WAVE OPTICS

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Lecture - 49: Fresnel's diffraction for a rectangular aperture

Hello, student. Welcome to the wave optics course. Today, we have lecture number 49, and in this lecture, we will discuss Fresnel's diffraction for a rectangular aperture. So we have lecture number 49 today. Before going to the rectangular aperture, let me go back once again to the circular aperture that we had done in the last class. So, our problem was, we have a structure like this, and then we have a circular aperture like this, and then we have a screen here and try to figure out what was the intensity at this point over 0 axis. So this is my small x, small y and this is z equal to 0, but this is z equal to some point z and we calculated in the last class that for this system if I want to find out the intensity at the point over this z-axis, this intensity was I, which is calculated over this z-axis. So 0, 0, z, that is something i naught and sine square pi by 2 p, a parameter called p, where the parameter p was a square divided by lambda z, where z is a distance, and a is the radius of this aperture and lambda is a wavelength. So that was the final expression we got: what is the value and then we plotted that a typical-looking plot was there and the plot was divided by I naught. So, that is why this value was. And then we plot it as a function of 1 by P, not P, but 1 by P to show how it changes as a function of z. And then we have maxima and minima, then maxima and minima, something like this we got. Now let us go back to this problem and try to understand these things. Yesterday we tried to discuss this. So suppose this is the aperture we are talking about and this is the z axis, and this is the point p over the z axis. Now if I join this, so this is 0 or o, and if I join this then, this is the value a, which is the radius of this aperture, and this point if I write a, then we can have a relation

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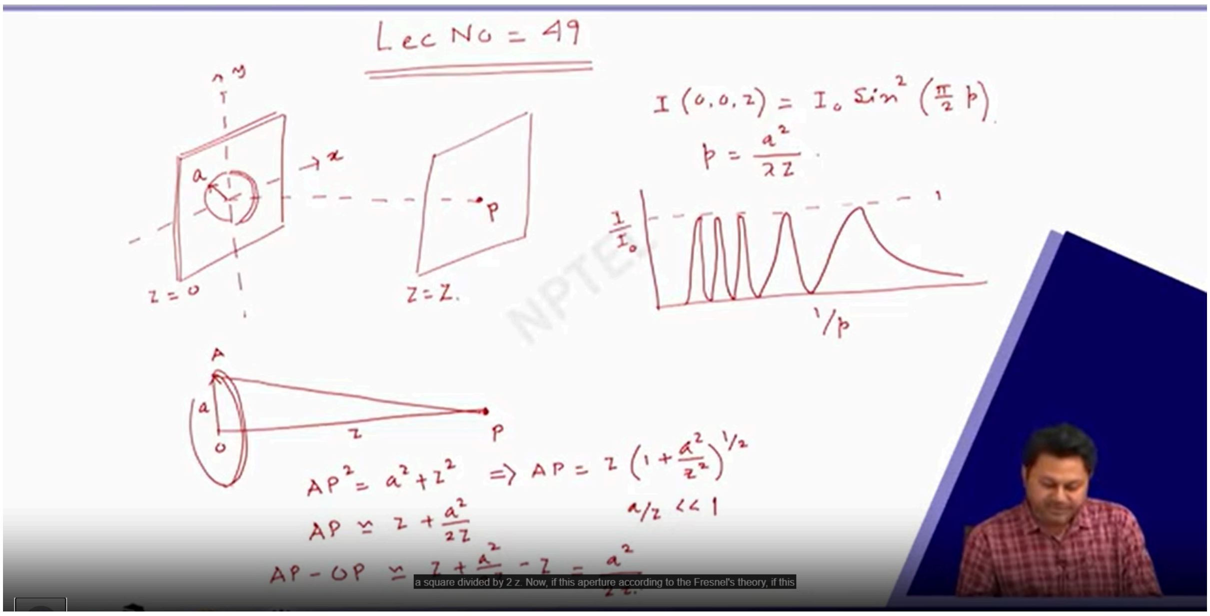

a p square of that is equal to a square plus, op square which is our z. So essentially it is z square, now a is much smaller than z, so with this, I can write ap is nearly equal to z plus a square divided by 2 z, the usual relation we use. From here, we can write AP is equal to Z1 plus A square Z square whole to the power half. And then we make an expansion of these things and we are going to get this under the condition that A divided by Z is much much less than 1. Now, if I want to find out what is the path difference between AP and OP, which is nearly equal to z plus a square by 2z minus z, then this value is essentially a square divided by 2 z. Now, if this aperture, according to Fresnel's theory, if this aperture contains n number of half period zones, then this path difference, whatever the path difference we have, is AP minus OP should be equal to N of lambda by 2. This is according to Fresnel's theory, and this value is simply a square divided by 2 of z. So, from this calculation, what we get is very interesting, and that is n the value, which is a number of zones is equal to a square divided by lambda Z, which is essentially equal to the parameter P we already defined during the calculation. So, what we get in the previous calculation P, essentially represents the number of half-period zones S that is contained by this aperture, whatever the aperture we have. Now note that the intensity distribution was related to this number. So it is I naught sine square pi by 2, then p that was the relation we got. Now we can see from this expression that when p goes to 2, 4, 6, etcetera, that means the aperture, which essentially means that, when this is 0 then i is essentially 0. So let me put that first i is essentially 0 under those conditions, and for that actually, that means the aperture contains an even number of zones which is true. When the apertures contain an even number of zones, then this individual zone should cancel out their individual contribution and essentially, we get a very low intensity that we also get from this calculation. When again when P is equal to 1, 3, 5, then I is equal to I0, which is the maximum value. So that means here the aperture contains the odd number of zones and we also calculate from that the radius of the in half period zone and the nth of the period zone that is R n square divided by 2 z, which is nearly equal to n lambda by 2 and from here we can see that Rn which is already calculated is root over of n lambda z to the power half. So that was the radius of the nth half pure zone.

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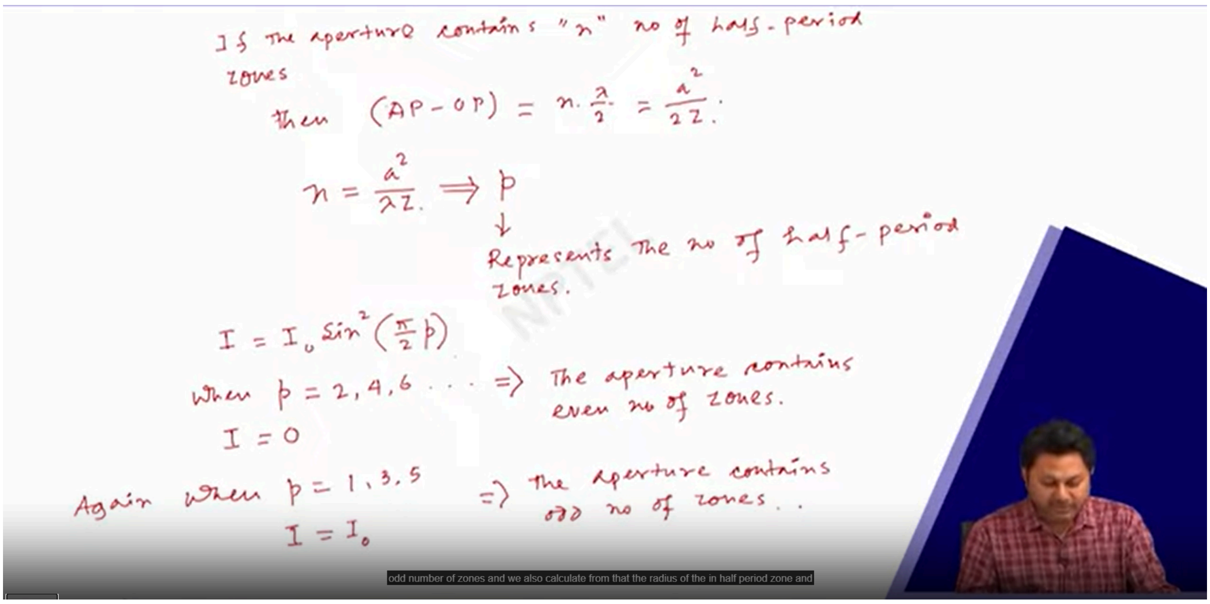

So that means if p is related to the number of zones and p is a square divided by, so p is here a square divided by 2 of z that is this A. So, if I reduce this Z, then P will increase and if P increases, that means the number of zones will increase. If the number of zones increases from 2 to 4, then what happens, we will get consecutively dark and bright fringes. Z decreases if I go to this side, then P increases. But if I go to that side, the Z increases, P is going to decrease. So, I can get the value of the P value from some value to decrease. So, we will get either integer or in certain points either 1, then 3, then 5, then 1 if I go in this direction. So, that means p is increasing by 1, and then we get 2, then 3, then 4. So, this value gradually we, this value is going to change and at each point, we have a different value intensity in one case it is maxima, in other cases it is minima, exactly the same thing we calculated by expanding the theory where I get the intensity is really changing with certain function and this function reads this with that value of p. So these things are correlated to each other, here during the calculation we got the value of p but p is essentially the number of zones that one can have for a given z. So that is the thing I wanted to discuss today before discussing the diffraction for rectangular aperture. So let me go back to today's topic: what happened to a rectangular aperture? So the aperture is now not circular but something like this, and we have, like before we have x and y coordinates and we have a screen here. And then, due to this aperture, we try to find out what the distribution here one can expect with a different coordinate big X, big Y. Now, before going to this calculation, we need to know a few things, and we will first introduce and the thing is called the Fresnel integral and this Fresnel's integral by definition suggest curly c, which is a function of nu is equal to integral 0 to nu cos function pi by 2 x square dx, that is differential integral, by definition we call it c and another is called s, and by definition, it is 0 to nu sine pi by 2 x square dx. So these two integrals are called the Fresnel integral and this is by definition that is the function. Now if I look carefully at this function, a few things that we can comment on and that is the integrals are even functions of, S nu. So I can write c of minus nu is equal to minus of c nu and s of minus nu is equal to minus of s nu, that is the relation we have.

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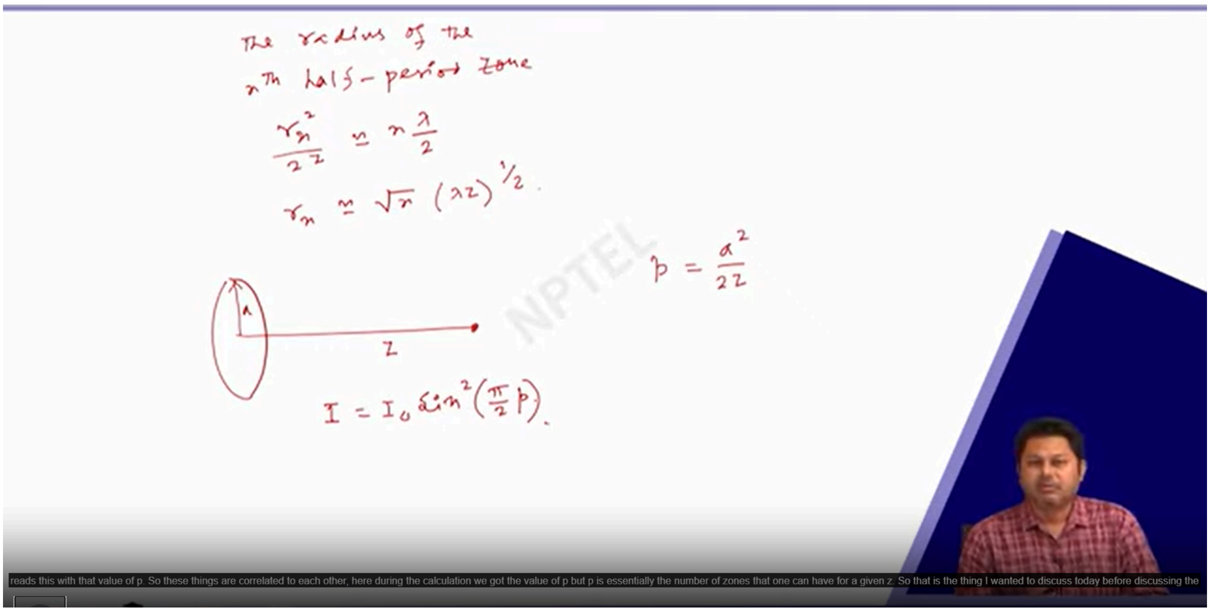

Also, I like to show these things that are an integral part of this, we have already shown. So minus infinity to infinity e to the power of minus mu x square dx is essentially root over of pi divided by mu, from the knowledge of gamma function one can do this integral very quickly, and they can understand very nicely how this integral has this form, but this is the value of this integral. Now if that is the case we can write it as minus infinity to infinity e to the power of i pi by 2 x square then we have dx that value essentially should be pi divided by minus i pi by 2, or it is root 2i. If I multiply 1i in the denominator and numerator, then this value minus i into i will become 1, pi and pi will cancel out, so it should be 2i. Now, this 2i is essentially root over of 2, then e to the power root over i is written by e to the power of pi by 4. And we have root over 2, 1 by 2, 1 over root over 2, then i this which is essentially 1 plus i. Now, if I look at the left-hand side, then the left-hand side is interesting because the left-hand side gives us this. If I expand this, this gives us the value of Fresnel's integral. So, let me do that. So, minus infinity to infinity, e to the power of i pi by 2 x square dx that is essentially 2 0 to infinity e to the power of i pi by 2 x square dx, and that quantity if I have it is 2 0 to infinity cos of pi by 2 x square dx plus i integral 0 to infinity sine pi by 2 x square dx bracket close. So essentially this is 2, and this value is Fresnel's integral C nu, not nu because here the limit is infinity, so it should be C infinity. This is the identity we are doing here C infinity plus I s infinity. And that quantity is essentially equal to 1 plus I. So that means if I write it C of infinity plus I S of infinity that is essentially half plus i of half, so I get a value for this integral extreme value c infinity is equal to half and s infinity is equal to half, that is one set, another also c of 0 will be 0 and s of 0 will also be 0. So, this identity we show in today's class. I do not have much time to go beyond that. So, in the next class, I will start from here and try to discuss more about Fresnel's integral. And then what will we do? We are going to exploit this expression to find out what the intensity distribution is, and how the intensity distribution looks for a rectangular aperture. With that note, I would like to conclude here. Thank you very much for your attention and see you in the next class.

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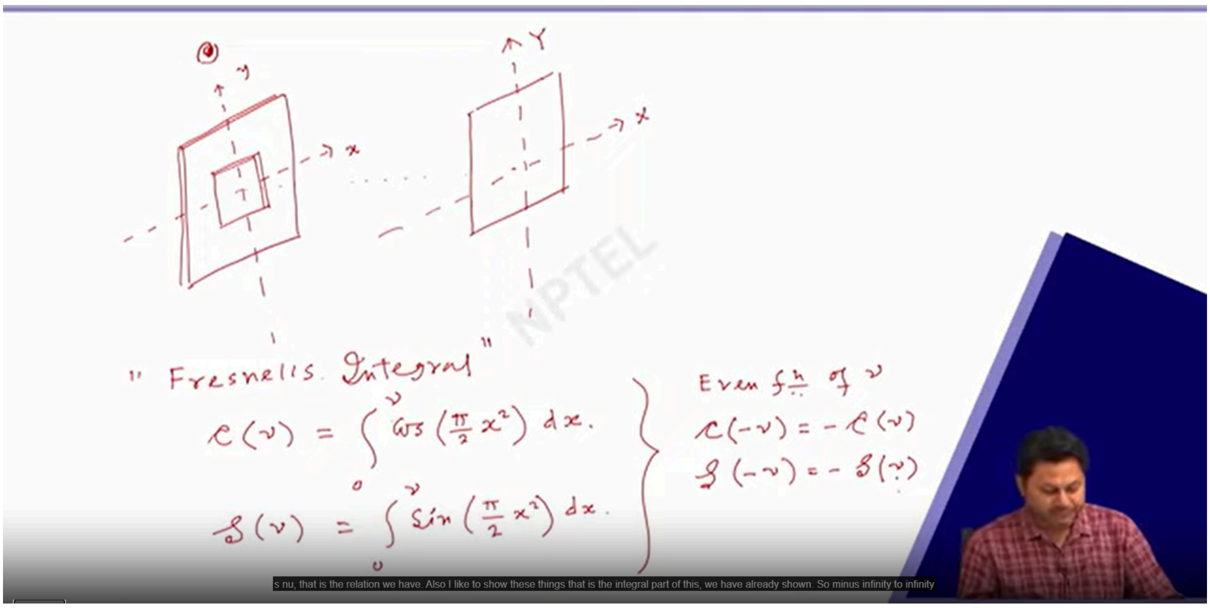

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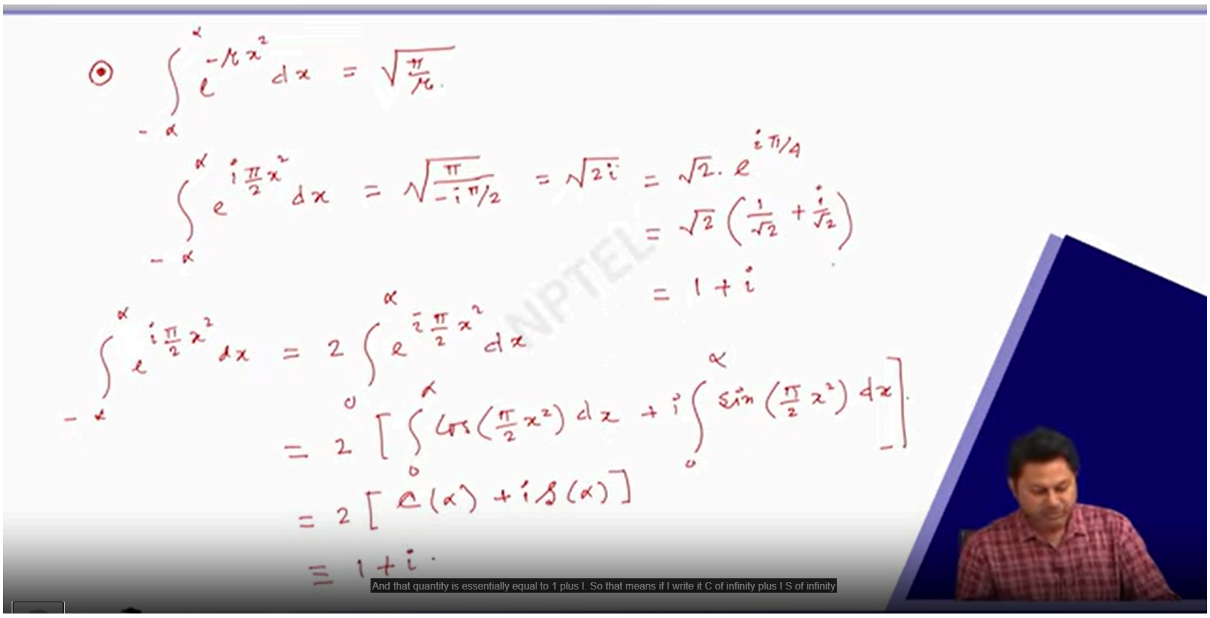

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