WAVE OPTICS

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Lecture - 43: Fresnel Diffraction

Hello, student; welcome to the wave optics course. Today, we have lecture number 43, and today, we are going to start a new topic, which is called the Fresnel diffraction. So today, we have lecture number 43. I'm going to start today with a new topic, as I mentioned, which is Fresnel's diffraction. So before that, let me quickly remind you that the phenomena of diffraction were broadly divided into two types: Fraunhofer diffraction and Fresnel diffraction. So, in Fraunhofer-type refraction, you may remember that the source and the screen are both at infinity because the structure is something like this. So essentially, we deal with this as the source, where the light is emitting, and then we place a lens here. We can have a parallel ray that makes the incident like the source infinity, then we have some aperture here, and then again we place a lens here and on the screen. So suppose light is diffracted, and then we accumulate all the light here at some point P, and we find a pattern here, but essentially, the source and screen are both at infinity. This is the source, and that is the screen. Fresnel diffraction, on the other hand, is the structure; it is simple. If I have a source here, it emits light and interacts with this aperture, and then we will get some pattern here out of that. In this case, what happens is that light that falls to the aperture will be a spherical wave, and then that coming here and reaching here is also a spherical wave since both are infinite. The setup is simple, and we will get a pattern here. On the other hand, in Fraunhofer's case, the light that is coming here, that is heating to this aperture, is not a spherical wave; rather, this is a plane wave, and whatever I get is captured here in this screen. This case is the source, and it is screened. Different cases are discussed under Fraunhofer's diffraction problem. For example, we discussed the single-slit problem. We discussed the double slit problem. We exchanged this idea to discuss the multi-slit problem, and then we discussed also what happened for circular aperture and rectangular aperture. So, we discussed these five problems in detail with all the calculations for Fraunhofer diffraction cases. Now, we are going to do something similar in Fresnel's diffraction cases, but you will see that the treatment will be completely different here. The basic reason behind that is the light that is coming here is no longer considered to be a plane wave; rather, it will be considered to be a spherical wavefront that is coming, and based on that, we are going to do all the treatment. So, before solving the problem here in the Fresnel diffraction case, we will try to develop the theory and the background mechanism to get an intensity at point P. So, let us start with that. So, in Fresnel's case, what we do is this. So, we will start today with a basic understanding of Fresnel's diffraction. So, as I mentioned here, the source and the screen are in the finite distance, they're in the finite distance. So let me draw here that if this is a source and what happened from this source spherical wavefront I'm going to emerge and let me draw a portion of this spherical wavefront, it will emerge like this I am just drawing a portion of this wavefront which is no longer plane but spherical in nature and then we have an aperture here through which the light will fall. So, let me draw that first. So this is the window, and here we

have a window like this is O, so this is a point, say, this is a small cross-sectional area, and the light that is coming to this point is, say, R prime, and this is my S and the light that is coming through this window is some point here that is P and this is R. This cross-section is DA, the cross-sectional area. This is the point O, and this is the structure we have in our hand. Okay, so the electric field at point o in the aperture at this point here the electric field. So let me write about this point. So this o, that is the origin, and this is another point say o prime. So the electric field, O prime that is the area having the small area D that patches in the aperture is E naught is equal to E s divided by r prime e to the power of i k r prime minus omega t. So note that this is a spherical wavefront that's why the 1 by r prime term is there, which was not before in the case of Fraunhofer diffraction problems. Now this quantity e is divided by r prime e to the power i k r prime is considered to be a complex amplitude that we know that E, I can write it as minus omega t and this E 0 tilde is basically the complex amplitude. This is a complex amplitude. Now, the contribution to the resultant field at point P is due to all small areas. So, due to this small area, what is the contribution if I try to find out that contribution here, I can write that is, the contribution to the resultant field at P due to the small area dA that should be dEp is equal to d e naught divided by r e to the power of i k r minus omega t, since p is in the finite distance. So the wave that is coming from this d area will also be a spherical wave. And if it is a spherical wave, then that should be the form 1 by r e to the power i k r minus omega t, which is the form of the spherical wave. And d e naught is a small amount of magnitude that is there. So, note the amplitude here. So, the amplitude, which is d e naught divided by r, which is the amplitude that is proportional to the area dA. That is, if dA is large, then the contribution of the electric field will be large at the point P. So, I can write it as dE0 by R is essentially, I just write Ea by R dA in this way, where Ea is the field amplitude per unit area. If dA is a field amplitude per unit area, then dA multiplied by area is a field amplitude. So, this is a field amplitude divided by R. On the left-hand side also, I have an amplitude divided by R. So, dimensionally, it is fine. So, now, Ea, is again proportional to the complex amplitude of the original electric field that is falling here.
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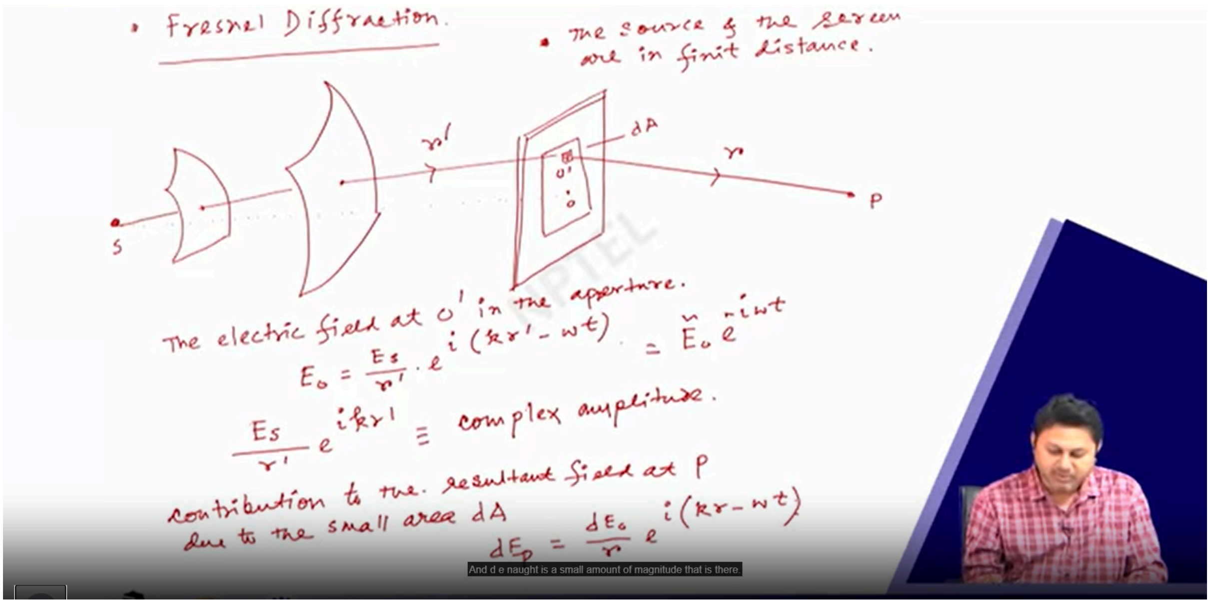

So, if I go back and see that dE0 is somehow related to the amount of field that is coming from S, we can have that Ea should be proportional to A, and if that is the case, I can write that Ea is proportional to the complex amplitude, that we had earlier and that is e naught tilde and what was e naught tilde where let me write down e naught tilde was e s divided by r prime e to the power of i k r prime that we already had. So e a is essentially a constant alpha, this is proportional science, and this is a constant alpha that will be multiplied by this it's a proportionality constant here, so e s by r prime, e to the power of i kr prime that we have. So if E is known then from this expression that dEp is E0 by R. So, that I can write. Okay So, let me see what notation we use here. Here we use the small area y E0 by A, I write Ea, and Ea is this quantity. So, essentially, we can have this. So, dEp, which is equal to E0 divided by R is EArdA, and EA is this, so I can write alpha directly, then ES divided by rr'. Then I have E to the power of ikr". multiplied by e to the power of ikr, minus omega t, I just put the value and that value if I simplify because one is e to the power ikr prime and another is e to the power ikr. So I can write it as alpha Es r r prime e to the power of i, then I have k r plus, r prime minus omega t and then I should have a dA here. So I correlate the source, and then I have an aperture here with a small area and a point here p, and the light is going like this So, I correlate the electric field here to whatever I have here, which is EP, and it is related to this area dA. So, now I want to find out what should be the total field here, then, what we are supposed to do is we just need to integrate over this aperture, and that we do. So, the total field at P is ep and essentially, that should be the integration over dm. So it is equal to alpha es, then we have e to the power of minus i omega t term, which should come outside because that has nothing to do with the integral over an area, then we integrate over the entire aperture, and we have 1 divided by r r prime e to the power of i k r plus r prime over dA. So that is the total expression we have and that expression suggests how the total field at P is related to the field at S. And if I know everything, then by doing this integration, I can find out the field that is there at P. So that is the structure. But in this calculation, a few things are neglected. Let me, in the book, you will find these factors. And that thing we are going to neglect here.

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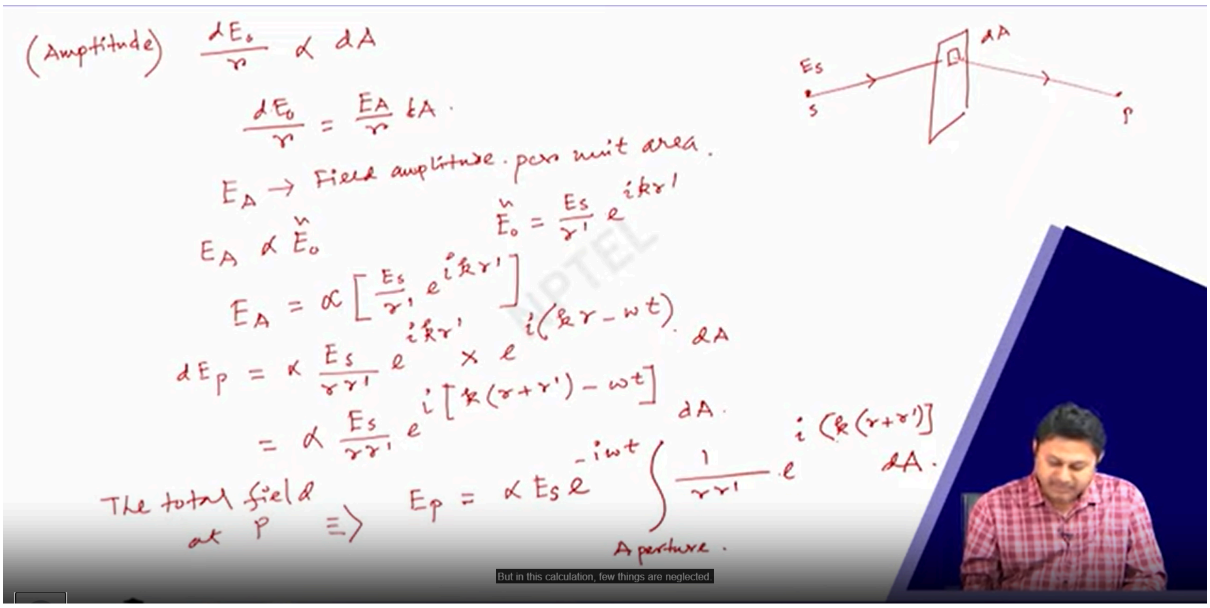

So, E p, let me write down once again that E p is equal to alpha E s, then e to the power of minus i omega t, and then over the aperture we have 1 divided by rr prime e to the power of ik r plus r prime dm, that is the overall integral. But a few things as I mentioned are neglected. The first thing is this expression does not take account of the obliquity factor, normally this obliquity factor we written as F theta, and expression-wise F theta is half of one plus, cos of theta. So what is the obliquity factor? If I have a source here, if I want to find out something here then theta is zero. From this, I can get the F theta value, which is 1. But if I move to another point here, this is my P and if I move to another point P1 then I am with this horizontal line, I am making an angle theta, and as soon as I am making angle theta the factor. So, we are going to reduce the amplitude we are going to reduce here because of this obliquity factor, and if I move more and more wide angles then this value will change. So, that value is essentially called the obliquity factor and we neglect it here in this calculation. We consider that if you go back to the figure we consider that for whatever the ray I draw has nothing to do with the angle theta but which is not essentially the case. But we neglect it here for simplicity. Also, another thing is neglected here in this expression, when we calculate the total field by taking into account the integral over this aperture does not take. So, pi by 2, there is a pi by 2 phase shift of the diffracted wave related to the primary diffracted waves relative to the primary incident wave, and that pi by 2 factor is not. We didn't take this into account. So, we will discuss this in the coming few classes about how these things are there. So, let me first turn. So, do not take it into account. Let us now take into account the pi by 2 phase shift of the refracted wave relative to the primary incident wave we're going to discuss this in the coming few classes. So that condition is also not taken into account in this treatment so, that is the overall thing. So i today I don't have much time to go forward with this idea but the point is today I try to develop from a very straightforward expression, which is a spherical wave that is emerging from a source and if it is hitting some aperture then it get diffracted by this aperture and we are going to get some some pattern at the screen. So if this pattern emerges then what should be the electric field that we calculate with this expression, that is shown here. So in the future classes, we will be going to exploit this expression and try to find out what the total field is by doing certain integration etcetera. But there are other simple methods that we need to learn before and in the next class we are going to do exactly the same thing and we will learn something called Fresnel's zone construction, that how by not doing that rigorous calculation. By simply using the concept of zone, we try to find out the intensity or at least get the idea of the intensity at some point due to some aperture. Okay, so with that note, I would like to conclude here in today's class. Thank you very much for your attention and see you in the next class.

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