WAVE OPTICS

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Lecture - 42: Fraunhofer diffraction for a rectangular aperture

Hello, student; welcome to our wave optics course. Today, we have lecture number 42, and in this lecture, we will extend the idea that we developed in the last class, how a circular aperture gives a different Fraunhofer diffraction pattern. Now, we extend this idea, and we will understand how a rectangular aperture will give you the diffraction pattern. So, we have lecture number 42, and in this class, we try to understand if I have a rectangular aperture and what the intensity distribution should be. So, like the previous case, let me draw a 3D picture of the aperture. So, here we should draw the aperture which is not a strip, but it is a substantially big aperture like this. So we have an aperture like this, and there will be an XY coordinate. This is x and we have a plane here where we want to find out the intensity distribution pattern. Okay, so let us define this. So this is B, and this is, say, A, and here, at some point, we want to find out the intensity. Suppose this point has a coordinate big X, Y, Z, and this is the point. From here to P, if I join, this is R and some other coordinate point. Here is a small strip I take dx, dy say, and I want to find out this. So, this is a big R, and this is a small r. So the small r here, so let us first understand the geometry, small r is x minus, small x square plus, y minus small y square plus big z square. Assuming that this plane is at z equal to 0, these are small x, and small y. Then similarly, R is, so we can have R as this, I expand this, so the whole to the power half is there. So this is x square, big x square plus big y square plus, big z square plus, x square plus, y square minus 2 of small x, b x plus small y, big y bracket close. So that is small r. Now note that x squared plus y square is the length; I am talking about this length because I consider x and y in this coordinate here.

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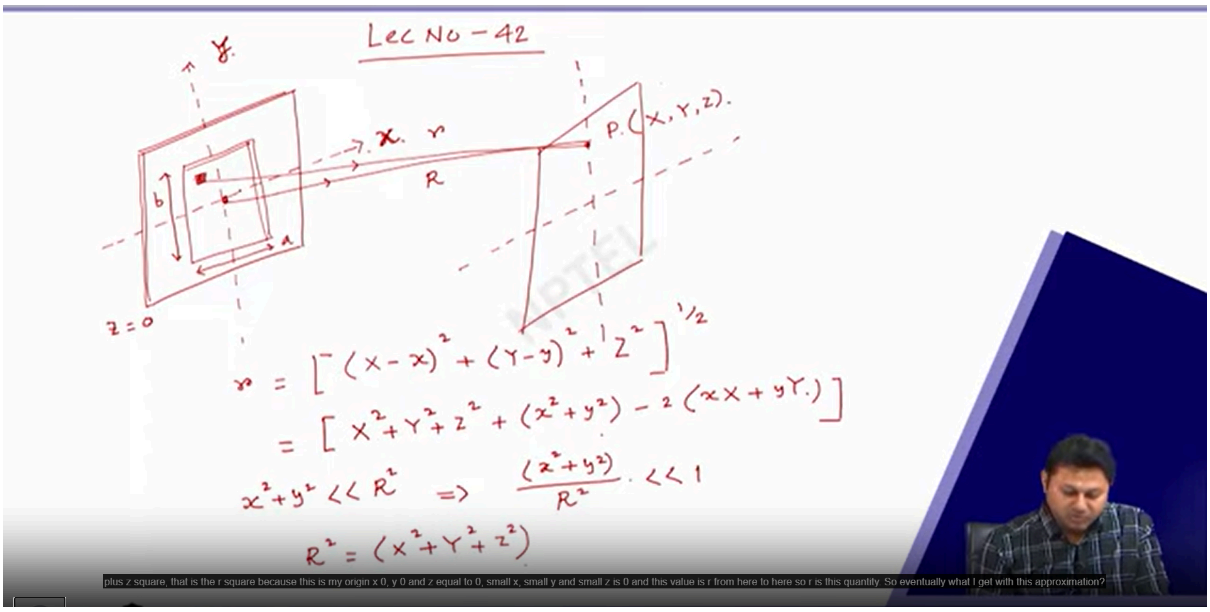

So, I am considering from here to here this length, which is x square plus y square, that length will be much much less than this r. That is the length, almost the distance between the aperture and this screen. So that means we can write here that x squared plus y squared divided by r, r square rather, r square is very very less than 1. So if that is the case, we can neglect it because, again, r square is x squared plus y squared plus z square, that is the r square because this is my origin x 0, y 0 and z equal to 0, small x, small y, and small z is 0, and this value is r from here to here so r is this quantity. So eventually, what do I get with this approximation? That is small r is nearly equal to r 1 minus x plus yy by r square bracket close. Now, the path difference here is interesting, and this path difference is simply delta equal to big R minus small r, the mod of that rather. But let us consider this as the path difference. So, if I look back to this, whatever the path r and whatever the path smaller is, the difference is simply r minus r. So, if this is the path difference r minus r, the quantity I can get from here and that quantity gives me delta is essentially nearly equal to or this quantity is equal to because I already have this expression is small x, big X plus, small y, big Y divided by R. If I multiply this R square will be simply R. So once we have these path differences then we know what should be the field there like before the case of circular aperture that field at P due to the small area dA. What do we expect? We expect the expression like this: dEp is E naught divided by r like before e to the power of i kr minus omega t and then e to the power of i k delta path difference small k delta dA. So the total field will be, suppose total field I write ep that is the sum over d ep and that is e naught divided by r e to the power of i k r minus omega t, this term you find that essentially does not have any effect because, at the end of the day, we are taking the intensity. So, the mod square term will come, the mod square ep we are taking, so the exponential term will no longer be there, but it is better to have this for the completeness, and then we have the integral over the area, and it is e to the power of i small k delta and dA. So dA, the small area for the Cartesian coordinate system is simply dx dy, and if we exploit this expression of dx dy and put it because k delta has small x and small y,

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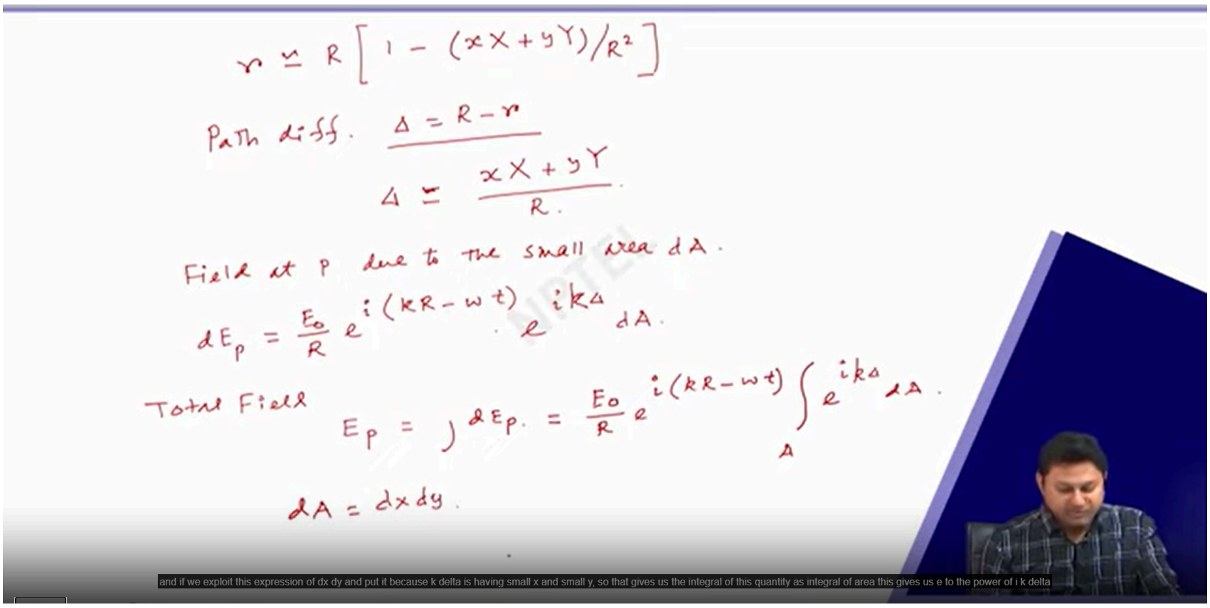

so that gives us the integral of this quantity as integral of the area this gives us e to the power of i k delta dA and that is equal to integral, two integral, one is for dx and another for dy e to

the power of ikx plus y divided by r dx dy and that thing I can write as dx, if you remember this aperture whatever the aperture we had this length was b and width was a, so the integration of dx will be minus a by 2 to a by 2, we have e to the power of i k x by r x then dx and then we have integral minus b by 2 to b by 2, e to the power of i k y by r small dy. So this integral are very straightforward, this expressions it will be e to the power of i K big X by R small x divided by i K big x by r evaluated these two at a by 2 minus a by 2 point multiplied by e to the power of i k then y by r r evaluated at point b by 2 and minus b by 2, in those two points they are evaluated. So this expression is straightforward and one can have. So when we put a by 2 here, these things will be e to the power ik by rx a by 2 and another case it is a by 2. So if I multiply a here in the denominator, so I have sine of, say, beta x divided by, beta x with a multiplier a, where my beta x is simply k by 2 then x by x a by r, that is my beta x multiplied by b into sine of beta y, divided by beta y, just to bring the same form, sin beta by beta just to bring this form I write it, where beta y similarly is k by 2 and then we have big Y b by r. So, if you just put b by 2 minus b by 2, you are going to get this expression and just manipulating this expression, we get this. Now, what is the intensity? So, once we have this integral, then my intensity is proportional to E p square. So, I can write intensity here is I naught sine square beta x by beta x square into sine square beta y by beta y square. Note that this is a similar expression that we have for a single slit. So, for a single slit expression what was there? So, let us draw it. Suppose that was the window and we have a single slit like this. That was a single slit and for a single slit the value of a was very very small. The value of A was very very small compared to B. If this is B, then this was A, but we neglected that. So, A was very, very small compared to B. Since A was very, very small compared to B, inside this beta x. So, let me write down what was beta x once again. So, beta x was how much? It was k by 2, then we have a multiplied by x divided by r. And what is beta y? Beta y is k by 2, b y divided by r when a is very very small. What happened, we can say that beta x almost tends to 0. so beta x tends to 0, a is small and r is high so a by r this ratio is small a tends to 0 here.
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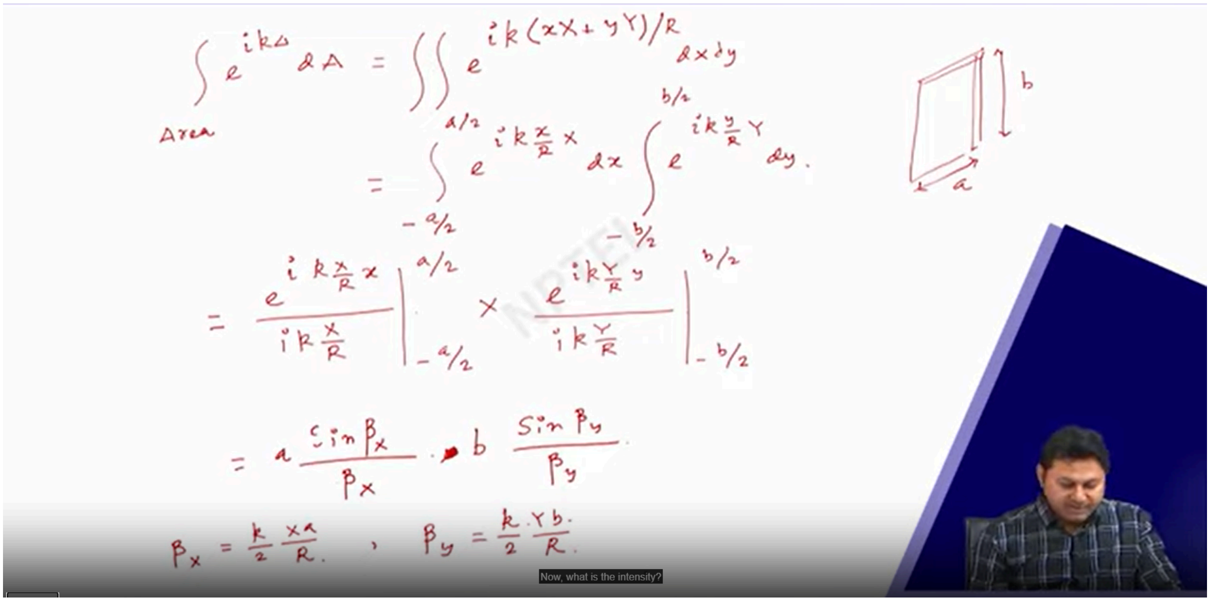

So, if I put just a tends to 0, then this quantity basically goes to 0 or false or very small value. And then sin square beta x divided by beta x square, this quantity goes to 1. So, whatever the general expression we had, this general expression simply I can write it as I naught sin square beta divided by beta square. Also note that the sin theta was if I put a very small very nearly equal to 0, then the condition of the b, if I put then y by r that quantity comes out to be sin theta. So, that is why beta during that time was nothing but k by 2 a, B rather than sine theta that was for single slit. So one slit, the slit problem. So this is a single-slit problem. But here this is generalized and no longer A is neglected instead of having a strip. Here what we get is a window, a big window, or a rectangular aperture. So in rectangular aperture, this is my B and this is my A. A and B are large. Large means I can't write the condition like A is much smaller than B. So, they are legitimate values, big values. So, in that case, the results are converted to a general form and this general form I can write is I naught sine square beta x divided by beta x square, into sine square beta, y divided by beta y square. So that means in the x and y in both directions we have a distribution that is something we need to discuss here as well whatever the time we have. So for a single slit, we have the distribution like this. It is sine square beta divided by beta distribution and distribution was like this. Only one coordinate was there because A tends to 0 and that is the intensity distribution and this is a function of theta. But in the case of rectangular aperture, we have two coordinates x and y and the distribution is over two coordinates. So, one distribution will be like this along x direction. If I call this is my x direction and this is my y. So another distribution will be maybe I can put a different color will be along this direction. So in this case it is purely one dimension. So I am going to get an intensity distribution right here. We have a maximum, then we have a minima, we have a minima, we have a minima, we have a minima here, we have a minima, we have a minima. And then minima means secondary maxima. But in this case, we have one maxima here. And then secondary maxima like we have from a single slit. But also in this direction, we have symmetric patches like this. Both the directions, this direction, and this direction. In 3D I like to show in this way. So that you can understand the x and y axes and how this intensity distribution takes place. And here in one dimension how it takes place. So here I, is I naught sine square beta divided by beta square. Exactly a similar expression here, I, is I naught. But there are two expressions for two different directions. One is beta x divided by beta square and beta y divided by beta x square. This is beta y square. Okay, so with that note I would like to conclude today because we don't have time and in the next class we will try to understand more about these spectra thank you for your attention, and hope you are enjoying the course. See you in the next class.

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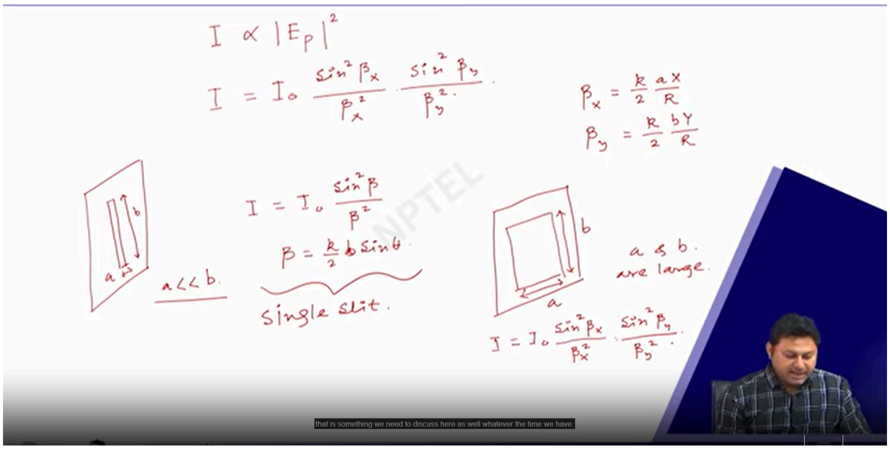

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