WAVE OPTICS

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Lecture - 40: Resolving power of grating

Hello, student, welcome to the wave optics course. Today we have lecture number 40 and in today's lecture, we will discuss the resolving power of the grating. So we have lecture number 40. I am going to discuss resolving power. So, let me remind you what we have done in the last class, if we have a grating spectra for two different wavelengths then let me draw it. So, this is the spectra for getting spectra and so on. This is a spectrum for one particular wavelength. And if I have another wavelength, then there will be a slight shift and I'm going to get something like this. So these spectra, two spectra are for two different wavelengths. If this is for one wavelength and this is for another wavelength, this colour is for one wavelength, say, for lambda 1 and the other is for another wavelength lambda 2. So the angular dispersive power is defined by d theta d lambda and that which is d theta d lambda and we find that this quantity d theta d lambda essentially is p divided by a and then cos theta. So for a given p and theta, the resolving power or the dispersive power sorry the dispersive power proportional to 1 by a. So if the number of the ruling is very high, then what happens, we will have a grating, having more dispersive power, it can disperse the separation between two wavelength, the principal maxima with a larger amount compared to the grating, which is having less number of line per unit length. So that means if two wavelengths are there and if I do the experiment for two different gratings, in one case the number of the ruling is hundred per centimetre and in other cases thousand per centimetre.

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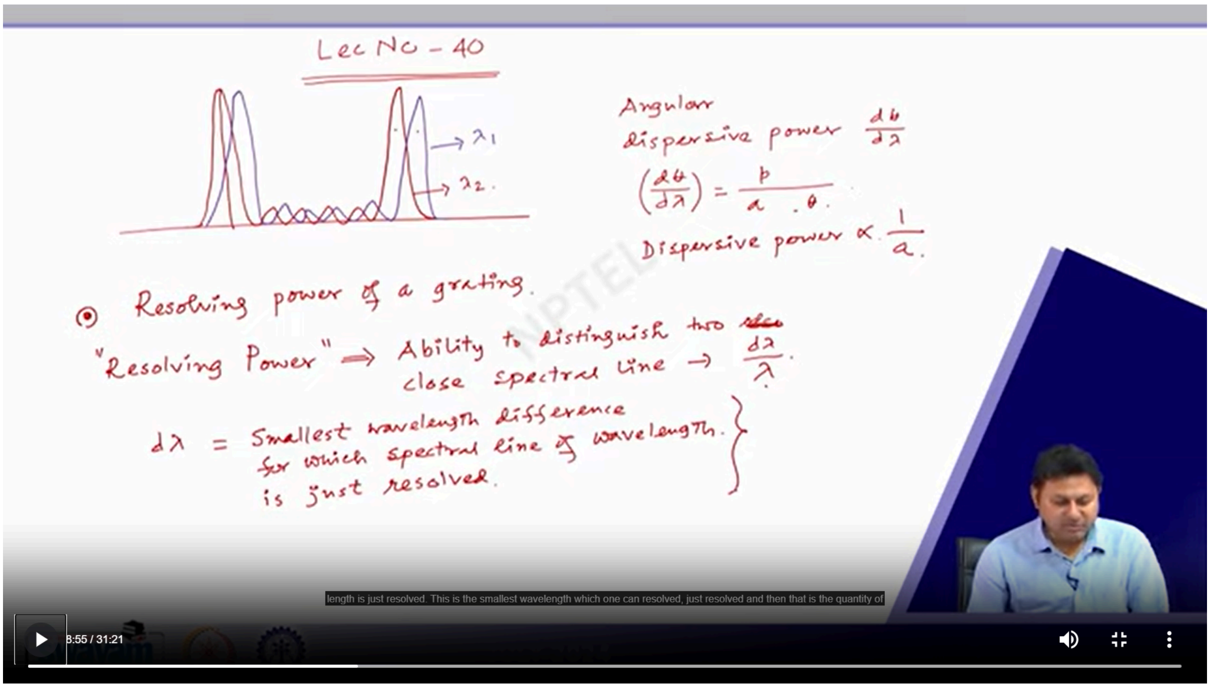

In the case of a thousand per centimeter for the same order we will see the spatial angular separation between a given order will be much more, in case of the grating having more number of rulings. So after that, we try to understand the next property or next quantity, which is called the resolving power of a grating. So, what is resolving power? Resolving power is the ability to distinguish two close spectral lines, which is D lambda by lambda by definition. So what is D lambda here? D lambda is the smallest wavelength difference for which spectral lines of the wavelength spectral line of wavelength are just resolved. This is the smallest wavelength which one can resolve, just resolved and then that is the quantity of resolving power. What is the meaning of that just resolve? So let me quickly draw the spectra once again that we have done earlier. So for one wavelength, the spectral line will be something like this and for other wavelengths, it is like this. So two wavelengths are here associated, say, this is lambda plus this d lambda and the other wavelength is lambda. So, lambda and lambda plus d lambda if we put these two together and then allow to diffract to a grating in the spectra we will see these two principal maxima placed side by side. But what is the condition for this delta lambda, that is the minimum separation that one can have if they two are far apart then the resolving of these two is very simple but if it is not that if it is close enough, then it is difficult to resolve. So what is the condition for that? Let us try to understand. So in terms of theta, if I want to do this plot. So let me plot it, then this is for red, say, we have a peak here and I am not plotting here the secondary Maximus and for other wavelengths, the condition is something like this, that the minima of this wavelength should be the maxima of another. So it should be like this okay. So these are the two wavelengths that we want to resolve and how the theta is measured. So this is the coordinate we have and this is the way the distribution one can have and I measure from this point this is my theta and this small is d theta, so this is the peak of e mth maxima for wavelength. lambda And this is minima for lambda, okay so this thing is also the peak of the mth maxima, for the wavelength lambda plus delta lambda okay, so that is the condition.

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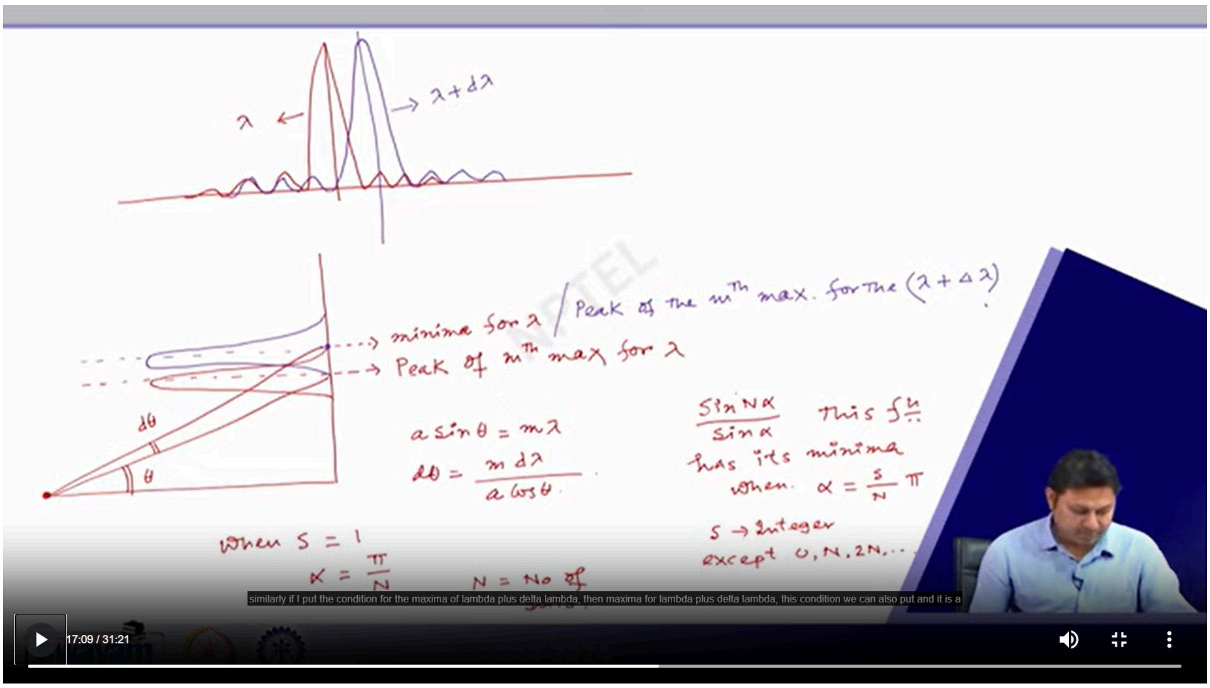

So if I use now the condition for maxima then we have a sine theta is equal to m lambda and d theta is equal to m d lambda divided by a cos theta that we have. Now, note that when we have this function because this is the function we are plotting sin n alpha divided by sin alpha. So, this function has its minima when alpha is equal to s divided by n multiplied by pi that we derived in an earlier class, where s is an integer, except these values 0, then n, then 2 n integers multiple of n is prohibited this value should not be there. So when s is equal to 1 then alpha I can write is as pi by n, n is a number of, mind it this is a number of slits. So similarly if I put the condition for the maxima of lambda plus delta lambda, then maxima for lambda plus delta lambda, this condition we can also put and it is a sine, theta is changed, now theta plus d theta, that is equal to 1 by n lambda plus m lambda because m lambda is the condition for the maxima and other order, the minima condition is 1 by n. So, the next order is 1 by n s. So, I am going to get this. T This is the condition for the maxima also for lambda plus e lambda. So, once we have this, we can write as m plus 1 by n lambda that we have and we already have another expression in our hand a sine theta that is equal to our m lambda. So from these two, we can have sine theta plus d theta divided by sine theta that is m plus 1 by n lambda whole divided by m lambda, just divide these two expressions that we get, this is 1 and this is 2, dividing 1 by 2 I get this is simply 1 plus 1 by m n. Now for the small theta, for small d theta rather, sine theta plus d theta is nearly equal to, sine theta plus d theta cos theta. So here we have sine theta plus d theta, cos theta divided by sine theta is equal to 1 plus 1 by m n. So we have d theta is equal to something like 1 by m n tan theta. So that is one equation, again, also we have a sine theta is equal to m lambda and from there we also calculate that d theta is equal to m d lambda divided by an of cos of theta. So, this is equation 2, again I write from 1 and 2. Equating these two conditions leads us to something like 1 divided by mn. Then tan theta I divide as sin theta by cos theta. And that side we get md lambda divided by a of cos theta, cos theta cos theta will cancel out and eventually, I have 1 by mn sine theta equal to m d lambda by a, where sine theta again is m lambda divided by a from the condition.

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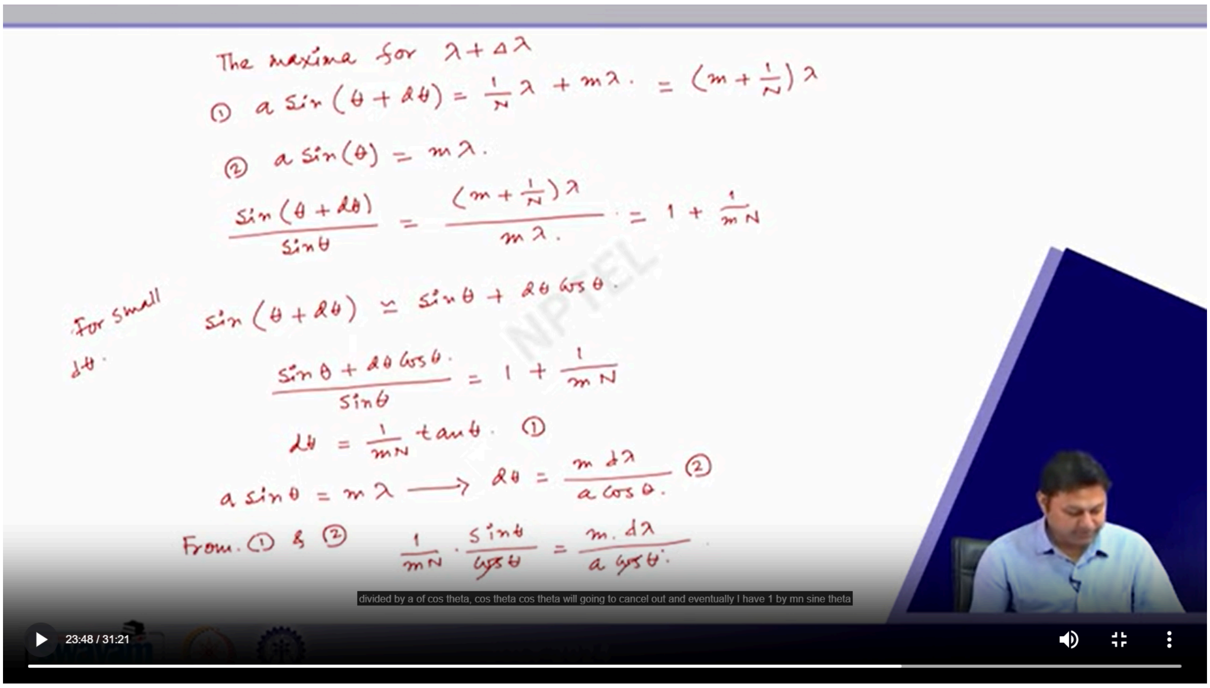

So, we are just playing with the expression to get what the expression is. So, our goal is essentially to find out what is the expression of d lambda divided by lambda. That is the resolving power of the grating and we need to find out what is this value under that condition, under just a resolving condition. And what is the just resolving condition? I am going to explain again. So, let me finish this calculation. We are almost done. So, sin theta is this, if I replace this, eventually we have 1 by m n multiplied by m lambda divided by a equal to m by a d lambda. So this, m m will cancel out, a and a cancel out, so we have d lambda by a lambda, so lambda divided by d lambda is equal to m multiplied by n, okay that we are getting. Let me check once again, Mn is lambda divided by d lambda. So this is the quantity we are looking for and if I go back to our original thing, what is the spectral ability to distinguish two closely spaced spectral lines and that is the quantity we are looking for? And so from this expression, we can see that if this quantity d lambda by lambda is 1 by Mn. So, if n is very high, if this is the grating number or the number of the ruling, this is the number of slits. So, the number of slits, if it is very large, then what happened? We have this d n d lambda by lambda quantity very small, which means we can resolve even a very small wavelength by using that particular wavelength whose number of slits is very large, number of slit is very large means, the grating, the ruling per unit length is large. So how do we resolve that? What is the condition for resolving? The resolving condition just about to resolve to spectra is this. If I have a spectra peak, central peak, and here we have a minima on that and exactly at that point if I have the maxima for other wavelengths, then that is the condition for just resolving. So, here that condition we exploit, and when we exploit that condition, we find that is the condition for maxima of lambda and that is the condition for minima of lambda plus d lambda. That is the condition for the maxima of lambda plus d lambda, but that is also the condition for the minima of lambda. So, these two conditions we combine together and after combining these two we find that this quantity d lambda divided by lambda, that quantity is 1 divided by m n, that means, if the number of slits is very large then we can have small wavelength separation and that small wavelength separation can be resolved. This is the amount of the wavelength separation that we can resolve compared to a given wavelength lambda. So that we are able to resolve this delta lambda will reduce more if the number of slits is more. So, delta lambda will be small, I mean even a smaller wavelength can be resolved with the grating for which we have the number of slits n is very high. So, today we have that much. So, we discuss in detail two important properties of grating, one is its resolving power and another is the angular diffraction. And this resolving power, we find, is directly related to the number of slits. And if we increase the number of slits in a given grating, then that grating essentially will have a more resolving power. So, with this note, I would like to conclude here. Thank you very much for your attention and see you in the next class.

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