WAVE OPTICS

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Lecture - 47: Fresnel's diffraction from an aperture

Hello, student. Welcome to the wave optics course. Today we have lecture number 47, and in today's lecture we are going to discuss the diffraction phenomena for an arbitrary aperture and how it happens in Fresnel's case. So, today we have lecture number 47, and today we are going to discuss how Fresnel diffraction happens for an arbitrary aperture. So, before doing the specific case, we need to do this to understand a few aspects. So let us do that. So suppose we have a system, where this is an arbitrary aperture and this is a coordinate system. This is the x y coordinate system, where we have an aperture that is placed like this and then we have a screen here. This is the screen where the pattern will go to form, and also, there is a coordinate here, which is big x y. So if I have, here z is 0, it is some z point, so the coordinate of any point p over this screen is simply X Y, and Z. Now I have a point here, X and Y coordinate a small section here and if I join this point P, this is say our prime and from this center to the point P this is r, this is the total picture we have. Now if a plane wave is incident normally on this diffracting aperture which is shown here, we say it is a, for example, this aperture is a. So when the plane wave is falling here then what happened? The light will be going to diffract here and the field at point p due to the small area dx dy, is a small area over this aperture. I write it dx dy. It should be dEp. This is the field due to this small area dx dy at the point P, we did it earlier. It should be e 0 divided by r e to the power of i kr and dx dy, where our r is simply x minus small x square plus, and to the power half, that should be the value of r if the coordinate of this point is x y and z equal to 0.

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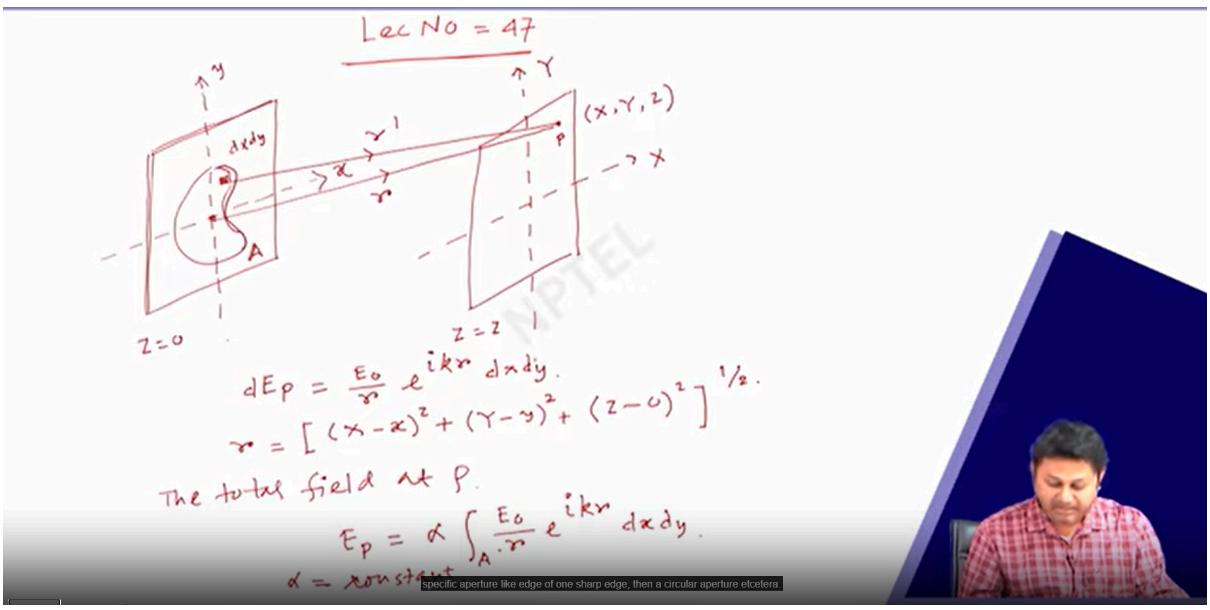

Now, the total field, if I want to calculate at P, will be simply the total field is EP that will be some proportionality constant and then I need to integrate where alpha is a constant. So

essentially, we need this integration over this full aperture area A. So the essential thing is to execute this integral for this particular problem. But before that, we can try to find out what should be the alpha with sudden boundary conditions, and that is an interesting kind of approach; we're going to use these things for other structure regular apertures. This is an arbitrary aperture but a specific aperture like the edge of one sharp edge, then a circular aperture etcetera. But before that, it is interesting to find this proportionality constant alpha. So now in this class, we are going to find out how to get this alpha. So the determination of alpha, so we have ep equal to this constant, that we had. Now note that in the absence of aperture A, suppose there is no aperture at all. The plane wave simply moves through this system and then EP will be simply a plane wave, and EP can be written as E naught, e to the power of i kr, this is a plane wave, and we can have this case when there is no aperture, that is. I just simply remove this aperture that is placed in the x, y, z equal to 0, it is removed. That means I gradually increase the area of this aperture. This area tends to be infinite, which is the condition I put here. So that means if that is the case, then we can simply write from here that alpha integration, I change this area whatever the area from minus infinity to plus infinity, that means there is no aperture at all physically, then this integration will be minus infinity to infinity, minus infinity to infinity, please note that this is alpha and this is infinity looks same then we have E naught divided by r, e to the power of ikr, then we have dx dy. Based on that condition, I can say that this value, the left-hand sight, should be equal to simply e0 e to the power of ikr because at the end of the day, what we have is not kr, it is kz because along the z direction the plane wave is moving. So that is the boundary condition we have, and from this condition whatever the condition is written here, we can calculate the value of alpha. Exploiting this expression, this condition, or this equation, I can calculate alpha. So, R I can write, let us write R here. If you remember, it was big X minus x whole square, plus big Y minus y whole square, plus Z square. So, if I take Z common, then it should be Z 1 plus bx minus x square divided by z square, plus whole to the power half.
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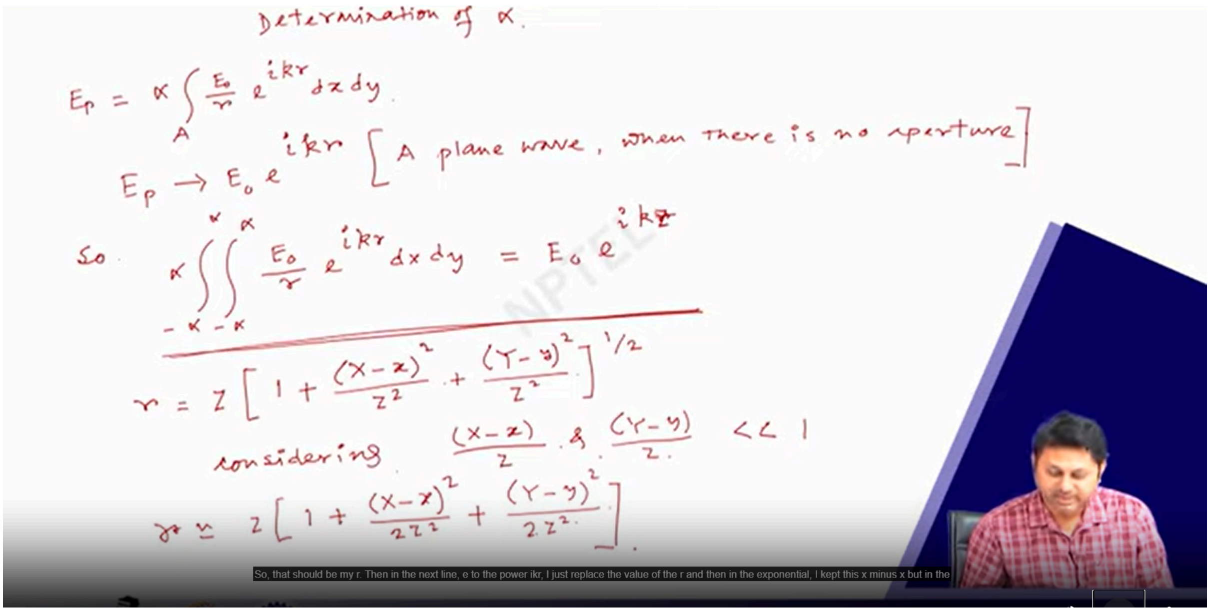

Now, this is much, I mean, considering x minus x divided by z and y minus y divided by z, is much much less than 1. So x minus x is the difference between two points, and z is the difference distance between, see if I go back to this figure x minus, this is the difference between this coordinate and whatever the coordinate we have. So for all the points here this value x minus x or y minus y should be very, very less compared to the z. If I try to get it in the far distance, then z is much much bigger than this. So if I consider this condition, maybe we can go to, then we can have e to the power of i kr divided by r is nearly equal to e to the power of i kz divided by z into, so 1 by r term I just simply replace by z, with that condition but the rest of the term I keep it in exponential. p So, If I go back to what we did here? That then r will be equivalent to under this condition, r let me write it here is nearly equal to z, and then I put this value as 1 plus x minus x square divided by 2 of z square, plus y minus y square divided by 2 of z square. So, I just expanded this, considering the condition that x minus x divided by z and y minus y divided by z is much, much less than 1, and then I expanded. So, this half term will come here as 1 by 2, 1 by 2. So, that should be my r. Then in the next line, e to the power ikr, I just replace the value of the r, and then in the exponential, I kept this x minus x, but in the denominator for simplicity, I simply replace r as a z. So this value is essentially e to the power of i k z divided by z, then we have if I expand this into e to the power of, this is not x, it looks like x, so then e to the power of i k by 2 z and I have x square y square term. So, I have a big x square plus a big y square term. I wrote it and then we have small x square and small x big and big x y terms that are there. So I write the rest of the term like I k 2 z. Then I have small x square minus 2 small x bx which is one term, and also I have e to the power of i k divided by 2z this. So after that, I put this condition to here this integral and we can write it alpha, then e to the power i k z divided by z, this term should come out, and also e to the power of i k by 2 z big x big y term should come out because this is nothing to do in the integral. Integration is over small x, small y and then we have integration minus infinity to infinity one term, which is e to the power of i. This is a small x square minus a by z small x. This is one term in the integration with dx and other similar-looking terms over dy, and it should be b to the power of i k by 2 z y square minus i k by z small y big Y integral small i. The calculation looks big, but it is very straightforward. At the end of the day, we will find the value of alpha that is the left-hand side and that should be equal to something in the right-hand side and that value if I put here look to this in this equation, the right-hand side it should be e naught, e to the power i k z, that is in the right-hand side. So that thing is equal to, so if I go back, I integrate this entire thing and right hand side I get, e naught, e naught essentially will cancel out. So we will come to this later. Now let us execute this integral first, then we will come to this. So I would like to note here one very standard integral, that is a very, very useful integral and that is this, minus infinity to infinity, e to the power of minus alpha t square plus beta t dt, that integral is equal to root over of, this is a very famous integral, and we are going to use this several times e to the power of by 4 alpha. That is the result of this integral. This is a very famous integral, I suggest that students should note it. And if I exploit this integral here, you can see this is exactly the same form, alpha here is i k divided by 2 z, and beta here is i k z x. So if I exploit this integral here, then this integral x square minus i k by z, x x, dx, d small x rather, that value if I put alpha and beta in that particular form it should be simply root over of pi by alpha, alpha is this quantity. So I have pi divided by k, and then we have 2, and then there is a minus sign here, and if I absorb this minus sign, it should be 2 and then z and then i, that should be the value, then e to the power of beta is this value. So I can have e to the power of minus i k. Then I have a big x square divided by 2 of z because this value is beta square divided by 4 alpha. And if I execute this beta square divided by 4 alpha, it will come like this. So, let me do that. Mind it, this alpha I am trying to find out here. This is the general notation I am using alpha and beta. This alpha has nothing to do with this alpha, so please note that. So, here is how I get this. I can quickly see what the beta square is. Beta square is the minus of k square divided by z square and big X square, which is my beta square divided into beta square and then divided by 4. So 4 by alpha was this. So it is divided by 4 into K divided by 2 of Zi. So, this 2 will cancel out with 4, we have 2, 1K will cancel out and it should be 1Z is also going to cancel out. So we have K multiplied by X square divided by, so this Z cancels out, there will be I here, so this quantity we want to get. So essentially, what we have is this that alpha e to the power of i k z divided by z e to the power of i k divided by 2 z x squared plus y square that value multiplied by e to the power of whatever we had here, I need to put here we have this quantity and e to the power i k x square divided by 2 z. So, for y also we have this term. So, if I put these two terms together, we will get a very interesting-looking term and that is minus i k of k divided by 2 z big x square plus y square, exactly the same term that we had here but with a negative sign. So this will cancel out and then the multiplication of 2 pi divided by k z i, which we can get from the first term, was the root of this thing. So for x and for y, this same term will come pi divided by k 2 z i. So this root over will no longer be there because two same terms will come from x and y integral and this value is equal to k. We have alpha here, alpha multiplied by this. So that is equal to E naught, e to the power of ikz. So, you note that this term will simply cancel out, e to the power ikz will cancel out, E naught will also cancel out because here in this equation, this E naught and this E naught was there. So, essentially, this is E naught here. So, what I get is alpha equal to, very interesting result, alpha equal to minus of, this term will go down. So, alpha is equal to minus of k of i is divided by 2 pi, k is 2 pi divided by lambda. So, this is also written as minus of i by lambda. So, this is a very interesting expression we have. We managed to get the boundary condition of what alpha is. So, next time when we calculate for a regular structure, maybe in the next class we will do that.

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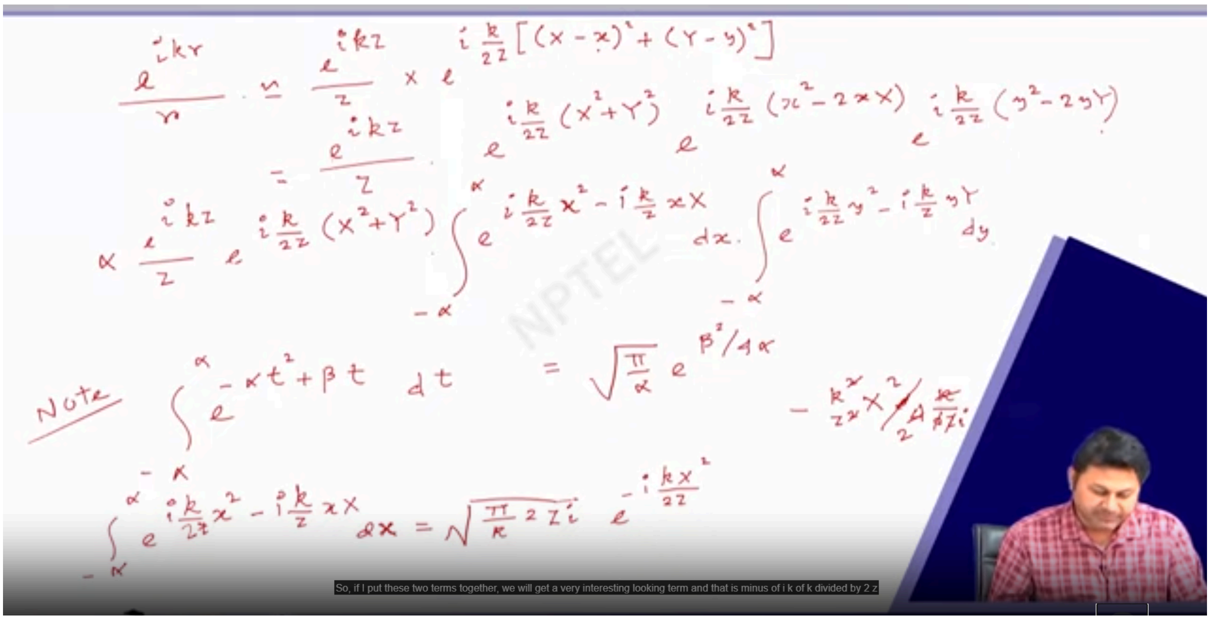

Instead of writing this field, so that means my E p, the total field will be, in the previous case we wrote that this is this integral of E naught divided by r e to the power of i k r and over the entire aperture area that was the form and after doing a bit lengthy calculation we find that this alpha is essentially minus of i k divided by 2 pi. This proportionality constant is a value and this is the value we have after doing this calculation. So, today I do not have much time. So, today what we learn is very important, how to tackle the problem when we try to find out what should be the total field due to some given arbitrary aperture say in the case of Fresnel diffraction. As I mentioned the Fresnel diffraction problem is a little bit simple to execute practically but mathematically, its calculation will be a bit tricky compared to the Fraunhofer case. The only reason is that in the Fraunhofer case, the wavefront that is coming is a plane wave which is easy to deal with but in this case, we have spherical waves where we have additional term 1 by r. So, with that note, I would like to conclude here. Thank you very much for your attention and see you in the next class for more detail about how to calculate the Fresnel's diffraction for a specific case. Thank you.

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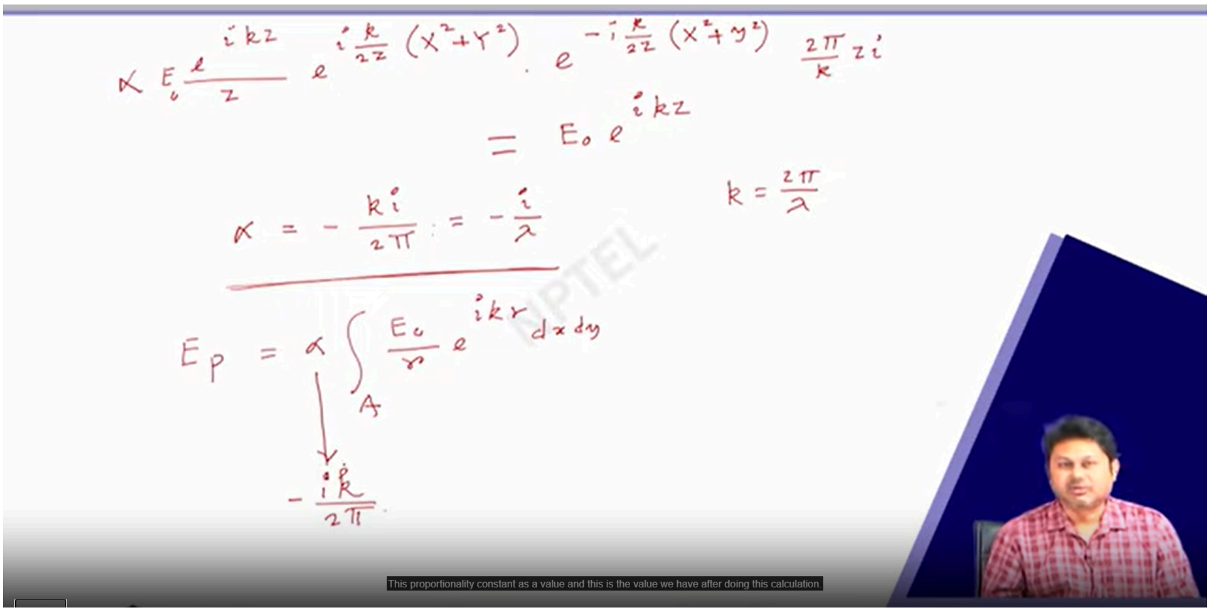
$\alpha E_0 \frac{e^{ikz}}{z} e^{i\frac{k}{2z}(x^2+y^2)} \cdot e^{-i\frac{k}{2z}(x^2+y^2)} \frac{2\pi}{k} z i$
$= E_0 e^{ikz}$
$\alpha = -\frac{ki}{2\pi} = -\frac{i}{\lambda}$
$k = \frac{2\pi}{\lambda}$
$E_p = \alpha \int_A \frac{E_0}{r} e^{ikr} dx\,dy$
$-\frac{ik}{2\pi}$
NPTEL
This proportionality constant as a value and this is the value we have after doing this calculation.