WAVE OPTICS

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Lecture - 46: Zone Plate

Hello, student, welcome to the wave optics course. Today we have lecture number 46 and in today's lecture, we are going to discuss something called the zone plate. So, we have lecture number 46 and today's topic is called zone plate. So what is a zone plate? Let me define first, a zone plate is an optical device based on Fresnel's theory of half-period zones, it consists of parallel glass plates having concentric circles of radii proportional to the natural number that is 1, 2, 3 etcetera. So, this concept has already been discussed in the last class, that if I have these zones like this, these are potentially the zones. Then the radius of the rth, nth zone was root over of n multiplied by lambda r naught. So you can see that, essentially rn for a fixed lambda and r naught, rn is proportional to root over of n, where n is the integer number. So, based on that if I prepare the zones, the concentric circles by these glass plates, then it will behave like a zone, it will construct the zone and we're going to get a similar result that one can expect with Fresnel's theory. So let me calculate how one can do that. So suppose this is my cross-sectional view of this zone plate, we have zones concentric circles but the radius has a relationship and their integer is proportional to the root of the consecutive number over that. This is the way the zones are formed and now what do we do? We make a cross-sectional view that this is a source point here and I place the zones here, so this is my source point S. Light is coming and then falling on this plate. Some images are formed here at point P. This is the location. This AB is the location of these zones. I am drawing the perpendicular view of that. So if I now join this radius it should be like this.

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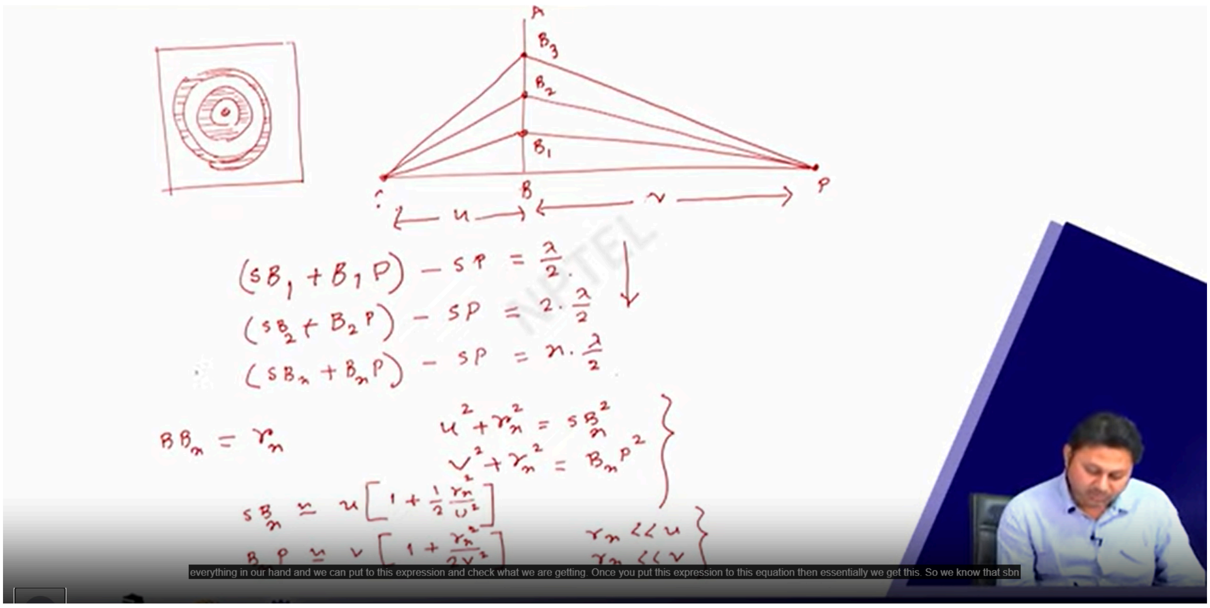

These are the points of this radius I am joining, such that this point is b1, this point is b2, this point is b3, and from here to here this is u, from here to here, this length is v. Now we know that the point B 1, B 2, B 3 should be such that the path difference. From S to P is lambda by 2, so that means S B 1 plus b1 p minus sp will be lambda by 2. Similarly, Sb2 plus b2p minus sp is 2 into lambda by 2. SBn plus similarly for nth one bnp minus sp will be n multiplied by lambda by 2 following this trend okay. Now BBn is essentially the radius of these zones so I write as r n and from this structure, we can have u squared plus rn square is equal to u sbn square and similarly, V squared plus Rn square is equal to Bn P square of that. Simply using the Pythagoras rule, we can find this. Just put in 1, 2, 3, and you will get this structure. Now SBN can be written as nearly equal to this quantity. u and r if I compare this normally u are much much bigger than r, so I can write it this is something like u 1 plus half rn square divided by u square. Similarly, bn p is nearly equal to v multiplied by 1 plus rn square divided by 2 v square, with the fact that rn is much much less than u, rn is much much less than v as well. So this is the condition and based on this condition, we can get this. Now we have sbn, bnp everything in our hand and we can put to this expression and check what we are getting. Once you put this expression to this equation then essentially, we get this. So we know that sbn plus bnp minus sp is n multiplied by lambda by 2, that is the thing, and now I put the approximate value u 1 plus, half R n square, u square minus v 1 plus, no, first this plus, so this is plus and then half rn square v square and then minus sp, sp is u plus v, u plus that quantity is n lambda by 2. t So, from here, we can simply find that u plus v, u plus v is going to cancel out. So, essentially, we have half R n square. If I take this common, then we have 1 by u square plus 1 by v square, that quantity is equal to n lambda by 2, or in other words Rn square is equal to n multiplied by u okay here this will be 1 by u 1. So, this square term will not be there because there is a multiplication of u. So, this term and this term will not be there. So, it is u v r square essentially, if I put this on that side, half-half will cancel out. So, n multiplied by uv divided by u plus v multiplied by lambda. So, essentially, the similar kind of result we have that Rn will be proportional to root over of n, here u and v are associated because this is the way the plates are placed and this is the source point we have u here and this is from here to here and here this is the point where image is formed,

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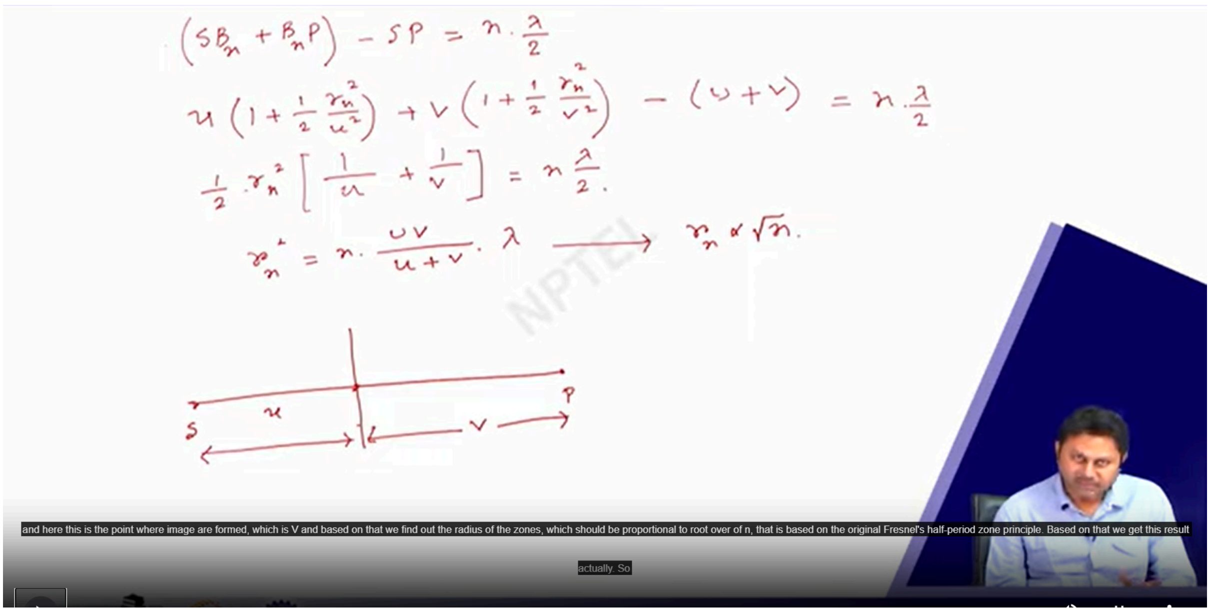

which is V and based on that we find out the radius of the zones, which should be proportional to root over n, that is based on the original Fresnel's half-period zone principle. Based on that we get this result actually. So this is the way the zones are constructed. Now if I calculate the radius, the area of the nth zone, it should be simply a n, that is pi r n square, minus r n, minus 1 square, so that value is essentially pi into lambda into u v all divided by u plus v and then we have n minus, n plus 1. So essentially this value is pi lambda uv all divided by u plus v, that is the area one can have. So based on this information, one can construct these zones. Now if I place this zone on point b and allow the light to fall on that. So, what do we get? We get to this point P, the intensity. So, this is the point P. So, the resultant amplitude at P will be u P equal to a1 minus, a2 plus, a3 because of these zones, minus a4 this alternative plus, minus will go on. So if we now intercept the wavelet from an even number of zones. So I just do the experiment with an even number of zones, that is second, four, six etcetera. Then the resultant displacement at the point p will be, so if we intercept the wavelet from the even number of zones that is second, fourth, sixth, I just block that. Then, all these negative terms will cancel out. So, then, the resultant displacement at P will be uP equal to a1 plus, a3 plus, a5, and so on. This is the way the intensity is going to change here because I am just intercepting this. So, in a similar way if odd numbers are intercepted then uP will be a2 with a negative sign obviously. But at the end of the day when we calculate the intensity this negative sign will not come into the picture it should be a4 plus a6 etcetera. So again we have a maximum. So depending on which zone you are intercepting or cutting based on that we will get the intensity. So, you can see that this R n square is equal to n lambda u v divided by u plus v, that is the equation we have. And from that, we also calculate in rewriting this expression, one can have 1 by u plus, 1 by v is equal to n lambda by Rn that is equivalent to 1 by Fn, if I consider this equation to be the same as the equation of a lens. So, some sort of lensing effect one can get. So, we know that if we have this lens and this is the object and here we get so if this is u, if it is v then 1 by u plus, 1 by v is equal to 1 by f, that is the equation for a lens, the similar expression we get here but interestingly, the focal length Fn is now the function of n.

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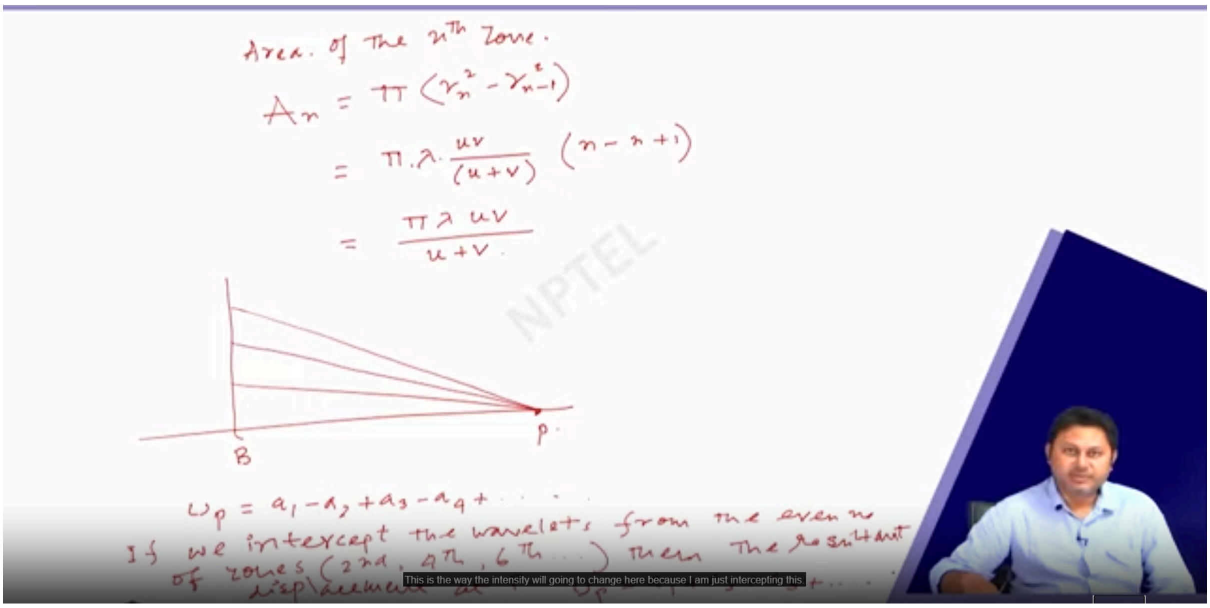

So Fn is equivalent to Rn square. So here we have a Rn square actually. Rn square divided by n lambda. Now, note that if I put it Rn is proportional to n itself. So, the focal point F1 can be R1 square by lambda, that is R2 square divided by 2 lambda, and so on. So, this is called the primary focal length. F So, this is happening because as I mentioned R n is proportional to root over of n. So, that is why you can see that R n square by n this is essentially a constant. So what happens if I place a zone? Then it will convert the light to a particular point, behave like a lens and the focal point will be calculated with this and if I know Rn, then we can also calculate what the lambda is using if I find out the primary focal length. There are also secondary focal lengths where if I put Fn equal to 2, then I am going to get the secondary focal length, and so on. So focal length fn for a fixed n is what we can calculate with this ratio. So calculating this primary focal length with this zone plate construction one can find out what is the corresponding lambda in the experiment. So this kind of zone plates are there, you have a light source and then you plate the zone. And then what are we getting? We are getting the focal point and then if we change u and v. For example, if I change u and v what happened? From this formula, you can see that Rn will change. And we are going to get another focal point and so on. So with this, we have primary and secondary focal lengths and we can calculate using this data. We can calculate the wavelength that is used in the experiment. So I don't have much time to discuss more about this zone's construction. So in the next class what we do is, we are going to discuss more about how using the aperture we can get the field distribution in other points. A similar kind of calculation we done in the Fraunhofer problem, where we have an aperture and when we have an aperture then how it is formed is the intensity distribution that one can get in the screen. So similar kinds of things we will be doing but the approach of this calculation will be slightly different. And that we are going to do step by step. So, with that note, I would like to conclude today. Thank you very much for your attention and see you in the next class.

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