WAVE OPTICS
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Lecture - 04: Plane wave, Spherical wave

Hello, student to lecture number four. For this course, wave optics, today we will discuss what is called plane wave spherical wave, cylindrical wave, etc. So let me go back to what we have done so far and then gradually we discuss different kinds of waves. So we had first the wave equation and the wave equation was in this form in 1D. So it is lecture number 4 and first, we had the wave equation and the wave equation in 1D. We had the form like del 2 y del x square is equal to 1 by v square del 2 y del t square and we have a solution here of the form where we can write the solution y equal to so let me erase this. So y is equal to f of x plus minus vt which is the form of the solution we have in 3d. We have the wave equation in this form and here the psi should be a function of r and t now we also describe what is called the harmonic waves. The harmonic wave is something in one dimension if it is propagating along x direction is a sine k x minus v t or I write a sine kx minus omega t, in complex form I can also write this harmonic wave,

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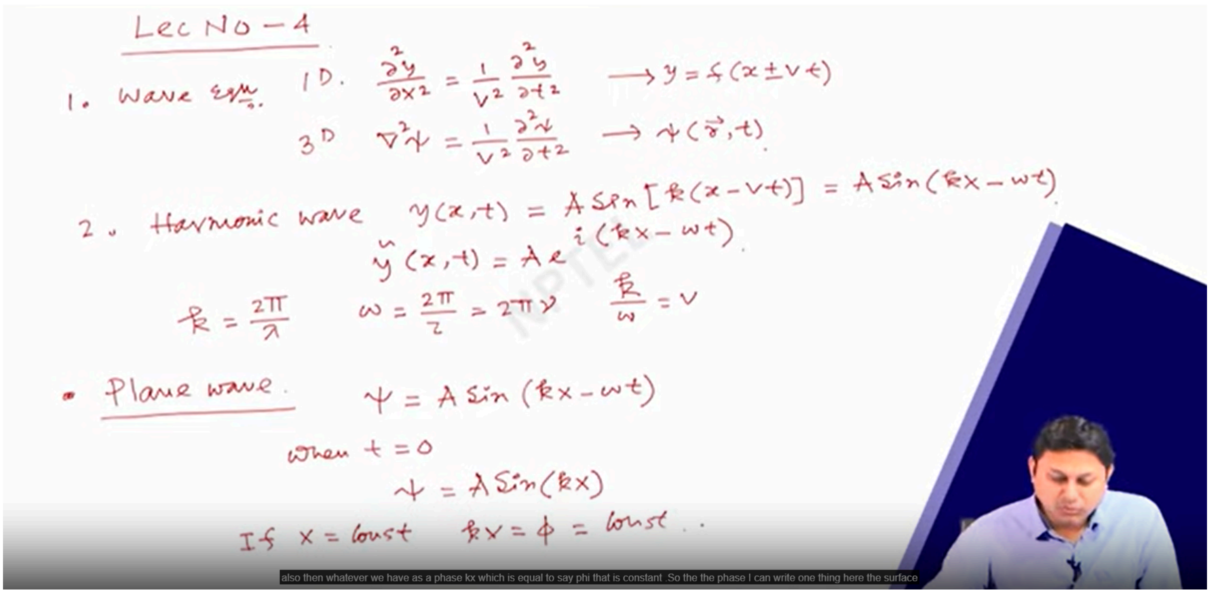

and the form will be simply a e to the power of i kx minus omega t here, k is a propagation constant having the magnitude 2 pi divided by lambda and omega is angular frequency and we can write it as 2 pi by tau temporal frequency, temporal period and 2 pi new, k by omega is essentially the velocity of the wave. These are the things we learned in the last class. So today we will expand on all these things and try to understand more about them. So the next topic that we are going to explore is called the plane wave. So, the wave displacement is defined by in general I am writing a different notation phi here. Psi is equal to A sine in 1D kx minus omega t. Now, when t is equal to 0 that is at some or t equal to constant for simplicity let us take t equal to 0. You can do that for t equal to constant as well. Then the form of psi is simply a sine of kx. Now if x is constant also then whatever we have as a phase kx which is equal to say phi that is constant. So I can write one thing here: the surface of the constant phase essentially gives us the wave front which means if the wave supposes I am illuminating a light here is a light source and generally the waves are going outward and forming a wave front. This wavefront we will discuss in detail later but it will produce some kind of wavefront and that wavefront is normally in this way it is propagating so these are called wavefronts. Now the sinusoidal wave that we are discussing here, if I solve this equation phase equal to constant then that means the surface of the wavefront we're going to get, from this expression from this equation, that phase equal to constant.

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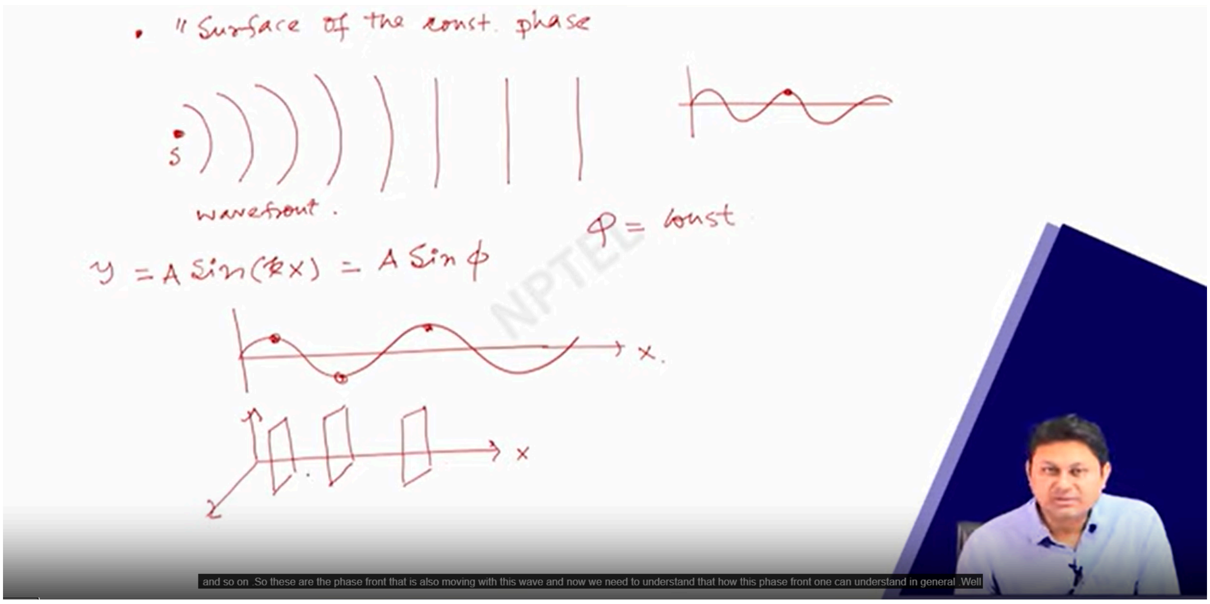
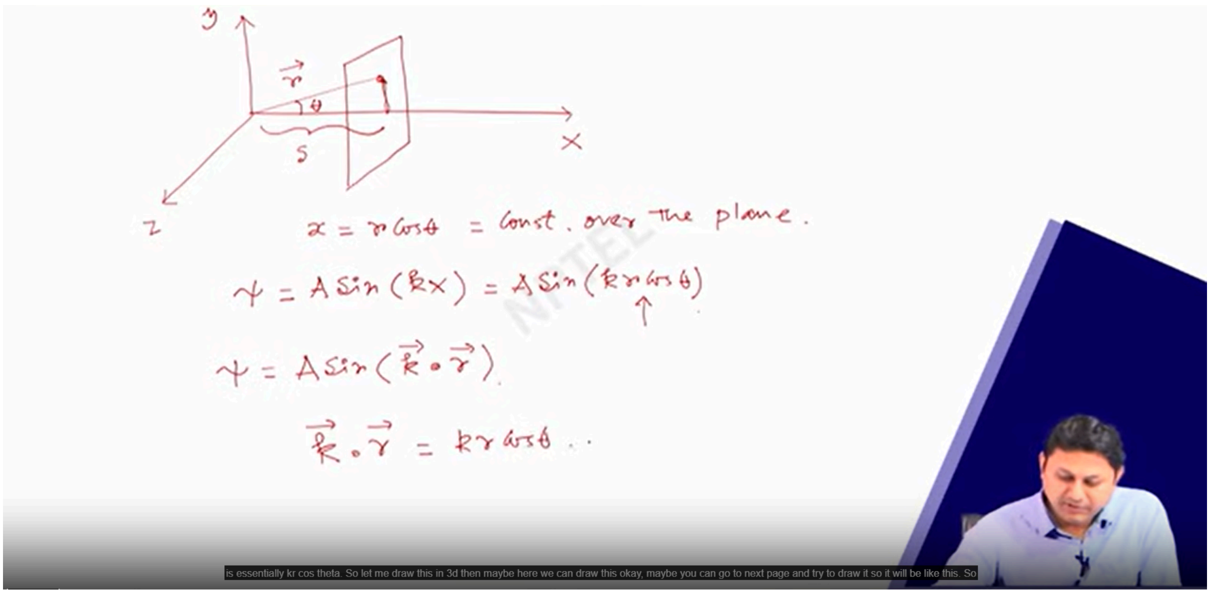


If this wave front moves a very large distance one can understand that it becomes essentially a plane wave that is forming initially. It was curved but as it is propagating to a very large distance it is forming like a plane but the point is from where we are going to get this plane. As I mentioned, the phase constant phase. So the wave that is moving here is like this. So if I find any particular point here which is the phase of this wave and this phase, if this phase is constant then we will get a set of an equation and phase equal to constant that equation basically tells us what should be the surface of this wavefront. So, the surface of the constant phase is given to us here in this case when we write a sign for this sinusoid this harmonic wave say a sine kx or a sine phi this is my phase and if I write this is equal to some constant then this gives us a family of a plane which is perpendicular to x. So that means this is the way the wave is moving along x direction, if I have a fixed x that is the phase is equal to constant, and try to understand what is the shape of the corresponding face front then we're going to see that it is essentially forming a surface and that is a plane. So over this direction x is propagating and each point . Suppose I now go to this point here also it is forming a surface and so on. So these are the phase fronts that are also moving with this wave and now we need to understand how this phase front one can understand in general. Well, let me draw it once again. So this is the plane we are talking about and this is my X and this is the point on the surface of this face front and this is say r vector along this direction say this is z and this is y. So if I have the perpendicular from this point over x this length is say s and this angle is theta.(Refer slide time: 13:55)



So here x is equal to that means this is a point along this direction it is x is r cos theta and over this plane, this r cos theta is constant over the plane. Now psi is equal to a sine kx and in place of x if I put this r cos theta this is essentially an of sine k r cos theta and this quantity is entire quantity is basically constant over this phase so if I write k x equal to constant then x become constant because k is constant and I find that if it is constant then that gives a surface which is essentially a plane like this. So, the phase front should be a plane when we solve the equation that the phase is equal to constant. Well, this is for one-dimensional propagation. In three-dimensional propagation, we can also have the same thing. But in 3D, for example, the wave functions can be written as a sine k dot r instead of writing kx because it is moving in an arbitrary direction. So it should be k dot r if that is the case then we can have k dot r is essentially kr cos theta. So let me draw this in 3d then maybe here we can draw this okay, maybe you can go to the next page and try to draw it so it will be like this. So the K will be perpendicular to the wavefront, this is Z. Say this is X and this is Y, and any point we have R over this plane and if I draw a perpendicular on this R this is S. So, phi if I write it as A sine of k dot r minus omega t then a particular time t if the phase is constant then I have the equation k dot r equal to constant at fixed t, then k dot r constant basically gives you as I mentioned this phase equal to constant gives you the equation of the wavefront.

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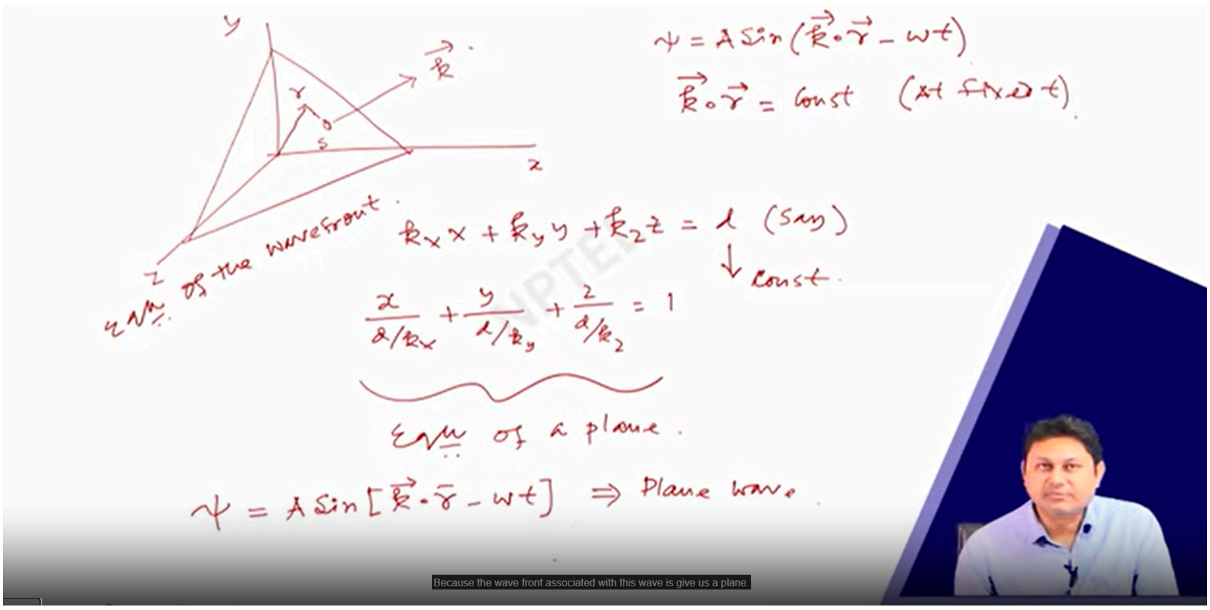

So the equation of the wavefront is essentially k x x plus k y y plus k z z is equal to say d some constant where this is constant. So that is essentially the equation of the wavefront if the mathematical form is like this then we have. We can readily find that this is nothing but I can write it in this well-known form which tells us that this equation essentially gives us an equation of a plane. This is the equation of a plane. So the point is, whenever you have a harmonic wave in the form sine or cosine k dot r minus omega t that represents a plane wave. Because the wavefront associated with this wave gives us a plane. Now, let us go back to the figure that we had a few minutes ago if we have a source, then initially the wave front moves in this way. But you can see that this is not a plane rather this is spherical. So, that means we should also have the spherical kind of wave. So, the next thing that is why we are going to discuss it is spherical waves. So, in fact, if you have a point source, it illuminates light and the light is propagating as a wave. But the wavefront that is produced is basically spherical, that is basically spherical in nature. So, in order to understand that, we need to write down the 3D wave equation, which is this. And now, this is a spherical wave. So, we need to use this Laplacian, whatever the Laplacian we have in the equation, in this wave equation, in r theta phi in this coordinate. Because the psi will be a function of r theta phi and t for spherical waves. Now from the symmetry we can see that if this is a source and if the spherical waves are coming out like this then from the symmetry, we can see from spherical symmetry. From spherical symmetry, we can write that psi will be only a function of r.

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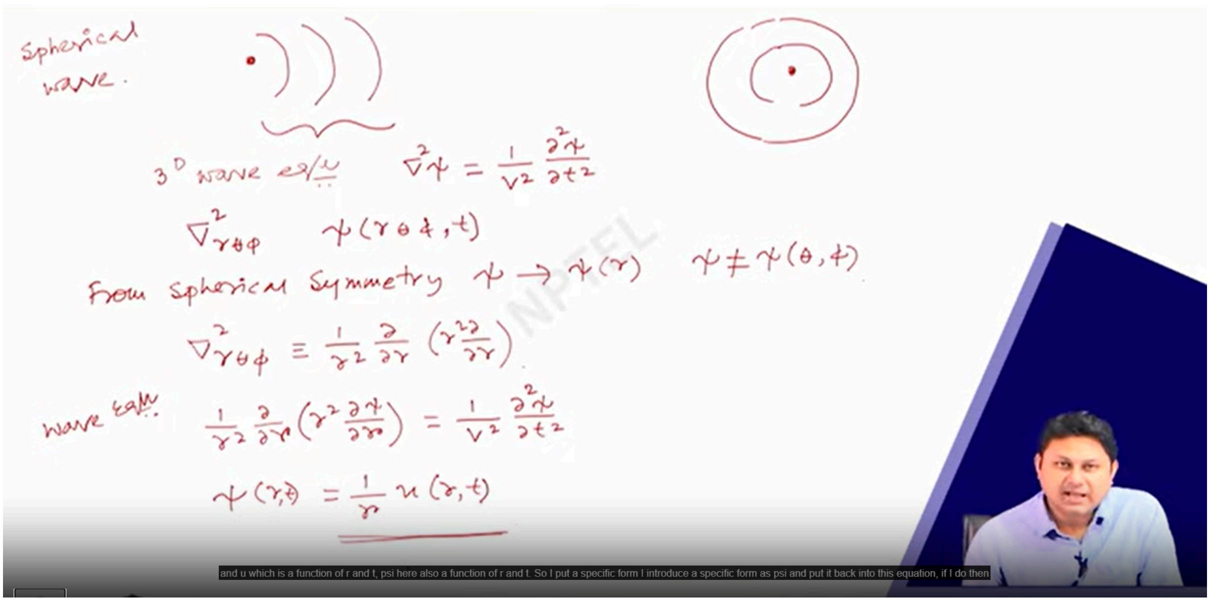

It will not be a function that is independent of, psi will not be a function of theta or phi, because the theta phi symmetry should be there. In that case, life will be easy because this Laplacian, whatever the Laplacian we are talking about here, r theta phi, will have a very straightforward form 1 by r square and then del, del r and then r square del, del r.Now if I put my wave equation in a form like this the wave equation will have a form like this. If I write in terms of this r it will be like this; r square, del psi, del r that is equal to 1 by v square, del squared psi, del t square so that equation is a 3d wave equation. But at the same point, we impose the spherical symmetry on that so we expect that the psi whatever the psi we will get from the solution of this differential equation gives us a spherical wave. So we will do one thing without solving this in a rigorous manner. Rather we check something easily that lets us put phi which is a function of r in this form 1 by r and u which is a function of r and t, psi here is also a function of r and t. So I put a specific form. I introduce a specific form as psi and put it back into this equation, if I do then I can have this operator, this Laplacian over psi and that will give us 1 by r squared. This is the left-hand side of the wave equation I am calculating in fact, del del r, and then we have r squared del del r and in place of psi we have ru. Because psi I put as this is 1 by r u that is a function of r and t also when I put it back here in this equation then I'm going to get this. Let us do this calculation to state forward the partial derivative then we have, r square and I have one.

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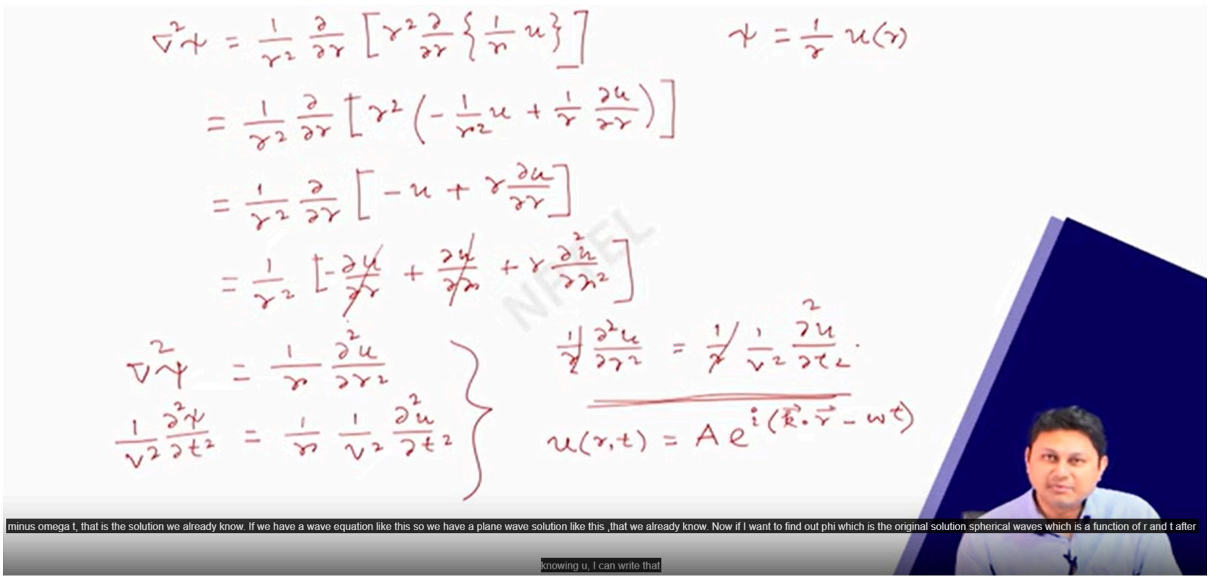
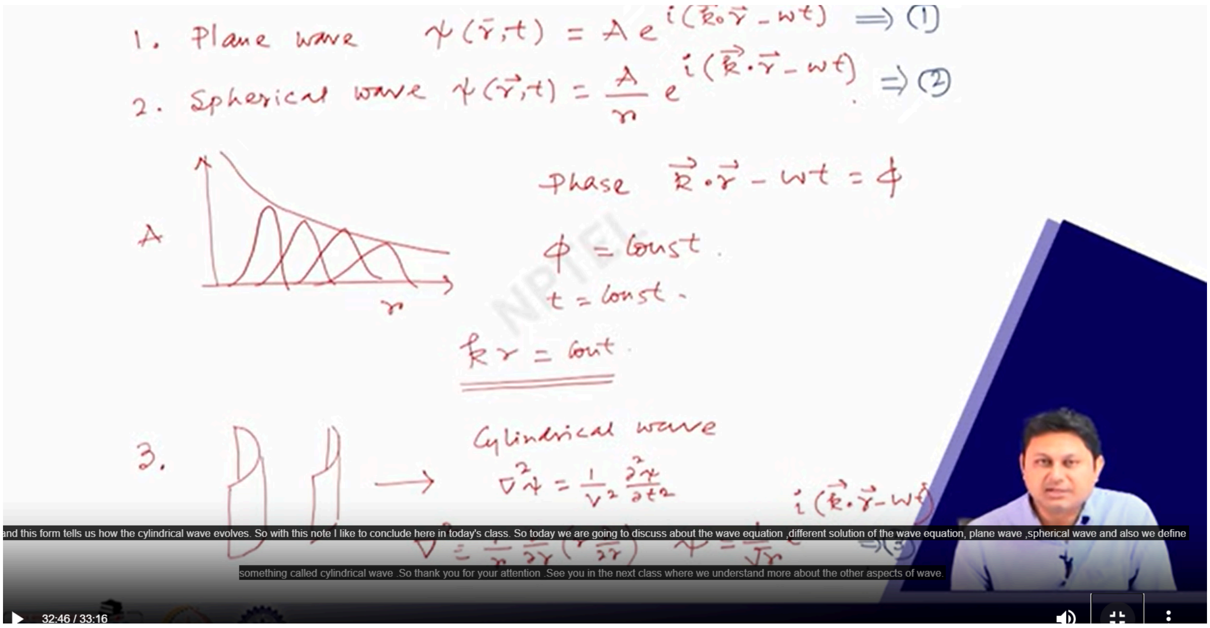


The derivative with respect to r for 1 by r square will be minus 1 by r square u and then plus 1 by r del u del r. This thing is 1 by r square del del r minus of u plus r, del u del r, this is 1 by r square minus of del u del r then plus of del u del r, I am just doing the calculation with this r del 2 u del r square. So this term seems to be cancelling out, so essentially what I get is 1 by r del u del r square and that is this quantity. So, I can also calculate 1 by v square, del 2 psi, dt square and it will be simply 1 by r,1 by v square, del 2 u del t square. Now, if we equate this we are going to get 1 by r, del 2 u, del r square that is equal to 1 by r, 1 by v square, del 2 u, del t square. So 1 by r, 1 by if I cancel out we can see that u is following a standard wave equation. Which is d2 u, dr square is equal to 1 by v square, d2 u, d square. So, readily we have the solution for u as u, which is r and t. It should be written as some constant a e to the power of i k dot r minus omega t, that is the solution we already know. If we have a wave equation like this we have a plane wave solution like this, that we already know. Now if I want to find out phi which is the original solution of spherical waves which is a function of r and t after knowing u, I can write that it should be divided by r e to the power of i k dot r minus omega t. So, what I found is for plane waves. The solution phi which is a function of r t will have the form a e to the power of i k dot r minus omega t for spherical waves. The solution is almost the same except a 1 by r term is sitting here it should be k dot r minus omega t. (Refer slide time: 32:46)



So that means over the distance in a spherical wave what happened that over the distance, The amplitude will decay like this because of this 1 by r term present in the denominator the wave that is moving, the spherical wave that is moving the amplitude is going to decay as a function of 1 by r so there will be a variation in amplitude. Not only that from here that is the phase. So, the phase is k dot r minus omega t which is phi. So, phi is equal to constant if I put this equation as I mentioned earlier also that gives you phi equal to constant, this equation gives you the equation for the wavefront. We already know that this is a spherical wave so for a given time t is equal to constant at this given time we have the equation that, k r is equal to k dot r equal to constant or k r equal to constant because k and r will be in the same direction. So that is the equation of a sphere, k dot r equal to constant or r equal to constant is the equation of it is the equation of a constant phase and that will give you a sphere. Well, we can also expand this calculation for another kind of wave which we call the cylindrical wave, but I am not going to do this calculation in detail. So a cylindrical wave is something where the wavefront is forming a cylindrical shape like this. So it is moving like the wavefront is moving like this in 3d it will look like this. So this is the way the wavefront is going to move so for cylindrical waves, Vigorous calculation can be possible and I am not going to do that. The wave equation, whatever the wave equation we have, this Laplacian will be simply replaced by the cylindrical coordinate and from the cylindrical symmetry. We know that the phi and z dependent will not be there so this operator will be replaced by 1 by r, del del r is a Laplacian operator and if I reduce it to only r variable only for the derivative for r variable it will be something like this. So we're going to get an equation and if I do the rigorous calculation we will find that the wave phi will have a form like this, root over r e to the power of i k dot r minus omega t. So this is another kind of form. So the plane wave we discuss in detail is having a form like this. Let me do that in a different colour.Say this. So we will have a form like this. Equation 1. That is for the plane wave. This is for a spherical wave. We calculate in a rigorous way how this kind of form can be obtained if the wavefront is spherical in nature and also one can have a cylindrical wave which I didn't calculate in a rigorous manner, but if somebody does this calculation by using separation of variable then one can get a solution of this particular form for r tends to infinity for a very large distance, one can have roughly this form and this form tells us how the cylindrical wave evolves. So with this note, I would like to conclude here in today's class. So today we are going to discuss the wave equation, different solutions of the wave equation, plane wave, and spherical wave, and also we define something called cylindrical wave. So thank you for your attention. See you in the next class where we understand more about the other aspects of waves. Thank you and see you again.