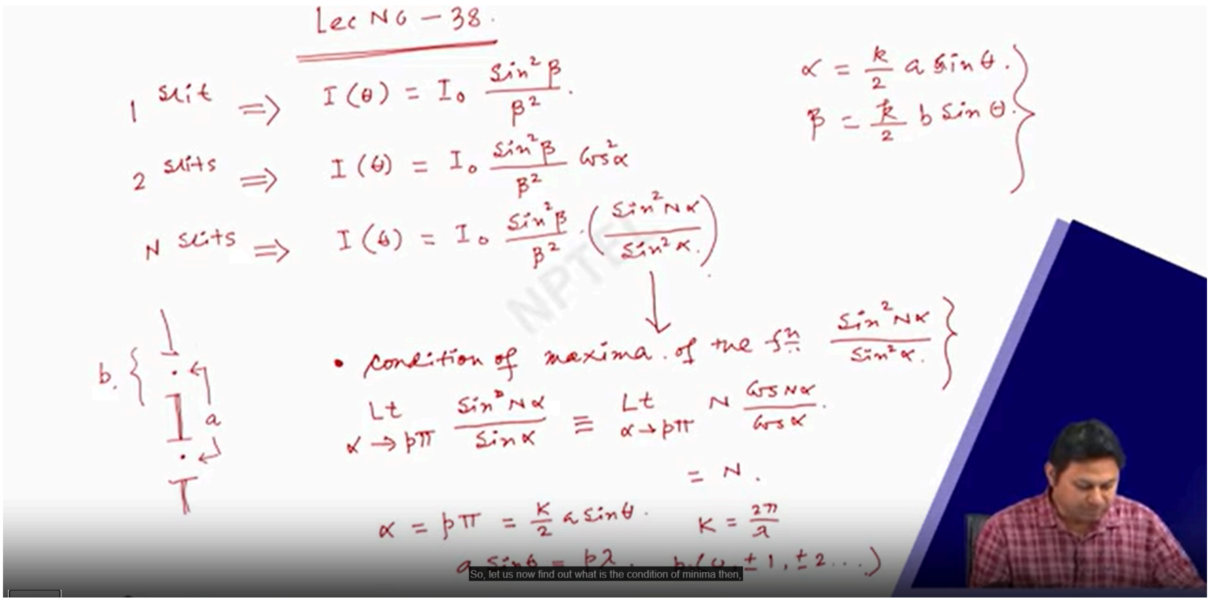
WAVE OPTICS
Prof. Samudra Roy
Department of Physics
Indian Institute of Technology Kharagpur
Lecture - 38: Grating spectra

Hello, student, welcome to our wave optics course. Today we have lecture number 38, where we are going to discuss the grating spectra or the intensity distribution that we derive for n number of slits. So, we have lecture number 38 today and discuss the grating spectra or the n number of slits. So for one slit or the single slit the intensity distribution was I which is a function of theta was I naught sine square beta, divided by beta square, for two slits we calculate that intensity distribution which is a function of theta is I naught, sine square beta, divided by beta square and cos square alpha that was for two slits and for n number of slits the intensity distribution, that we derived is I function of theta will be I naught sine square beta, divided by beta square, then sine square N alpha divided by sine square alpha. So these three distributions we calculate rigorously starting from the first principle and from that you can see that if I put n equal to 1, we get back from this expression, if I put n equal to 2, I can get back the expression for the second one that is for two slits, so for three slits, four slits what should be the value you can put by just putting the value of n. Also, you should note that here alpha is k by 2, a sine theta that is apparent, that is the value of alpha and beta is k by 2 b sine theta, what is a and b, that we do not need to define also that comes from the geometry for single slit. We have one slit width and that slit width is b but for multiple slits, we have another parameter and that is the separation between the consecutive slits. So the separation between the consecutive slits is defined by a, so that is a, and the width of the individual slits is defined by b. So, those are the two parameters I defined from the beginning, and based on that we calculate the intensity distribution which is shown here. So, this is the basic structure we have. (Refer slide time: 09:19)



So, what we like to find today is that we concentrated on the n slit problem and concentrated on this term, what is the condition to get a maximum and minima for this term because of the diffraction condition we know but the interference condition we have this contribution is coming due to the interference. So we try to find out what are the maxima and minima of this function. So the first thing we calculate here is the condition of the maximum function. Let me write explicitly the function sine square n alpha divided by sine square alpha, I want to find out the maximum of this function. Now from here, you can see that when alpha, if I put alpha tends to 0 here then the numerator and denominator both vanish. So I need to do the limit in a different way. So the limit, if I put alpha instead of 0 if I put p pi, p is an integer, so any integer of pi if I put here, then the limit of this quantity, rather so sine n alpha, let us remove this square because I'm trying to find out the limit for this quantity. It will be simply equivalent to, using the law hospital rule, the derivative of these things multiplied by, divided by derivative. So it will be N of cos of N divided by cos of alpha. So, if I put p pi here and if I want to find out what then this value is essentially equal to n, and n is a large number and that is the maximum value of that. So, alpha equal to p pi is a condition for the maxima of this function. If that is the case, then I can write it as because alpha is k by 2 and this is the condition of maxima. So, alpha is k by 2 a and sin theta. So, k is 2 pi divided by lambda. So, simply by putting, we have A of sine theta is equal to P lambda, that is the condition when the value of the P can take 0, plus minus 1, plus minus 2, etcetera. So, this is the value of these angles or this is if I write a sin theta is equal to p lambda. So, this is the value at which this function should have a maximum. So, let us now find out what is the condition of minima then, because minima are also, so, we want to track this function, so, condition of minima of the function, which function? This sin n alpha is divided by sin alpha. It is a square, but I want to find out what is the condition of minima for this function. So, note that when, it is interesting to note that when n alpha is equal to, so this function vanishes, when, n alpha is equal to some integer s pi, whereas s is some integer. Under that condition what happened? The sin of N alpha will vanish but there is a restriction over S, because as soon as the value of the s is equal to, such that it is a multiplier of the n,

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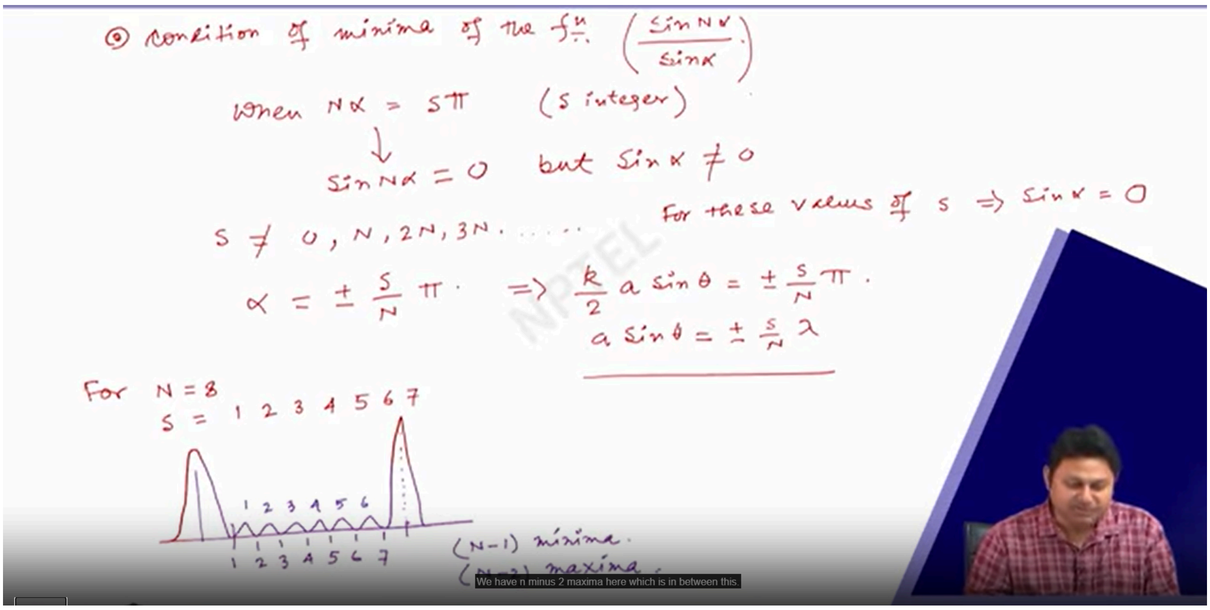
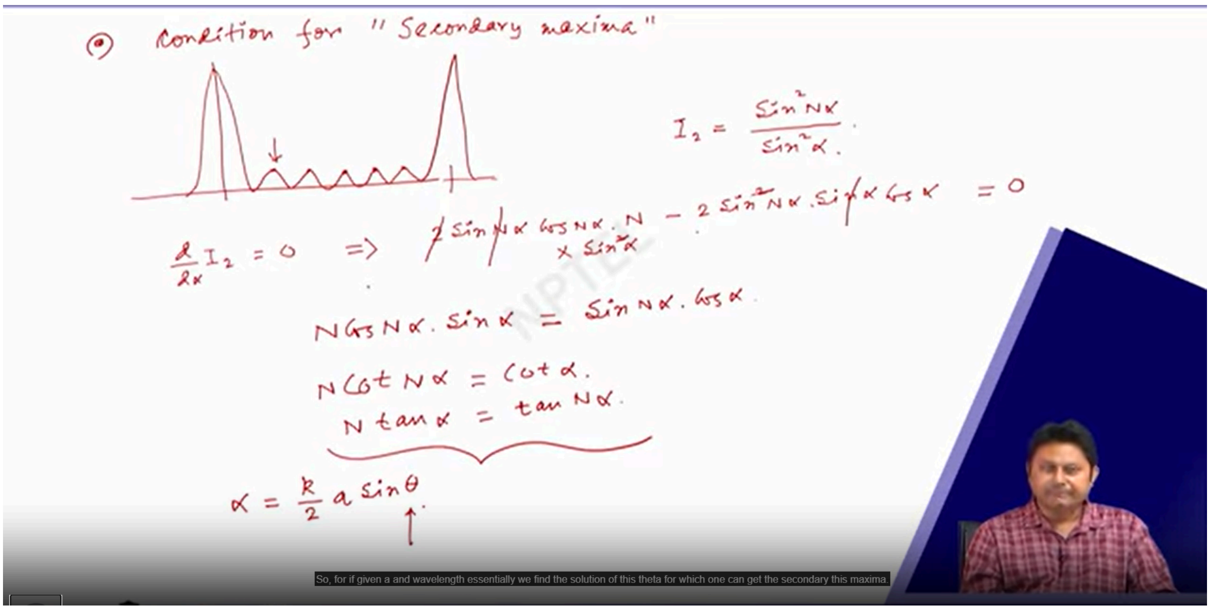


then alpha should be some integer multiplier of pi and then the denominator should have an issue. The denominator of the condition is 0. So, the condition of the minima is the sin 3, this will be 0, but for the value of that particular alpha, sin alpha should not be 0, that condition should be there. So, as soon as I put sin alpha not equal to 0, that means the value of the s should not contain 0 or n or 2n etcetera because at this point the function goes to maximum 3n and so on because these values of s, what we get is sin alpha will be 0. Because as soon as I put s equal to 0, alpha will be 0, s equal to 1, alpha will be pi, s equal to 2n, I s equal to n, alpha equal to pi, s equal to 2 n, alpha equal to 2 pi and so on and sin alpha will going to vanish. So, this is the value that should not be there, and apart from this, we will get the value of s such that the sin n alpha is 0. So, alpha is essentially plus-minus because the minus sign is also there, s divided by n pi is the condition, and alpha is the condition of minima rather. So, that condition tells us that k a, sorry, k by 2 a sine theta is equal to plus minus s by n pi and we essentially get a sine theta is equal to plus-minus of s by n lambda, that is the condition for minima. Now what will the function look like? Suppose, n is 8 that I know, n is 8 means, s should be one, two, three, four, five, 6, and 7 it should not have the value 8, it should not have the value 16 etcetera. So that means at n equal to 8 that is if I have 8 slits because n determines the number of slits, then we get a spectra like this, so I have a peak at some value of n and there should be 7 minima before 2 peaks. So here we have 1 minima, 1, 2, 3, 4, 5, 6, 7 and then we have a maxima again. So, here we have a minimum that is 1. Here we have a minimum of 2. Here we have a minimum of 3. Here we have a minimum that is 4, 5, 6, 7 okay. It is for n equal to 8. So, n minus 1 number of minima will be there n in the spectrum for two consecutive principal maxima. So, these are the principal maxima we have. And we are going to get, so it depends on the number of slits. So if we know the number of slits we know, then we can define how many minima there are. So how many maxima, in between these two minima, there are maxima also. So, there are 1 maxima, 2 maxima, 3 maxima. These are the secondary maxima 4, 5, 6. So, we have n minus 2 maxima here. We have n minus 2 maxima here which is in between this. So, now this secondary maxima that one can also find the condition as we get for double slit problem is how to get the secondary maxima. (Refer slide time: 23:34)



So, now we are going to find out the condition for the secondary maximum. So, essentially what we try to find is that we have two principal maximas sitting here and then suppose we have this. So we try to find out what is the condition of this secondary maxima. These are the principal maxima and these are the secondary maxima. And there is a distribution over theta, and we try to find out what are the conditions to get these peaks. What is the value of the theta for which I am going to get these peaks? So the condition is straightforward the d, d alpha of i2 is equal to 0. What is I2 here? I2 is that function sine square n alpha divided by sine square alpha i, I write this function as I2. So you can use I instead of I2, you can use some different but anyway, that is the thing essentially. So this derivative gives us simply 2 of sine of n alpha and then cos of n alpha, multiplied by n minus 2 of sine square 1, sine square will be multiplied here, multiplied by sine square denominator you have a sine square alpha, then 2 of sine square n alpha function and the derivative is sin alpha cos alpha whole divided by sin to the power 4 alpha is 0. So, essentially this quantity is 0. So, that means if I take a few commons like here we have sin n alpha, cos sin n alpha, cos n alpha multiplied by n. So, 2n so, 2, 2 let us first cancel out this 2, 2. Then what else do we cancel out? What else is there? So, sin 1, sin n alpha will cancel out with this. Then cos n alpha is there, it will remain there but this sin alpha 1 will cancel out to this sin. So, essentially we have n multiplied by cos n alpha, multiplied by sin alpha, that is a condition we have, that is equal to sin alpha into cos of alpha. So, that means we have a condition that n of cot of n alpha is equal to cot of alpha or n tan alpha is equal to tan of n alpha that is the condition we have. So, we need to solve this transcendental equation; one can solve this graphically only. So, if somebody solves this graphically then he or she can find out the value of the alpha for which one can find out the secondary maximum. What is alpha? Essentially we get alpha is k by 2 a and sine theta. So, if given a wavelength essentially we find the solution of this theta for which one can get the secondary maxima. So, that is the condition for principal, that is the condition for this secondary maxima. So, what is the condition for principal maxima? That we were supposed to calculate earlier, but anyway. So, let us find out the condition for principal maxima that is rather important.

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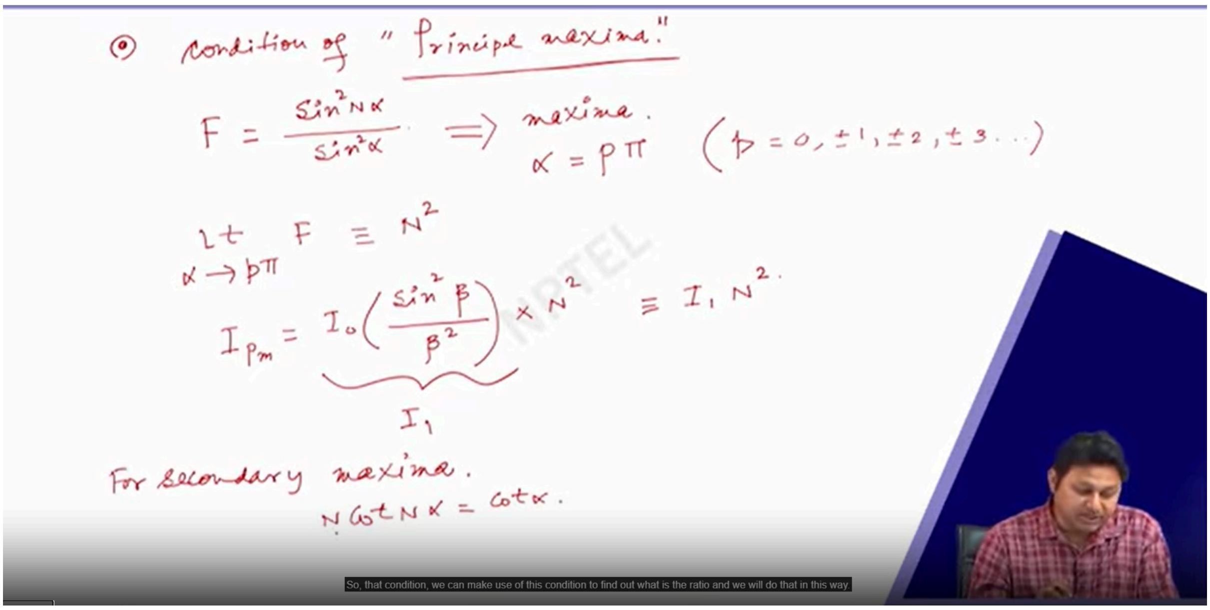

So, condition of principal maxima. So the function is, our function f is sine square n alpha and then sine alpha and v sine square alpha and we show that these become maximum. Condition of maxima arises when alpha is equal to p pi where p is some integer, p can take the value of 0, plus minus 1, plus minus 2, plus minus 3, and so on. So limit alpha tends to p pi, which we calculate of this function, f is essentially n square where f is this function. So the principal maxima I write Ip m is equal to I naught, sine square beta divided by beta square that was already there, multiplied by n square. And if I write this entire term as some intensity I1, so this is equivalent to I1 multiplied by n square. So, n square times increment of the intensity I 1 will be there for the principal maximum. So, we can also find out what is the ratio of the intensity of the secondary maximum that we can do. For the secondary maximum, we find n of cot of n alpha is equal to cot of alpha that is the condition we have. So, in that condition, we can make use of this condition to find out what the ratio is and we will do that in this way. So, I can write n square, say, cos square n alpha divided by cos square alpha is equal to sine square n alpha divided by sin square alpha. This condition I get with the condition of maxima and that is n of tan alpha is equal to tan of n alpha or n of cot n alpha is equal to cot alpha both are same. So, tan, I write sin by cos tan n alpha, I write sin n alpha divided by cos alpha and then I write these things. Again I can write like n square into cos, I write 1 minus sine square n alpha cos, I write 1 minus sine square alpha. This site is sine square N alpha right, sine square alpha which is equivalent to N square, sine square N alpha divided by N square, sine square alpha, just multiply n square both sides. Now we know that if two ratios a by b are equal to c by d then I can write this is equal to a plus c divided by b plus d. By doing so one can have for this quantity for secondary maxima sin square n alpha divided by sin square alpha and that I calculate under the condition of secondary maxima, this is equal to n square whole divided by 1 plus, n square minus 1 sine square alpha. So secondary maxima intensity of the secondary maxima is equal to i of I naught sine square beta divided by beta square and this quantity sine square n alpha divided by sin square alpha, but this quantity we need to calculate under the condition of secondary maximum. So, this quantity we calculated here.

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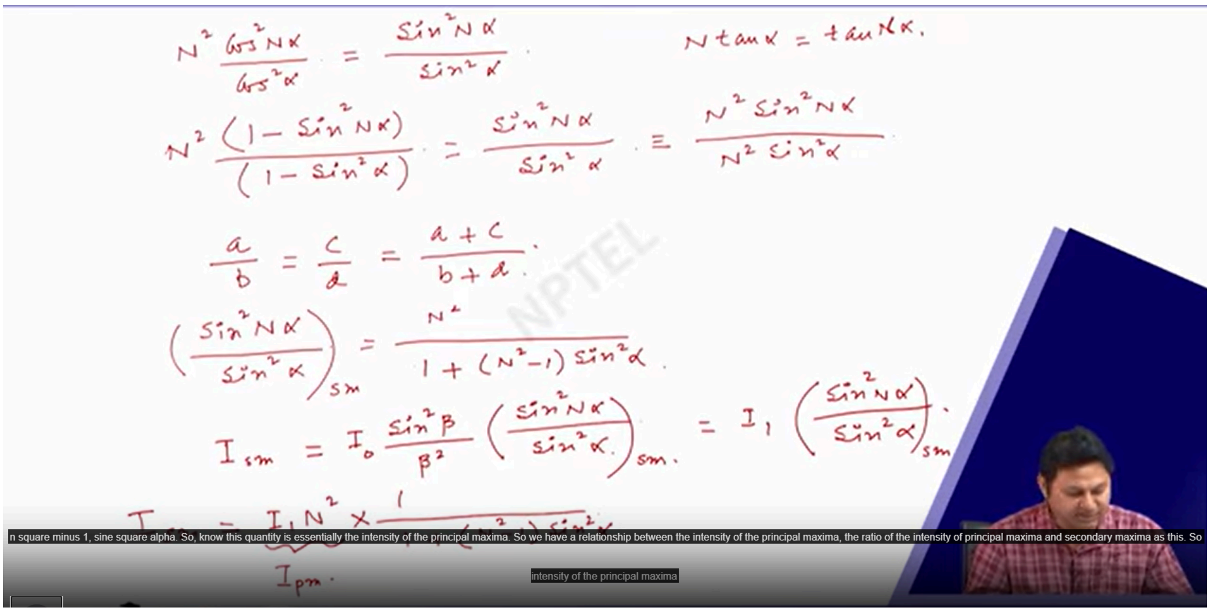

So, this is equal to something called I 1 and that sin square n alpha whole divided by sine square alpha secondary maximum. Now I just replace these things. So that I can have, i secondary intensity of the secondary maxima is equal to intensity i1 multiplied by n square, multiplied by 1 divided by 1 plus n square minus 1, sine square alpha. So, now this quantity is essentially the intensity of the principal maxima. So we have a relationship between the intensity of the principal maxima, the ratio of the intensity of the principal maxima, and the secondary maxima. So the intensity of the principal maxima we have is i1 n square that we calculated and the intensity of the secondary maxima is i1 n square multiplied by 1, divided by 1 plus n square minus 1, and then sine square alpha. So the ratio, if I find, i secondary maxima divided by i principal maxima, this ratio will be 1 divided by 1 plus n square minus 1 sin square alpha. Now, that is the ratio of the two maxima and based on the value of n and alpha, one can find out what value. So, that means I essentially calculate that this is the principal maxima and these are the secondary maxima, there are many secondary maxima. So the ratio of the intensity between the principal and secondary maxima is Ipm and here this is Ism. So I calculate the intensity of the principal and secondary maxima and that comes out to be this. By putting the value of the n and others, one can find out what the ratio is. So, I like to stop here because today I don't have the time to go forward. In the next class, we will discuss these grating spectra and if time permits we also discuss the resolving power of this multiple-slit system, which we in general call grating. So with that note, I would like to conclude here, thank you very much for your attention and see you in the next class

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