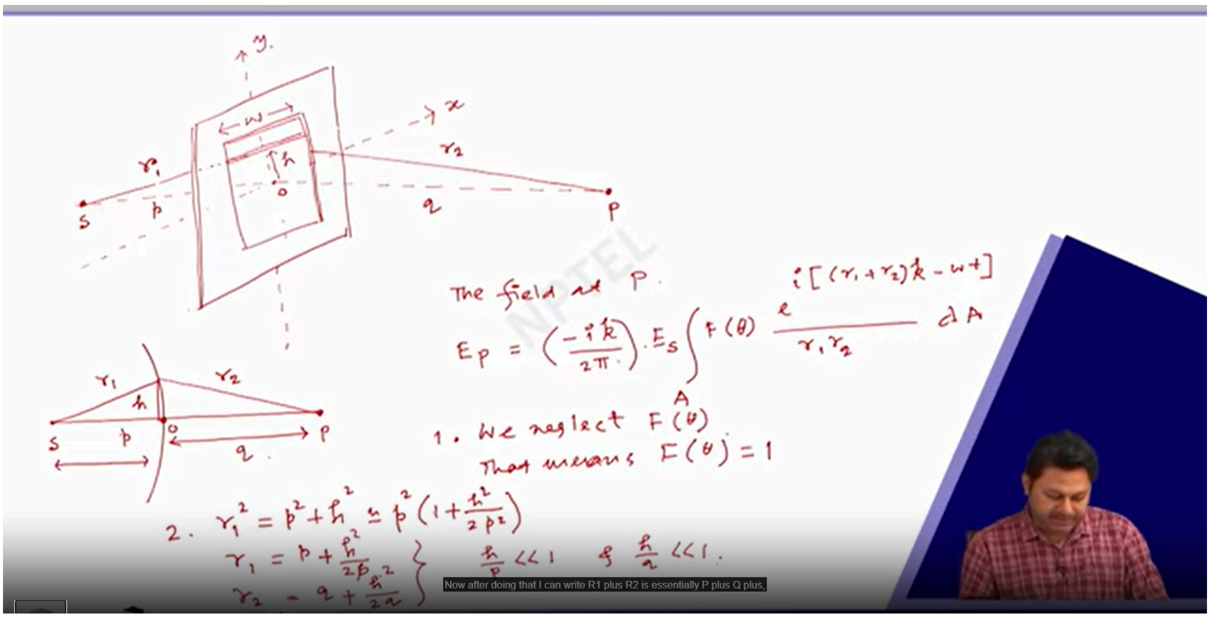

So now, after having that the EP in this notation will be this minus of i k divided by 2 pi, that proportionality constant we figure out, then there should be intensity at the source point s they feed at the source points s, then I have integral over this factor. So, e to the power of i k, then we have this quantity. So, I can simply write d plus h square divided by 2L minus i of omega t, which is a time part. Anyway, it will have nothing to do with the integral, so it will come out then pq over the area, whatever the area, so I didn't put so far the condition of rectangular aperture. So, just the general thing we are doing. So I can write this all together, say, I k 2 pi multiplied by ES divided by PQ, put everything outside and I write it as total field ET then also I take a few terms outside which is not related to this DA like I k D minus omega t. So that is the term I take outside, and in the integral only term that is there is e to the power of i k then h square divided by 2 l d a. So, E t, as I mentioned, E t contains all these terms, minus i k divided by 2 pi, then E is divided by p q okay. These things we consider as E t. Now let us go back to the structure because I need to do this integral here. We only required the structure, and the structure is like this. So we have a rectangular kind of aperture. So suppose this is my aperture, and this aperture is the x-axis, this is y, and we have a small segment here if you remember and that segment is better I said it is distance H and this was W, and these things is DH. After having this geometry, we can write that da is a small area here, which is w into dh, so the integral will be over h. Now let me go back to the expression of the ep, ep was e to the power of i k d minus omega t, that term was there and inside the integral. Now I need to do this integral over h, so I can write h1 to h2 then I write e to the power of i k h square by 2L, then dA, dA is W dH, that is the integral we finally get. Now, I can manipulate a few things here. I can do a few things, for example, here k h square divided by 2L. I write it as 2 pi divided by lambda, which is the value of the k, then h square I write and I put 2 L here. So, this essentially gives me an interesting thing, and that is pi h square divided by lambda L, the same thing I write in a different way. Now let h square is equal to something I write y square lambda L divided by 2, that I replace. Why I replace this specific term one can understand easily, then that gives me h is equal to y root over of lambda L by 2, from here I write 2 h d h is equal to 2 y lambda L by 2 d y or d h is essentially y by 2 h lambda L dy,

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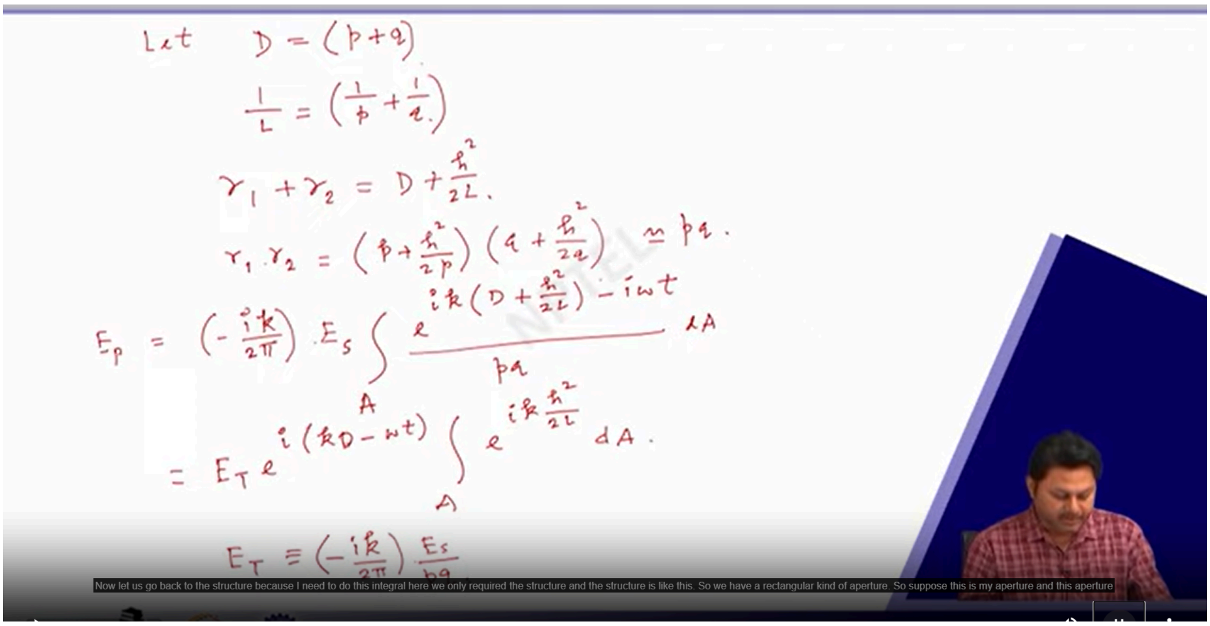

which is y divided by 2 of y because now h, I replace from this value, then I have root over of 2 then root over of lambda L and then we have 1 lambda L here with dy. So from here, I can get my dH is equal to the root over of lambda L by 2 dy, that I get. Now I'm going to replace everything here, and we see a very interesting looking term if that is the case then my ep will be et e to the power of i k d minus omega t, and then we have w outside then this term root over of lambda L by 2 also outside and the integral will be some y 1 to y 2 e to the power of i pi by 2 y square dy. Now, essentially we have the Fresnel's integral. So, Ep, I write this entire thing as A p, another at the amplitude multiplying e t, multiplying w all these things and I write Ap by putting all the terms inside the Ap, then e to the power of i whatever is there k d minus omega t and then I write y1 to y2 cos of pi by 2 y square dy plus i y1 y2 sine of pi by 2 y square dy. So this limit y1 to y2 cos of pi by 2 y square dy can be written as y1 to 0 plus, 0 to y1 so after having this stuff I can change it and that thing it will be 0 to y1 and cos of pi by 2 y square dy plus, so here it is 0 to y 2 pi by 2 y square dy, so essentially, this is C of y1 with a minus sign plus C of y2. So we are almost there. W So after having this, this is the same for sine also. So for rectangular aperture the field that I get is finally, this ap e to the power of i kd minus omega t and then we have c of y2 minus, c of y1 plus i s of y2 minus s of y1. So if that is the field, the intensity at point P, intensity IP will be mod of it is proportional to the mod of EP square then I can write IP is I 0 so when I take mod of this square then the exponential term will go away and essentially what we have is this c of y2 minus c of y1 square of that plus s of y2 minus s of y1 square of that bracket close, so we finally figure out. So we don't have much time to discuss more about this. In the next class, we will exploit this and try to understand a few more stuff about what happened to this aperture. So what we get is the intensity distribution for this aperture. Whatever the aperture we have, say, this is the aperture. So, the intensity distribution here is related to this expression. That expression suggests that the intensity distribution is related to this integral value, which is called the Fresnel's integral. So, in the Fresnel integral, we know that we have C1, C, and S functions. And we have a qualitative idea of how this function behaves. In the next class, we will show, not today, because today my time is up, that using this general expression, what should be the intensity pattern for a rectangular aperture or one-edge system in a sharp edge. How the diffraction pattern will look is not simple because it deals with this function C and S and also here the concept of Cornu's spiral will come into the picture because here you can see this is coming like cy2 minus cy1 whole square and sy2 minus s1 whole square. So, if I put y1 to be 0, then it should be simply cy2 and sy2 square of that. So, that is exactly in Cornu's spiral these two parameters are there. So, with that note, I would like to conclude in today's class. In the next class, we will start from here and try to understand more about how the diffraction pattern depends on this function called the Fresnel's integral. On that note, thank you very much and see you in the next class.

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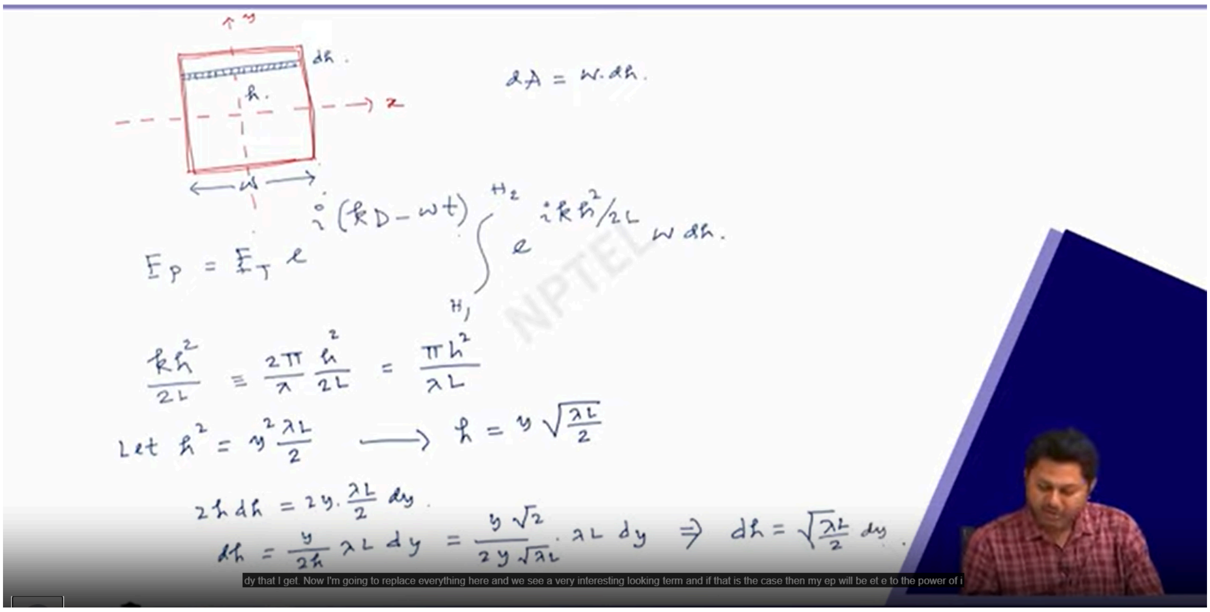

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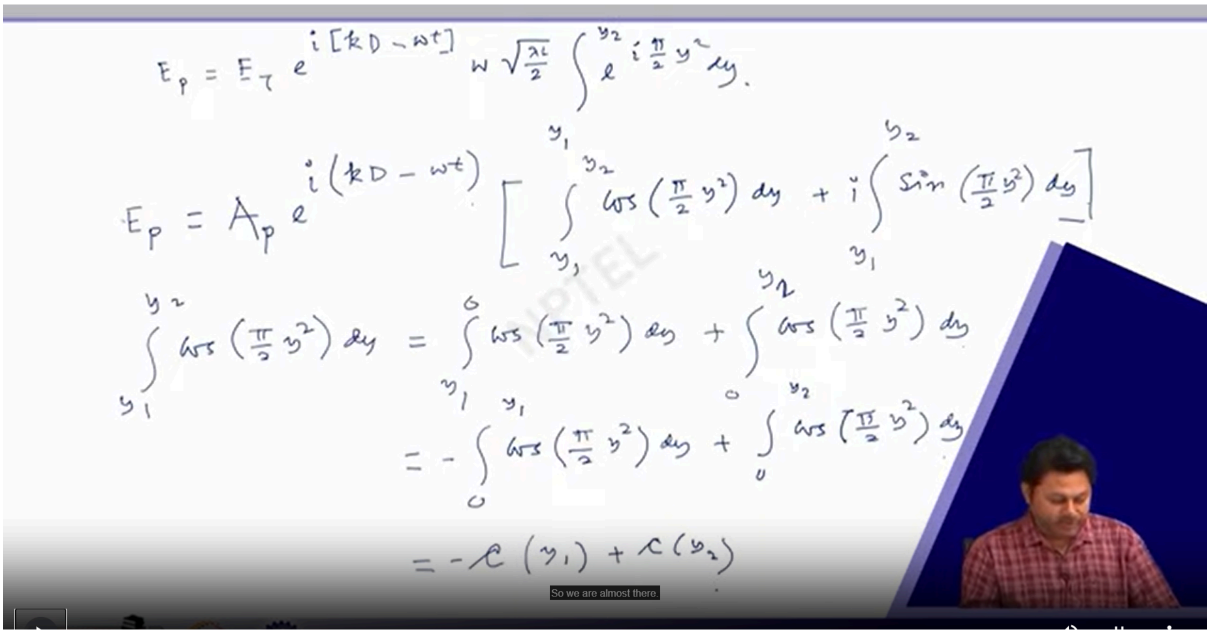

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