WAVE OPTICS
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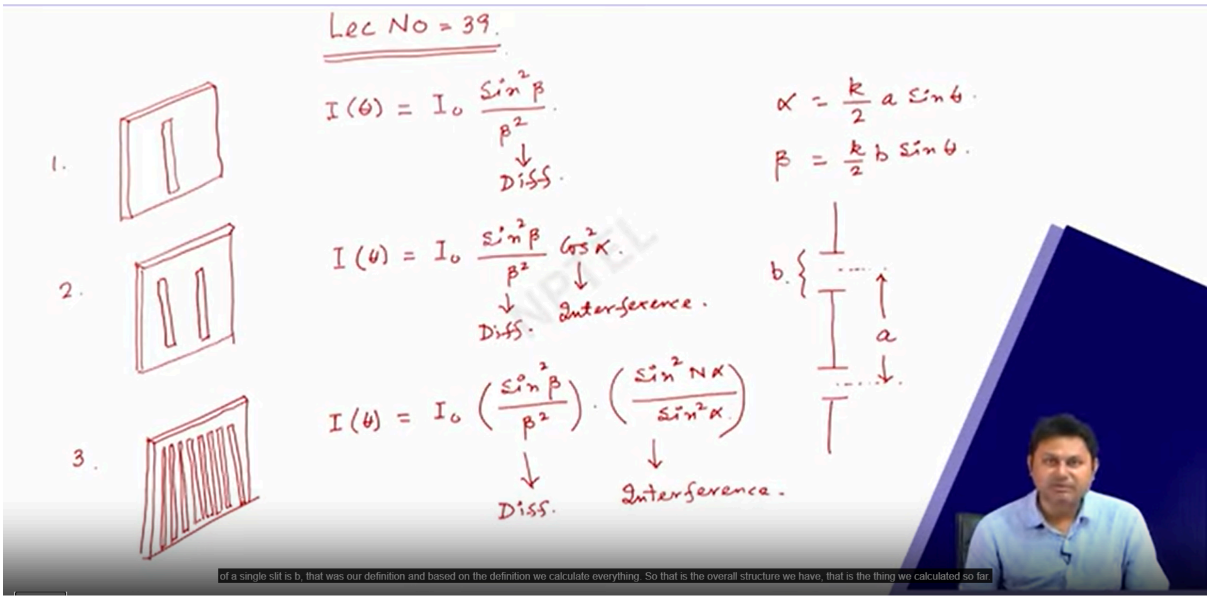
Lecture - 39: Grating spectra (Cont.)

Hello, student, welcome to the wave optics course. Today we have lecture number 39 and in this lecture, we are going to discuss the grating spectra that we discussed in the last class. We continue this discussion. So, this is lecture number 39 and we are going to discuss grating spectra. Before going to the grating spectra discussion, let me remind you what you have done so far. So, first, we discussed the single-slit problem. Let me draw the physical structure of the single slit we have a single slit, so this is a single slit, and for that, we have intensity distribution expansion, intensity distribution was this, then this is one, then we had double slit, have an aperture like this instead of having one window. Now we have two windows placed side by side. So this is a double slit and for this, we had the expression of intensity at some point in the screen, and finally, for grating, we have multiple slits. So not only one aperture, we have multiple tiny apertures, multiple sets there. Okay, for this we calculate I theta, the most general form that one can have is this. We have sine square beta, divided by beta square, then sine square n alpha, divided by sine square alpha. So in the first case, this term is totally due to diffraction, second case this term is due to diffraction and this term is due to interference, this case this term is due to diffraction and this term is due to interference. Here alpha is k by 2 a sine theta and beta was k by 2 b sine theta. What is a and b that we also defined? So if I draw a slit, for example, two slits, this is the structure. So the separation between the two slits is A and the width of a single slit is b, that was our definition, and based on the definition we calculate everything. So that is the overall structure we have, that is the thing we calculated so far. Now we try to understand the spectra for the grating structure. (Refer slide time: 06:48)



So we already had this spectral distribution that is coming due to the sine square, n alpha, divided by the sine square alpha term, that we discussed. So we have a principal maxima and then we have some secondary maxima and also some minima. So these are the spectra and this is due to the function sine square n alpha divided by sine square alpha. So these are the maxima. How many maxima and how many minima we have we can calculate? This is 1, 2, 3, 4, 5, 6. So we have 6 minima here. If we have 6 minima then the number of slits n is 7. And we have, if it is, then the number of maxima should be 5. So, here we have 1, 2, 3, 4 and 5. That is the number of secondary maxima in between two principal maxima and that is the condition we have. And also we calculate what is the ratio between this and this. If this peak intensity is of the principal maxima, and if the intensity here is, say, secondary maxima, then the ratio we calculated in the last class, and that is I secondary maxima divided by I principal maxima, Pm. That quantity is 1 divided by 1 plus, n square, minus 1, and then sine square alpha. Note that, in the expression we have alpha. So that means this secondary maxima intensity is a function of theta as well because alpha is k by 2 multiplied by a and sin of theta, that means the secondary maxima will have the effect, it will not be the same for all the cases. There will be some variation, but this variation is small. So eventually it looks like the same peak. Well, now further we try to understand the spectra because the total intensity pattern not only contains this term then the diffraction term is also there. So that means sine square beta divided by beta square and sine square n alpha, divided by sine square alpha. So this term is also there, which I write in the bracket. So the total intensity will be the multiplication of these two and if I do that we know that there will be an envelope that is generated by, so it should be something like this. So, let me draw it here. So, maybe on the next page because this is a big picture. So, let us use the next. So, we have spectral distribution for this function and this is the change, say, alpha or theta whatever. Now on top of that, if I put the spectra at the intensity of the diffraction pattern, then it should be like this. So, this dotted line corresponds to sine square beta divided by beta square. So, what will the overall spectra look like? So, it seems we have something like this, an overall spectrum one can expect which is a combination of these two.

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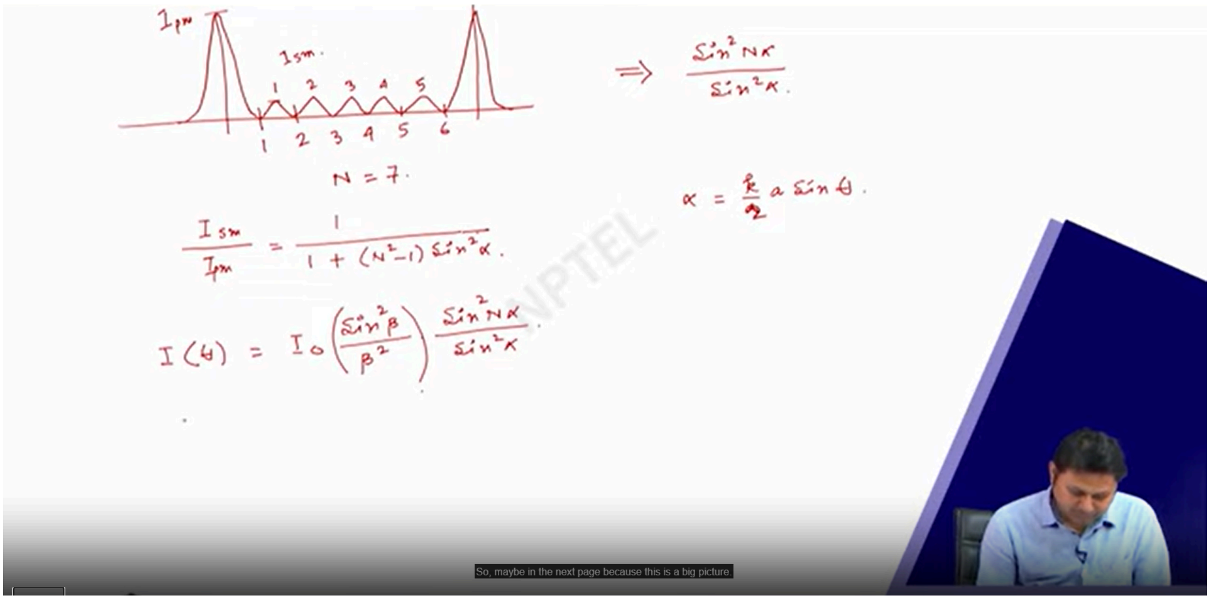

So, I plot here, that should be a function of theta with this form and this is the combination of this. So, here also there is a possibility like in this case the maxima of one spectrum one. A is coinciding with the minima of another. So that means there is a possibility that we can have something called absent spectra or missing spectra lines. What is absent spectra? Suppose P-th order principal maxima for interference that is for this term, that one can find when we have alpha equal to p pi and then we get a sine theta is equal to p lambda, that is the condition for principal maxima and alpha, then let me write it is k by 2 a sine theta, several time I wrote this, by that time you should familiar with this notation, this pi by lambda a sine theta and beta is k by 2 b sine theta, which is pi by lambda b sine theta, these two we know and then from the condition for principal maxima we can have this. Now for the same theta that we have, if we have the mth order diffraction minima, that is coming for the term sine square beta divided by beta square. From that term, one can get this. Then we have another expression for the same theta and we have here I can write that beta is equal to, actually I am looking for the minima, this is m pi, where m is not equal to 0, that is the condition of the minima. So that means we have b sine theta, same sine theta we have m lambda, so we have one equation, let us put this equation one and this is equation two. So, we have equation 1 and equation 2 and they should be for the same theta. T So, I can write from equations 1 and 2, we have sine theta is equal to P by lambda a, that is equal to m lambda by b, again we have a relationship between a and b. So that means a by b is p by m the ratio of two integers. So that is the condition of missing order as we have for double slit exactly the same thing. This is the condition for missing orders. That means when A is this. Let me draw what is A. So A is this and this is B. When A is, for example, 2 of B, then this ratio, so when suppose A is 2B, then the A by B ratio is 2. So, we have P by M equal to this. So, P is the order for maxima. So, P will be 2 of M, which means when m is equal to 1, 2, 3 etcetera, then p will be 2, 4, 6 etcetera and these will be the missing order, that should come through the interference pattern, for interference. So exactly like for double slit, for multi-slit, also we can have this spectrum, that is what we call the missing order. The next thing that we need to understand regarding the grating spectra is something called the angular dispersive power of grating.

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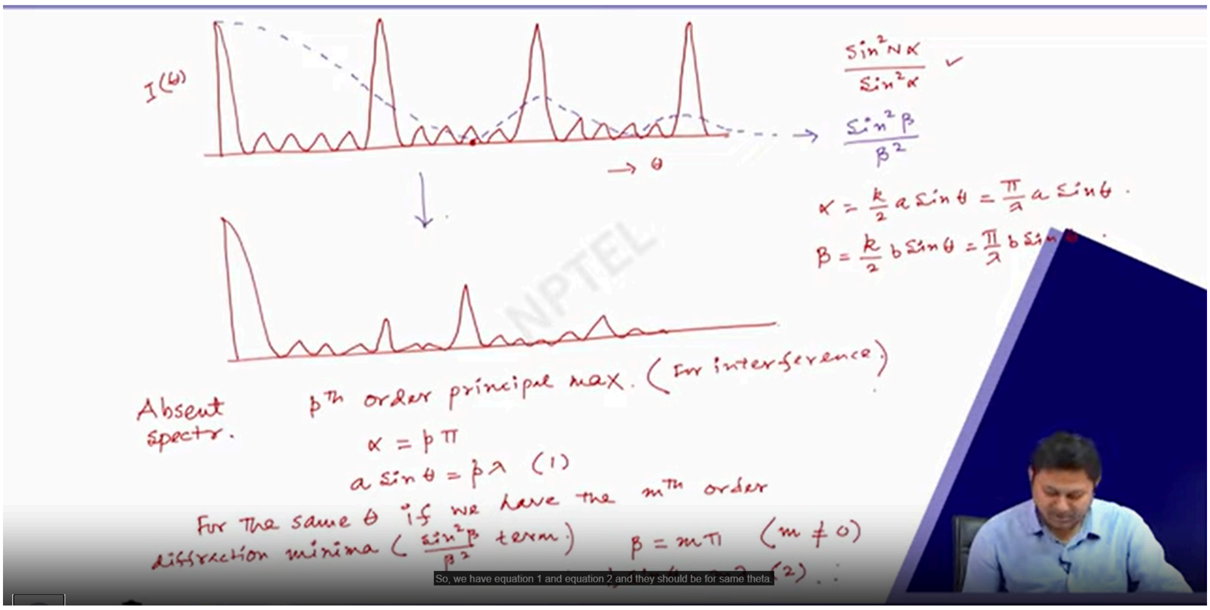

So, angular dispersive power means, how we can separate with respect to wavelength, and how the separation is there. So let me first define and then let me quickly. So this is the quantity we want to find dispersive power. So what is the meaning of that? So we know that this is the spectra for grating. So let me draw the spectra, for one given wavelength. So we have a maximum here and then we have secondary minima and then a maxima here and then secondary maxima then, maxima here and this is for a given wavelength. Now if I have another wavelength and for that wavelength, if I like to draw the same curve here, there will be a slight shift of the same theta because the condition, for maxima we know and that condition is A sine theta is equal to m lambda. So, for different lambda for a given theta, the condition mth order will change. So, maybe for other wavelengths, we have a spectrum something like this. So that means there is a shift of this principal maximum whatever the maximum we have there is a shift and that shift is expected because I am dealing with another wavelength so order and both the wavelengths are obeying this equation. So there will be a shift now if there is a shift then what happened? So, with the same theta, there will be a separation. So, how much separation with unit lambda one can have the angular separation that basically measures the dispersive power of a grating? So, the rate of change of the angle of diffraction, means I am talking about the theta with wavelength lambda. How with wavelength lambda, there is a shift of theta in order to get the same order of principal maxima. So, we have I is equal to I naught sine square beta divided by beta square, multiplied by sine square n alpha divided by sine square alpha, that was the form we have. Now for p-th order principal maxima, we have a sine theta equal to p lambda that we already mentioned here. That is the condition we have. So here I can write a of cos theta, then d theta, d lambda is equal to p or in word d theta d lambda is equal to p by a sine theta. So that means d theta d lambda, which is proportional to 1 by a for a given p and theta. So 1 by a means, what is 1 by a? 1 by a is essentially the number of rulings per length, typically the grating has a ruling per millimeter few hundred or few thousand so if the number is high then the resolving power of the grating should also be high. So, the angular dispersive power of the grating increases with the increasing number of rulings per unit length. So if the number of rules is very high then, that basically increases the dispersive power of the system. So with that note, I don't have the time to discuss more about the grating spectra or other properties of the spectra. So I'd like to conclude here today because of the lack of time. So in the next class, we will start from here and try to understand another quantity, which is called the resolving power of the grating. So, with that note, I would like to close my topic here. I like to stop my discussion here. So, see you in the next class and thank you for your attention.

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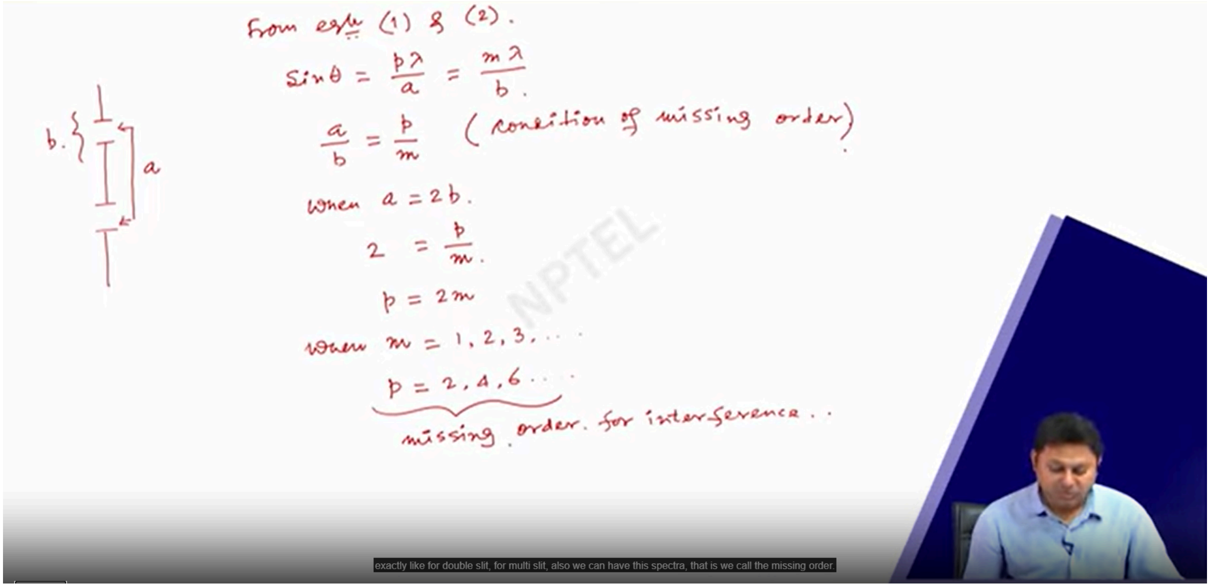

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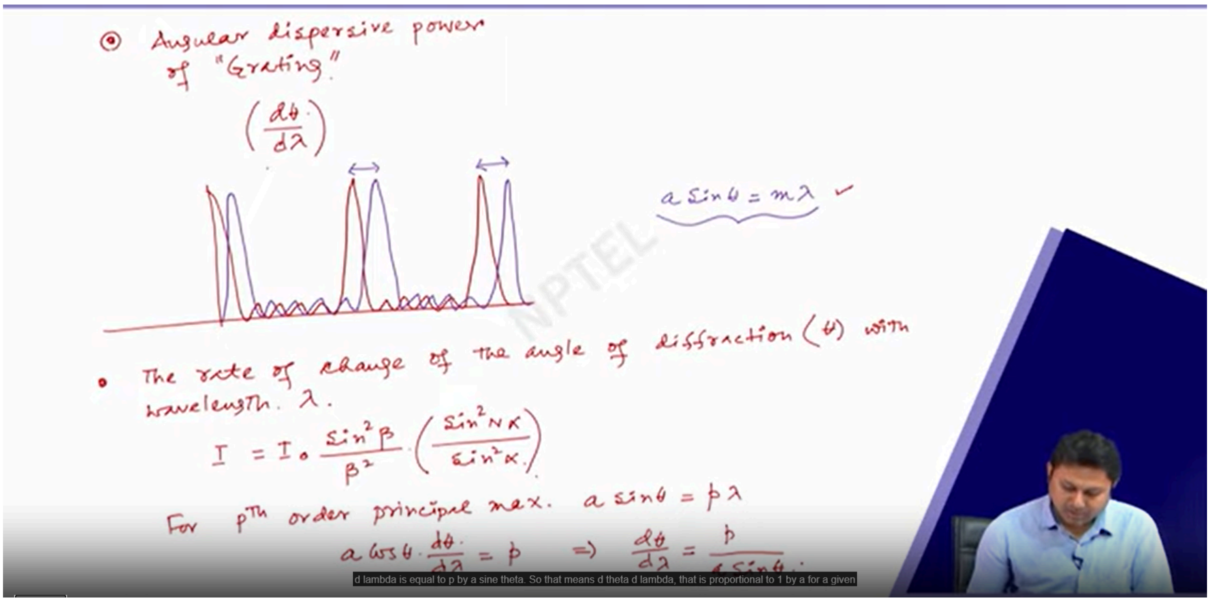

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