WAVE OPTICS

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Lecture - 45: Fresnel's half period zone (Cont.)

Hello, students, welcome to the wave optics course. Today, we have lecture number 45 and we will discuss Fresnel's half-period zone and a few more aspects of that. So today we have lecture number 45. So we have already started the discussion of Fresnel's half-period zone. So let me do that once again as a reminder that, suppose we have a wavefront that is propagating and this wavefront can be divided according to Fresnel's theory with few zones like this. Such that if I have a point here P, the distance between the nth zone and the central to central point, if I write this R naught, it is R naught plus n lambda by 2. That is the basic structure. So what we are doing is that if I want to find out the intensity at this point P, instead of considering the entire wavefront, we can divide this wavefront into a few zones, and these zones are situated or are located in such a way that if I join the nth zone to point P, and if I join the first zone, the central point to P, if it is r naught, if the r naught distance, that is the perpendicular distance from the plane wavefront to the point P, the point of observation P, this is r naught, then it is r naught plus n into lambda by 2. So, with this notation, it is easy to show. So, let me draw these zones once again. So, that we discussed in the last class, but I am doing this once again. So, suppose these are the concentric circles forming the zones, and each circle should have some area. I am shading the alternative zones so that one can understand how this is formed. A This is the way the zones are formed here and this is the distance P and this is R naught. So it is easy to show that the area of each zone is almost the same.

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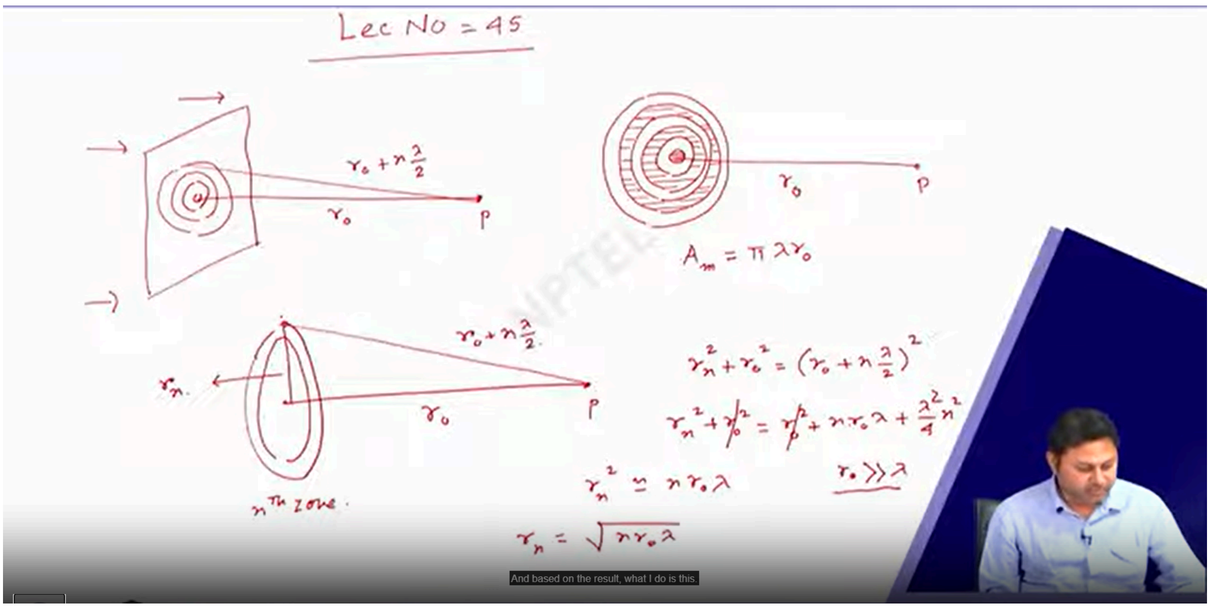

So the area, for example, of the mth zone is simply, so yesterday we calculated that the area of the mth zone is roughly pi into lambda into R naught, that is the area of the individual zone, and it is approximately the same for all the cases because this right-hand side doesn't depend on m. We can also calculate the radius of these individual zones, which we didn't calculate precisely. So today we will do that. So let me draw that. So suppose this is the tip, this is the nth zone I am drawing, and this is the central point. So, according to, this is the nth zone, and this is point P, this is R naught, and if I join this line from here to here, that value is, say r naught plus n lambda by 2. So that is how we define it. So, if I want to find out the radius of the nth zone, then we can also calculate that with very easy steps. And that is, we have an equation. So, suppose the radius here is Rn. So I have simply Rn square plus R0 square is equal to R0 plus n lambda by 2, a whole square of that or Rn square plus R naught square is R naught squared plus n R naught lambda plus lambda square by 4 n square. Now this R square, R square is going to cancel out and essentially what I get is Rn square is nearly equal to n R naught lambda or in other words Rn because R naught in general is very, very large compared to the wavelength lambda. So that is why I can neglect this lambda square term and essentially Rn is equal to root over n R naught lambda. That is an interesting result we have that this is the way the radius is defined for the nth zone. Based on the result, what I do is this: Before that, let me remind you what we had, and that was if I want to find out the intensity and we get a remarkable result here that this is the wavefront that is containing, that is a propagating wavefront, the same figure that I had used. So this is a propagating way front containing zones like, this my drawing is not with that period scale, obviously I'm drawing this by hand but you should understand the concept and then what happened that if I want to find out what is the intensity or the amplitude at this point P naught what we find that the total disturbance that we get due to the entire wavefront that is a n is simply equal to a 1 divided by 2, that is the contribution of the entire wave front is summed up at the point P, and we find it is nothing but whatever the contribution we have for the first zone that is this zone divided by 2, that is a remarkable result. Now what we can do is this, suppose we have a wavefront that is coming, and we have an aperture here.

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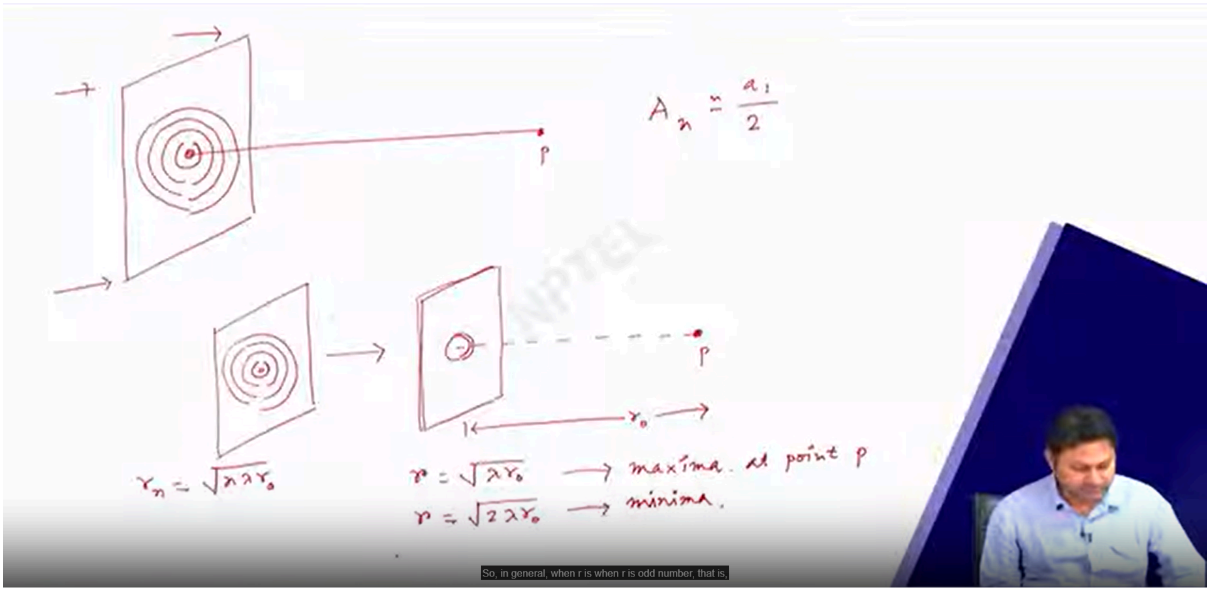

This is the wavefront that is coming with this kind of zone, and it is moving, and we also have an aperture here this is the system where we have an aperture, the light is passing through this aperture, and this is p and suppose here to here this distance is as before r naught. So now, if I increase the radius of this aperture, what happens is that the p point will be gradually illuminated, and it will be eliminated most when the aperture r will is equal to because here we know that r n is equal to root over of n lambda r naught that is the result just derived. So when this aperture radius is exactly equal to root over of lambda r naught, that means I am only allowing the first zone to pass through this aperture, then we get a maxima here in the point P. So, we get a maxima. Now, interestingly, if I increase this aperture to such that this aperture is allowing not one zone, but two zones, then what happens? This aperture will have a value like 2 of lambda of R naught. That means I am increasing the aperture to lambda r0 root over 2 lambda r0, then here only one zone is allowed to pass through this aperture, but here in the next case, two zones are allowed to pass through, and we know that when there are two zones that are passing through even number of zones, that is passing through the contribution of the amplitude is such that they are canceling each other. And if that is the case, here we will get a minimum. That is a very interesting kind of result, which means if I increase the aperture in general, it suggests that we should get more intensity at point P, but here it is not the case, and that is happening entirely because of the wave nature of these things. So we get a maxima at point P. And if I increase more, then we are going to get a minimum. In a similar way, if I increase more, then we are going to get maxima and minima. So, in general, when r is an odd number, that is, 2n plus 1 multiplied, this is the aperture with aperture radius we are talking about circular aperture. For example, lambda r naught then we get maximum and when r is an even number this integer is an even number that is 2 n lambda r naught, then we have a minimum here where n can go to 0 1, 2, etcetera. H So, this is the way we can see that even if we increase the aperture, the value at some point doesn't increase the intensity but it is decreasing and it is changing in an alternative way. We will show how it is changing. In fact, another important thing is if I change the location point of the distance, then we will see a similar kind of change. So, for example, before that, let me also make a few notes. So note 1 is that if I increase the aperture, I discuss this issue; another issue is if instead of having an aperture, I have an opaque region, then what happens?

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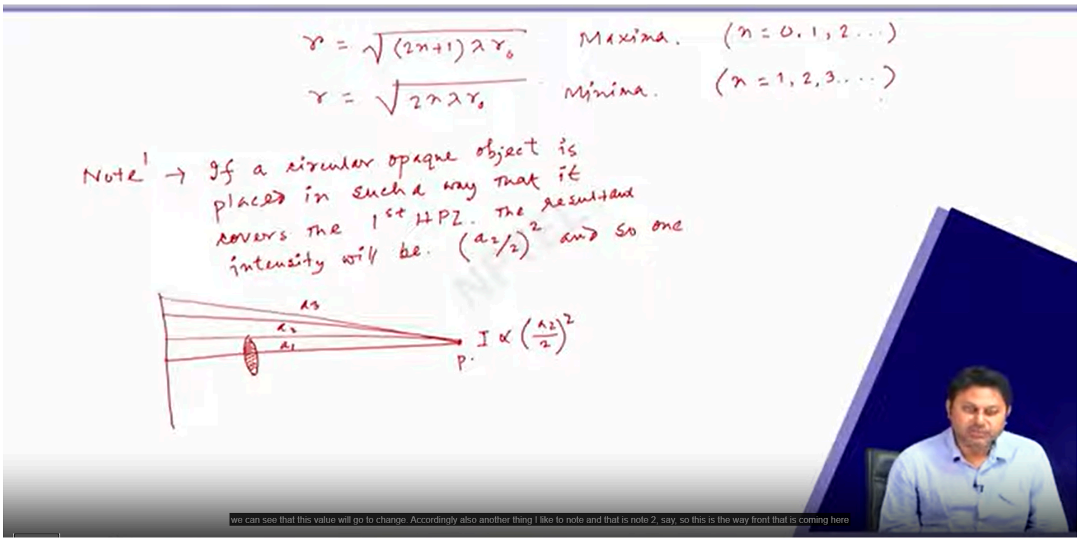

If a circular opaque is placed in such a way that it covers the first half period zone, I write in short hpz then the result in intensity will be the result and intensity will be A 2 by 2, we already calculated earlier, that what should be the resultant intensity square. If everything is open. Then what happened? We have a resulting intensity a1 by 2 whole squares. Now what I am doing, is the way the wavefront is moving. Let me show the figure. And we place the opaque object here. So, this is the way the other rays are contributed and come to this point. So the intensity of these things is a1 amplitude, for this, it is a2, for this it is a3, and so on. So the resultant amplitude, if I cover this, then the intensity will be proportional to the next one which is A2 divided by 2 squares. Now, if I cover another in such a way that two zones are now covered, then the intensity will be A3 by 2 whole squares, and so on. So, it will be this and so on. So, in the previous case, we show that we have an aperture and in this case, we have a opaque object that we are placing in front of the wavefront in such a way that it covers the entire first zone and then the intensity I am calculating at this point P and if I increase this point we can see that this value will go to change. Accordingly, also another thing I like to note is note 2, say, so this is the way front that is coming here and over here I have several points, say, p1, p2, I am moving towards or away from this whatever the aperture we have. Suppose at a distance p on this axis what happened that the aperture transmitted, only the first half period zone. So at this point, if I come forward, then the value of R0 will change and since the value of R0 will reduce, we have reconstruction of the zone because the zone area and the zone radius both depend on this value R naught. So, if I move from this point to this point, this value will change. So, if the distance is such that only the first half period zone of the wave is coming here at this point, that should be the value of I p, at this point, the intensity that will be proportional to a 1 square. In a similar way if I wider the aperture for the same value, if I come and we need to wider that these things but for a wider aperture or for a near point of P1 on the axis in such a way that lets the axis in the aperture it transmit only the first two half period zone, instead of one. Then what happened? The intensity IP1 will be, now proportional to a1 minus a2 square. So that is for another point, if I move to another point, the zone will be constructed accordingly because, as I mentioned, the zone construction depends on what is the distance from this point. So I am moving over these points, and accordingly, this intensity is going to change, and you can see that this value is very close to 0. Again, if I move to another point so, this is IP1, IP2, and IP3 that is proportional to A1 minus A2 plus A3, and this is again a high value because A1, A2 will cancel out, and this is a high value. So, the point is if I move along P1, P2, P3, the intensity variation should also come as a, change as a periodic way. Sometimes it is less, sometimes it is large and that is also an interesting aspect of the Fresnel's zone. So if I plot this stuff, so two cases are there one, if I increase this aperture and we keep fixed the detector here which is capturing intensity but we find that there will be a variation of the intensity and this intensity variation is to some extent period. I am just drawing a very rough thing. So, the intensity will go down and up and down and up and down and up something like this. If the radius of the aperture r is changing, if r is increasing the interesting fact is this the intensity at point p, the interesting point is minimal. That means even though we are moving, we are increasing the aperture the intensity is reducing. In a similar way we get a similar kind of figure if I just plot how the intensity is changing, if I change my r naught, that is if I change this distance then also we are supposed to get a variation something like this. So how is this variation, what is the functional form of this variation? It shows that like a sinusoidal kind of curve, but there is also something more to it. So we're going to discuss this in the later part of the lecture. But here also we have, if you go towards the aperture or go away from this aperture, we will get not only the maxima but also the minima, these periodic variations are there. So today I like to discuss these two aspects of Fresnel's zones. I don't have much time to discuss further an instrument called the zone construction or an instrument through which we can prepare this. So, in the next class, I will try to discuss these zones' construction do detailed calculations, and show how the relationship is there with distance and wavelength and we will find a nice-looking expression as we get in the case of lenses. It acts like a lens where light can be confined to a particular point. So with that note, I like to conclude here. Thank you very much for your attention and see you in the next class.

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