WAVE OPTICS

Prof. Samudra Roy
Department of Physics
Indian Institute of Technology Kharagpur

Lecture - 48: Fresnel's diffraction for a circular aperture

Hello, students, welcome to the wave optics course. Today we have lecture number 48 and today we are going to do a specific problem, where Fresnel's diffraction is calculated for a circular aperture. So we have lecture number 48 and in today's lecture, we're going to calculate the Fresnel's diffraction for a circular aperture. So let me draw the setup, In the last class we had demonstrated a similar kind of setup but the aperture was arbitrary and I figured out a very important parameter which is the constant. So this is small x, small y and here we have the aperture. So this is the aperture but this aperture has a specific shape and is circular in nature. So some sort of symmetry we can expect from this problem and then we have a screen here, and there is a point P over the screen. This coordinate is x y and z, z coordinate here is zero, exactly the same treatment I'm going to follow. So if that is the case, we know that my dep was proportional to e naught divided by r e to the power of i kr, I can also include the omega t term because this is essentially a propagating wave, but in the last calculation we omit that even if you put this there will be no effect but here for the completeness, let us put and dx dy r we calculated. So r was this z plus x minus, small x square divided by 2 of z plus. So, DEP is essentially, now if I remove this proportionality constant, this proportional sign to make a proportionality constant and that proportionality constant we calculated in the last class, which was minus of i k divided by 2 pi, an interesting looking term and then we have E naught, then executing all these we get exactly the same calculation that we put forward in the last class except this omega t term but doesn't matter this even if you put the omega t term here it will give you the same thing and

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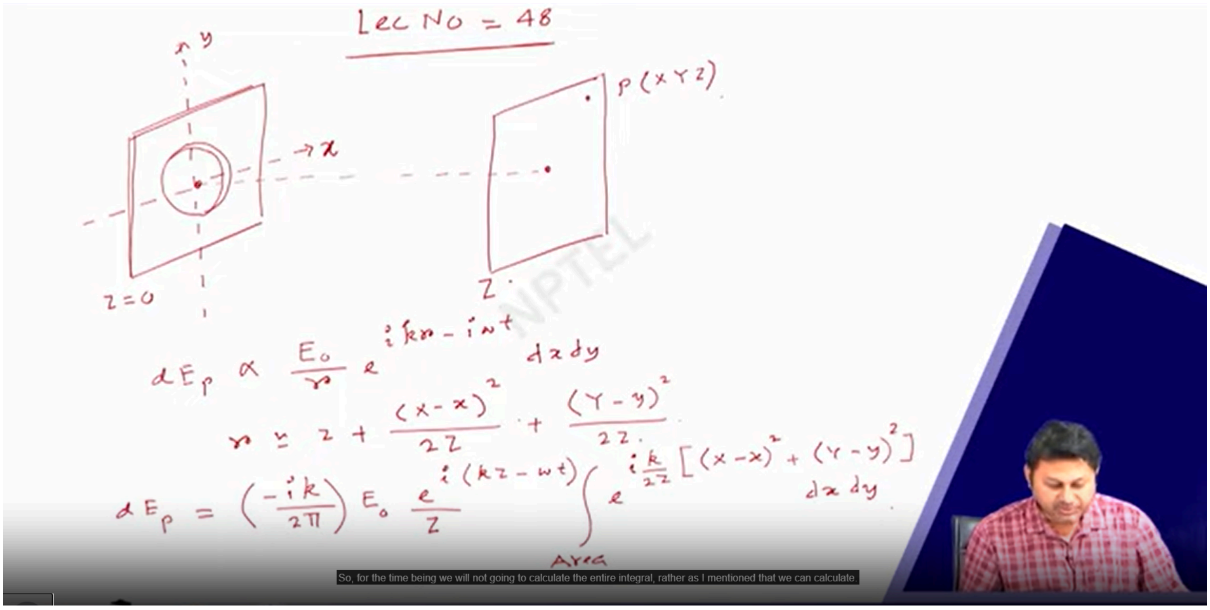

over an area and then we have e to the power of i then that thing we need to execute. So, this integral, whatever the integral we have, is a bit difficult to perform analytically. So, this is the integral. So, to make life simple, what do we do? We calculate everything over the z-axis. That is the simplification we can make. So, this integral is not going to be calculated because everything here is going to change. So, instead of that, we restrict our calculation to the variation of the z. So, this is the point where we have x y is equal to 0, and so if I put a dotted line towards this point, so here the x y is 0, but z is equal to z. So this is a z plane. So we want to calculate over z, if I move the screen over z how these things are going to change? So, for the time being, we will not be going to calculate the entire integral; rather as I mentioned we can calculate. So, the intensity variation we will look over the z-axis, this intensity variation over the z-axis we want to check first. So what we can do by making that means what we are doing is x putting this coordinate x and y equal to zero because I am looking only over this z-axis. As soon as I do that, my integral becomes very simple, and I can write that the total E for the point 0, 0, z is essentially i k divided by 2 pi with a negative sign. However, then E naught, and then we have e to the power of i k z minus omega t over z, and then we have the integral e to the power of i k divided by 2z and small x square plus y square and dx dy. That is the thing we need to execute. Now, for circular symmetry, we have a few advantages, and that is this. So, that was the aperture we had. So, over the aperture, the x, y point, we can write as, so this is my phi and this is my rho. So, x can be simply rho cos phi and y can be simply rho sin phi such that x square plus y square become simply rho square and the area will be rho d rho, d phi. So, this area integral will be like this e to the power of i k divided by 2 z and then rho square, and then we have rho d rho, d phi, that is the area. So I can divide it into two parts one is the integral of phi, which goes 0 to 2 pi, which is d phi, and another integral, which we can go 0 to some area. If the radius of the circular aperture is a to 0 to a e to the power of i, then k is divided by 2z and then we have rho square, rho d rho that we have. Now, this first term is simple. It is simply 2 pi, but I can simplify it by taking k rho square divided by 2 z as some new variable say q. So, we have k rho divided by Z rho will be dq such that rho d rho is equal to Z by K and dq.

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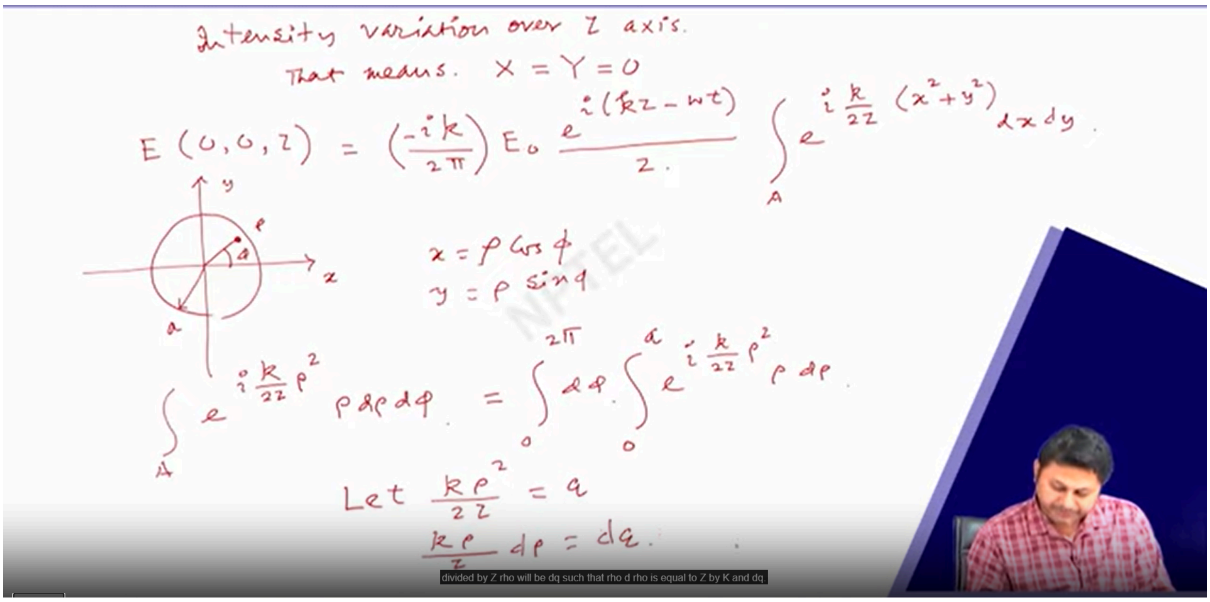
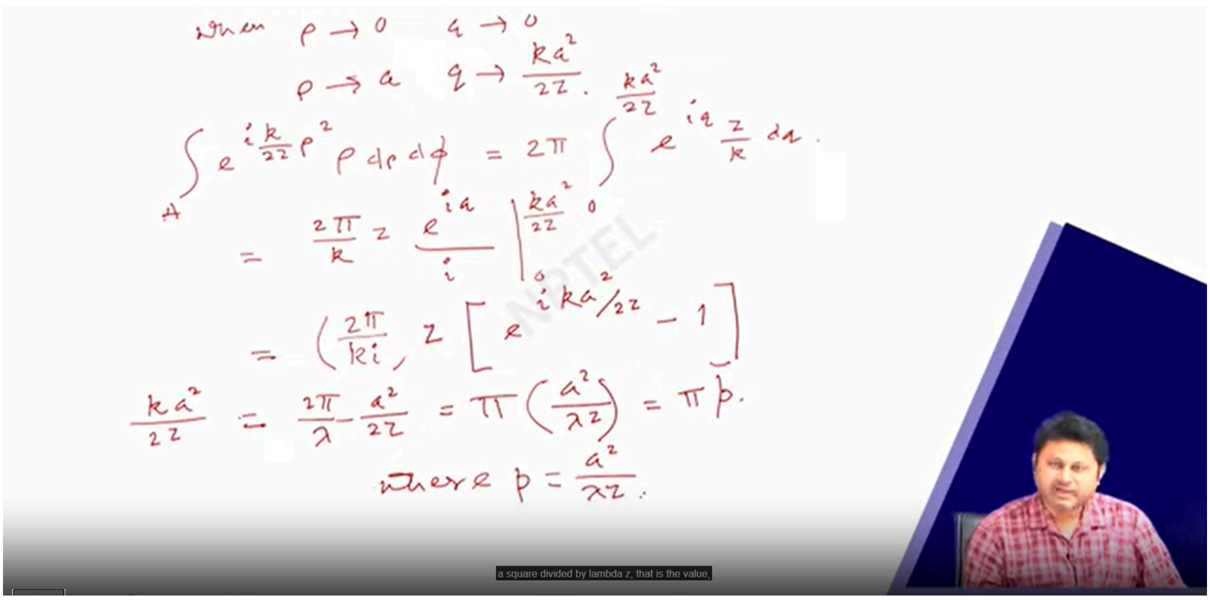


So, also the limit needs to be changed. So, when rho tends to 0, q also tends to 0. When rho tends to an upper limit, q tends to simply k a square divided by 2 of z, so that is the value we have. Now, putting all this together this area integral e to the power of i k 2 z rho square, rho d, rho d phi that is essentially 2 pi integration of 0 to this value k a square upper limit divided by 2 z and then e to the power of iq then z by k d cube. So this is simply 2 pi divided by k and z then e to the power of i q divided by i with the limit 0 to k square divided by 2 z. So, this is 2 pi divided by k i. Note that this beautiful term again appears here which is 1 by alpha actually with a negative sign, then z is here. Then we have, e to the power of i k square by 2 z minus 1 just put the limit that's all, so let us put k a square by 2 z is essentially k is 2 pi divided by lambda and then we have a square divided by 2 z. So these two will cancel out, and essentially we have pi multiplied by a square divided by lambda z, and I write it pi into another variable p , where p is a square divided by lambda z, that is the value, I just put the new variable. Under this new variable, however, this expression that is the integral of this area is simply comes out to be this 2 pi divided by i k, then we have z multiplier, then we have e to the power of i pi p minus 1 with the new variable p, and we can manipulate e to the power ip and we get a something, so let me do that, so this is 2 pi divided by ik z, and I can take e to the power of i pi p by 2 common and if I do, then I will going to get e to the power of pi p by 2 minus e to the power of minus ip by 2, which is essentially 2 of i of sine pi P by 2, that we get. So once we have this integral for this area, mind it from where we started this calculation and this calculation was started here, so this area calculation we were doing so far. So, E essentially comes up to be this. So my E is over the z-axis which means 0, 0, z this is the minus of i k divided by 2 pi which is the value we calculated in the last class. That is the value of the alpha proportionally constant, then we had e naught e to the power of i kz minus omega t propagating wave and then finally the integral due to this area. So we have 2 pi divided by ik, and then we have a z here, and then we have 2 of i here, and e to the power of i pi p divided by 2 is also there, and we have a term sine pi p by 2, this is the term we have. Now intensity, so you can see that, 2 pi i k, i k 2 pi will cancel out, 1 minus i will be there but intensity i at the point z this is the point we are talking about 0, 0, z is proportional to the mod of e evaluated as 0, 0, Z mod square of that. (Refer slide time: 17:50)



If that is the case we have a very interesting-looking expression, and that is I is equal to some constant I naught, and all this term which is having I will cancel out, and essentially we have sine square here. We have divided by the z term because that is important. Let me go back and check it because here we have divided by z. So, then this z actually whatever is there will cancel out. And then, if I take a mod sign, everything will go away except this sine square pi by 2 p term, mind it, where P is equivalent to a square divided by lambda z that we replace. We replace this term. Let me check, yeah, P is a square divided by lambda z. So, that term, so, here if I plot this, we are going to get an interesting kind of plot and how the intensity will change over Z point that is I am plotting now. I which is a function of Z only divided by I naught make it normalized. So this is the value of 1 which is the highest value of sine. So the variation of the intensity is like if I plot as a function of 1 by P because 1 by P corresponds to the Z lambda with some scaling, so that is equivalent to the lambda divided by S square and Z. So that means by making these apertures the same lambda S square is constant and I'm moving this point. So what I'm getting is that the intensity distribution has a curve like this. It is a different curve to plot actually, gradually these periods are going to change, it will be something like this. So the variation of the intensity over this, now what is the meaning of that? Let me go back and check so, so what we are doing is, this aperture which has a circular structure and here we are placing the point P and now we are changing this point to this problem. However, we have already discussed qualitatively when we are discussing the Fresnel's zone. So what we are doing when we are changing this point p these whatever the value was there the distance r naught if you remember with this notation r naught then the area of the zones were depending on r naught and what happened that if I move towards this aperture or move away from this aperture what happened that these zones will be going to change and because of that we will going to get a circular a bright, maxima, minima, maxima, minima, this kind of result for different points. If I move here, maybe we have more. So the zone area for example, the radius of the zone, if you remember, was the root over of n pi and n r0, sorry, n r0 lambda, that was the value.

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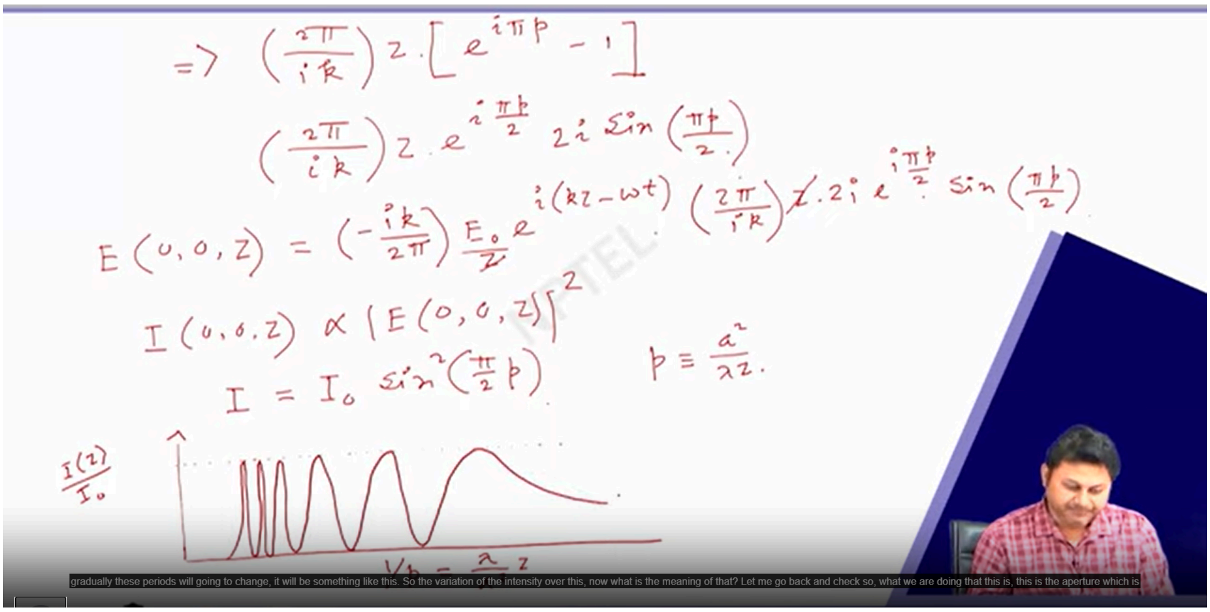

Now, if I change my r0, every time then the number of zones, the area of the zones also going to change and if it changes, then we have sometimes an odd number of zones and sometimes we have even number of zones depending on which point we are talking about suppose this is p1, this is p2 etcetera. We move towards the aperture or away from this aperture. So based on what we are getting different zones will be exposed and in this case, we can get maxima and minima alternatively this kind of point is exactly a similar result at least qualitatively we derived today. Where, when you plot I z that is intensity over this, this is my z axis divided by I naught over 1 by p which is proportional to z actually. So, this is actually lambda z by a square. We are getting a result that corresponds to a similar feeling. So, we have maxima, then minima, then maxima, and minima over this distance. So, that means this is the distance over which we are moving and alternatively maxima and minima we are getting. So, this result is very nice whatever the result. So, I do not have much time to discuss more. Maybe in the next class, I will do it. So, what I tried to show in today's class is that by doing a rigorous calculation based on Fresnel's law or Fresnel's expression that is finding the intensity by doing the integral, we come across a similar qualitative result that already discussed in this particular class, when we are discussing Fresnel's zones. So zones are a different concept where we divide this particular wavefront to different zones and allow the wavefront to propagate. Then essentially from a given point, we are looking at those zones, all odd and even zones are adding up. So, if there are odd even zones together, then we find that the intensity goes minima and we calculate that qualitatively. In today's class what we find is exactly whatever the concept we have for zones, a similar kind of thing one can calculate by doing the integral thing. However we don't do the integral for the entire region, only we consider what is happening over the z axis and we find a very concrete result showing a similar kind of structure. With that note I would like to conclude, so, in the next class, we will do more. We will calculate a few more things related to that. Thank you very much for your attention and see you in the next class.

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